Spatial Social Community

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Overview

1. $\sqrt{e}$
   - Listing all triangles in a Graph
   - Time Complexity
   - Truss Decomposition
   - Time Complexity

Listing triangles

Procedure `Tree()`

Find a rooted spanning tree for each nontrivial connected component of G;

If any tree edge is contained in a triangle the procedure terminates(Problem);

Delete the tree edges from G;

Algorithm `Triangle()`

Repeat Tree until all edges of G are deleted

Time Complexity

- Let c denote the number of connected components. During the execution of Triangle, the value of c increases (c=1 at the initialization).
- If $c \leq n - e^{\frac{1}{2}}$: each iteration of Tree causes the deletion of $n - c$; $n - c \geq n - (n - e^{\frac{1}{2}}) = e^{\frac{1}{2}}$ edges; since there are total e edges in G, the Triangle may at most call $\frac{e}{e^{\frac{1}{2}}}$ times of Tree.
- If $c > n - e^{\frac{1}{2}}$: the degree of each vertex is at $n - c$; $n - c \leq n - e^{\frac{1}{2}} = e^{\frac{1}{2}}$; since each iteration of Tree decreases the degree of each non-isolated vertex, there may be at most $e^{\frac{1}{2}}$ such iterations.

Truss Decomposition

$k \leftarrow 2$;
compute sup(e) for each edge $e \in E_G$;
sort all the edges in ascending order of their support;
while $\exists e$ *such that* $sup(e) \leq (k-2)$ **do**
    let $e = (u, v)$ be the edge with lowest support;
    assume, w.l.o.g., $deg(u) \leq deg(v)$; **for** *each* $w \in nb(u)$ **do**
        if $(u, w) \in E_G$ **then**
            decrease $sup((u, w))$ by 1;
            decrease $sup((v, w))$ by 1;
            reorder (u, w) and (v, w) according to their new support;
        end
    end
    $\tau(e) = sup(e)$;
    remove e from G;
end
if *not all edges in G are removed* **then**
    increase k by 1;

Time Complexity

- Let $nb_{\geq u}(u)$ be the neighbors of u that have degrees no less than degree of u
- Prove that for any $u \in V_G$, $|nb_{\geq}(u)| \leq 2\sqrt{m}$
- If $deg(u) \leq \sqrt{m}$, then $|nb_{\geq}(u)| \leq 2\sqrt{m}$
- If $deg(u) > \sqrt{m}$ and suppose $|nb_{\geq}(u)| > 2\sqrt{m}$, then $\sum_{u \in V_G} deg(u) > 2E$, which is impossible($\sum_{v \in nb_{\geq}(u)} \geq |nb_{ge}(u)| \times deg(u)$).