EL-2320 Applied Estimation Course Project Dual state and parameter estimation of a four wheel vehicle

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***Abstract*—With the advent of autonomous systems, knowing both states and parameters is essential for the system. In this work, the example of 4-wheeled vehicle is used to investigate some parameter estimation methods, while the states are being estimated simultaneously. In this article, we deal only with mass, position of centre of mass, tyre coefficients among the parameters to be estimated along with the states of lateral and longitudinal velocity, and yaw rate. Since in a vehicle, the parameters and states are coupled together in extremely complex ways, we have used a simple model(kinematic) to estimate some of the parameters and a more engaging model(dynamic) to estimate the rest, while using information derived from the kinematic model. The methods investigated in this article are Dual Extended Kalman Filter (Dual EKF), Unscented Kalman Filter (Dual UKF), and UKF + particle filter. The performance of the estimators are also compared based on convergence time, region of convergence, stability and accuracy.**

Keywords : State and parameter estimation, vehicle model, EKF, UKF, Particle Filter

I. Introduction

Besides state estimation, parameter estimation is gaining more and more importance because of the emergence of the autonomous system, since the goal is zero human intervention, so the machine should calibrate itself. This has applications from load lifting cranes, autonomous vehicle, spacecrafts, satellites etc. In the current work, we explore methods for parameter estimation for 4-wheeled vehicle. In the automotive domain, the vehicle safety is one of the most basic requirement during their development. Vehicle dynamics control systems have been developed to improve the control and safety of the vehicle, user and the other elements around them. They seek to prevent any undesirable behaviour, due to terrain, or external factors or rash driver's input, by use of active control systems. But the performance of these controllers depends heavy upon how accurately they know the vehicle's states and parameters. The dynamics of a vehicle heavily relies upon parameters like moment of inertia, tyre coefficients etc but they can change over time and also there is no direct observation method for measuring them. All of this makes this study very useful.

In order to decide on the vehicle model to be used, we weighed the benefits and limitations of various models, from simplest 2-wheel bicycle model to complex tyre models. In the works by other authors as well, Wenzel et. al [1] utilized a full 4 wheel 4-dof vehicle model, however were able to estimate only 3 parameters, namely mass, moment of inertia in z-direction and longitudinal position of the COG. They employed 2 tyre models to show their effect on state and parameter estimation. Erik et al. [2] also demonstrated in his thesis work, the role of different tyre models in estimating rest of vehicle parameters. In this work 2 models have been used in cascaded fashion to identify 5 parameters. A 2-wheel bicycle model(kinematic model) was employed to estimate the position of COM which is then fed to the dynamic model based on 4-wheel simplified track model using linear approximations to measure the tire forces and mass ofthe vehicle. This dynamic model is then used to estimate the vehicle mass, and the longitudinal and lateral stiffness coefficients of tires.

For parameter estimation several approaches were analysed in the literature study. Erik et al. [2] modelled the physical system as the grey box model, and estimated the parameter values by minimizing an error function with respect to the parameters. A version of Gauss-Newton algorithm was used in the thesis for the same.

Wan et al [3] presents comparison between various dual estimation approaches such as dual EKF, dual UKF, dual UKF/EKF, joint EKF and joint UKF. In the dual filter approach, two simultaneous filters are made to run and current estimate of variables are used in the other filter. Wherasin joint estimation approach, states and parameters are combined into single vector and single filter is made to run. However, the paper presents results from experiments on chaotic series used in autoregressive machine training. Wenzel et al [1] implemented a model-based vehicle estimator, but only using dual EKF and were able to estimate only 3 parameters.

In this work, we have tried to estimate more number of parameter, 5 to be exact, by decomposing the vehicle model into 2, to get a better estimate with lower number of measurements. Also, Dual UKF based estimation had not been reported to large extent. Further, a novel method coupling UKF with Liu West particle filter [4] has been presented. The comparison of these techniques is done for dual state and parameter estimation of non-linear models.

Outline: The following paragraphs contain the detailed methodology of the work. The kinematic and dynamic model of the vehicle is discussed in the section **Vehicle Model**. The **Estimation Framework** section provides detailed information about the working of the dual filters used. The results of the estimation and effects due to noise etc and comparison of the estimators with respect to accuracy, stability and convergence is presented in the **Results and Discussion** section.

II. Methodology

A. Vehicle Model

Since the parameters are highly coupled, even in the bicycle model [5], this can lead to incorrect estimation of the parameters. Therefore, after careful consideration, two separate models were used in a cascaded fashion in this work. The parameters estimated using kinematic model are used by the dynamic model to estimate the rest of the parameters.

