An Article Title That Spans Multiple Lines to Show Line Wrapping

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Abstract

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I. Introduction

With the popularity of quantum computing spreading in the investors and researchers,quantum error correcting has also been much under scrutiny.Though in the early time, people have the scepticism that weather quantum computing can keep and manipulate a quantum state error-free for a long time ,the development of quantum error correction making it more optimistic.

Among so many prospective codes of quantum error correcting,the most promising classes is the surface code,witch is proposed by Alexei Kitaev in 1997.(2,3)the topological concept of the surface code or toric code is to encode the qubit in a 2D lattice of physics qubits system which has some specific boundary condition(3).In this case ,we can influence and correct a wide range of qubits by applying a given way on the torus.Although challenges of fabricating a large-scale quantum memory still exist ,the progresses on experimental advances(4) in recent years give us hope to make it a reality.

Another significant period of a quantum error correcting task is to decoder.It is natural that we want to find an optimal solution to figure out the most probably error and correct it for every given measurement,witch is called the Maximums Like Algorithm.But it seems unpractical for its exponential operation time.So how to balance between performance and running time of the algorithm becomes the priority .A lot of solution have been designed to solve such problems,such as face code and neural network.

In this paper,we will take an insight of a kind of modified Maximum Like Algorithm ,Blossom Algorithm,as well as we will

II. Background on Quantum Error Correction

A standard error correction usually consists of four step:

1. **Encode** : encode the quantum state to another state state:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow |\psi_E\rangle = \alpha|000\rangle + \beta|111\rangle$$

2. **Detect** : detect the quantum errors by measurement
3. **correct** : correct those errors and decode the qubits to get the corresponding logical state
4. **compute** : compute logical operations on the state by redefining a universal gate set on the code.

Here we will focus on the first three steps and outline our work by the following steps:

1. **Introduce errors**
2. **Plot syndromes**
3. **Match the syndromes**
4. **Get the result**

A. toric code

For a better understanding of the toric code,we assume a L×L lattice,where every single qubit lives on the stage,so we have L^2 qubits.The stabilizer group of toric code is generated by two types of operator A_s and B_p:

$$A_s = \bigotimes X_q, B_p = \bigotimes Z_q$$

The X and Z are the Pauli matrix the Hamiltonian of the system is the linear combination of the tow operator:

$$H = -A_s - B_p$$

We show the stabilizer and the lattice in Figure 1.

B.Quantum Error

Generally speaking,the noisy and environment disturbance is inevitable in real quantum system.It is impossible for us to clarify the procedure of the coupling between the system and the environment.But

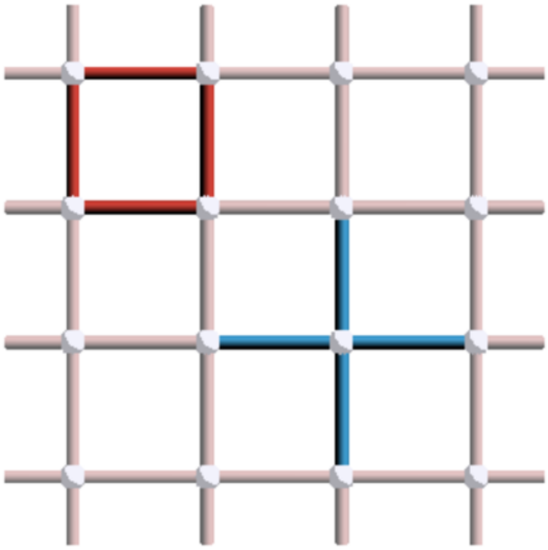


Figure 1: *An illustration of the lattice.The unit marked by the red line or the blue line shows the A_s operator and B_p operator respectively*

in fact,we can easy the challenge by simplifying all the quantum errors into two types of error:

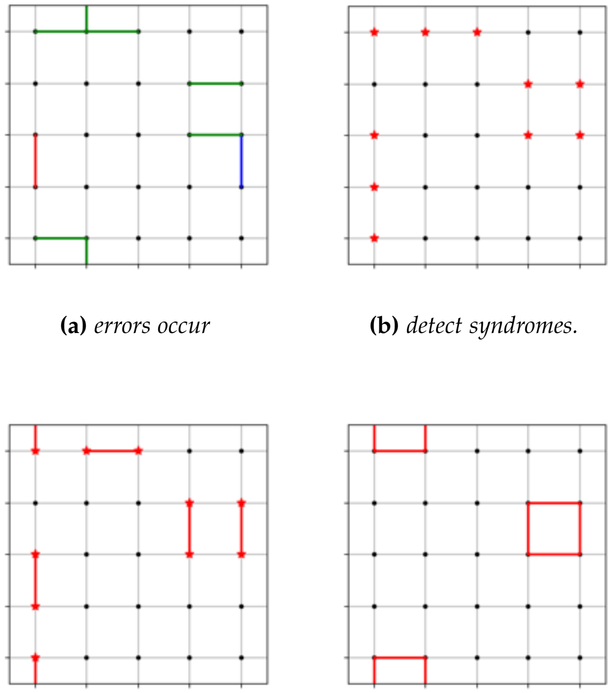
$$\text{Bit-flips error:}|\psi\rangle \rightarrow \hat{X}|\psi\rangle$$

