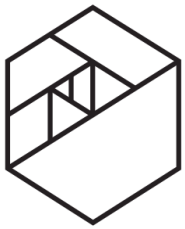
Model Selection II



Training and test sets

	Date	Title	Budget	DomesticTotalGross	Director	Rating	Runtime
0	2013-11-22	The Hunger Games: Catching Fire	130000000	424668047	Francis Lawrence	PG-13	146
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2	2013-11-22	Frozen	150000000	400738009	Chris BuckJennifer Lee	PG	108
3	2013-07-03	Despicable Me 2	76000000	368061265	Pierre CoffinChris Renaud	PG	98
4	2013-06-14	Man of Steel	225000000	291045518	Zack Snyder	PG-13	143
5	2013-10-04	Gravity	100000000	274092705	Alfonso Cuaron	PG-13	91
6	2013-06-21	Monsters University	NaN	268492764	Dan Scanlon	G	107
7	2013-12-13	The Hobbit: The Desolation of Smaug	NaN	258366855	Peter Jackson	PG-13	161
8	2013-05-24	Fast & Furious 6	160000000	238679850	Justin Lin	PG-13	130
9	2013-03-08	Oz The Great and Powerful	215000000	234911825	Sam Raimi	PG	127
10	2013-05-16	Star Trek Into Darkness	190000000	228778661	J.J. Abrams	PG-13	123
11	2013-11-08	Thor: The Dark World	170000000	206362140	Alan Taylor	PG-13	120
12	2013-06-21	World War Z	190000000	202359711	Marc Forster	PG-13	116
13	2013-03-22	The Croods	135000000	187168425	Kirk De MiccoChris Sanders	PG	98
14	2013-06-28	The Heat	43000000	159582188	Paul Feig	R	117
15	2013-08-07	We're the Millers	37000000	150394119	Rawson Marshall Thurber	R	110
16	2013-12-13	American Hustle	40000000	150117807	David O. Russell	R	138
17	2013-05-10	The Great Gatsby	105000000	144840419	Baz Luhrmann	PG-13	143

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9	2013-03-08	Oz The Great and Powerful	215000000	234911825	Sam Raimi	PG	127
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17	2013-05-10	The Great Gatsby	105000000	144840419	Baz Luhrmann	PG-13	143

Training set

Test set

Training and test sets

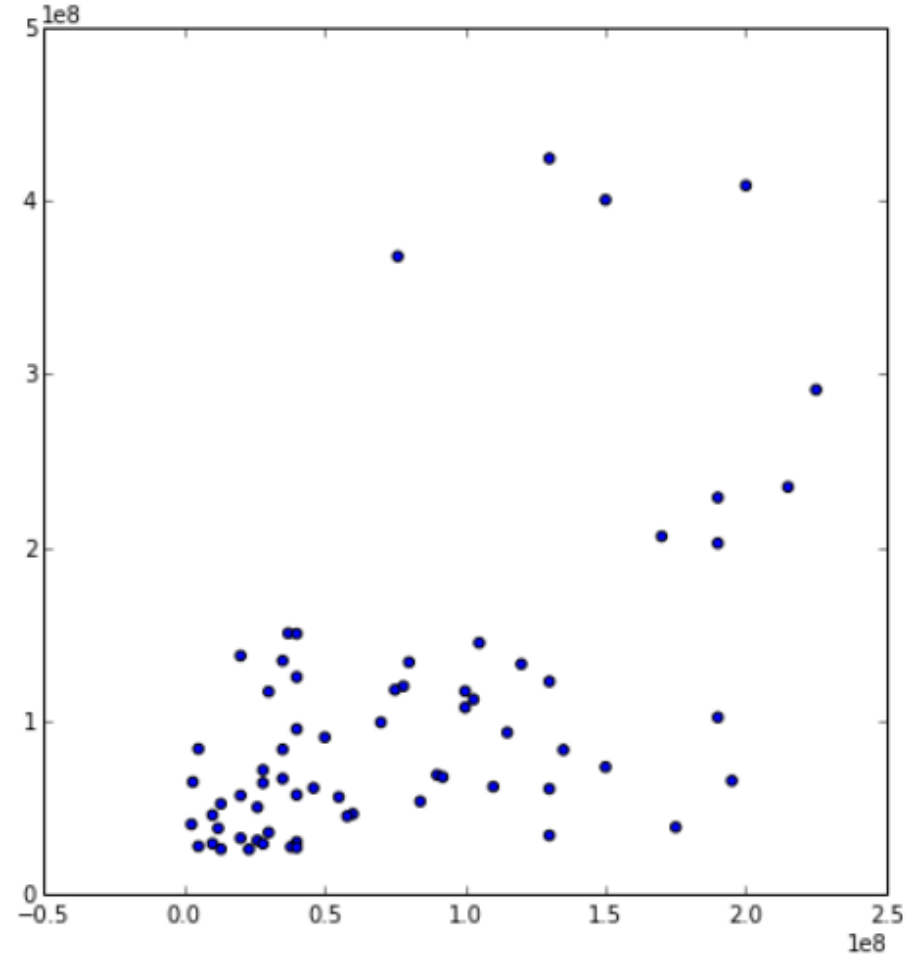
Training set

fit the model

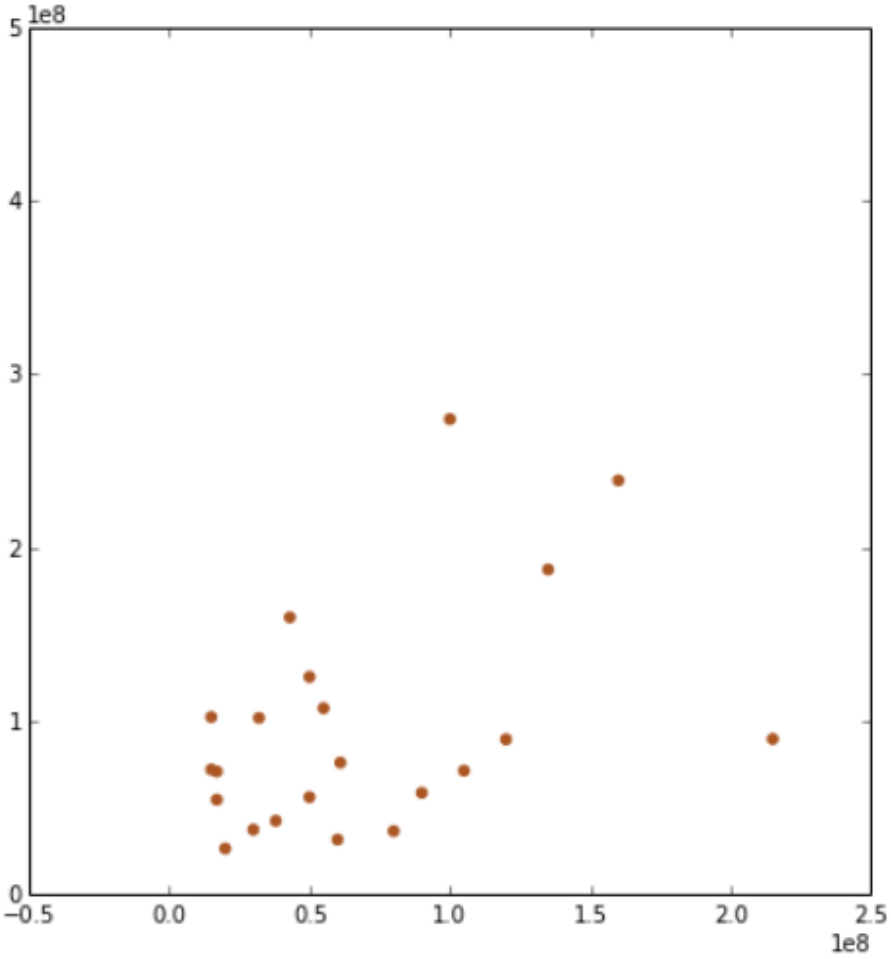
Test set

measure performance
- predict y with model
- compare with actual y
- measure error

