Linear algebra problems

1. Consider

$$R = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

   a. What is the determinant of R
   b. Is R an orthogonal matrix?
   c. What is the inverse of R?
   d. Use matlab to find the eigenvalues and eigenvectors of the above matrix. What are the eigenvalues and eigenvectors of the above matrix?

2. Consider the rotation matrix

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

   a. What is the determinant of the above matrix?
   b. Is R an orthogonal matrix? Prove that it is an orthogonal matrix by multiplying it by its transpose.
   c. This matrix performs a rotation of any vector in the x-y plane by an angle theta . to illustrate this, consider the vector

$$v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

      Rotate the above vector by an angle =45 degrees (θ=pi/4).

   d. Draw the vector v in the x-y plane
   e. Draw the vector after rotation in the x-y plane. Confirm that the rotated vector points in the direction $[1\ 1]^T$.

Probability problems

1. Illustration of central limit theorem: Consider a uniformly distributed random variable with pdf

$$x = \frac{1}{5} * [1\ 1\ 1\ 1\ 1]$$

Assume iid random variables, x1, x2, …xN all with the above pdf.

   a. To find the pdf of x1+x2, convolve the above pdf with itself in matlab. Plot the results
   b. To find the pdf of x1+x2+…+xn, convolve the above pdf with itself in matlab 10 times (use the conv function). Plot the result. Do you notice that the pdf of the sum is closer to a Gaussian than the original uniform PDFs?

2. Consider the lottery example presented in class
   a. If the number of numbers drawn were 5 instead of 6, what would the expected value of winnings be?
   b. How much would the lottery winnings have to be in order to purchase a ticket?

Chap 2 problems

1. Review solution to problem 1
    a. Ellipse shaped classes will turn out to be important when we do Bayesian classification with Gaussian PDFs
2. Problem 6
3. Problem 7

Chap 3 problems

1. Review solution to problem 1
    a. Recalculate the change posterior probabbilyt if the P(d=1|t=0)=1e-6
    b. This indicates that if a false test is unlikely, then a positive test results indicates the disease is likely
2. Problem 6
3. Problem 8