JAMES CHHUN

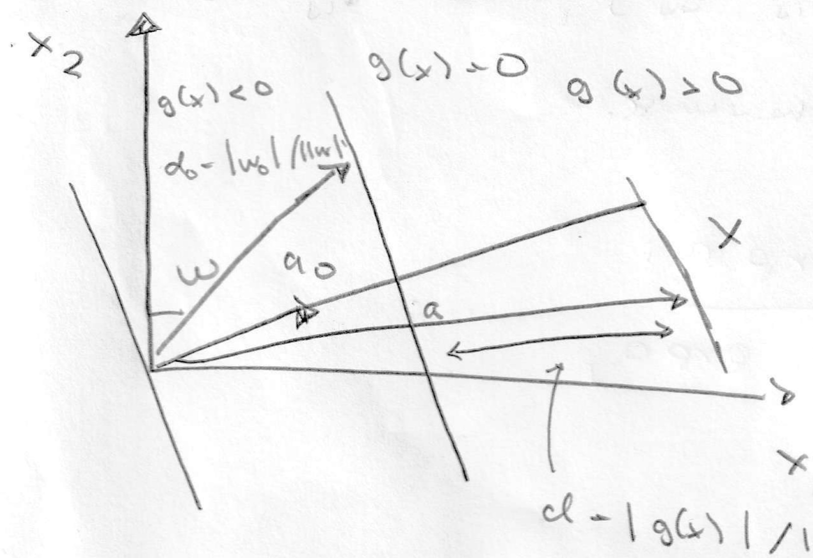
T/TH 530-645

2.) FOR TWO-DIMENSIONAL CASE OF FIGURE 10.2

SHOW EQUATIONS 10.4 and 10.5

SOLUTION

- USE FIGURE 10.1



- take x_0 on the hyperplane, the angle between x_0 and w is a_0 and because it is on hyperplane,

$\rightarrow$ $g(x)=0$.

$\rightarrow$ $g(x_0) = w^T x_0 + w_0 = \|w\| \cdot \|x_0\| \cos a_0 + w_0 = 0$

$\rightarrow$ $d_0 = \|x_0\| \cos a_0 = \dfrac{|w_0|}{\|w\|}$

- For any $\underline{x}$ w/ angle a to ω,

$\rightarrow g(x) = \omega^T x + \omega_0 = \|\omega\| \cdot \|x\| \cos a + \omega_0$

$d = \|x\| \cos a - \frac{\omega_0}{\|\omega\|} = \frac{g(x) - \omega_0 + \omega_0}{\|\omega\|} = \frac{g(x)}{\|\omega\|}$

3.) Show derivative of softmax, $y_i = \exp(a_i) / \Sigma_j \exp(a_j)$

is $\partial y_i / \partial a_j = y_i (\delta_{ij} - y_j)$, where δ_{ij} is

1 if $i = j$ and 0 otherwise.

Sol

$\rightarrow$ given $y_i = \frac{\exp a_i}{\Sigma_j \exp a_j}$

for $i = j$, we have

$$\frac{\partial y_i}{\partial a_i} = \frac{\exp a_i (\Sigma_j \exp a_j) - \exp a_i \exp a_i}{(\Sigma_j \exp a_j)^2}$$

$$\Rightarrow \frac{\exp a_i}{\Sigma_j \exp a_j} \left(\frac{\Sigma_j \exp a_j - \exp a_i}{\Sigma_j \exp a_j} \right)$$

$$\Rightarrow y_i (1 - y_i)$$

for $i \neq j$,

$\rightarrow$ $$\frac{\partial y_i}{\partial a_j} = \frac{-\exp a_i \exp a_j}{\left(\sum_{j'} \exp a_{j'}\right)^2}$$

$$= -\left(\frac{\exp a_i}{\sum_{j'} \exp a_{j'}}\right)\left(\frac{\sum_j \exp a_j}{\sum_{j'} \exp a_{j'}}\right)$$

$$= y_i(0 - y_j)$$

$\rightarrow$ can combine equations

$\rightarrow$ $$\frac{\partial y_i}{\partial a_j} = y_i(\delta_{ij} - y_j)$$