Training set



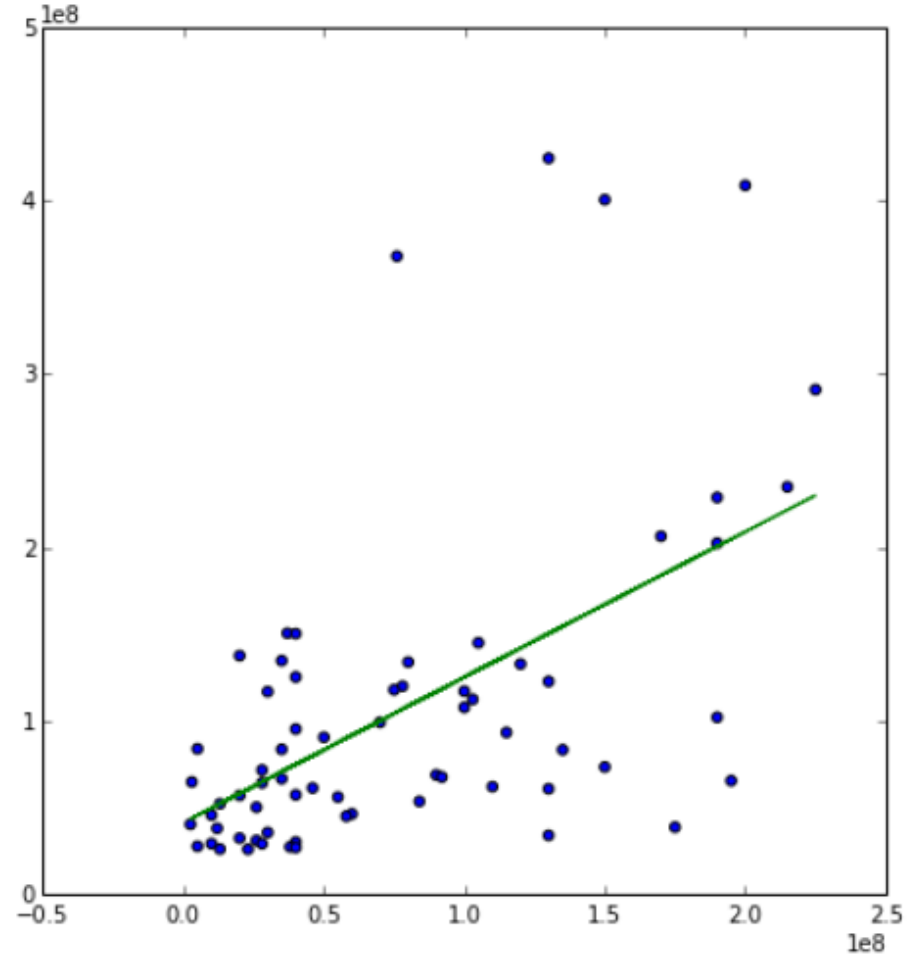
Test set

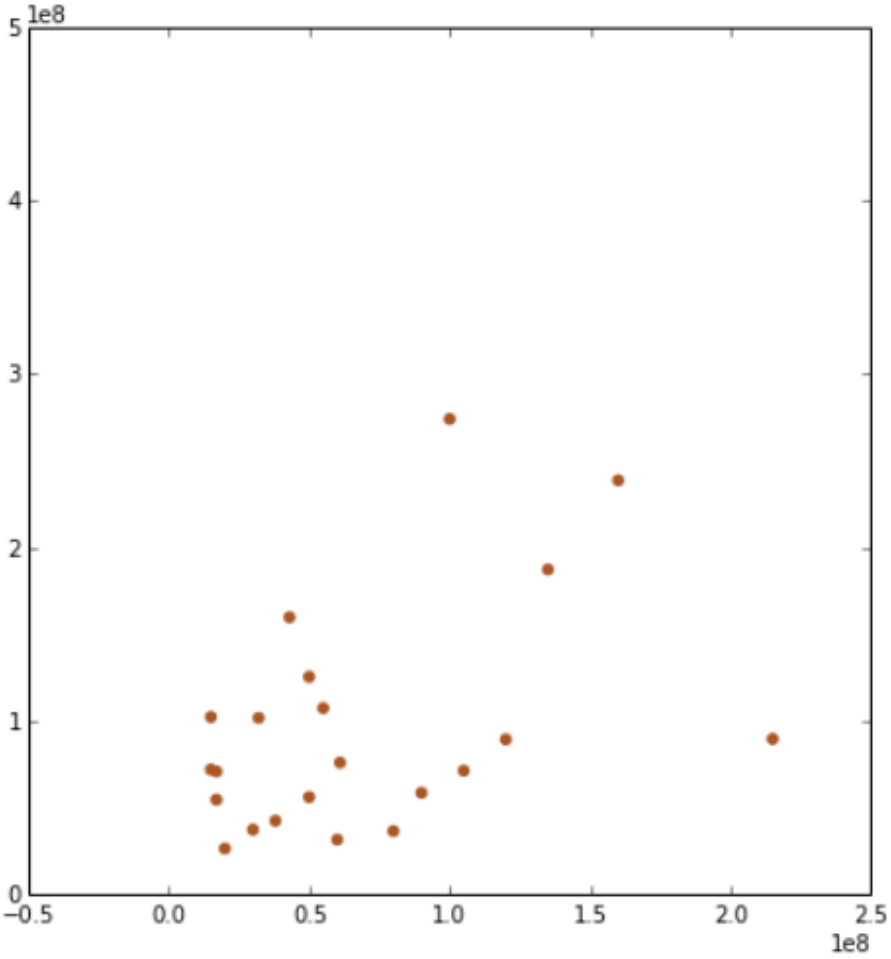


Training set

Test set

Fit the model

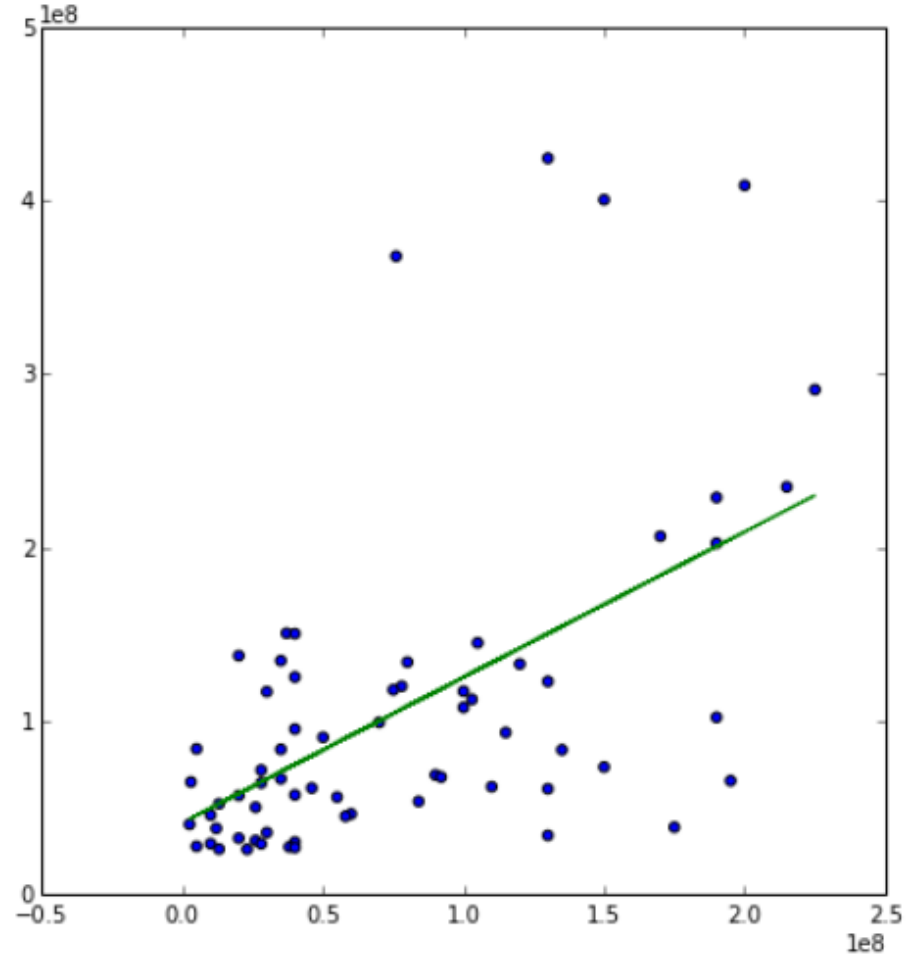


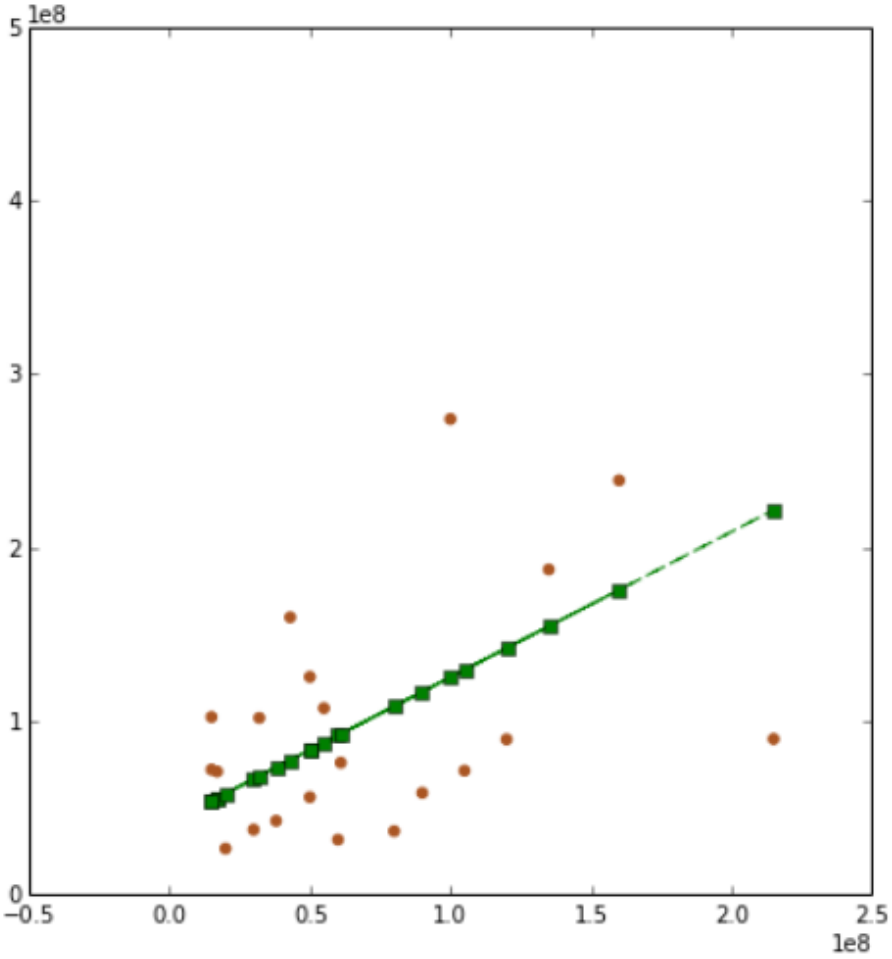


Training set

Test set

Use the model
to predict y from x

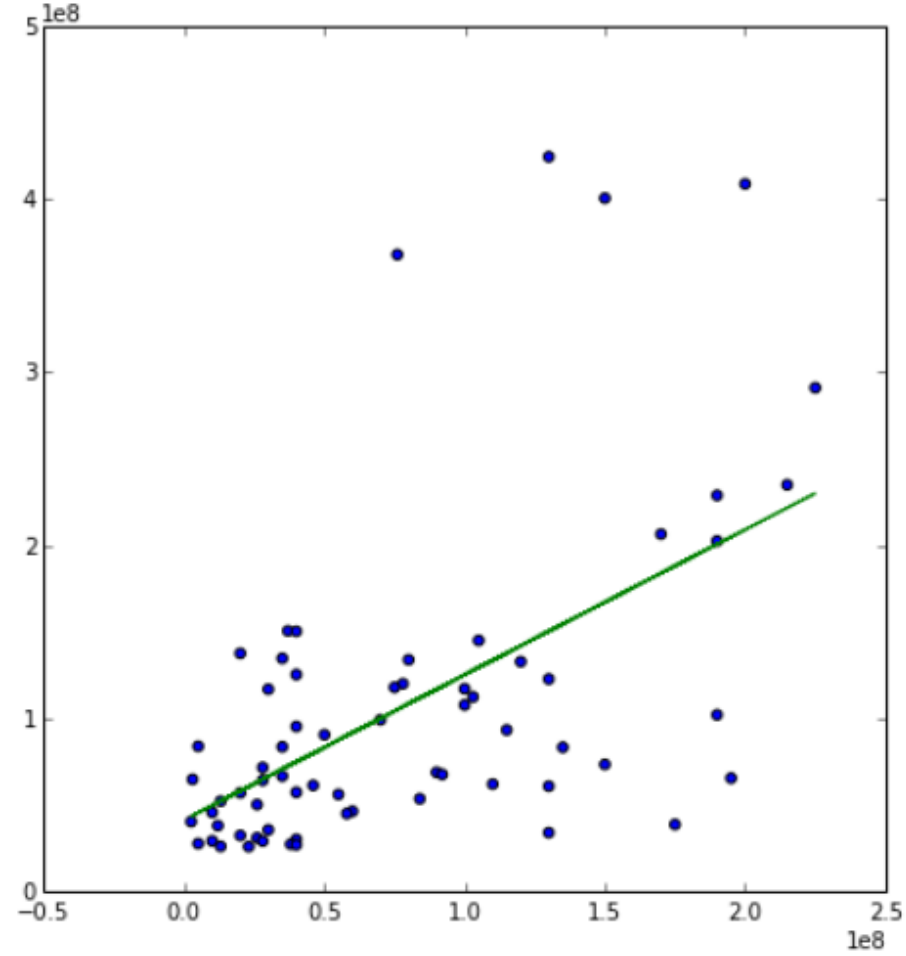


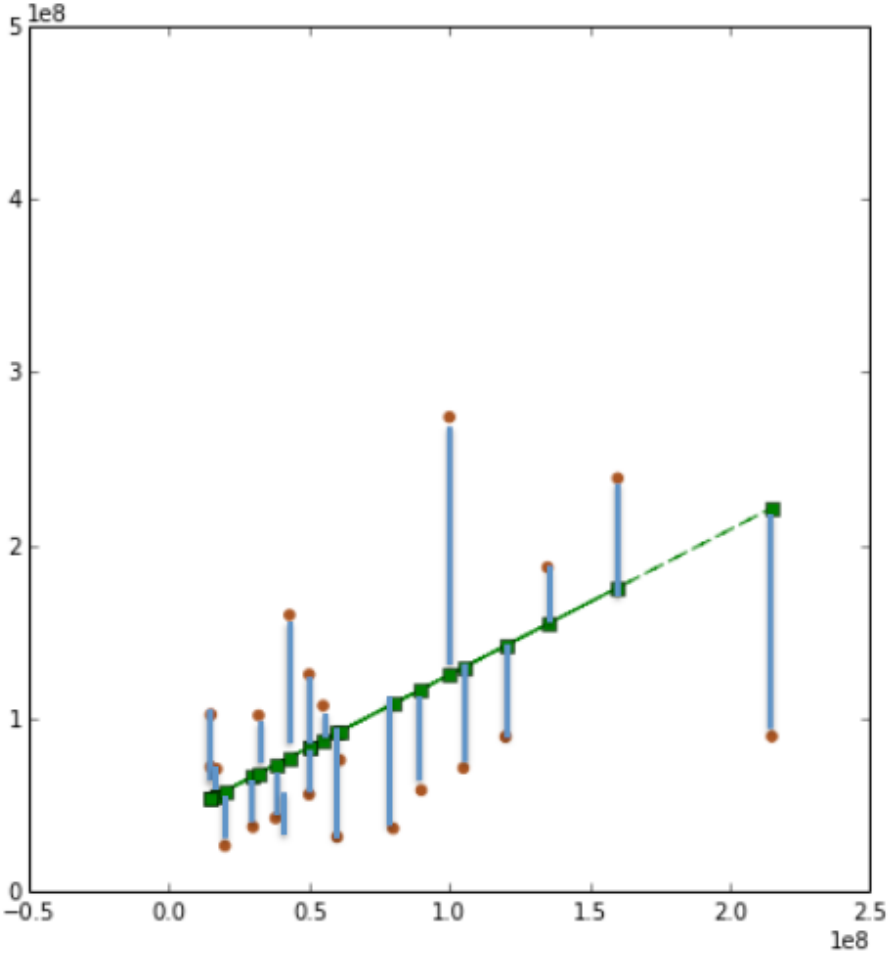


Training set

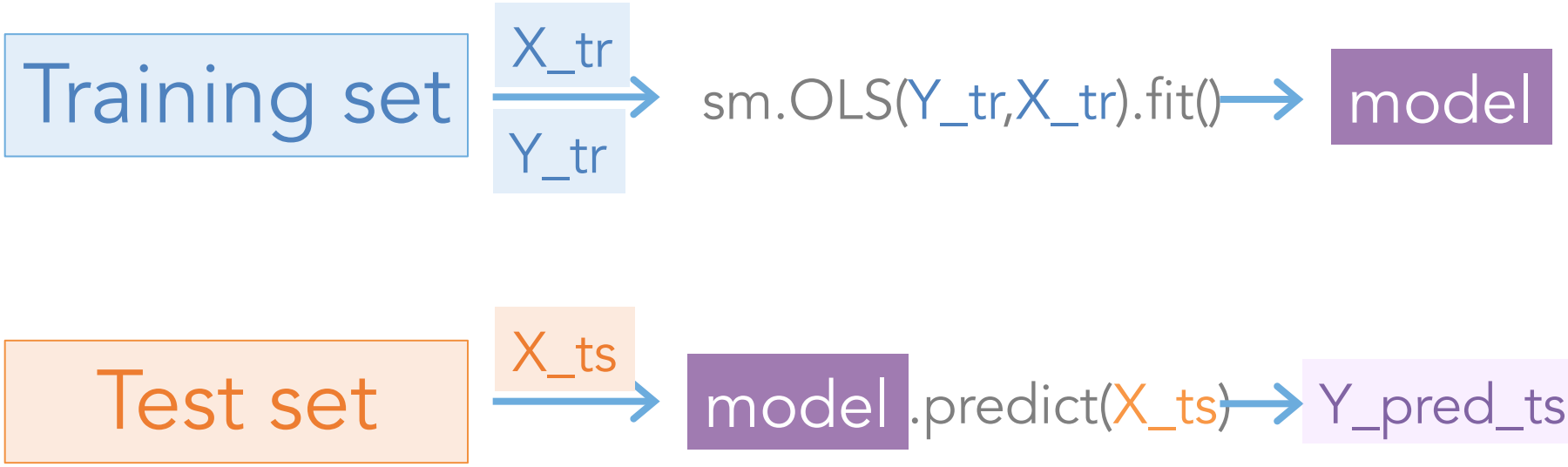
Test set

Measure the error

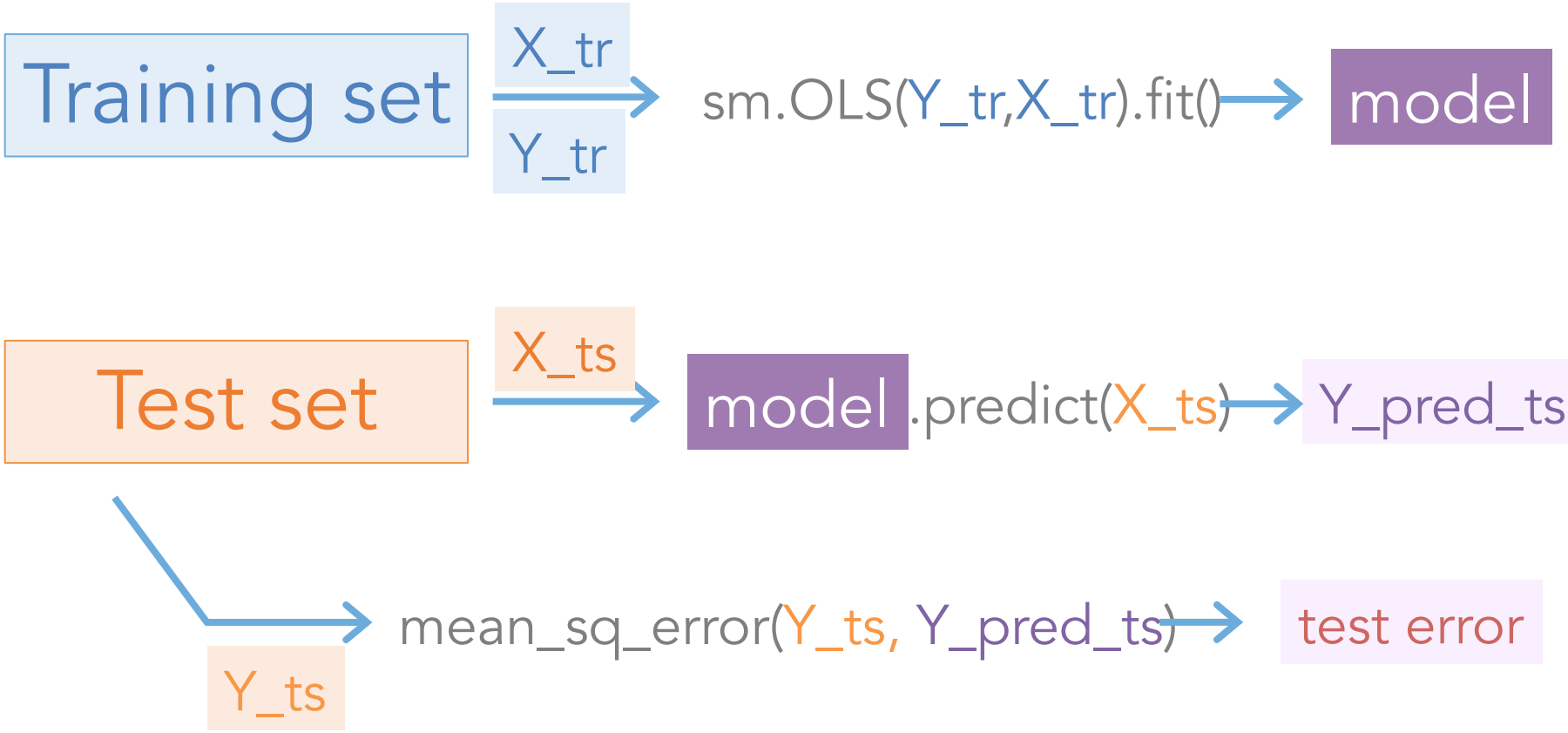




Training and test sets



Training and test sets