1) Kinematic Model: The kinematic model is a simplified bicycle model of vehicle dealing with the kinematics of the vehicle. The states in this model are position and velocity and the equations are governed by the non-holonomic constraint as shown in the Fig.1

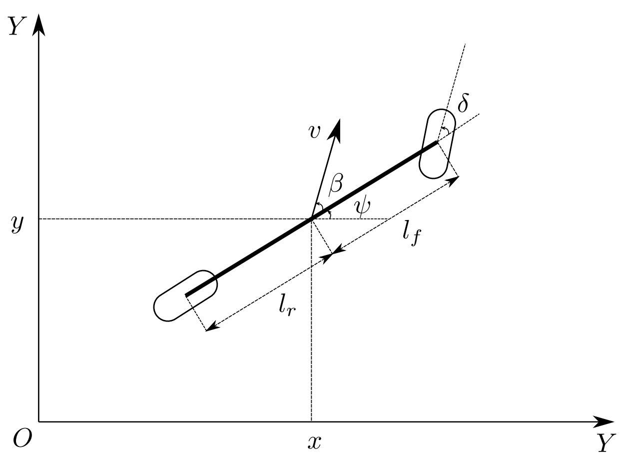

Fig. 1. Simplified Bicycle Model from [6]

The equations in the inertial frames are as follows -

$$\beta = tan^{-1}(\frac{l_r}{l_r + l_f})tan(\delta) \quad (1)$$

$$\dot{x} = v\cos(\psi + \beta) \quad (2)$$

$$\dot{y} = v\sin(\psi + \beta) \quad (3)$$

$$\dot{\psi} = \frac{v}{l_r}sin(\beta) \quad (4)$$

$$\dot{v} = a \quad (5)$$

where, x and y are the position of vehicle along x and y coordinates with respect to the inertial frame O. v is the vehicle speed. ψ is the heading angle and δ is the input steering angle. β is the angle of the current velocity of the center of the mass of the vehicle with respect to the longitudinal axis of the car. l_f and l_f represents the distance from the center of the mass of the vehicle from the front and rear axles of the car. These distance are crucial for control and may vary based on the distribution of load. Through the kinematic model, these distances are estimated which are further utilised in the dynamic model.

2) Dynamic Model: The dynamic model used in this work is the simplified bicycle model based on the one presented in [6]. The vehicle model is derived from standard rigid body mechanics, wherein only planar model of four wheel is considered ignoring roll and pitch motions as shown in Fig. 2 resulting in 3 degrees of freedom.

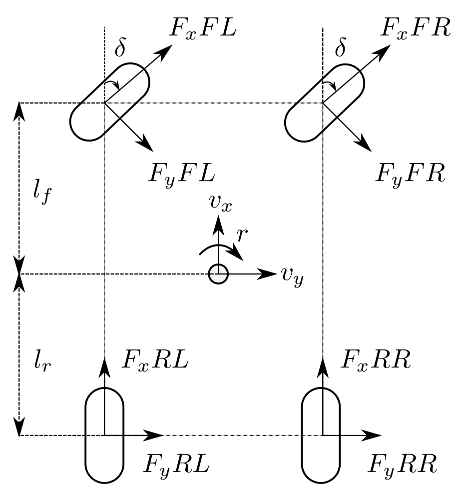

Fig. 2. 4-wheel Dynamic Model from [6]

In x direction -

$$\dot{v_x} = v_y r + \frac{1}{m}(F_{xFR} + F_{xFL})cos\delta - (F_{yFR} + F_{xFL})sin\delta + F_{xRR} + F_{xRL} - C_a {v_x}^2 \quad (6)$$

In y direction -

$$\dot{v_y} = -v_x r + \frac{1}{m}(F_{xFR} + F_{xFL})sin\delta + (F_{yFR} + F_{xFL})cos\delta + F_{yRR} + F_{yRL} \quad (7)$$

and yaw motion is governed by

$$\dot{r} = \frac{1}{J}(l_f((F_{xFR} + F_{xFL})sin\delta + (F_{yFR} + F_{xFL})cos\delta) - l_r(F_{yRR} + F_{yRL}) \quad (8)$$

Here, v_x and v_y are the vehicle's longitudinal and lateral velocities respectively at the vehicle's Center of Gravity. r is the yaw rate.

The forces F_{xXX} and F_{yXX} depends on the tyre model. In this work, piecewise continuous tire models are considered both in longitudinal and lateral direction.

The equation of the forces are determined by the following equations

$$\begin{aligned} F_{yFX} &= C_y \alpha_{FX} \\ F_{yRX} &= C_y \alpha_{RX} \\ F_{xFX} &= C_x(s_{FX}) \\ F_{xRX} &= C_x(s_{RX}) \end{aligned} \tag{9}$$

Where C_y and C_x are the tyre coefficients in y and x direction respectively which have been assumed to be same for all 4 tyres. s_{FX}, and s_{RX} are the slip inputs on the front and rear tyres respectively, for either left or right tyres. If we want to operate the vehicle as 2-wheel driven, we can give respective slip inputs as 0.

In our case, we have considered a front-wheel driven vehicle and therefore s_{RR}, $s_{RL} = 0$.

α_{FX} and α_{RX} are the slip angles for front and rear tyres respectively and calculated as below.

$$\begin{aligned} \alpha_{FX} &= \delta - \arctan \frac{v_y + l_f r}{v_x} \\ \alpha_{RX} &= - \arctan \frac{v_y - l_r r}{v_x} \end{aligned} \tag{10}$$

,where l_f and l_r are the distance of COG from front and rear axle.

