Assignment 2 Report
State Feedback Control Design

EL2700 – Model Predictive Control

September 10, 2020

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Part 1: Inverted Pendulum Model

Inverted Pendulum Model

1. A higher number of sample points leads to a more accurate numerical solution. However, more sample points also increase computational effort.

2. The point $\pm\pi/2$ is excluded (by substracting / adding a small ε from the grid, since the angle of $\pm\pi/2$ would mean that the pendulum reaches horizontal orientation. Once, it is at this orientation it can not be lifted be moving the cart anymore.

3. For every point in the discretized x_1-x_2-grid, the optimal cost-to-go value $J_k(x)$ and the corresponding optimal control action $u_k^\star(x)$ are computed. This is done for every time step. Doing so, it is later on possible to obtain the optimal state trajectory by forward simulation.

4. Adding constraints on the states speeds up the computation greatly. The constraint on the variable $x_1 = \theta$ does not change anything in the computation, because the system would be uncontrollable anyway once the pendulum is horizontal. The constraint on $x_2 = \dot{\theta}$ must be well chosen, in order to not lead to physically not meaningful solutions. (In physical systems the angular velocity $\dot{\theta}$ is clearly bounded but this bound is not as straight forward to quantify as the one on the angle θ its self.)

5. Increasing the R value, which represents the input cost, leads to a controller reacting slower but with a lower input signal. Increasing the Q matrix, representing the cost of not being at the equilibrium state, leads to a faster controller.

Performance Comparison

The comparison between the linear model and the nonlinear model with feedback linearization is given in table 1, where the convergance rate is measured as the time it takes for both states to get below 0.01, i.e. $|x_i(t)| < 0.01$ $i = \{1,2\}$.

	Linear model	Nonlinear model
Computation time	28.06 s	29.01 s
Accumulated cost	9445.38	9994.9
Spent control energy	321.82	491.57
Convergence rate	3 s	3 s

Table 1: Comparison of the linear model and the nonlinear model with feedback linearization.

Part 2: Cart-Pendulum Model

By adding the states of the cart to the model, the grid increases a lot as can be seen in the following equation

$$\text{Grid size} = n_1 \cdot n_2 \cdot n_3 \cdot n_4 \ (\cdot T),$$

where n_i represents the number of discretization points of the state variable x_i and T represents the number of time steps. By adding two variables to the problem, the grid increases by the factor of $n_1 \cdot n_2$. Therefore also computation time increases at least by the same factor.

We expected the controller to drive the system states nevertheless to the reference point. However, this could not have been achieved.