Cross validation

What's better than a single test set?

	Date	Title	Budget	DomesticTotalGross	Director	Rating	Runtime
0	2013-11-22	The Hunger Games: Catching Fire	130000000	424668047	Francis Lawrence	PG-13	146
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6	2013-06-21	Monsters University	NaN	268492764	Dan Scanlon	G	107
7	2013-12-13	The Hobbit: The Desolation of Smaug	NaN	258366855	Peter Jackson	PG-13	161
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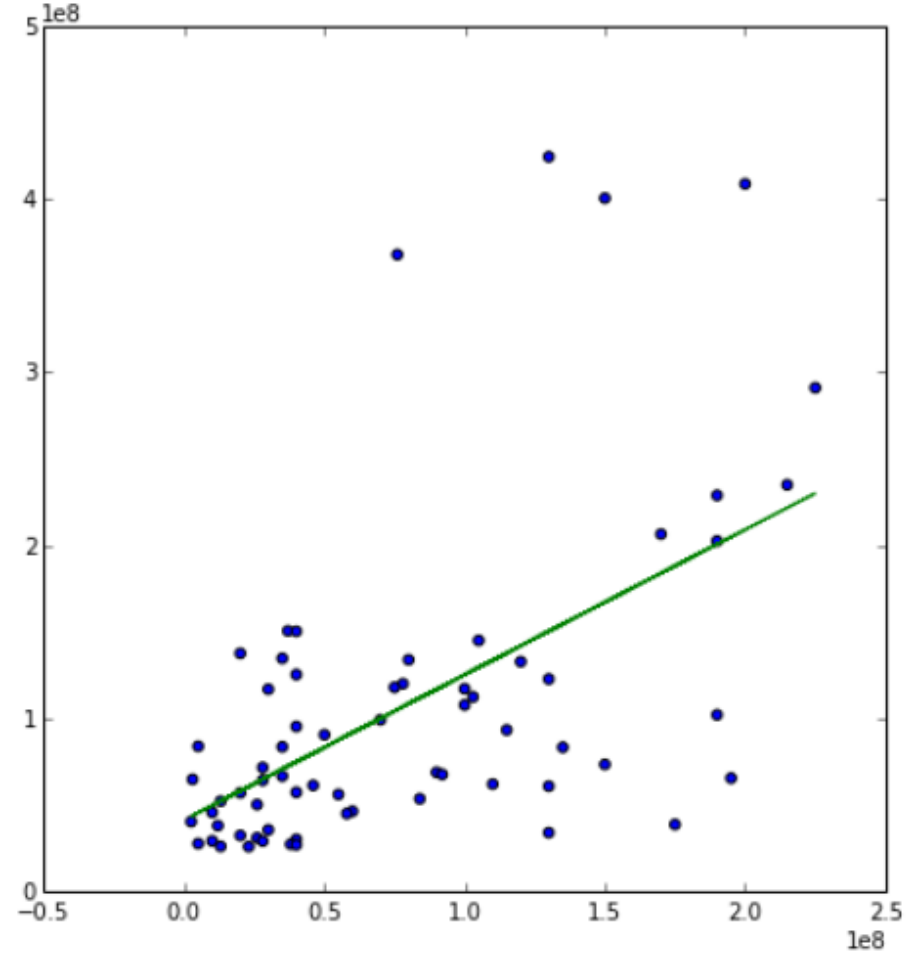
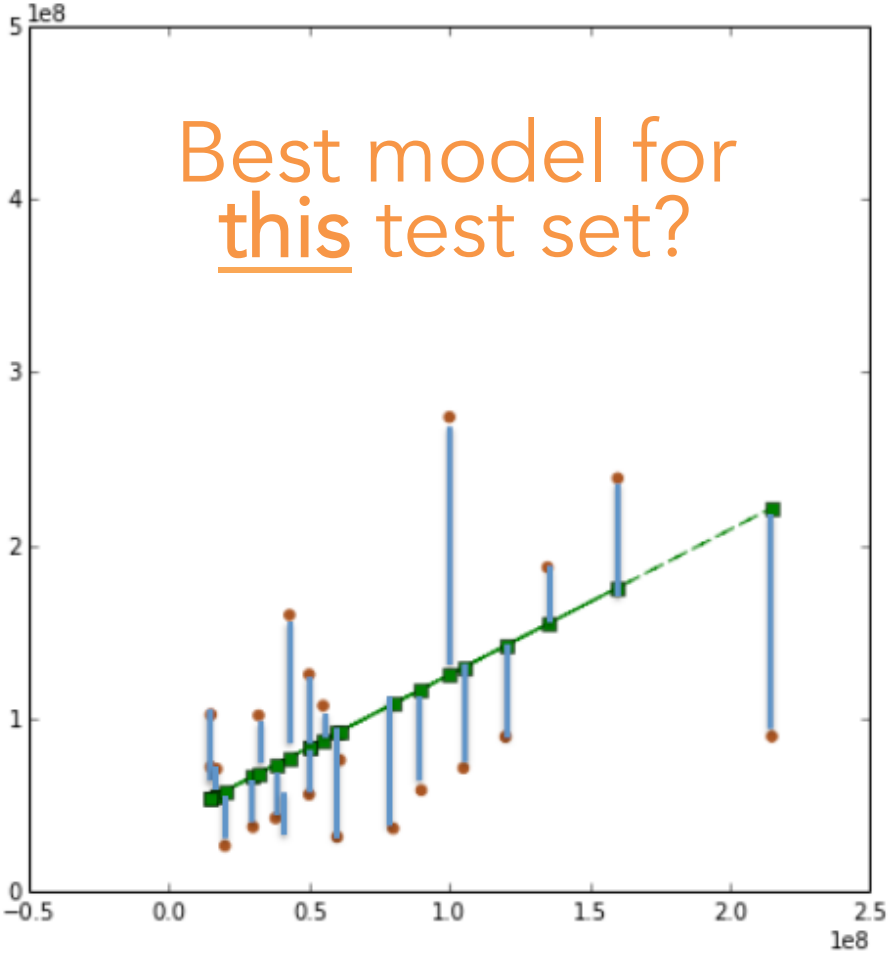
Training set

Test set

Training set

Test set





Cross validation

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16	2013-12-13	American Hustle	40000000	150117807	David O. Russell	R	138
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Training set

Test set

Cross validation

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15	2013-08-07	We're the Millers	37000000	150394119	Rawson Marshall Thurber	R	110
16	2013-12-13	American Hustle	40000000	150117807	David O. Russell	R	138
17	2013-05-10	The Great Gatsby	105000000	144840419	Baz Luhrmann	PG-13	143