So, for our dynamic model we have 5 inputs:

$$\begin{bmatrix} s_F L & s_F R & s_R L & s_R R & \delta \end{bmatrix}, \tag{11}$$

3 states :

$$\begin{bmatrix} v_x & v_y & r \end{bmatrix}^T \tag{12}$$

,and 5 parameters :

$$\begin{bmatrix} l_f & l_r & m & C_x & C_y \end{bmatrix}^T \tag{13}$$

B. Estimation Framework

A general flow of the algorithm is presented in the fig 3. The Bayesian filters follow this framework but there is no general description for non-parametric filters. So it could be an outline for dual EKF and dual UKF, but not for the particle filter. But all the estimation algorithm make use of the model in the same way, employing kinematic model at first and then dynamic model. As stated, kinematic model is used to estimate the position of centre of mass of the vehicle from the front and rear axle. Dynamic model is then used to further estimate the remaining parameters of mass and tyre coefficients.

Following three estimation framework was utilised to perform state and parameter estimation.

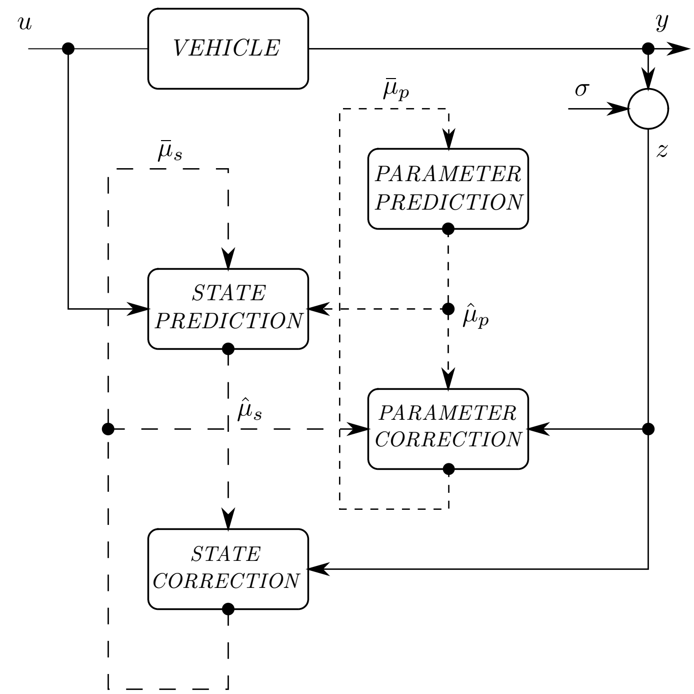

Fig. 3. Estimation algorithm outline for Bayesian filters

1) Dual EKF: The estimator is implemented using the dual extended Kalman Filter(DEKF) technique, which makes use of two simultaneously running Kalman filters for estimating states and parameters, as presented in [1].

An advantage of this technique is that parameter estimator can be switched off after we are able to reduce the uncertainty sufficiently in its estimation.

Unlike the joint EKF, which utilizes a combined vector for state and parameter, which was proposed by Wan and Nelson [3] and also used by Best and Gordan [8], this variant runs 2 EKF simultaneously instead of just one.

The algorithm followed is presented in algorithm 1, and as shown state estimation is followed by parameter estimation. The EKF for state estimation is quite conventional, and the only deviation in the parameter estimation EKF from the conventional method is the jacobian for parameters, $\boldsymbol{H_p}$, which is given as,

$$\boldsymbol{H_p} = \boldsymbol{H_s} \frac{\partial f(\boldsymbol{\hat{x}_s}, \boldsymbol{\hat{x}_p})}{\partial \boldsymbol{\hat{x}_p}}$$

The innovation term $z_t - H_s \hat{\mu}_{st}$ is the same for both state and parameter correction.

In order to get better results, the model belief uncertainties can be decreased manually, since the better the parameter estimation gets, the model uncertainty should become smaller. A method described in [9], uses a factor $\gamma < 1$ to decrease the covariances at each time step,which is depicted as the last step of the algorithm.

Algorithm 1 Dual EKF algorithm

procedure DUALEKF($\mu_{p_{t-1}}, \mu_{p_{t-1}}, \Sigma_{p_{t-1}}, \Sigma_{s_{t-1}}, z_t, R_{p_{t-1}}$)

$$\hat{\mu}_{p_t} = \mu_{p_{t-1}}$$
$$\hat{\Sigma}_{p_t} = \Sigma_{p_{t-1}} + R_{p_{t-1}}$$

$$\hat{\mu}_{s_t} = nextState(\mu_{st-1}, \hat{\mu}_{p_t}, u)$$
$$G_s = getStateJacobian(\mu_{st-1}, \hat{\mu}_{p_t})$$
$$\hat{\Sigma}_{s_t} = G_s \Sigma_{s_{t-1}} G_s^T + R_s$$

