Joint Routing, Channel Assignment, and Scheduling in Wireless Networks with General Interference Models

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***Abstract*—Throughput optimization in wireless networks with multiple channels and multiple radio interfaces per node is a challenging problem. For general traffic models (given a set of source-destination pairs), optimization of throughput entails design of "efficient" routes between the given source-destination pairs, in conjunction with (i) assignment of channels to interfaces and communication links, and (ii) scheduling of non-interfering links for simultaneous transmission. Prior work has looked at restricted versions of the problem. In this article, we design approximation algorithms for the joint routing, channel assignment, and link scheduling problem in wireless networks with general interference models. One of the unique contributions of our work is addressing the above joint problem in the context of physical interference model. In addition, we also address the above problem with the constraint that the designed routes between each source-destination pair be single-path flows. To the best of our knowledge, ours is the first work to address the above joint problem in such general contexts. For each version of the problem, we design approximation algorithms that guarantee near-optimal throughput. We demonstrate the effectiveness of our algorithms in general contexts through simulations.**

I. Introduction

One of the central questions in communication networks is: Given a collection of source-destination pairs, what is the maximum rate (throughput) at which the network can transfer data from the sources to the corresponding destinations? The above throughput maximization problem is challenging in the context of wireless network due to the presence of wireless interference. In addition, it has been shown that one can significantly increase the network network by equipping each node with multiple radio interfaces that operate on multiple orthogonal channels. Availability of multiple channels and interfaces poses the additional challenge of determining efficient assignment of channels to links and interfaces. In general, the throughput maximization problem in wireless networks entails solving the joint problem of efficient routing, effective channel assignment (which channel should each interface/link should operate on), and interference-free scheduling of links (when should each link be activated).

Prior works on the above throughput maximization problem have only addressed restricted versions of the problem: single channel [14], static channel assignment [2], channel assignment and scheduling for a predetermined set of possible paths [15]. Finally, [12] derives upper bounds on the achievable throughput, without designing approximation algorithms. The main shortcomings of the prior works are three fold. *Firstly*, [2] considers only a static assignment of channels to interfaces. On the other hand, dynamic channel assignment offers more flexibility and improved capacity [12], and incurs minimal overhead using improved hardware technology [15]. The work in [15] does consider dynamic channel assignment, but for a predetermined set of paths between each source-destination pair. *Secondly*, all the prior works are for simple (pairwise) interference models, wherein the model is represented as a set of *pairs* of links that interfere with each other. On the other hand, the physical interference model (based on the requirement that signal to interfering-noise ratio being above a threshold) is less restrictive, and in general yields higher capacity than pairwise interference model in scenarios that do not use CSMA techniques [5]. *Thirdly*, all the prior works consider multiple paths between each source-destination pair (unless predetermined single-path routes are used). Multipath internetworks are more complex to configure, while single-path routing infrastructure has simplified routing tables. Moreover, in single-path routing, the problem of packet-reordering (needed in multi-path routing) does not exist. Indeed, in conventional networks (e.g., Internet), application-level flows generally use single path routing.

Motivated by the above considerations, in this article, we address the joint routing, channel assignment, and scheduling problem for throughput optimization in wireless networks in the following more general contexts: (i) multiple channels with dynamic channel assignment, (ii) physical interference model, and (iii) unsplittable flows (single-path routes) between each source-destination pair. In particular, the main contributions of our work are:

- We present a $(c + 2)$-approximation algorithm for the joint routing, channel assignment, and scheduling problem with dynamic assignment of channels in networks with pairwise interference model. Here, c is a small constant (generally referred to as the network-interference degree [15]), which depends on the precise interference model.
- For the same problem in the context of physical interference, we design a $(\mathcal{C}+3)$-approximation algorithm. Here, $\mathcal{C}$ is a appropriately defined network parameter which can be bounded under reasonable assumptions.
- Finally, we extend our techniques to the setting where the traffic flow between each source-destination pair must be on a single path. Here, we make use of classic randomized rounding technique [18] in combination with our above results to derive probabilistic performance bounds.

In addition, for the joint problem in networks with static channel assignment, we give constant-factor approximation algorithms for both pairwise and physical interference models.

Our algorithm is *much simpler* that the involved result in [2] for the pairwise interference model.

Paper Organization. Rest of the paper is organized as follows. In the next section, we present our models of network and interference, informally describe the problem addressed, and discuss related work. In Section III, we address the problem in the context of pairwise interference model under various settings. Section IV considers the problem in the context of physical interference, and Section V extends the techniques for unsplittable flows. We present concluding remarks in Section VI.

II. Models, Problem Description, and Related Work

In this section, we present our model of the network, informally describe the problem address in the article, and discuss related work. More formal treatment to the formulation of the various problem versions will be given in the following sections.