Training set 2

Test set 2

Cross validation

	Date	Title	Budget	DomesticTotalGross	Director	Rating	Runtime
0	2013-11-22	The Hunger Games: Catching Fire	130000000	424668047	Francis Lawrence	PG-13	146
1	2013-05-03	Iron Man 3	200000000	409013994	Shane Black	PG-13	129
2	2013-11-22	Frozen	150000000	400738009	Chris BuckJennifer Lee	PG	108
3	2013-07-03	Despicable Me 2	76000000	368061265	Pierre CoffinChris Renaud	PG	98
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15	2013-08-07	We're the Millers	37000000	150394119	Rawson Marshall Thurber	R	110
16	2013-12-13	American Hustle	40000000	150117807	David O. Russell	R	138
17	2013-05-10	The Great Gatsby	105000000	144840419	Baz Luhrmann	PG-13	143

Test set 3

Training Set 3

Cross validation

	Date	Title	Budget	DomesticTotalGross	Director	Rating	Runtime
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1	2013-05-03	Iron Man 3	200000000	409013994	Shane Black	PG-13	129
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3	2013-07-03	Despicable Me 2	76000000	368061265	Pierre CoffinChris Renaud	PG	98
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5	2013-10-04	Gravity	100000000	274092705	Alfonso Cuaron	PG-13	91
6	2013-06-21	Monsters University	NaN	268492764	Dan Scanlon	G	107
7	2013-12-13	The Hobbit: The Desolation of Smaug	NaN	258366855	Peter Jackson	PG-13	161
8	2013-05-24	Fast & Furious 6	160000000	238679850	Justin Lin	PG-13	130
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14	2013-06-28	The Heat	43000000	159582188	Paul Feig	R	117
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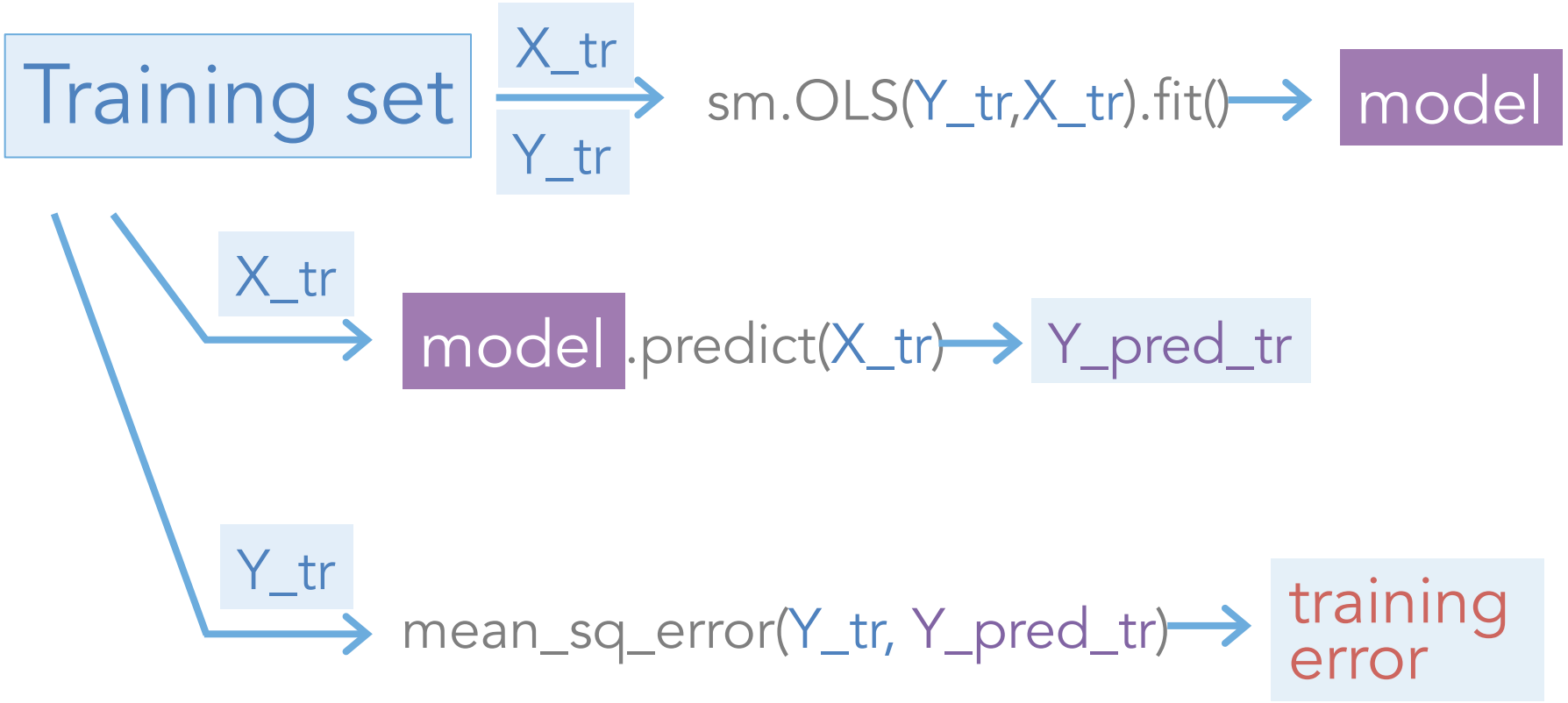
16	2013-12-13	American Hustle	40000000	150117807	David O. Russell	R	138
17	2013-05-10	The Great Gatsby	105000000	144840419	Baz Luhrmann	PG-13	143

