1. Consider the word `unusual`. How many unique subsets of 5 letters (of the 7) exist? How many different strings could be made from 5 of those 7 letters?

Since there are a total of 7 letters and the letter u is repeated 3 times. So the number of possible combinations for the subset of 5 letter word is

$$\frac{7!}{3!} = 840 \text{ possible subsets}$$

As for how many strings, the total number of strings that are possible is

$$7! = 5040 \text{ total number of strings possible}$$

2. Using a standard deck of playing cards, how many ways are to form a 5-card hand with 2 pairs (i.e. pair of one value, a pair of a different value, and a fifth card of some other value)?

The first two cards will be the first card each of the 2 pairs. We have 13 cards to choose from so,

$$C(13,2) = \frac{13!}{2!(11!)} = 78$$

We need to pick 2 cards from each suit for each card and we need to do this twice

$$C(4,2) = \frac{4!}{2!(2!)} = 6$$

Last, we need to account for the 5th, there are 11 cards to choose from and we need one

$$C(11,1) = \frac{11!}{1!(10!)} = 11$$

Additionally, from that one card we need 1 suit

$$C(4,1) = \frac{4!}{3!(1!)} = 4$$

And so, there are $78 \times 6 \times 6 \times 11 \times 4 = 123{,}552$ combinations to form a 5 hand with 2 pairs.

3. A violinist serenades couples at a romantic restaurant. She will play 16 songs in an hour and there are 7 couples. One couple is having a fight and will allow at most 1 song to be played to them before they ask the violist not to return to their table. If we care only about the number of songs each couple receives, how many ways can the songs be distributed amongst the couples.

16 # of songs that can be played by the violinst
7 couples

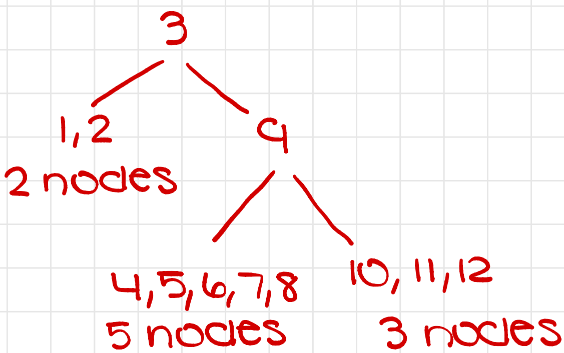
She can play 16 songs for 7 couples

Total number of ways $= C(16,1) \times C(15,6)$
$= 16 \times 5{,}005$
$= 80{,}080$

There are 80,080 ways the songs can be distributed amongst the couples.

4. There is a Binary Search Tree with 12 nodes. Each node has a distinct value between 1 and 12. The root has value 3, and its right child has value 9. How many possible Binary Search Trees could this be? Tip: *Try to define how many ways there are to form a BST of 2 nodes. Then try to define how many ways there are to form a BST of 3 nodes (think about the possible insertion order based on rank: smallest, medium, largest)* ***in terms of 2 node trees*** *for certain cases. Continue to do this for 4 node trees (in terms of 3- and 2-node trees for various cases of insertion ordering based on rank) and 5 node trees.*

root → 3
right child → 9



i) 2 nodes $= \dfrac{2(2)C_2}{2+1} = 2$

ii) 5 nodes $= \dfrac{2(5)C_5}{5+1} = 42$

iii) 3 nodes $= \dfrac{2(3)C_3}{3+1} = 5$

Total possible Binary Search Trees
$= (2)(42)(5)$
$= 420$ possible trees

5. 10 friends arrive to get their COVID vaccine during a particular time slot. During that time slot there are 4 identical nurses administering shots, but 1 of the nurses **may** (or **may not**) be scheduled for a break during the time slot in which the friends arrive. Also, how long it takes the nurses to administer a shot varies wildly, so the nurses working during the time slot are guaranteed to serve at least 1 person, but how many additional people they are able to serve is arbitrary. How many different combinations are there for the number of patients served by the nurses?

There are two cases to consider, one case where the 4th nurse is on break and one where the 4th nurse isn't on break

case 1:

(7,0,0)
(6,1,0)
(5,1,1)
(5,2,0)
(4,3,0)
(4,2,1)
(3,2,2)
(3,3,1)

8 possibilities

For 3 nurses, 3 people are already taken care of so 7 are left

case 2:

(6,0,0,0)
(5,1,0,0)
(4,1,1,0)
(4,2,0,0)
(3,1,1,1)
(3,2,1,0)
(3,3,0,0)
(2,2,2,0)
(2,2,1,1)

9 possibilities

For 4 nurses, 4 people are already taken care of so 6 are left