Network Model. We consider multi-hop wireless networks. A network is modeled as a directed graph $G(V,E)$, where V is the set of network *nodes* and E is the set of directed communication *links* each connecting a pair of network nodes. A directed link (u,v) denotes that u can transmit to v directly (in absence of other interfering transmissions). Each edge e in $G(V,E)$ has a capacity of $c(e)$ bits/sec, which denotes the maximum data that can be carried on e in a second. There are a certain number of frequency channels available, and each node may be equipped with multiple radio interfaces. Throughout this article, we use the term *interface* to mean a radio interface. We use K to denote the number of orthogonal frequency channels available, and $I(u)$ to denote the number of interfaces on a node u. Note that having more number of interfaces than channels is not useful; thus, we can assume that $I(u)$ is less than K for all u.

Interference Models: Pairwise and Physical. Due to the broadcast nature of the wireless links, transmission along a communication link (between a pair of wireless nodes) may interfere with transmissions along other communication links in the network. Interfering links cannot engage in successful transmission at the same time. An *interference model* defines which set of links can be active simultaneously. In this article, we consider two types of interference models, viz., pairwise and physical. A pairwise interference model is represented by a set of pairs of links that interfere with each other, when transmitting on the same channel. In particular, the pairwise model is represented as a conflict graph, wherein the vertices are the communication links of the network and the edges in the conflict graph identify pairs of links that interfere with each other. In the physical interference model, successful reception of a packet by a node u to node v depends on the signal-to-noise ratio (SINR) at v. The above interference models will be formally described in the following sections. Note that the interference models only models interference in a common channel; links operating on different (if we assume them to be orthogonal) channels do not interfere.

Time Slots. In our model, the system operates synchronously in a time slotted model. In any time slot, each interface is tuned to a certain frequency channel. In this article, we will consider static as well as dynamic assignment of channels to interfaces; in the static model, the assignment of channels to interfaces is fixed across time slots (for the same traffic load), while in the dynamic model, a interface can choose different channels in different time slots. In each time slot, a set of non-interfering links are active. Each active link needs to use a pair of interfaces I_u and I_v at u and v respectively such that I_u and I_v are operating on the same channel. We assume unicast transmissions; thus, an interface can be used by at most one link. However, in a time slot, a link (u,v) may support multiple simultaneous communications using multiple pairs of interfaces (each pair operating on a separate channel to avoid interference) between u and v. Thus, in a time slot, a *multiset* of links may be active, wherein each *instance* of a link is associated with a different pair of interfaces. The above constraints will be more formally presented in the following sections.

Problem Description. In this article, we address the "joint routing, channel assignment, and scheduling" (JRCAS) problem in the context of pairwise and physical interference models. One of the unique contributions of our work is solving the above problem with single-path flows. An informal description of the problem is as follows. Problem Input: We are given a wireless network graph $G(V,E)$, number of available channels, number of interfaces on each node, and an interference model. We are also given a set of source-destination pairs (possibly, with desired load). Output: Given the above, the JRCAS problem is to output a (periodic) interference-free schedule of link transmissions into time slots that guarantees maximum total data rate transfer between the given source-destination pairs. The above may be based on multi-path or single-path traffic flow between each source-destination pair; we look at both versions of the problem. In either case, the flows must observe edge-capacity and other constraints. Notes. (i) Recall that, each active link (u,v) uses a pair of interfaces on u and v that operate on the same channel, and each interface can be used by only one active link in a time slot. Thus, design of a link schedule strategy also entails appropriate assignment of channels to interfaces and interfaces to links. In this article, we look at static (across time slots) as well as dynamic assignment of channel to interfaces. (ii) The JRCAS problem is a generalization of the classical maximizing multicommodity flow problem [1] with additional resources (channels and interfaces), constraints (interference and interface), and outputs (periodic link schedule and channel assignment).

Related Work. One of the first works that address the throughput maximization problems is the work by Jain et al. [9], where the authors give an LP formulation to the problem. However, their formulation requires enumeration of all interference-free sets of links, which can be exponential in number. Thus, their work does not provide any polynomial-time algorithm with performance bounds. The above shortcoming was first remedied in an insightful work by Kumar et al. in [14] who design a constant-factor approximation algorithm for interference-free scheduling of links for throughput maximization for single channel. Our work in this article borrows their key insight and technique, and extends their work to incorporate other aspects of the throughput maximization problem, viz., routing, static and dynamic assignment of channels, consideration of more general physical interference, and restricting to single-path flows. Alicherry et al. [2] address the JRCAS problem of our article in the setting of multiple channels and static assignment of channels to interfaces, and design a (cK/I)-approximate solution, where c is the network-interference degree, K is the number of channels, and I is the minimum number of interfaces on each node. However, their approach in unnecessarily involved and rather complicated. In fact, in this article, we show that one can derive the same approximation bound with a trivial generalization of [14]'s work.

Other line of closely related research stems from the seminal work by Tassiulas and Ephremides [20], who present an optimal link scheduling policy for arbitrary network models. However, their scheduling policy needs to iteratively solve an optimization problem (find the maximum-weighted interference-free set of links) that is NP-hard even for simple interference models. Also, their scheme cannot be extended to solve the *joint* routing and scheduling problem. Based on the above result, [6, 16, 19] design simple scheduling policies that guarantee near-optimal throughput for single channel. Recently, Lin and Rasool [15] extend the above ideas and consider the joint (dynamic) channel assignment and scheduling problem, for a predetermined set of possible paths. All the above extensions are for pairwise interference models. Finally, Dimkais and Walrand [7] study the performance of longest-queue-first (LGF) scheduling in a graph and interfering queues, and present sufficient conditions when LGF strategy yields optimal throughput.

