Assignment 3 Report
Linear Quadratic Regulator

EL2700 – Model Predictive Control

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Part 1: LQR Implementation

1. We want to penalize the states not being at zero. Therefore we first introduce a the tuning matrix $Q = I$ and $R = 1$, see figure 1. Then we tuned the entries by setting the penalties according to Bryson's rule and tuning it. The system response with the tuned matrices is displayed in figure 2). The position of the cart at time $t = 10$ with the tuned matrices is $x_1(10) = 10.24$m and a maximal angle deviation of $\max|x_3| = 0.157$rad, which satisfies the requirements.

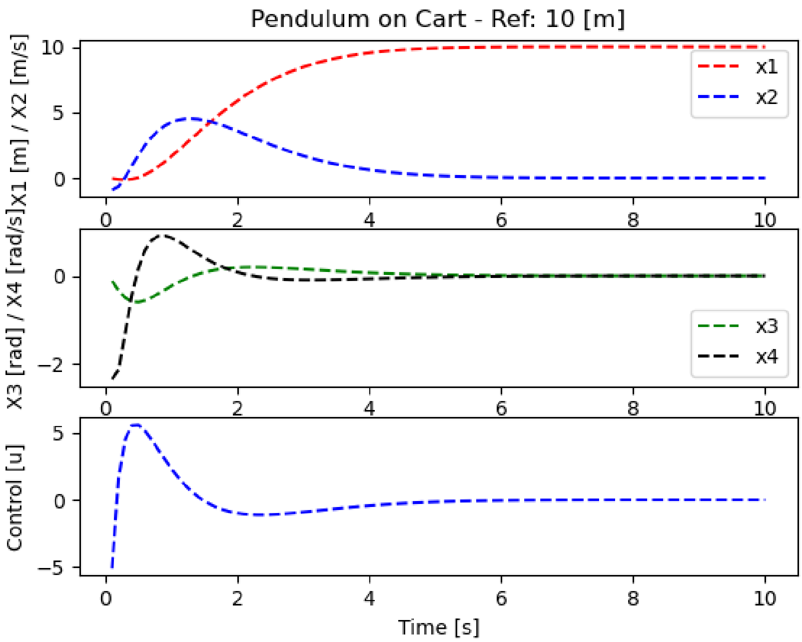

Figure 1: System response with $Q = I$ and $R = 1$

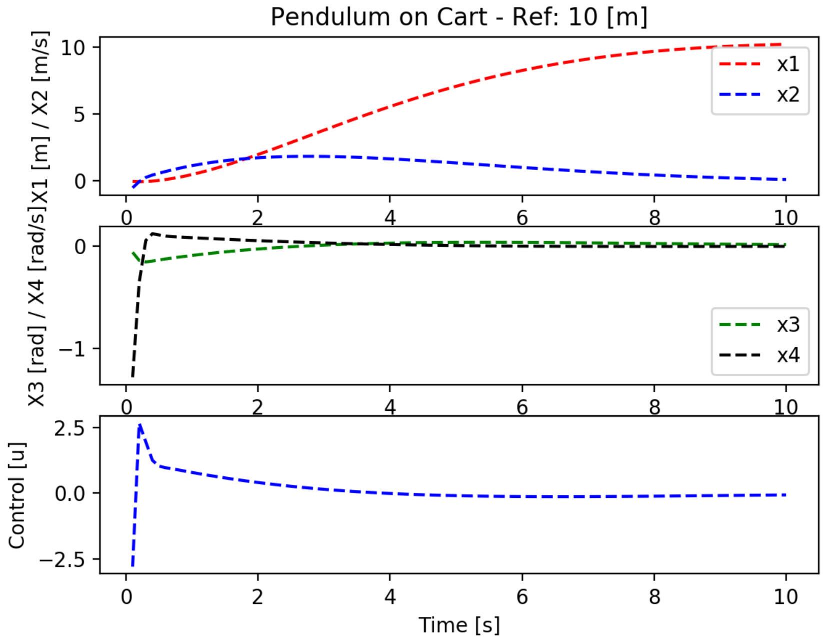

Figure 2: System response with $Q = diag(10^{-2}, 5^{-2}, 0.05^{-2}, 3^{-2})$ *and* $R = 10^{-2}$

2. The constant disturbance introduces a steady state error to the system. The cart position after 20 seconds with disturbance is at $x_1(20) = 9.58\text{m}$ (without disturbance it was at $x_1(20) = 10.00\text{m}$).

Part 2: Adding integral action

3. We start with Q_1 equal to the Q used in part 1 and $q_i = 1$ first. Since the system response is good and the steady state error could be removed ($x_1(20) = 10.01\text{m}$). However, the requirement of maximum angle is not satisfied anymore ($\max|x_3| = 0.347$). Increasing q_i leads to higher inputs and therefore a faster response of the system, however the maximum angle deviation also increases. Therefore we decrease it to $q_i = 10^{-2}$. Now all the requirements are satisfied (especially the angle requirement $\max|x_3| = 0.0496$) and the steady state error is removed ($x_1(20) = 9.91\text{m}$), as can be seen in figure 3.