$$\text{Phase-slip error:}|\psi\rangle \rightarrow \hat{Z}|\psi\rangle$$

The X error and the Y error are the fundamental mode of quantum error and they can appear at the same time,which we called Y error.In this paper,we assume that each error has the same probability $p/3$ to occur(in the code we implement ,the p is 0.15).

Decoder

As show in Figure 2(a)-(d),when a series of errors happen and we detect this error by the measurement and collect all the syndromes.The task of decoder is to restore the most possible error.The green line ,red line and blue line in Figure 2(a) indicate the X error ,Z error and Y error.It is showed in the figure 2(b) that we detect these error with the measurement we mention above.The next step and the most challenging part is to match the syndromes and decoder them to get result.We definitely hope the result we get is non-trivial,but sometimes the logical error is inevitable due to the limitation of decoder algorithm.In the next section,we will introduce the Blossom Algorithm and test its logical error and compare it with other decoder algorithm.



(a) *errors occur* **(b)** *detect syndromes.*

(c) *matching* **(d)** *decoder*

Figure 2: *the procedure of an error correction.(a)-(d) shows the different stage respectively*

III. Blossom Algorithm

Here,we implement a decoder algorithm called Blossom Algorithm,which is first proposed by Edmonds in 1965.Edmond's idea requires $O(n^2m)$

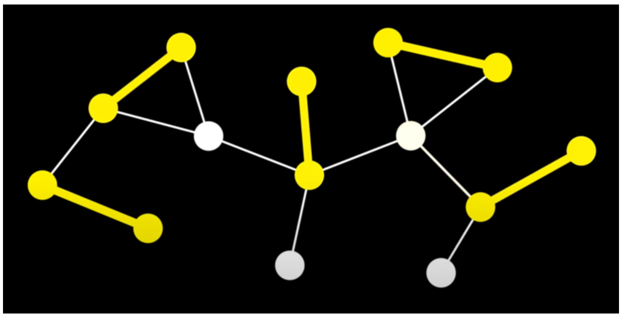
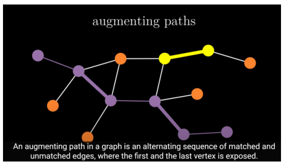


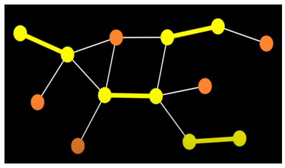
Figure 3: *An illustration of the graph.The balls represent the vertices and some are matched with each other and some are free.*

times,where n is the number of nodes in the graph and m is the number of edges.The Blossom Algorithm has been developed with the time and different improvement has been conducted.It is impossible for us to cover all the creative ideas.Before we discuss the kind of minimum weight matching Blossom Algorithm,we first get an insight of the oldest Blossom Algorithm.

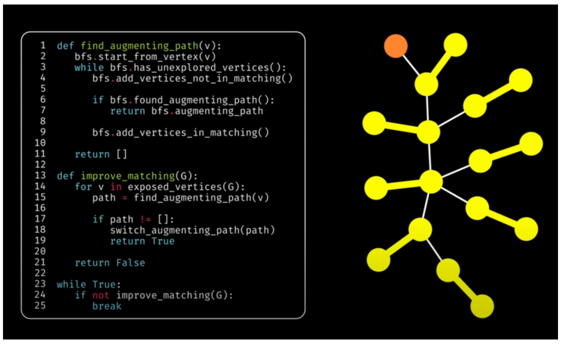
1. **Graph Representation**:First,you should represent the problem as a graph $G = (V, E)$,where V is the set of vertices and E is set of edges with associated weights,as show in Figure 3where each yellow ball represents the vertices and the segment represents the edge which has different weight.
2. **Augmenting Paths**:And then,an augmenting path is required.Like branches on a tree,an augmenting path is a path that stars and ends at unmatched vertices and alternate between unmatched and matched edges,as show in Figure 3(b).It can improve the size of the current matching by switching the matched and unmatched edges,as show in Figure 3(c).And the process of matching can repeatedly until no free vertices are left,at which point,we know the matching is maximum,as show in figure 3(d).