In other related works, [5, 21] address the problem of scheduling links in minimum number of time slots, [17] address the joint scheduling and power control problem, and [4] consider the joint routing and scheduling problem for power optimization. Finally, [11] presents approximate LP formulations for a class of scheduling problems, but they do not handle wireless interference constraints.

III. **JRCAS Problem in Pairwise Interference Model**

In this section, we address our JRCAS problem in the context of pairwise interference model. We start with a few definitions.

Definition 1: (Pairwise Interference Model; Conflict Graph) The *pairwise interference model* is represented by a set of *pairs* of communication links that interfere with each other, if scheduled on the same channel.

The set of pairs of interfering links can be represented by a *conflict graph* $G(V_c, E_c)$. The vertices V_c of a conflict graph are the communication links of the given network graph, and the edges E_c in the conflict graph connect the pairs of vertices that correspond to a pair of interfering communication links.

Most interference models (except for physical interference model) can be modeled as pairwise interference. For instance, transmitter model [22], protocol model [8], transmitter-receiver model [3, 13], etc. can all be modeled as pairwise interference models. □

We now define a concept of network-interference degree in a pairwise interference model. The performance guarantee (quality of solution) of our algorithms are expressed in terms of the network-interference degree.

Definition 2: (Network-Interference Degree.) *Network-interference degree* is the maximum interference degree of any link in the network, where the interference degree of a link (u, v) is the number of links that interfere with (u, v) but not with each other. In other words, *network interference degree* of a network is the size of the maximum independent set among the neighbors of any vertex in the conflict graph.

Network-interference degree can often be determined from the interference model, and thus is independent of the exact network topology. For example, under he so-called node-exclusive interference model for Bluetooth and FH-CDMA networks, the interference degree is 2. For the so-called bi-directional equal-power model that approximates IEEE 802.11 DCF, the interference degree is 8 [6]. □

Definition 3: ((Periodic) Link Schedule.) A *periodic link schedule* is a specification of a certain number of time slots, which is periodically repeated. For each time slot, we specify: (i) a *multiset* of active links, (ii) assignment of channel to interfaces, and (iii) for each active link instance e_i, the pair of interfaces (with the same assigned channel) used by e_i. We consider only interference-free link schedules, i.e., no two link instances active in the same time slot interfere with each other. Also, since an interface can be used by only one link instance, the number of link instances incident on any node u in a time slot must be less than $I(u)$, the number of interfaces on node u. The above constraint is referred to as the *interface constraint.* □

Definition 4: (Link Utilization ($\alpha(e, k)$ and $\alpha(e)$).) *Link utilization* of a link e for channel k in a given periodic link schedule is the ratio of (i) total number of instances (across all time slots) of link e being active on channel k, and (ii) the total number of slots in the period. We use $\alpha(e, k)$ to denote link utilization of a link e for channel k. Note that $\alpha(e, k)$ is cumulative across all source-destinations, since a link may be active to transmit packets for any source-destination flow.

Also, we use $\alpha(e)$ to denote $\sum_k \alpha(e, k)$, i.e., the total link utilization over all channels. □

Definition 5: (Link Flow ($f(e, k), f(e), f_i(e, k), f_i(e)$.) *Link flow* for a link e and channel k is the data rate carried by link e on channel k. That is, $f(e, k) = c(e)\alpha(e, k)$ where $c(e)$ is the data rate capacity of the link e. Also, we use $f(e)$ to denote $\sum_k f(e, k)$.

Finally, we use $f_i(e, k)$ or $f_i(e)$ to denote the link flow for a particular source-destination pair $\{s_i, d_i\}$. Thus, $f(e, k) = \sum_i f_i(e, k)$ and $f(e) = \sum_i f_i(e)$. □

Based on the above definitions and concepts, we now give a formal description of our JRCAS problem.

JRCAS Problem Formulation. The **input** to the JRCAS problem is a wireless network graph $G(V, E)$ (with link capacities and number of interfaces for each node), number of available channels, the conflict graph (representing the pairwise interference model), and a set of source-destination pairs. The **output** to the JRCAS problem is a periodic link schedule that results in maximum total data rate between the given source-destination pairs. The output link schedule should be such that the resulting link flows (due to the resulting link utilizations and given link capacities) satisfy the flow conservation constraints (given by Equations 3-5 later) at each node. Note that a periodic link schedule by definition includes assignment of channels to interfaces, is interference-free, and satisfies interface constraints.

The above JRCAS problem is NP-hard, since the special case of single channel and one-hop source-destination pairs is equivalent to interference-free scheduling, which is known to be NP-hard [21] even for very simple interference models.