$$K_{s_t} = \hat{\Sigma}_{s_t} H_s^T [H_s \hat{\Sigma}_{s_t} H_s^T + Q_t]^{-1}$$
$$\mu_{s_t} = \hat{\mu}_{s_t} + K_{s_t}(z_t - H_s \hat{\mu}_{s_t})$$

$$H_p = getParameterJacobian(\hat{\mu}_{s_t}, \hat{\mu}_{p_t})$$
$$K_{p_t} = \hat{\Sigma}_{p_t} H_p^T [H_p \hat{\Sigma}_{p_t} H_p^T + Q_t]^{-1}$$
$$\mu_{p_t} = \hat{\mu}_{p_t} + K_{p_t}(z_t - H_s \hat{\mu}_{s_t})$$
$$R_{p_t} = \gamma R_{p_{t-1}}$$

return $\mu_{p_t}, \mu_{p_t}, \Sigma_{p_t}, \Sigma_{s_t}, R_{p_t}$

2) Dual UKF: Dual Unscented Kalman filter was first presented in [7]. In Dual UKF, similar to that of dual EKF, two dependent UKF's run in parallel. The crucial part of the step is interlinking of these two parallel UKF's. The algorithm 2 gives a step-by-step description for DUKF. Only difference between DEKF and DUKF is that instead of linearizing around the calculated mean of the system, sigma points are calculated from the gaussian distribution, at which the non-linear function is evaluated.

In this framework, both state and parameter estimator follow the conventional UKF approach apart from the differently tuned noise levels and the weights α, β, γ for each.

3) Particle Filter(Liu West) + UKF: Inspired by classical Particle Filter, the aim was to implement particle filter for both, state and parameter estimation. But, it was observed that this approach ran into particle deprivation problem too often. One of the key reason behind such behavior might be high prediction inaccuracy due to unknown parameter. Therefore, a new combination of UKF + particle filter, based on Liu West variation [4], was designed. The combined algorithm is presented in Algorithm 3. In the algorithm parameter distribution is represented by θ, M denotes the number of particles. Rest of the notations follows []. Weight is updated based on the Gaussian likelihood of measurement.

This technique uses UKF for state estimation. Parameter estimation, however, is done by a particle filter. In Liu West's particle filter, in order to handle the particle deprivation, instead of re-sampling the parameter posterior distribution is approximated through a mixture of multi-variate Gaussian distributions.

This variation of the particle filter by Liu-west finds its most application in the cases where the quantity to be estimated does

Algorithm 2 Dual UKF algorithm

procedure DUALUKF($z_t, u, [\alpha, \beta, \gamma]$)

$$w_{mp}, w_{cp} = generateWeights(\alpha_p, \beta_p, \gamma_p)$$
$$\chi_{p_{t-1}} = generateSigmaPoints(\mu_{p_{t-1}}, \Sigma_{p_{t-1}})$$
$$\chi_{p_t} = \chi_{p_{t-1}}$$
$$\hat{\mu}_{p_t} = \textstyle\sum_{i=0}^{2m} w_{mp}[i] \chi_{p_t}[i]$$
$$\hat{\Sigma}_{p_t} = \textstyle\sum_{i=0}^{2m} w_{cp}[i](\chi_{p_t}[i] - \hat{\mu}_{p_t})(\chi_{p_t}[i] - \hat{\mu}_{p_t})^T + R_{p_{t-1}}$$

$$w_{ms}, w_{cs} = generateWeights(\alpha_s, \beta_s, \gamma_s)$$
$$\chi_{s_{t-1}} = generateSigmaPoints(\mu_{s_{t-1}}, \Sigma_{s_{t-1}})$$
$$\chi_{s_t} = nextState(\chi_{s_{t-1}}, \hat{\mu}_{p_t}, u)$$
$$\hat{\mu}_{s_t} = \textstyle\sum_{i=0}^{2n} w_{ms}[i] \chi_{s_t}[i]$$
$$\hat{\Sigma}_{s_t} = \textstyle\sum_{i=0}^{2n} w_{cs}[i](\chi_{s_t}[i] - \hat{\mu}_{s_t})(\chi_{s_t}[i] - \hat{\mu}_{s_t})^T + R_s$$

$$\hat{\chi}_{s_t} = generateSigmaPoints(\hat{\mu}_{s_t}, \hat{\Sigma}_{s_t})$$
$$\chi_{z_{st}} = \hat{\chi}_{s_t}$$
$$\hat{z}_{st} = \textstyle\sum_{i=0}^{2n} w_{ms}[i] \chi_{z_{st}}[i]$$
$$S_z = \textstyle\sum_{i=0}^{2n} w_{cs}[i](\chi_{z_{st}}[i] - \hat{z}_{st})(\chi_{z_{st}}[i] - \hat{z}_{st})^T + Q_t$$
$$\hat{\Sigma}_{sz} = \textstyle\sum_{i=0}^{2n} w_{cs}[i](\hat{\chi}_{s_t}[i] - \hat{\mu}_{s_t})(\chi_{z_{st}}[i] - \hat{z}_{st})^T$$

