1. A professor has 15 students and during lecture will (uniformly) at random choose a student to answer a question. The professor asks 8 questions during the lecture. What is the probability no student will have to answer more than one question?

P(E) = likely outcomes / sample space

sample space = 15

students cannot answer more than 1 question:

8-1=7 likely outcomes

P(E)= 7/15 = 46%, there is a probability of 46% of no student having the answer to no more than one question

2. An integer from the range 00000 - 99999 is generated uniformly at random. We are interested only in even integers that start with 2 odd digits where all digits are unique. If we randomly generate 8 of these numbers in succession, what is the probability we get exactly 5 numbers that meet our criteria?

From 100 - 1000

The number of required integers: $\underset{\text{5 odds}}{5} \times \underset{\text{4 odds}}{4} \times \underset{\text{5 evens}}{5} = 100$

From 1000 - 10000

The number of required integers: $5 \times 4 \times 7 \times 5 = 700$

From 10000 - 99999

The number of required integers: $5 \times 4 \times 7 \times 6 \times 5 = 4300$

Total num of required integers = 5000

Total num of integers from 0 - 99999 = 10000

Therefore, the probability of getting a required integer is $\frac{5000}{10^5} = 0.05$

$$\therefore P(x=5) = C(8,5) \cdot (0.05)^5 (1-0.05)^5$$

$$= 1.5 \times 10^{-5}$$

3. You roll 3 six-sided, fair dice. Let A be the event that at least 2 dice show 4 or above. Let B be the event that all 3 dice show the same value. Are A and B independent?

P(A) = P(2) or more dice show 4 or more

$P(x=2) + P(x=3) = C(3,2) \cdot (3/6)^2 \cdot (3/6) + C(3,3) \cdot (3/6)^3 \cdot (3/6)$

$P(x=2) + P(x=3) = 1/2$

P(B) = P(all three dice shows same num) = P(6 such ways are there since there are 6 different numbers)

$P(B) = 6/6^3 = 1/36$

$P(A \cap B)$ = P(all 3 same and 4 or above)
= P(all 3 num are 4) + P(all 3 num are 5) + P(all 3 num are 6) $= 1/6^3 + 1/6^3 + 1/6^3 = 3/216 = 1/72$

Since $P(A) \times P(B) = P(A \cap B) = (1/2) \times (1/36) = (1/72)$, therefore events A and B are independent

1. In poker, a flush is any 5-card hand where all the cards of the same suit. For this problem we will not distinguish between an ordinary flush and special flushes (like straight and royal flushes), meaning we will call any hand that has all 5 cards from the same suit a flush. Poker-player Paul loves a flush. What is the expected number of hands of poker he has to play to get a flush. (We assume each hand is dealt from a new deck containing of randomly ordered cards).

P(E) = likely outcomes / sample space

sample space = C(52,5)
likely outcomes = 4 x C(13,5)

$$P(E) = \frac{4 \times C(13,5)}{C(52,5)} = 0.001980792$$

, probability of a flush in a single draw

now, let x: number of hands to play until one flush and since dealing is done from a new deck, x is geometric

E(x) = 1/0.001980792 = 504.8486 expected number of hands of poker he has to play to get a flush

2. A basketball team has a superstar. When their superstar plays, they win 70% of the time. When their superstar does not play they win 50% of the time. Entering a 5 game stretch, the superstar had been recovering from an injury and said the chance they would play the next 5 games was 75%. You go on a trip to the jungle (no internet access). When you return you find out the team won 4 of the 5 games. What is the probability the superstar played those 5 games? You may assume the superstar doesn't get injured during those games (either they play all or none of the 5).

P(win | superstar plays) = 0.7
P(win | superstar doesn't play) = 0.5
P(superstar plays) = 0.75 for next 5 games

Therefore,
$P(\text{win } 4/5 \mid \text{superstar plays}) = C(5,4) \times 0.7^4 \times 0.3$
$= 0.36015$
$P(\text{win } 4/5 \mid \text{superstar doesn't play}) = C(5,4) \times 0.5^5$
$= 0.15625$

$P(\text{win } 4/5) = 0.15625 \times 0.25 + 0.36015 \times 0.75 =$
0.309175

Therefore,
$P(\text{superstar plays} \mid \text{win } 4/5) = 0.36015 \times 0.75 / 0.309175$
$= 0.8737$

There is a 87.37% chance of the superstar playing in those 5 games