Overview of General Approach. Using [14]'s approach, we start with a linear programming (LP) formulation that incorporates constraints relating link flows, link capacities, and link utilizations. In addition, we have constraints due to number of interfaces and for the wireless interference. The objective function of the LP is to maximize the total data rate between the given source-destination pairs. The formulated LP can be solved optimally in polynomial time. However, the LP solution only gives us link utilization and link flow values, i.e., it does not give a periodic link schedule that can realize the obtained link utilizations and flows. In fact, since the JRCAS problem is NP-hard, it is unlikely that we can compute the optimal link schedule in polynomial time. Thus, we derive an approximation solution as follows. To get a near-optimal link schedule, we scale down the link utilizations of the LP solution by a certain factor, so that the link utilizations satisfy a certain condition which allows us to design a link schedule for the scaled-down link utilizations. Since, the total data rate of the LP solution is an upper bound on the optimal data rate for the given JRCAS problem, the above yields a near-optimal periodic link schedule.

Below, we consider various settings of the problem, viz., single channel, multiple channels with static and dynamic channel assignment.

Linear Programming (LP) Formulation for Single Channel. Below, we use the variable i to vary over the given pairs of source-destinations $\{s_i, d_i\}$, F_i to denote the data rate between the source-destination pair s_i and d_i, and V_i to denote $V - \{s_i, d_i\}$. We also use $N(u)$ to denote the set of links incident on node u; i.e., $N(u) = \{e | e = (u, v) \text{ or } (v, u), \text{ and } e \in E\}$.

$$\forall\ i, \qquad F_i \geq 0 \tag{1}$$

$$\forall\ i, e \in E, \qquad f_i(e) \geq 0 \tag{2}$$

$$\forall\ i, u \in V_i, \qquad \sum_{(v,u)\in E} f_i((v, u)) = \sum_{(u,w)\in E} f_i((u, w)) \tag{3}$$

$$\forall\ i, \quad \sum_{(w,s_i)\in E} f_i((w, s_i)) + F_i = \sum_{(s_i,v)\in E} f_i((s_i, v)) \tag{4}$$

$$\forall\ i, \quad \sum_{(v,d_i)\in E} f_i((v, d_i)) = \sum_{(d_i,w)\in E} f_i((d_i, w)) + F_i \tag{5}$$

$$\forall\ e \in E, \qquad \alpha(e) = (\sum_i f_i(e))/c(e) \tag{6}$$

$$\forall\ u \in V, \qquad \sum_{e\in N(u)} \alpha(e) \leq 1 \tag{7}$$

$$\text{Maximize} \sum_i F_i$$

Above, Equations 3-5 represent flow conservation constraints, and Equation 7 represents the interface constraint, In addition to the above constraints, we still need to add a constraint for the wireless interference. Consider a particular time slot τ, and let X_e represent the binary variable which is 1 if the link e is schedule in the time slot τ, else X_e is zero. Then, by definition of the network-interference degree c, $X_e + \sum_{(e,e')\in E_c} X_{e'} \leq c$. Recall that E_c is the set of edges in the conflict graph. Summing the above equation over all time slots, and dividing by the number of time slots, we get:

$$\alpha(e) + \sum_{(e,e')\in E_c} \alpha(e') \leq c, \quad \forall\ e \in E. \tag{8}$$

As in [14], we *add the above equation* to the LP to represent the interference constraint.

Near-Optimal Link Schedule. Solution to the above LP formulation gives a set of link flows and link utilizations, but not a periodic schedule of links. In fact, it is possible that a periodic link schedule that guarantees the link utilizations (obtained from the LP solution) may not even exist.[1] Let the link utilizations obtained from the LP solution be $\{\widehat{\alpha}(e)\}$, and let $\widetilde{\alpha}(e) = \widehat{\alpha}(e)/c$ for each e. Then, note that

$$\widetilde{\alpha}(e) + \sum_{(e,e')\in E_c} \widetilde{\alpha}(e') \leq 1, \quad \forall\ e \in E.$$

Based on the above equation, a periodic link schedule $\widetilde{L}$ that results in the $\widetilde{\alpha}$ link utilizations can be easily designed [14]. The total data rate due to $\widetilde{L}$ is at least $1/c$ of the total data rate of the LP solution. Since the optimal data rate for the JRCAS problem is at most that of the LP solution, the

[1] To see this, consider three links e_1, e_2, and e_3, that interfere with each other (i.e., form a triangle in the conflict graph). If they are no other constraints, then the LP solution may assign link utilizations of 1/2 to each of the links. However, the best achievable link utilizations for the three links is 1/3 each. Note that even if we add a wireless interference constraint for each *clique* in the conflict graph, the LP solution obtained may still be unrealizable by a link schedule. To see the above, consider five links that form a pentagon in the conflict graph [9].

designed link schedule $\widetilde{L}$ is a c-approximate solution to the JRCAS problem. We will prove the above argument formally in the more general context of multiple channels with dynamic channel assignment.