$$K_{s_t} = \hat{\Sigma}_{sz} S_z^{-1}$$
$$\mu_{s_t} = \hat{\mu}_{s_t} + K_{s_t}(z_t - \hat{z}_{st})$$
$$\Sigma_{s_t} = \hat{\Sigma}_{s_t} - K_{s_t} S_z K_{s_t}^T$$

$$\hat{\chi}_{p_t} = generateSigmaPoints(\hat{\mu}_{p_t}, \hat{\Sigma}_{p_t})$$
$$Z_{p_t} = nextState(\hat{\chi}_{p_t}, \mu_{s_{t-1}}, u)$$
$$\hat{z}_{pt} = \textstyle\sum_{i=0}^{2m} w_{mp}[i] \chi_{z_{pt}}[i]$$
$$P_z = \textstyle\sum_{i=0}^{2m} w_{cp}[i](\chi_{z_{pt}}[i] - \hat{z}_{pt})(\chi_{z_{pt}}[i] - \hat{z}_{pt})^T + Q_t$$
$$\hat{\Sigma}_{pz} = \textstyle\sum_{i=0}^{2m} w_{cp}[i](\hat{\chi}_{p_t}[i] - \hat{\mu}_{p_t})(\chi_{z_{pt}}[i] - \hat{z}_{pt})^T$$

$$K_{p_t} = \hat{\Sigma}_{pz} P_s^{-1}$$
$$\mu_{p_t} = \hat{\mu}_{p_t} + K_{p_t}(z_t - \hat{z}_{pt})$$
$$\Sigma_{p_t} = \hat{\Sigma}_{p_t} - K_{p_t} P_z K_{p_t}^T$$

return $\mu_{p_t}, \mu_{p_t}, \Sigma_{p_t}, \Sigma_{s_t}, R_{p_t}$

not evolve with time. Since, the parameters of a vehicle are fixed over a short period of time, therefore it makes sense to use this technique here. Distinct feature of this variation over conventional PF is that instead of re-sampling of the particles conventionally, it consolidates the particles around the mean. The particles are re-sampled from the gaussian with mean m and variance h^2V, where h is a smoothing factor. Since m pushes the distribution towards the weighted mean of the system, as is given by the equation :

$$m = a\theta + (1 - a)\hat{\mu}_{p_t}$$

Algorithm 3 Particle filter + UKF

procedure LIU_WEST PF($\bar{\mu}_{s_{t-1}}, z_t, u_t, [a, \gamma]$)

$\hat{\boldsymbol{\theta}}_t = \bar{\boldsymbol{\theta}}_{t-1}$

for $i = 1$ to M **do**

$\hat{\boldsymbol{\mu}}_{s_t} = nextState(\bar{\boldsymbol{\mu}}_{s_{t-1}}, \hat{\boldsymbol{\theta}}_t^{[i]}, \boldsymbol{u})$

$\boldsymbol{w}_t^{[i]} = \boldsymbol{w}_{t-1}^{[i]} e^{(-\frac{1}{2}(z_t - H_s\hat{\mu}_{s_t})Q^{-1}(z_t - H_s\hat{\mu}_{s_t})^T)}$

$\boldsymbol{w_t} = \frac{\boldsymbol{w_t}}{\sum_{i=0}^{M} \boldsymbol{w}_t^{[i]}}$

$\hat{\boldsymbol{\mu}}_{p_t} = \sum_{i=0}^{M} \boldsymbol{w}_t^{[i]} \hat{\boldsymbol{\theta}}_t^{[i]}$

$\boldsymbol{m} = \boldsymbol{a\theta} + (\boldsymbol{1} - \boldsymbol{a})\hat{\boldsymbol{\mu}}_{p_t}$

$\boldsymbol{V} = \sum_{i=0}^{M} \boldsymbol{w}_t^{[i]} \hat{\boldsymbol{\theta}}_t^{[i]} - \hat{\boldsymbol{\mu}}_{p_t}$

$\bar{\boldsymbol{\theta}} \sim \mathcal{N}(\boldsymbol{m}, \boldsymbol{h^2V})$

$\bar{\boldsymbol{\mu}}_{p_t} = \sum_{i=0}^{M} \boldsymbol{w}_t^{[i]} \bar{\boldsymbol{\theta}}_t^{[i]}$

$\boldsymbol{w_t} = \boldsymbol{\gamma w_t}$

return $\bar{\mu}_{p_t}$

procedure UKF($\bar{\mu}_{p_t}, z_t, u_t, [\alpha, \beta, \gamma]$)

$\boldsymbol{w_{ms}}, \boldsymbol{w_{cs}} = generateWeights(\boldsymbol{\alpha_s}, \boldsymbol{\beta_s}, \boldsymbol{\gamma_s})$

$\boldsymbol{\chi}_{s_{t-1}} = generateSigmaPoints(\boldsymbol{\mu}_{s_{t-1}}, \boldsymbol{\Sigma}_{s_{t-1}})$

$\boldsymbol{\chi}_{s_t} = nextState(\boldsymbol{\chi}_{s_{t-1}}, \hat{\boldsymbol{\mu}}_{p_t}, \boldsymbol{u})$