Test set 4

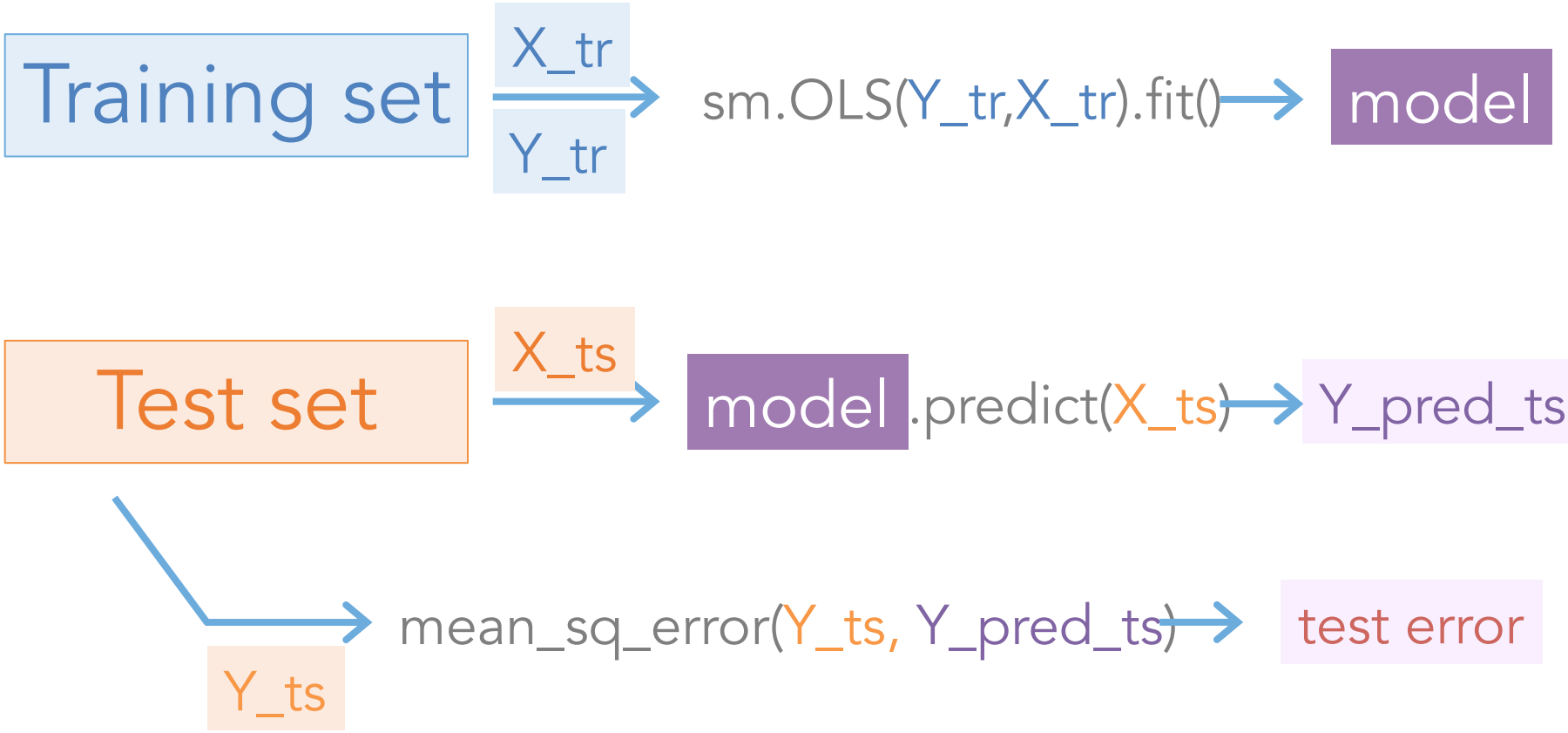
Training Set 4

Diagnostics with training and test errors

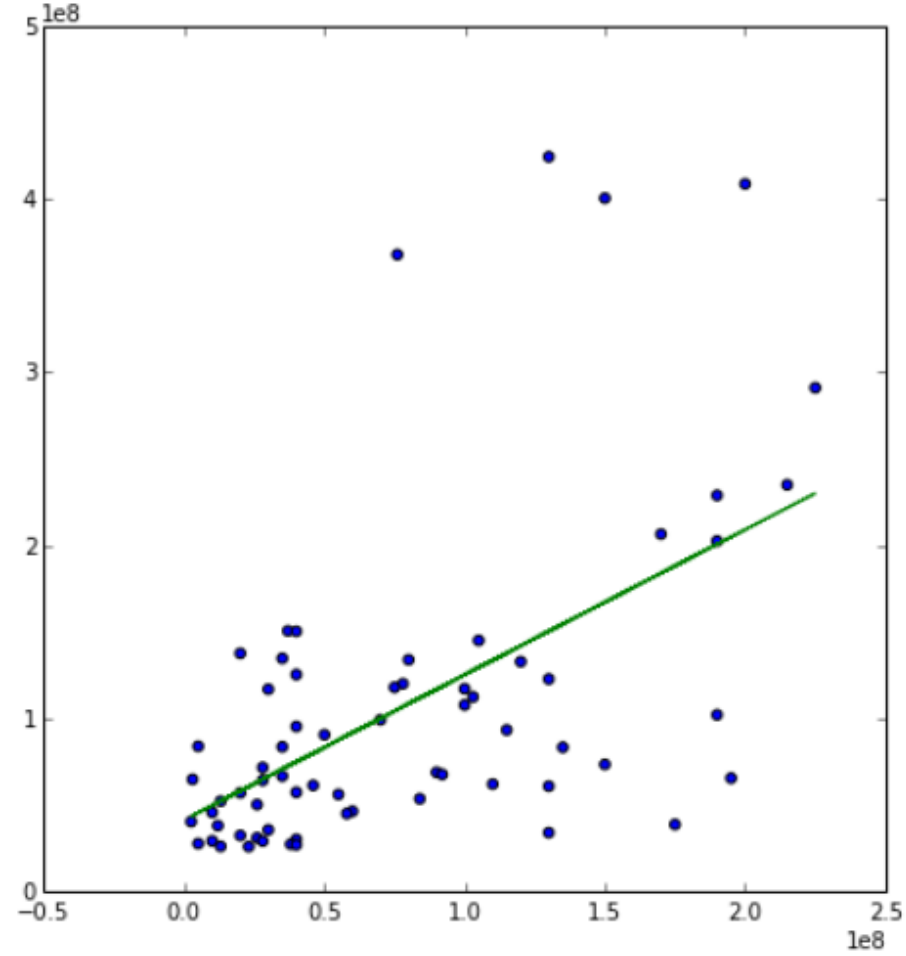
Calculating Training error



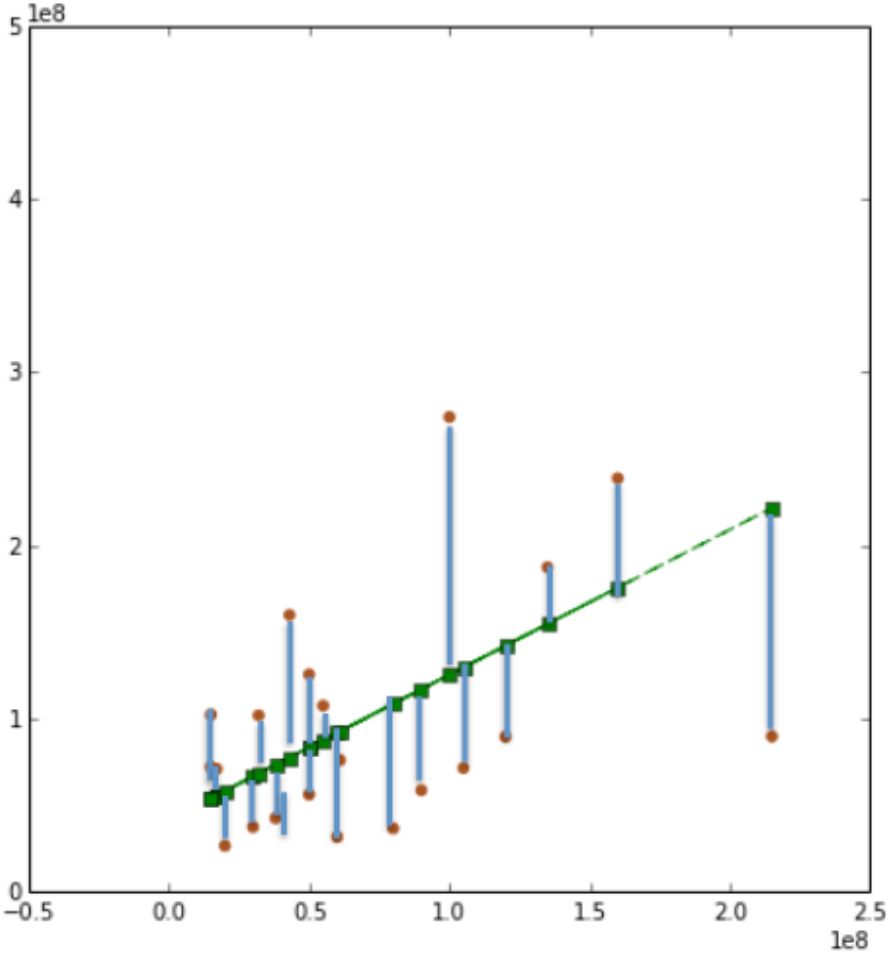
Calculating test error



Training set

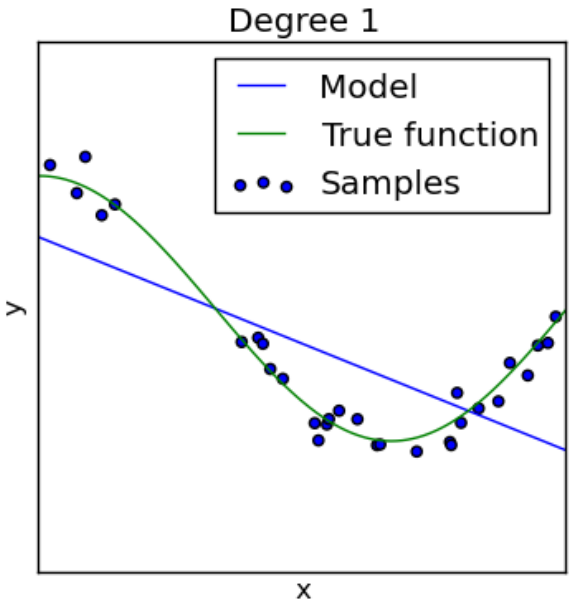


Test set

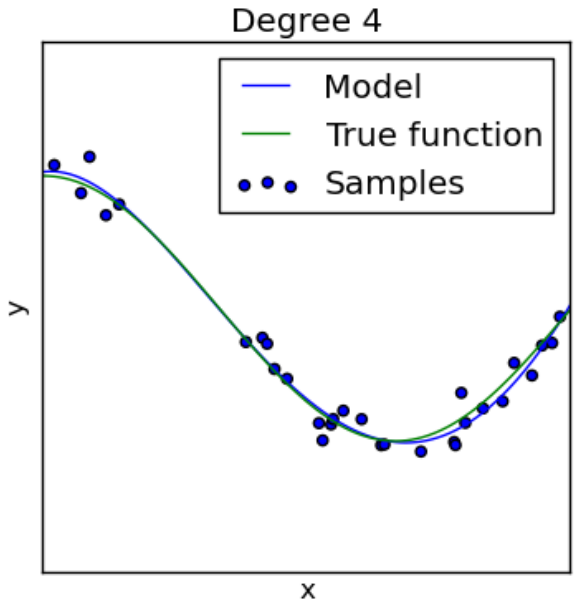


Diagnostics to detect under/overfitting

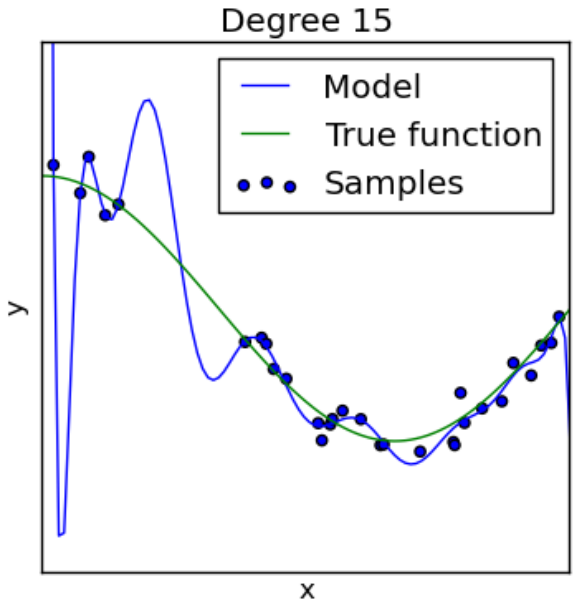
Underfitting



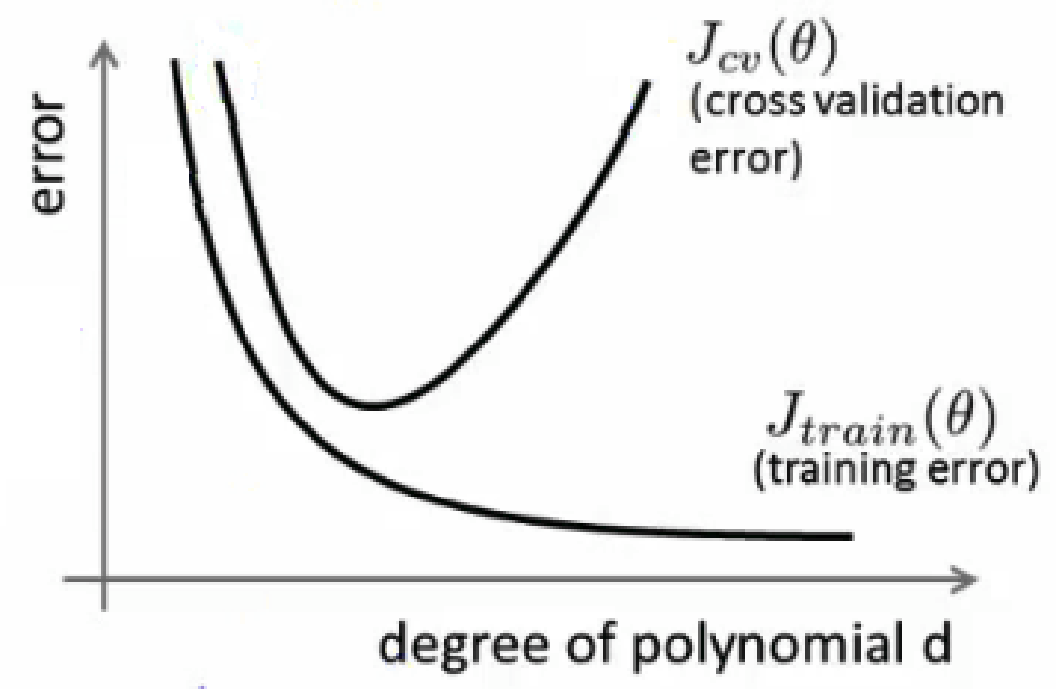
Just Right