Multiple Channels with Static Channel Assignment. We now show that the above approximation algorithm for single channel also yields a (cK/I)-approximation algorithm for multiple channels with static channel assignment. Here, K is the number of channels and I is the minimum number of interface per node. Essentially, the above designed c-approximate link schedule $\widetilde{L}$ for the single channel can be easily transformed (using I interfaces per node)[2] into a link schedule $\widetilde{L}_I$ whose total data rate is I times that of $\widetilde{L}$. Since the optimal data rate using K channels can be at most K times the optimal data rate using one channel, the transformed link schedule $\widetilde{L}_I$ has a total data rate of at least $1/(cK/I)$ times the optimal data rate using K channels. The above is a much simpler approximation algorithm (and simpler proof) than the result in [2]. As described later, the above techniques easily extend to JRCAS with more constraints and other objective functions.

Theorem 1: The above designed algorithm returns a (cK/I)-approximate link schedule for the JRCAS problem with multiple channels and static assignment of channels to interfaces. Here, c is the network interference degree, K is the number of available channels, and I is the minimum number of interfaces at each node. ∎

Multiple Channels with Dynamic Channel Assignment. We now address the JRCAS problem for multiple channels with dynamic channel assignment. The LP formulation for this case is shown below. As before, we use the variable i to vary over the set of given source-destination pairs $\{s_i, d_i\}$, F_i to denote the data rate between s_i and d_i, the variable k to vary over the set of available channels, and $N(u)$ to denote the set of links incident on u.

$$\forall\ i, \qquad F_i \geq 0 \tag{9}$$

$$\forall\ i, k, e \in E, \qquad f_i(e,k) \geq 0 \tag{10}$$

$$\forall\ i, e \in E, \qquad f_i(e) = \sum_k f_i(e,k) \tag{11}$$

$$\text{Flow conservation Equations 3 to 5} \tag{12}$$

$$\forall\ k, e \in E, \qquad \alpha(e,k) = (\sum_i f_i(e,k))/c(e) \tag{13}$$

$$\forall\ u \in V, \qquad \sum_{e \in N(u)} \sum_k \alpha(e,k) \leq I(u) \tag{14}$$

$$\forall\ k, e \in E, \qquad \alpha(e,k) + \sum_{(e,e') \in E_c} \alpha(e',k) \leq c \tag{15}$$

$$\text{Maximize} \sum_i F_i \tag{16}$$

Above, Equation 14 and 15 represent the interface and wireless interference constraint respectively. As in the case of single channel, we first solve the above LP optimally. Let $\{\widehat{\alpha}(e,k)\}$ be the link utilizations of the LP solution, and let $\widetilde{\alpha}(e,k) = \widehat{\alpha}(e,k)/(c+2)$ be the new scaled-down link utilizations. It is easy to see that, for each edge $e = (u,v)$ and channel k, $\widetilde{\alpha}(e,k)$ satisfies the following condition:

$$\widetilde{\alpha}(e,k) + \sum_{(e,e') \in E_c} \widetilde{\alpha}(e',k) + \frac{1}{I(u)}(\sum_{e \in N(u)} \widetilde{\alpha}(e)) + \frac{1}{I(v)}(\sum_{e \in N(v)} \widetilde{\alpha}(e)) \leq 1 \tag{17}$$

Above, $\widetilde{\alpha}(e) = \sum_k \widetilde{\alpha}(e,k)$. Based on the above condition, we can now design a periodic link schedule for the $\widetilde{\alpha}$ link utilizations as follows.

- Pick a large enough integer W such that $W\widetilde{\alpha}(e,k)$ is a positive integer for each e and k.
- Consider a period $\widetilde{L}$ of W time slots.
- Iterate through all pairs (e,k) in an arbitrary order, and place the link e with channel k in the first $\widetilde{\alpha}(e,k)W$ time slots of $\widetilde{L}$, wherein such a placement does not cause any interference or violate interface constraint due to the previously placed link instances.

The above algorithm results in a periodic link schedule of W time slots with link utilizations $\widetilde{\alpha}(e,k)$ for each edge e and k, due to the following facts. For any pair (e,k), the maximum number of time slots wherein (e,k) can *not* be placed due to interference with previously placed links is $W\sum_{(e,e') \in E_c} \widetilde{\alpha}(e',k)$ and due to interface constraint violation is $W(\frac{1}{I(u)}(\sum_{e \in N(u)} \widetilde{\alpha}(e)) + \frac{1}{I(v)}(\sum_{e \in N(v)} \widetilde{\alpha}(e)))$. Thus, by Equation 17, the pair (e,k) *can* be placed in at least $W\widetilde{\alpha}(e,k)$ time slots in a period of W time slots.

The designed periodic link schedule $\widetilde{L}$ results in a total data rate of at least $1/(c+2)$ times the total data rate obtained by the LP solution. Since the optimal data rate of the given JRCAS problem is at most that of the LP solution, the designed link schedule $\widetilde{L}$ is a $(c+2)$-approximate solution.