$\hat{\boldsymbol{\mu}}_{s_t} = \sum_{i=0}^{2n} \boldsymbol{w_{ms}}[i]\boldsymbol{\chi}_{s_t}[i]$

$\hat{\boldsymbol{\Sigma}}_{s_t} = \sum_{i=0}^{2n} \boldsymbol{w_{cs}}[i](\boldsymbol{\chi}_{s_t}[i] - \hat{\boldsymbol{\mu}}_{s_t})(\boldsymbol{\chi}_{s_t}[i] - \hat{\boldsymbol{\mu}}_{s_t})^T$
$+\boldsymbol{R_s}$

$\hat{\boldsymbol{\chi}}_{s_t} = generateSigmaPoints(\hat{\boldsymbol{\mu}}_{s_t}, \hat{\boldsymbol{\Sigma}}_{s_t})$

$\boldsymbol{\chi}_{z_{st}} = \hat{\boldsymbol{\chi}}_{s_t}$

$\hat{\boldsymbol{z}}_{st} = \sum_{j=0}^{2n} \boldsymbol{w_{ms}}[i]\boldsymbol{\chi}_{z_{st}}[i]$

$\boldsymbol{S_z} = \sum_{i=0}^{2n} \boldsymbol{w_{cs}}[i](\boldsymbol{\chi}_{z_{st}}[i] - \hat{\boldsymbol{z}}_{st})(\boldsymbol{\chi}_{z_{st}}[i] - \hat{\boldsymbol{z}}_{st}$
$+\boldsymbol{Q_t}$

$\hat{\boldsymbol{\Sigma}}_{sz} = \sum_{i=0}^{2n} \boldsymbol{w_{cs}}[i](\hat{\boldsymbol{\chi}}_{s_t}[i] - \hat{\boldsymbol{\mu}}_{s_t})(\boldsymbol{\chi}_{z_{st}}[i] - \hat{\boldsymbol{z}}_{s}$

$\boldsymbol{K}_{s_t} = \hat{\boldsymbol{\Sigma}}_{sz}\boldsymbol{S}_z^{-1}$

$\boldsymbol{\mu}_{s_t} = \hat{\boldsymbol{\mu}}_{s_t} + \boldsymbol{K}_{s_t}(\boldsymbol{z_t} - \hat{\boldsymbol{z}}_{st})$

$\boldsymbol{\Sigma}_{s_t} = \hat{\boldsymbol{\Sigma}}_{s_t} - \boldsymbol{K}_{s_t}\boldsymbol{S_z}\boldsymbol{K}_{s_t}^T$

return $parameters_t, w_t$

III. Results and Discussion

In this section the results of state and parameters estimati have been provided. In order to make analysis easier the resul have been studied according to the model.

A. Parameter estimation from Kinematic model

All the models estimate the states almost equally well, because of the simplistic nature of the model. So, those results are not presented here. Only the estimation of position of Centre of Mass, l_f and l_r, by different techniques has been discussed here.

In order to make the comparison easy between different techniques, same level of noise and number of iterations has been set:

$$\Sigma_s = diag([1, 1, 0.2, 2])$$
$$\Sigma_p = diag([5, 5])$$
$$R_s = diag([0.5^2, 0.5^2, 0.2^2, 1^2])$$
$$R_p = diag([1.25^2, 1.25^2])$$
$$Q = diag([1^2, 1^2, 1.5^2, 0.1^2])$$

```
simulation noise = 0.1
number of iterations = 1000
```

The results of parameter estimation from the three frameworks are compared in figure 4 and figure 5 for l_f and l_r respectively. The table I summarises the results to show a comparative difference in percentage error in the final values.

Estimation	$\hat{l}_f$	$\hat{l}_r$
Actual Value	1.5	1.5
Dual EKF	1.5914	1.5868
Dual UKF	1.4407	1.5463
Particle+UKF	1.7785	1.2431