(a)



(b)

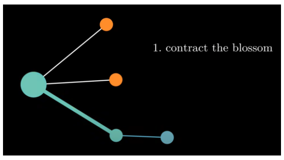


(c)

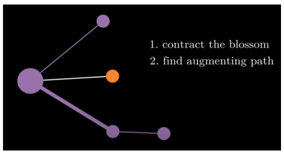
Figure 4: *The procedure of an error correction. (a)-(c) show the different stages respectively.*

3. **Blossoms**:But things don't always go well.When there is a blossom,a cycle containing an odd number of "pseudo nodes", where a pseudo node is either a vertex or another blossom,the algorithm fails,as show in Figure 3(e) and 3(f).We can see that since the augmenting path is longer than the shortest path,so is not found.
4. **Blossom Contraction and Expansion**:In this case,When an odd-length cycle (blossom) is detected, we should contract it into a single vertex, simplifying the problem and after finding a matching, expand the blossoms back to up-

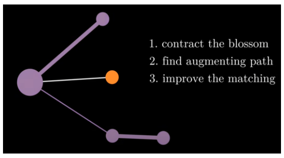
date the matching accordingly.The Figure 5(a)-(d) show the solution when a blossom is detected



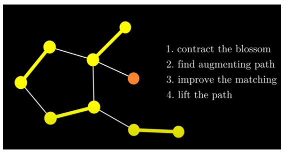
(a) *contract the blossom*



(b) *find augmenting path*



(c) *improve the matching*



(d) *lift the path*

Figure 5: *the procedure of an error correction.(a)-(d) shows the different stage respectively*

5. **Iterative Process**:Finally,repeat the search for augmenting paths, contraction, and expansion until no more augmenting paths can be found.The whole process and code is presented in Figure 6.

The minimum weight perfect matching algorithm is originated from the initial idea,but it has some improvement to handle with the practical problems. The implementation of the algorithm is the following:for every vertex i,it has a weight $l(i)$ and every edge is a coupling of the vertices i,j,it meets the condition:$w(i,j) \leq l(i) + l(j)$.Every perfect minimum weight matching,the sum of the weight edge is:

$$\begin{aligned}\mathrm{val}(M) &= \sum_{(u,v)\in M} w(u,v) \\ &\leq \sum_{(u,v)\in M} (l(u) + l(v)) \\ &\leq \sum_{i=1}^{n} l(i)\end{aligned}$$

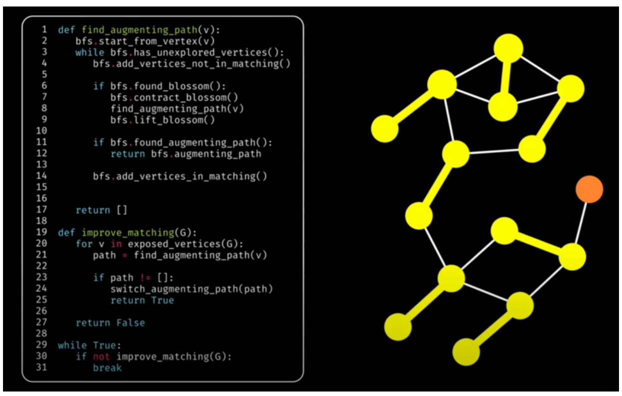


Figure 6: *the whole process and the code*

Defining z_u as the vertex labeling of u,and we also define $e(u,v)$ an equality edge if $(z_u + z_v = weight(e))$,and at this time the edge labeling of edge is called z_e.It requires $z_e = z_u + z_v - weight(e) = 0$. The augmenting path composed of "equality edges" is continuously expanded, and since all the edges used for expansion are "equality edges", the final maximum weight perfect matching obtained is still all "equality edges". In Figure 7,we demonstrate the whole process of this algorithm in detail.It consists of four step:

1. **Grow**:If edge(u,v) is tight,$l(u) = +$and $l(v) = \emptyset$ then the tree to which u belongs can be "grown" by acquiring node u and the corresponding matched node,as shown in the Figure 7(a)
2. **Augment**:If edge $(u,v)$is tight,$l(u) = l(v) = +$and u,v belong to different trees then the cardinality of matching x can be increased by "flipping" variable x_e for edges e along the path connecting the roots of the two trees,as shown in