Theorem 2: The above designed algorithm returns a $(c+2)$-approximate link schedule for the JRCAS problem with multiple channels and dynamic assignment of channels to interfaces. Here, c is the network-interference degree. ∎

Generalizations. Above techniques easily generalize for the following (i) directional antenna, (ii) varying power transmissions, (iii) incorporating other constraints in the JRCAS problem, (iv) solving the JRCAS problem with other objective functions. Directional antenna can be handled by defining "flavors" of each link (u,v) — each flavor corresponds to each "feasible" pair of directions of antennas of u and v respectively. A conflict graph is then appropriately constructed over pairs of $(link, flavor)$ as vertices, and the above techniques can then be applied appropriately. Different transmission powers can be handled in a similar way as directional antenna.[3] Finally, note that in the LP formulation (and hence

[2]Essentially, any I time slots of $\widetilde{L}$ can be combined into one time slot, using I interfaces per node.

[3]However, as suggested in [14], during design of the link schedule, we may need to consider links in the order of their lengths and redefine c, to achieve a constant-factor approximation.

the JRCAS problem), we can add any constraint (such as a "fairness" constraint) that is preserved when the link utilization (and flows) are scaled down by a constant factor. Similar, the approximation bound holds for any objective function that is a linear combination of the link flows or utilizations.

IV. JRCAS Problem with Physical Interference

In this section, we address the JRCAS problem in the context of physical interference model. As mentioned before, physical interference is less restrictive, and in general entails more capacity than pairwise model in scenarios that do not use CSMA techniques [5].

In the physical interference model, successful reception of a packet by a node u to node v depends on the SINR at v. To be specific, if $P_v(x)$ denotes the received power at v of the signal transmitted by node x, then a packet along link (u, v) is correctly received if and only if:

$$\frac{P_v(u)}{N + \sum_{w \in V_*} P_v(w)} \geq \beta,$$

where N is the background noise, V_* is the set of nodes in the network that are transmitting simultaneously, and β is a constant that depends on the desired data rate, the modulation scheme, etc. Below, we introduce the concept of weights that helps formulation of physical interference as a linear constraint.

Physical Interference Constraint using Weights. For a pair of links (u, v) and (r, s), let

$$w_{(u,v)}^{(r,s)} = \frac{P_v(r)}{\frac{P_v(u)}{\beta} - N}.$$

Recall that $P_x(y)$ is the signal strength received at x due to the node y. It is easy to see that transmission along a link e is successful in presence of a set E' of other links if and only if $\sum_{e' \in E'} w_e^{e'} \leq 1$.

Let $\mathcal{C}$ be the *upper bound* on $\sum_{e' \in E'} w_e^{e'}$ for any E' and $e \in E$. That is, let $\mathcal{C}$ be such that

$$\mathcal{C} \geq \sum_{e' \in E'} w_e^{e'} \quad \forall \ \ E' \subseteq E, e \in E. \tag{18}$$

The value of $\mathcal{C}$ can be bounded under certain assumptions, as shown later.

Based on the above definition of $w_e^{e'}$ and $\mathcal{C}$, we can represent the interference constraint in physical interference model as follows. Consider a particular time slot, and let $X_{e,k}$ represent the binary variable which is 1 if and only if the edge e is active on channel k in the given time slot. Then, we can see that the following holds for each e and k.

$$X_{e,k} + \frac{1}{\mathcal{C}} \sum_{e' \in E} w_e^{e'} X_{e',k} \leq 1 + \frac{1}{\mathcal{C}}.$$

The above is true, since when $X_{e,k}$ is 1, $\sum_{e' \in E} w_e^{e'} X_{e',k}$ must be less than 1; when $X_{e,k}$ is 0, $\sum_{e' \in E} w_e^{e'} X_{e',k}$ must be less than $\mathcal{C}$. Summing up the above equation over all time slots and dividing the result by the number of time slots, we get:

$$\alpha(e, k) + \frac{1}{\mathcal{C}} \sum_{e' \in E} w_e^{e'} \alpha(e', k) \leq (1 + \frac{1}{\mathcal{C}}), \quad \forall \ e \in E. \tag{19}$$

We use the above equation as the wireless interference constraint in the LP formulation.

LP Formulation. The LP formulation for physical interference JRCAS problem with multiple channels is same as the one for pairwise interference (i.e., Equations 9 to 16) except that we replace Equation 15 by Equation 19. As in previous cases, we first solve the LP optimally, and then scale down the resulting link utilizations by a factor of $(\mathcal{C} + 3)$. Let the scaled-down link utilizations be denoted by $\widetilde{\alpha}(e, k)$ for each edge e and channel k. Thus, the scaled-down link utilizations $\widetilde{\alpha}$ can be shown to satisfy the following condition.

$$\widetilde{\alpha}(e, k) + \sum_{(e,e') \in E_c} w_e^{e'} \widetilde{\alpha}(e', k) +$$
$$\frac{1}{I(u)}(\sum_{e \in N(u)} \widetilde{\alpha}(e)) + \frac{1}{I(v)}(\sum_{e \in N(v)} \widetilde{\alpha}(e)) \leq 1$$

Based on the above equation and using similar arguments as in the previous section, we can now design a periodic link schedule that guarantees the link utilizations of $\widetilde{\alpha}(e, k)$ for each e and k, using $I(u)$ interfaces at each node u.[4] Thus, we get a $(\mathcal{C} + 3)$-approximate solution for the JRCAS problem with physical interference. Also, using similar arguments as in the previous section, we also get a $(\mathcal{C}+1)K/I$-approximation algorithm for multiple channel with static channel assignment.