TABLE I
COMPARISON OF PARAMETER ESTIMATION FROM KINEMATIC MODEL

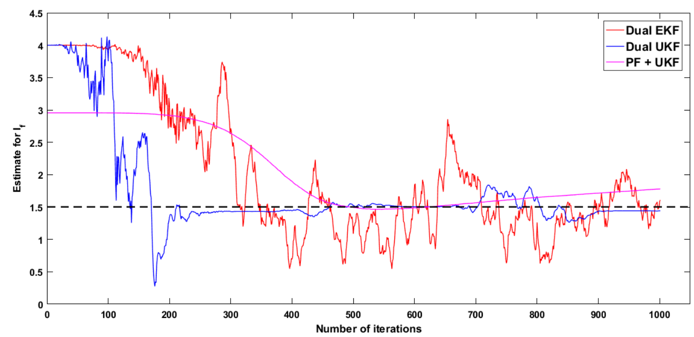

Fig. 4. Comparison of parameter l_f from kinematic model

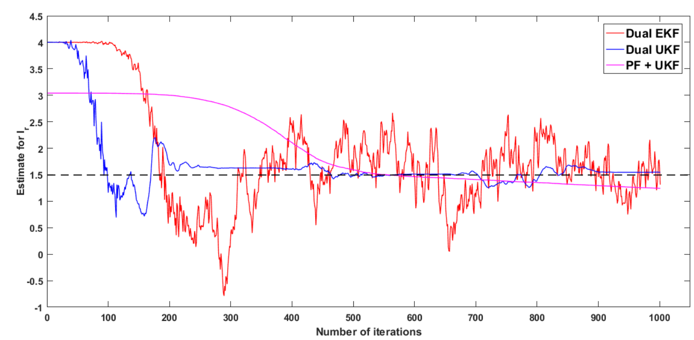

Fig. 5. Comparison of parameter l_r from kinematic model

The dual UKF is able to estimate the parameters with least error and least standard deviation as well. The dual EKF comes second in place, but with results sometimes totally unacceptable. The particle filter + UKF begins to diverge from the closest estimate from the actual value, and accumulates error in its estimate afterwards. So, even though the model is seemingly simple, linearization should be avoided as much as possible. The non-linear term are the trigonometric angles in this models, so as long as input steering angle is confined to small values, EKF could also run well.

The plots for covariance in estimated parameters for DEKF and DUKF are shown in figure 6 which shows large variance accumulated by EKF from which sometimes filter is never able to recover, especially if the noise parameters are not properly tuned.

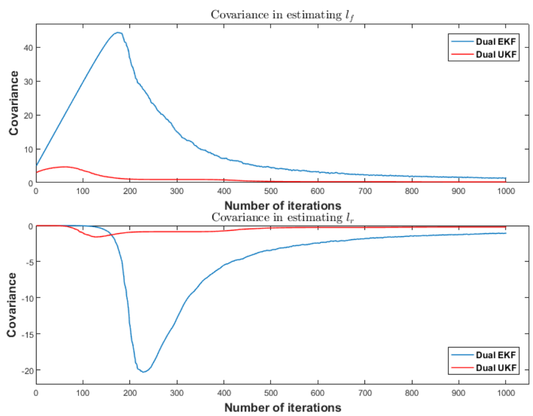

Fig. 6. Comparison of variance in parameter estimation between DEKF and DUKF

B. State estimation with Dynamic model

Dynamic model is seemingly complex with more coupled equations as compared to kinematic model. As a result linearization of the equations is not a great idea, and as a result the dual EKF does not converge to the right solution. Hence, only the results of Dual UKF and particle filter+UKF are discussed here. The input data used is the inbuilt Matlab dataset [10], used as an example for grey-box modeling.

```
load(fullfile(matlabroot,'toolbox',
'ident','iddemos','data','vehicledata'));
```

Initial beliefs for state is [0; 0; 0]. And the noise values and the initial belief uncertainties are kept as mentioned below :

$$\Sigma_s = diag([1, 1, 0.2])$$
$$\Sigma_p = diag([200, 2e4, 1.5e4])$$
$$R_s = diag([1.2^2, 1.2^2, 0.45^2])$$
$$R_p = diag([5^2, 5.75e2^2, 2.75e2^2])$$
$$Q = diag([0.2^2, 0.2^2, 0.5^2])$$

The state estimation has quite a satisfactory results for both dual UKF and PF + UKF estimators. The table II shows the values of mean error in different techniques for state estimation for v_x,v_y and yaw rate respectively. Instead of the evolution of states, the figure 7 shows the evolution of the errors in states of observers with times so as to provide a clearer difference between the performance of the two estimation frameworks. With same noise levels, PF + UKF takes slightly longer before stabilising and converging to the true states. Both the techniques have almost similar performance.

	$\hat{v}_x$	$\hat{v}_y$	$\hat{r}$
Dual UKF	0.0117	0.0546	0.0138
Particle+UKF	0.0316	0.1091	0.0412