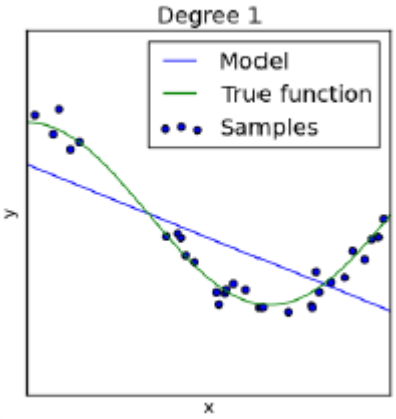
Overfitting

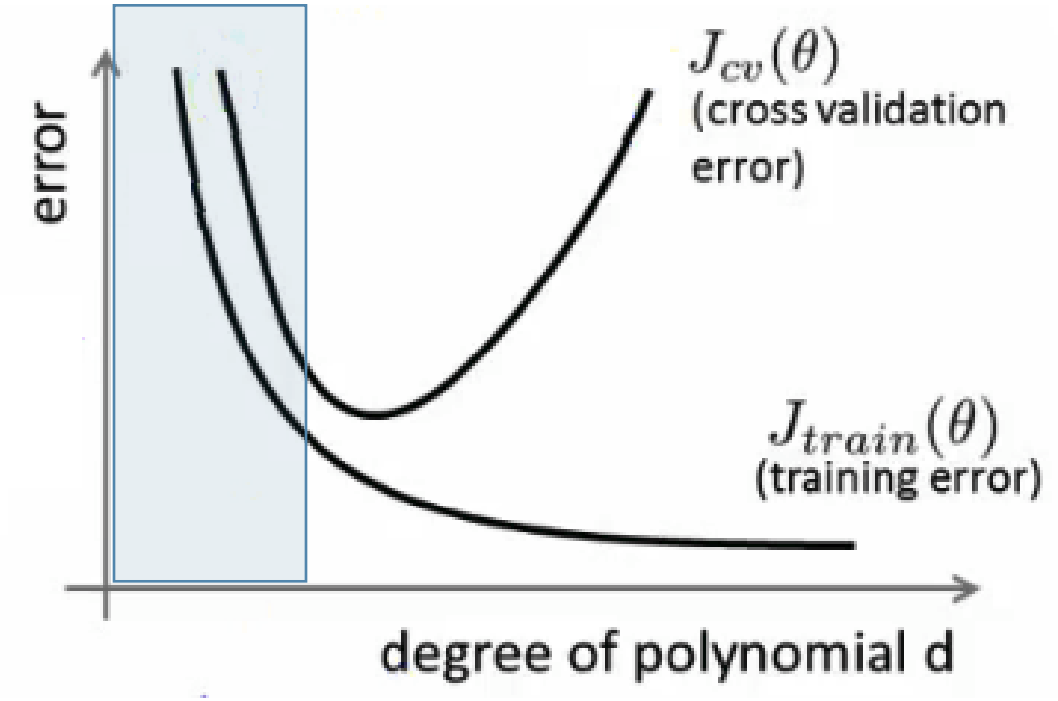


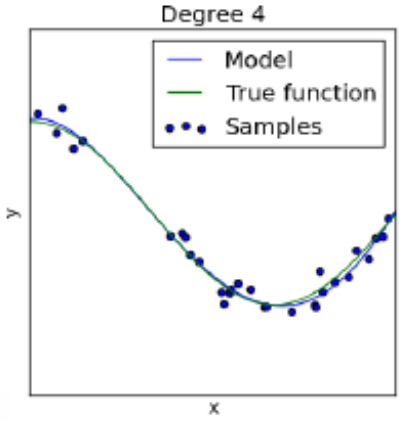
Model complexity vs error



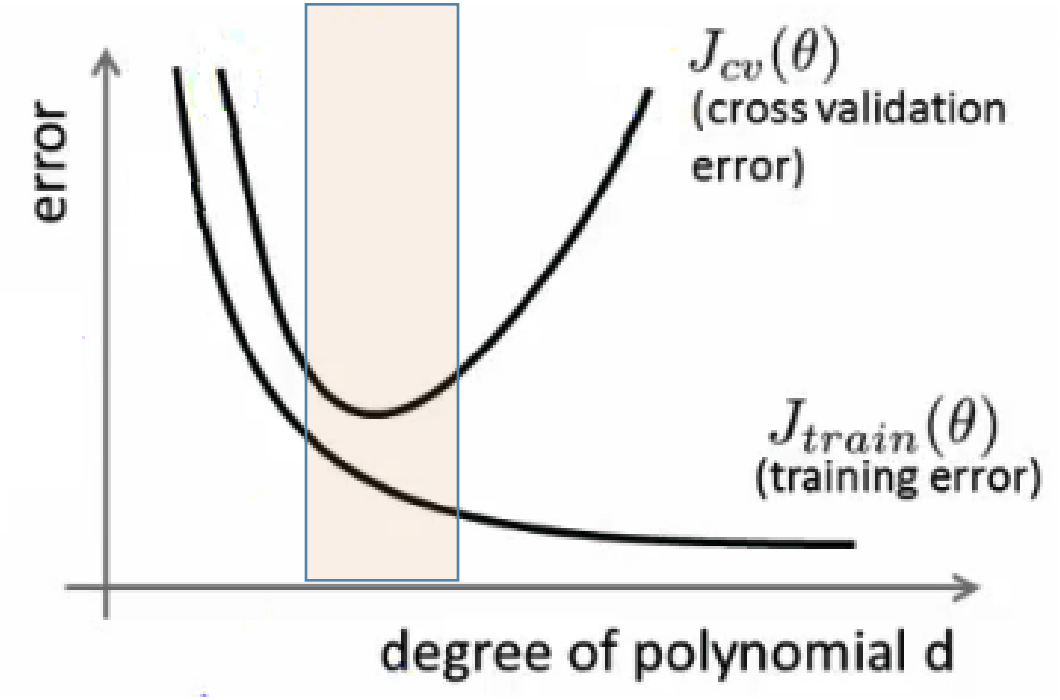
Underfitting



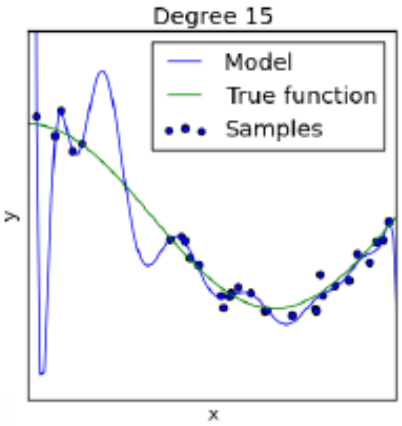


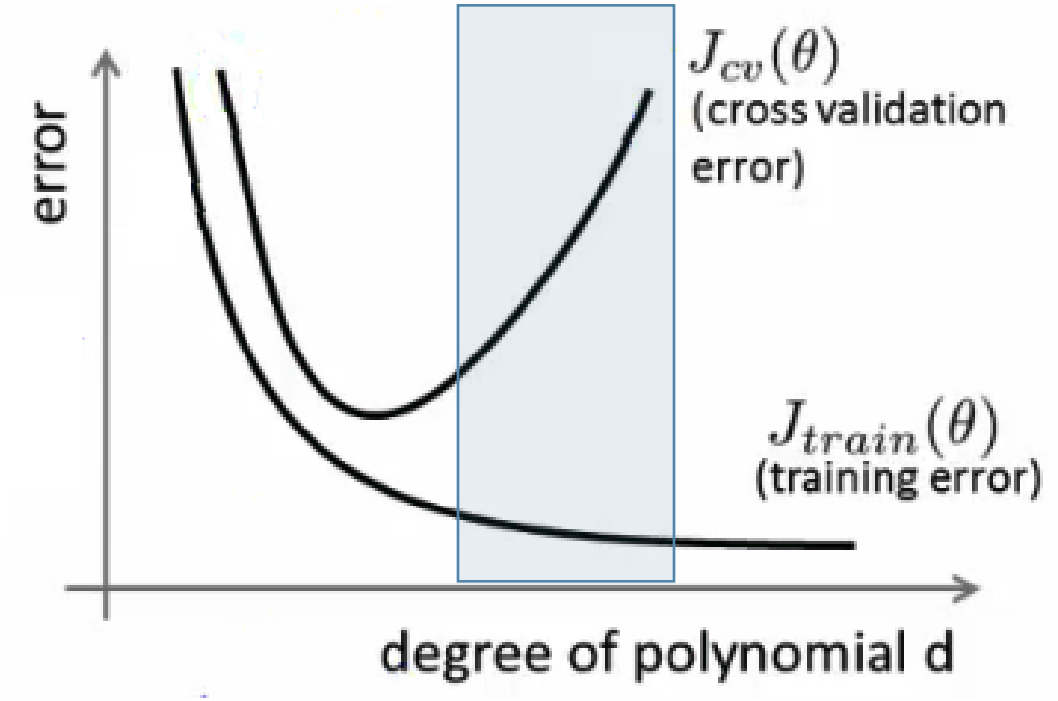

Degree 4
Model
True function
Samples
y
x

Just Right

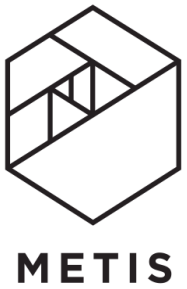

error
$J_{cv}(\theta)$
(cross validation error)
$J_{train}(\theta)$
(training error)
degree of polynomial d