Theorem 3: The above designed algorithm returns a $(\mathcal{C}+3)$-approximate solution for the multi-channel JRCAS problem with physical interference and dynamic channel assignment. For the case of static channel assignment, the above gives a $(\mathcal{C} + 1)K/I$-approximation algorithm. Here, $\mathcal{C}$ is as defined in Equation 18, K is the number of channels, and I is the minimum number of interfaces on each node. ∎

Bounding $\mathcal{C}$. We now bound the value of $\mathcal{C}$, as defined by Equation 18. Let us assume that the density of nodes in the given network is bounded by ρ and that the minimum distance between any pair of nodes is d_{min}. Also, we assume that radio signal propagation obeys the log-distance path model with path loss exponent γ. In other words, the signal strength at a distance d from a node transmitting with a power of P is assumed to be equal to P/d^γ. Now, we can bound the total signal strength at a node u due to all other nodes by integrating over the signal strength due to nodes in an annular disk of width dx at a distance of x from u as:

$$\int_{d_{min}}^{\inf} \frac{P}{x^\gamma}(2\pi\rho x) \ dx = 2P\pi\rho(\frac{1}{d_{min}^{\gamma-2}}).$$

[4] Note that in a periodic schedule of W time slots, when trying to place a link e with channel k in a time slot, the maximum number of time slots wherein (e, k) can *not* be placed due to physical interference with previously placed links is $W \sum_{(e,e') \in E_c} w_e^{e'} \widetilde{\alpha}(e', k)$.

Note that the lower limit of the above integration is d_{min}, since there are no nodes within a distance of d_{min} from u. Based on the above, the value of $\mathcal{C}$ can be bounded by (ignoring the ambient noise N):

$$\mathcal{C} \leq 2\beta\pi\rho d_{min}^2 (\frac{d_{max}}{d_{min}})^\gamma,$$

where d_{max} is the maximum length of a communication link.

V. JRCAS Problem with Unsplittable Flows

In this section, we generalize the techniques from previous sections to the JRCAS problem with unsplittable flows, i.e., wherein traffic flow between each source-destination pair is restricted to a single path (which may be different for different source-destination pairs). In this context, given traffic flow demands for each source-destination pair, we consider two objective functions: (i) maximize the number of source-destination pairs whose traffic demands can be completely satisfied, (ii) minimize the factor by which to scale-down the traffic demands, such that the scaled-down demands of all the source-destinations can be satisfied. We make use of the randomized rounding technique by Raghavan and Thompson [18] to solve our JRCAS problem with unsplittable flows. Recently, authors in [10] have also used the randomized rounding technique to find a single-path route for a time-varying traffic.

Maximizing Number of Satisfied Pairs. Given a wireless network graph and a set of source-destination pairs $\{s_i, d_i\}$ each with a traffic demand T_i, our objective is to maximize the number of source-destination pairs for which the demand can be *completely* satisfied, using a periodic link schedule that results in single-path flows. For uniform link capacities and traffic demands, we present a randomized approximation algorithm based on the randomized rounding technique of [18]. In particular, our approximation algorithm consists of the the following below steps. For now, let us assume dynamic channel assignment and pairwise interference model.

1) Solve the LP represented by Equations 9 to 16 and the added constraint: $F_i \leq T_i$.
2) The LP solution gives multi-path flows for each source-destination pair; let the data rates in the LP solution for each $\{s_i, d_i\}$ pair be $\widehat{F}_i$. Using a simple depth-first search, we divide each multi-path flow (of data rate value $\widehat{F}_i$) into a sum/combination of single-path flows, each of value $\widehat{F}_i x_{ij}$ where $\sum_j x_{ij} = 1$. Here, x_{ij} is the fraction of the total flow $\widehat{F}_i$ that flows into the j^{th} single-path flow.
3) Next, we randomly round off the fractional x_{ij} values to 0 or 1 as follows. For each i, i.e, source-destination pair $\{s_i, d_i\}$, we set x_{ij} to 1 with a probability of $x_{ij}(1-\delta)\widehat{F}_i/T_i$, for an appropriately chosen δ. The choice is done in an exclusive manner, i.e., for each i, either exactly one x_{ij} is set of one (with a probability of $x_{ij}(1-\delta)\widehat{F}_i/T_i$ each) and the rest set to zero, or all are set to zero (with the remaining probability of $1-(1-\delta)\widehat{F}_i/T_i$; recall that $\sum_j x_{ij} = 1$). Now, the entire demand flow of value T_i is routed unsplit through the j^{th} single-path represented by x_{ij} that was set to 1. For an appropriately chosen δ and uniform demands (i.e., $T_i = T$ for all i), it can be shown that with high-probability, (i) the constraints of the LP continue to be satisfied by the single-path flows constructed as above, and (ii) the number of source-destination pairs for which one of the x_{ij} gets set to 1 is "near-optimal." We refer the reader to [18] for details.[5]
4) Finally, the above constructed single-path flows are scaled down by a factor of $(c+2)$ to satisfy Equation 17 with high probability. A periodic link schedule can now be constructed for the scaled-down link utilizations, as in previous sections.