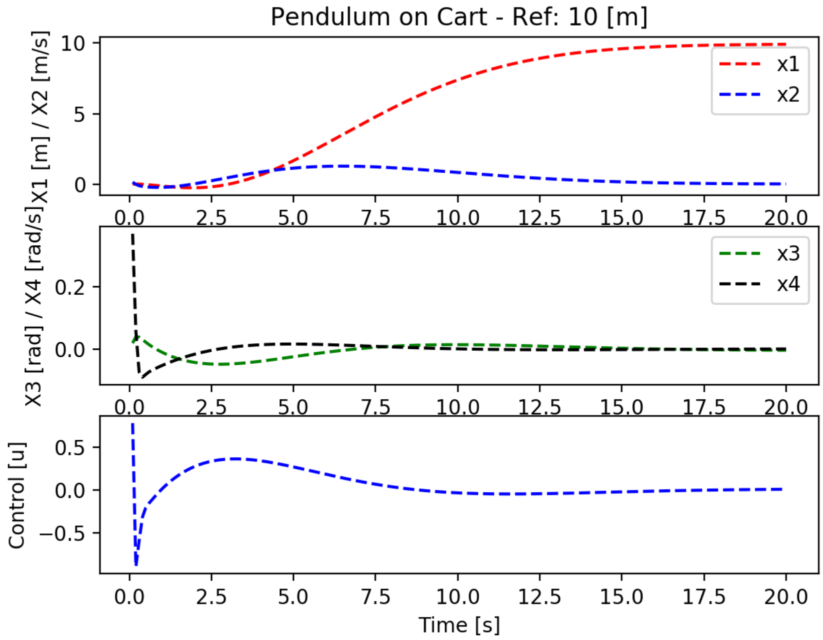

Figure 3: System response with $Q = diag(10^{-2}, 5^{-2}, 0.05^{-2}, 3^{-2}, 10^{-2})$ *(last entry corresponds to integral action) and* $R = 10^{-2}$.

Part 3: Output feedback controller

4. With $Q_p = R_n = I$ and the both choices of C the system response is much slower with the estimated system states compared to the state feedback controller. Choosing the output matrix C as $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ reduces the noise on the estimation of the state variables significantly, because since one more state is measured, less estimation is necessary.

   So, we decided to increase some bandwidth by adding artificial noise to the state equations :
   $$Q_p = Q_p^{nom} + \sigma B_i . B_i^T$$
   where Q_p^{nom} was chosen as $\text{diag}(0.01, 0.01, 0.01, 0.1)$ and σ as 1. The value for measurement noise was chosen as:
   $$R_n = \begin{bmatrix} 0.005 & 0 \\ 0 & 0.005 \end{bmatrix}$$
   The results are obtained as seen in figure 4, where we are able to follow reference of 10m up to 9.97m and maximum deviation in the pendulum's angle is 0.06rad.

5. Although the system with the output controller is not very smooth, increasing the noise and/or altering the initial point does not lead to unstable behaviour. However, the performance of the system worsens.

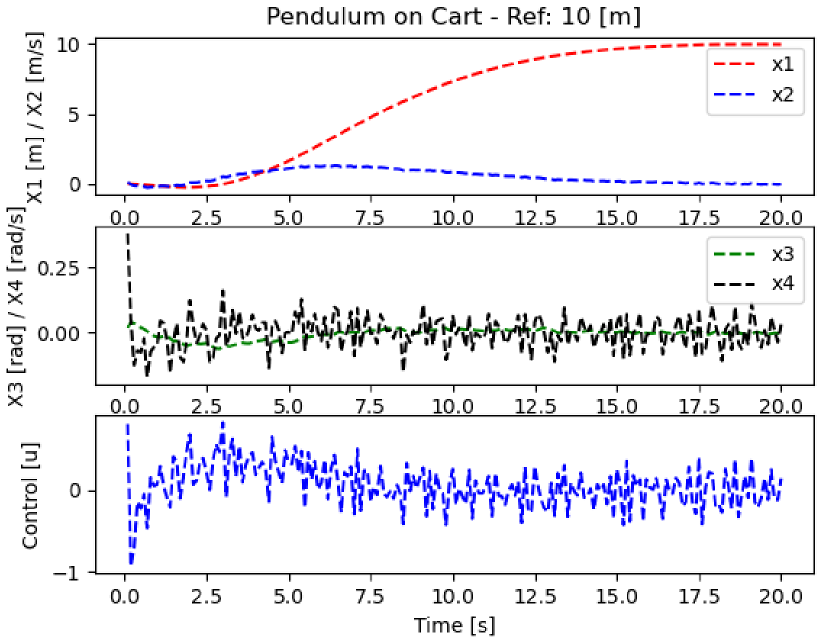

Figure 4: System response with controller tuning as in figure 3, but with output feedback and noise.

Updated Section: For the Kalman-filter we need deep knowledge about the model we use and the disturbances. The state feedback controller is very robust, but output feedback, not so much. If the accurate information about states is not available, the output feedback can only be as robust as the state prediction. Having more output signals (C-Matrix) clearly increases the information on the state(as we have seen in the last question), therefore the analysis here is made only with the C-matrix performing better. Increasing the noise on the measurements and on the model intuitively leads to higher fluctuations of the states and the control input, such that they eventually exceed the limits.