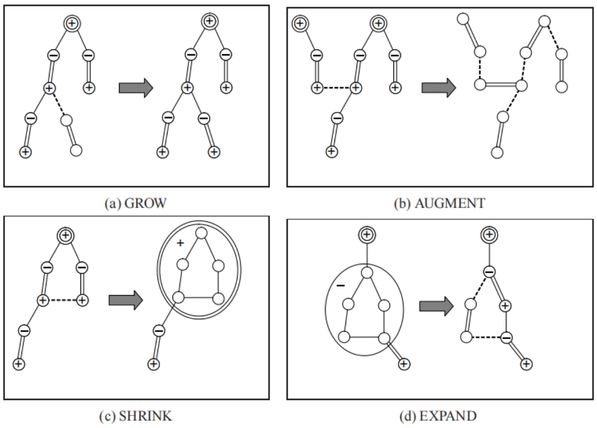


Figure 7: *An illustration of the minimum weight perfect matching*

Figure 7(b).All nodes in the trees become error-free.

3. **Shrink**:If edge $(u, v)$is tight,$l(u) = l(v) = +$and u, v belong to the same tree then there is cycle of odd length that can be shrunk to a blossom,as shown in Figure 7(c).The dual variable for this new blossom is set to 0.

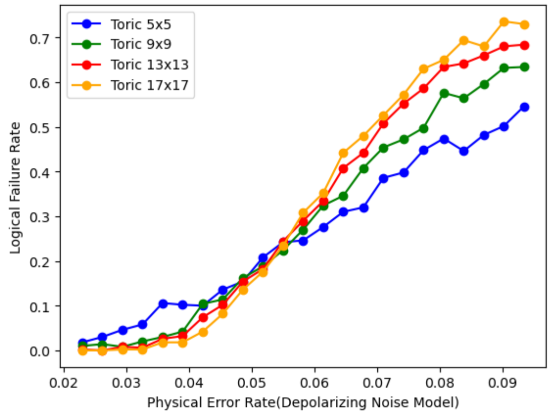
4. **Expand**:If node v is a blossom with $y_e = 0$ and$l(v) = -$ then it can be expanded.

the u^- represents odd vertex in an alternating tree,the u^+ represents the even vertex in an alternating tree and $u^\varnothing$ represents an vertex not in any alternating tree.

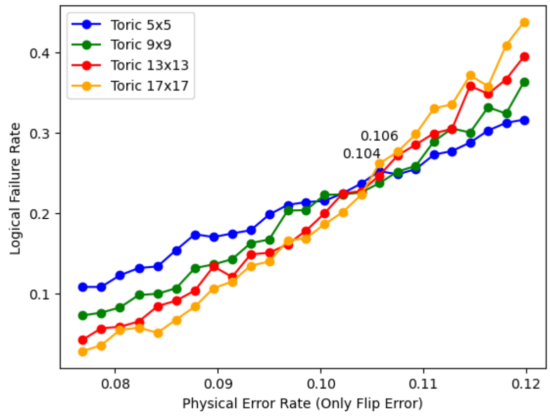
IV. Cellular Automation Decoder

Besides the Blossom Algorithm,we also get an insight of another decoder,the Cellular Automation Decoder.

V. Test and result



(a)



(b)

Figure 8

In this part,we will show our results and compare different decoders with their error threshold and time complexity .You can see more details in our github website,here,we just give a brief introduction. The error threshold is a critical parameter in the design and implementation of fault-tolerant quantum computation.It is related to the threshold theorem,which states that arbitrarily long quantum computations are reliable below a specific error rate that varies with the quantum decoders we use.Above

Table 1: *Time complexity of different decoders*

Decoder	time complexity
minimum weight perfect matching	$O(n^3)$
Union codes	$O(n)$

the specific error rate,the reliability of the quantum computation rapidly deteriorates,making it impossible to conduct a fault-tolerant quantum computation. Figure 8 (a)-(b) shows the logical error rate varying with the phase-flip and bit-flip error rate under the noise model respectively.For four different scale quantum memories, we set up 25 different groups of error rates,each group of error rates for 500 simulation tests, and calculate the logic error rate.We can calculate the error threshold by recording the intersection of four lines.And it shows the error threshold is about 0.104.As the time complexity ,shown in the Figure 9 (a)-(b) and table 1,the minimum weight perfect matching has the $O(n^3)$ time complexity.And the other,the Union Code represented by the previous group,has the $O(n)$ time complexity.Though,the minimum weight perfect matching is inferior in time complexity to previous work,it has an advantage over it by its higher error threshold.

VI. Conclusion