We note the following.

- In practice, we may end up using more than W time slots (even though the link utilization fractions are based only on W) for the constructed link schedule, since Equation 17 only holds with high probability. Use of more than W time slots essentially results in a further scaling down of the obtained link flows, which will happen with a low probability.
- The above approach (along with performance guarantees) easily extends to static channel assignment, directional antenna, variable power transmissions, as outlined in Section III.
- The above approach can also be used for non-uniform demands or link capacities or physical interference model, but the high-probability of performance guarantee cannot be shown.[6]
- Finally, the objective of the above approach results in an "unfair" solution, i.e., demands of a near-maximum number of source-destination pairs is satisfied, while the other pairs are assigned *zero* traffic. Below, we address the problem with another objective to derive an approximate and fair solution.

Minimizing Demand-Scaling Factor. Given a wireless network graph and a set of source-destination pairs $\{s_i, d_i\}$ each with a traffic demand T_i, here we wish to minimize the factor λ such that traffic demand of T_i/λ can be satisfied, using a periodic link schedule based on single-path flows. As before, we start with assuming uniform link capacities and traffic demands (i.e., $T_i = T$ for all i), dynamic channel assignment and pairwise interference model. Our below approach is based on the randomized rounding technique of [18] for the minimizing

[5] Basically, arguments in [18] can be used to show that with a very high probability, the obtained integral solution satisfies two things: (i) The LP constraints are satisfied, and (ii) The total resulting data rate (from the single-path flows) is at least $\widehat{F}(1-\delta) - O(\sqrt{\widehat{F}(1-\delta)})$, where $\widehat{F} = \sum_i \widehat{F}_i$ is the optimal (obtained from the LP solution).

[6] The difficult comes due to the fact that Chernoff's bounds on deviation from the mean hold only for *sum* of random variables, and not for a general linear combination of random variables.

congestion problem. Our approach consists of the following steps.

1) First, we formulate a LP to minimize the factor by which the "capacities" ($I(u)$ for each node and the network-interference degree c) should be increased so that the given traffic demands can be fully satisfied. This LP is essentially represented by Equations 9 to 16, but we multiply the right hand side of Equations 14 and 15 by a congestion-factor λ, add the constraint $F_i = T_i$, and change the objective function to "$\text{Minimize } \lambda$."
2) As in the previous approach, we divide the multi-path flows of the LP solution into a sum/combination of single-path flows for each source-destination pair. That is, for each $\{s_i, d_i\}$, we divide the multi-path flow of value T_i (obtained from the LP solution) into single-path flows each of value $T_i x_{ij}$ such that $\sum_j x_{ij} = 1$.
3) Next, we randomly round off the fractional x_{ij} values to 0 or 1 as before, but use $\delta = 0$ and $\widehat{F}_i = T_i$. That is, for each i, exactly one x_{ij} is set of one (with a probability of x_{ij} each) and the rest are set to zero. The total flow of T_i is now routed unsplit through the single-path represented by x_{ij} that was set to 1. Using similar arguments as in [18], it can be shown that for any ϵ such that $0 < \epsilon < 1$, the congestion-factor $\widetilde{\lambda}$ provided by the above procedure (after the rounding off) does not exceed $\widehat{\lambda} - \sqrt{2\widehat{\lambda} \ln(|E|/\epsilon)}$ with probability of at least $1 - \epsilon$, where $\widehat{\lambda}$ is the congestion-factor obtained from the LP solution and hence, is at most the optimal.
4) Finally, we scale down the above constructed single-path flows by a factor of $(c+2)\widetilde{\lambda}$ to satisfy Equation 17 with high probability. A periodic link schedule can then be constructed for the resulting link utilizations, as in previous sections.

As in the previous approach, in practice, we may end up scaling the link utilizations further when designing the periodic link schedule. This should happen with a probability of ϵ. The above arguments extend to static channel assignment and other generalizations. Above approach can also be used for non-uniform demands or link capacities or physical interference model, but the probabilistic guarantee cannot be shown.

VI. **Conclusions**

In this article, we have designed approximation algorithms for the joint routing, channel-assignment, and scheduling problem for throughput optimization in various settings. In particular, ours is the first work to consider the complete version of the above problem in the context of dynamic channel assignment, physical interference, and unsplittable (single-path) flows. For the single-path flow version of the problem, we use randomized rounding techniques from [18], and develop probabilistic algorithms that deliver a near-optimal solution with high probability. We evaluated the probabilistic algorithms through simulations, and observe that the degradation of the solution quality due to the rounding off process is within acceptable limits. For future work, we would focus on developing distributed implementations of the developed algorithms, possibly, by integrating them with the distributed network routing protocols.

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