TABLE II
MEAN ERROR IN THE ESTIMATED STATES

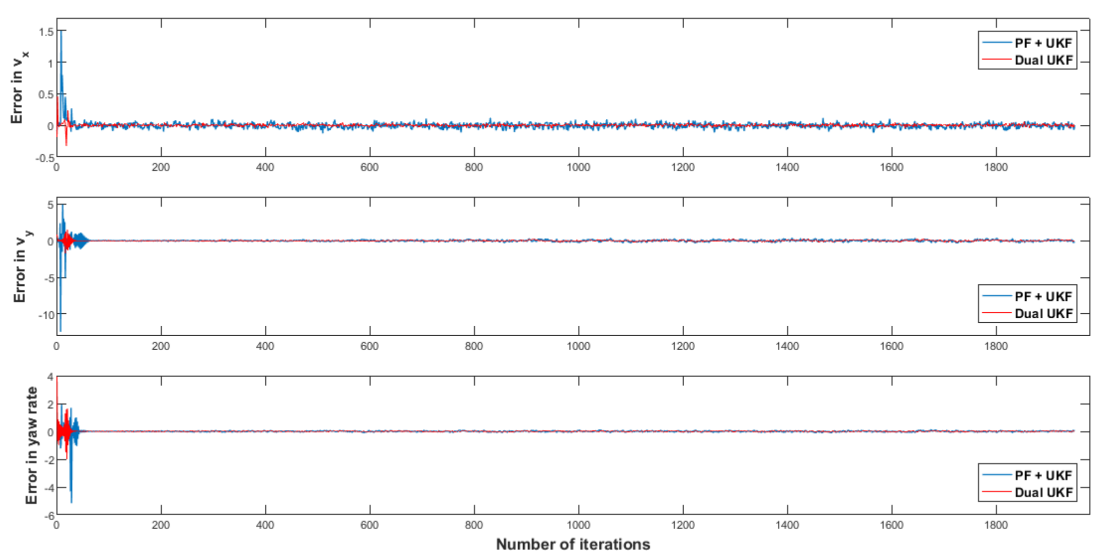

Fig. 7. Error between estimated states and true states

C. Parameter estimation with Dynamic model

In dual UKF, initial belief for the parameters are taken as [1500; 18e4; 4e4] for mass, lateral tyre coefficient and the longitudinal tyre coefficient respectively. Whereas, for PF + UKF, parameter estimator is initialized from zero.

The figure 8, 9 and 10 compares the estimation of parameters mass, lateral and longitudinal stiffness coefficient for tyres between Dual UKF and PF+UKF estimation technique. The table III provides final parameters calculated.

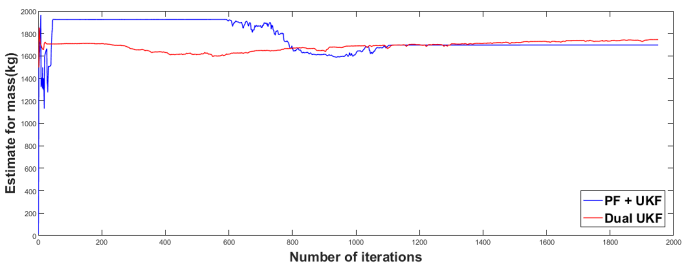

Fig. 8. Estimation of mass of vehicle by various estimators

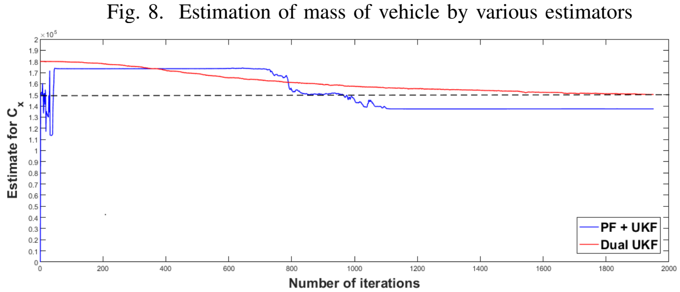

Fig. 9. Estimation of Lateral stiffness of tyres by various estimators

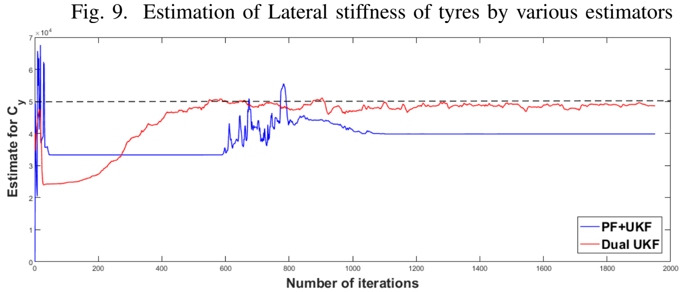

Fig. 10. Estimation of Longitudinal stiffness of tyre by various estimators

	Mass(kg)	C_x(N/rad)	C_y(N/rad)
Actual Value	1700	150000	50000
Dual UKF	1750	150290	48630
Particle+UKF	1700	137400	39850

TABLE III
PARAMETERS ESTIMATED BY PF+UKF AND DUAL UKF

For the dual UKF, the parameter estimation depends upon the process noise level for parameters, R_p set by the user, and should be set around $0.2 - 0.3\%$ of the parameter to be estimated. Minor tuning might be required to make the filter fast or slow by increasing or decreasing the process noise respectively, depending upon how far or close is the initial belief from the actual parameter value.

Whereas, PF + UKF has a bit more noisy and probabilistic output. Although it is less sensitive to the noise levels as compared to the dual UKF and gives more consistent results for different noise inputs.

IV. CONCLUSION

Dual state and parameter estimation have been attempted in this work using three different estimation framework. The dual EKF can work with simpler models, since approximation from linearization does not affect the integrity of the solution. Dual UKF method works well in most cases but requires some tuning with the noise levels. The novel method tried in this work was of Liu-West particle filter + UKF method, which is most resilient to noises and gives a decent estimation within small number of iterations. It can be further improved by improving the measurement model as it has been assumed that all the states can be directly measured. So a more general and real-life model for measurement can be considered. Also, to exploit all the benefits of the dual estimation framework, a check based on the variance of the parameter estimator can be set which switches off the parameter estimator once it has converged to a solution with sufficient precision. This leads to saving of the computational power and states estimator can then run alone.

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