Overfitting





Regularization



OLS Regression Results

Dep. Variable:	DomesticTotalGross	**R-squared:**	0.286
Model:	OLS	**Adj. R-squared:**	0.278
Method:	Least Squares	**F-statistic:**	34.82
Date:	Sun, 14 Sep 2014	**Prob (F-statistic):**	6.80e-08
Time:	21:59:46	**Log-Likelihood:**	-1738.1
No. Observations:	89	**AIC:**	3480.
Df Residuals:	87	**BIC:**	3485.
Df Model:	1		

	coef	**std err**	**t**	**P>|t|**	**[95.0% Conf. Int.]**
Budget	0.7846	0.133	5.901	0.000	0.520 1.049
Ones	4.44e+07	1.27e+07	3.504	0.001	1.92e+07 6.96e+07

Omnibus:	39.749	**Durbin-Watson:**	0.674
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	99.441
Skew:	1.587	**Prob(JB):**	2.55e-22
Kurtosis:	7.091	**Cond. No.**	1.54e+08

$$AIC = 2k - 2\ln(L)$$

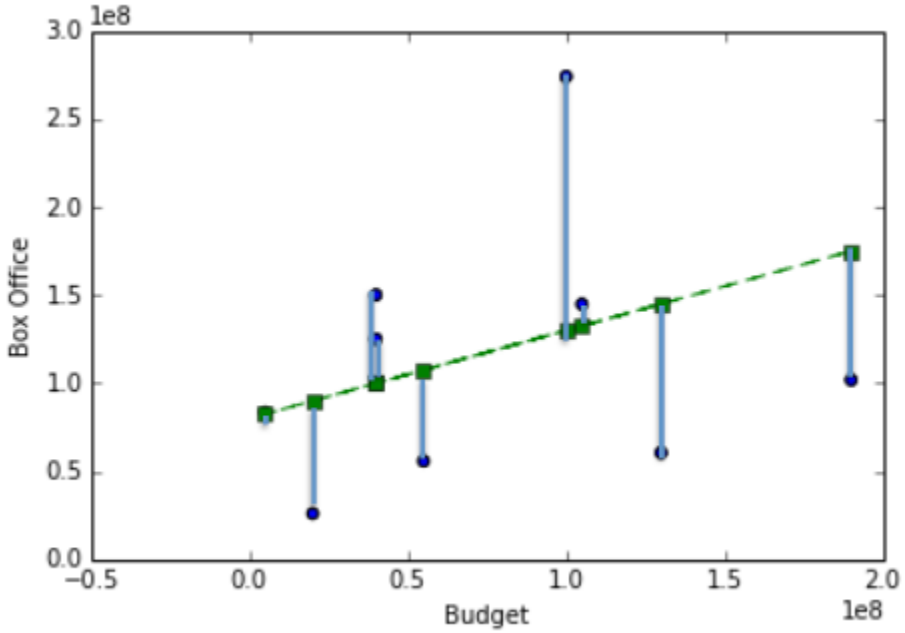
\# parameters

Log likelihood

While awarding goodness of fit, penalize model complexity

While awarding goodness of fit, penalize model complexity

Why not do that while we are fitting?

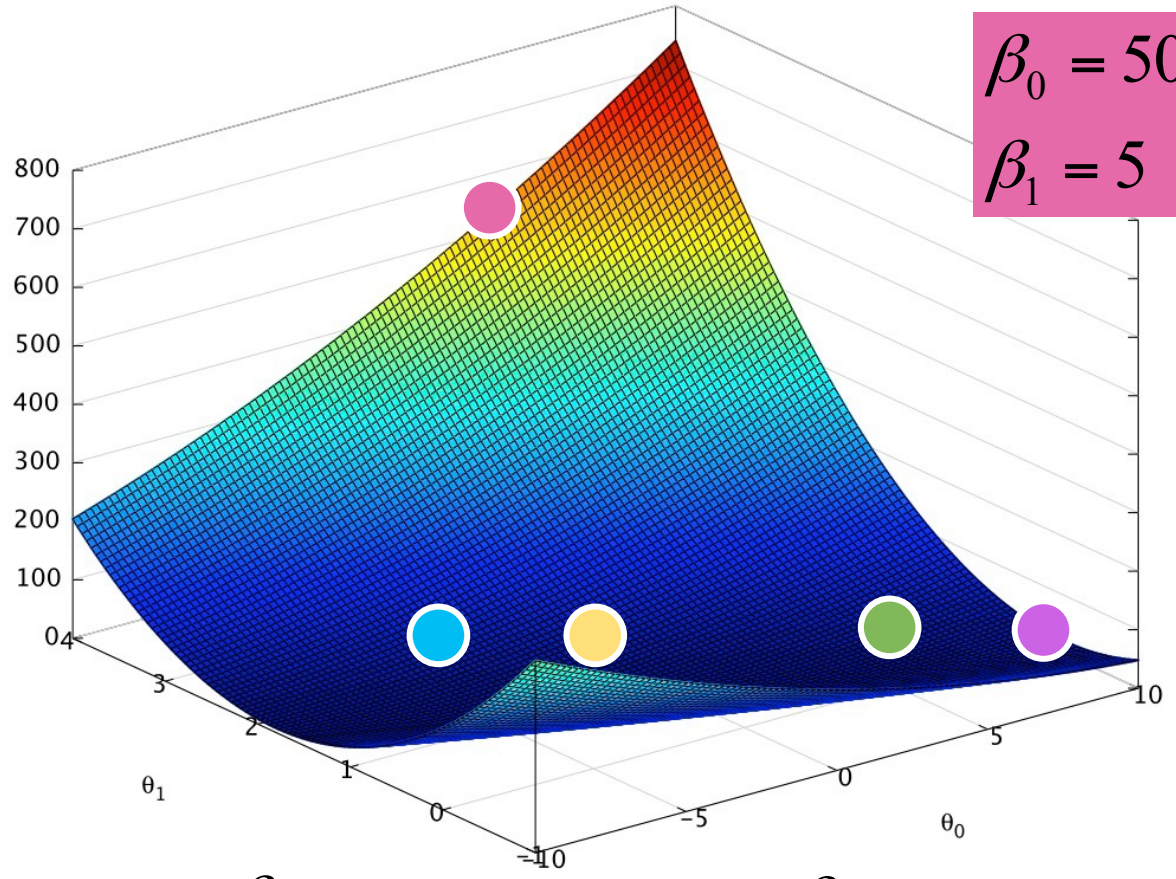


Cost function

Takes a model (specific parameter values), returns a score

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

$J(\beta_0, \beta_1)$



β_1

β_0

$\beta_0 = 50million$
$\beta_1 = 5$

$\beta_0 = 80million$
$\beta_1 = 0.5$

$\beta_0 = 0$
$\beta_1 = 1.5$

$\beta_0 = 120million$
$\beta_1 = 0.1$

$\beta_0 = 30million$
$\beta_1 = 2$

Cost function

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left((\beta_0 + \beta_1 x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2$$

Lower for better fits

Cost function

Add a penalty for the size of each parameter!

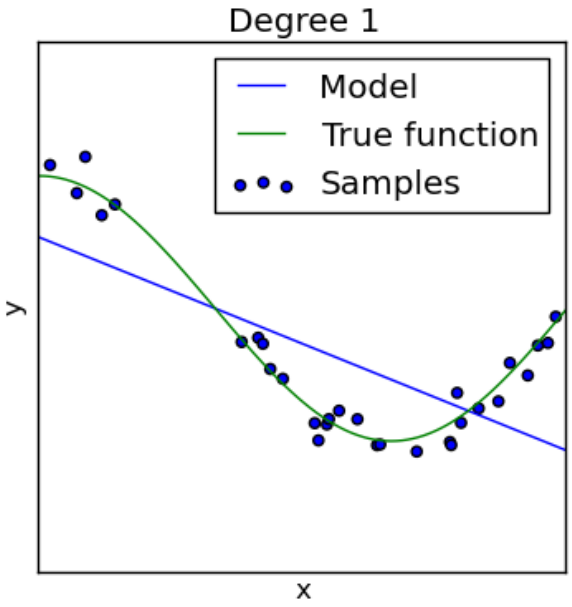
$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Low: good fit
High: bad fit

Low: simple model
High: complex model

Diagnostics to detect under/overfitting

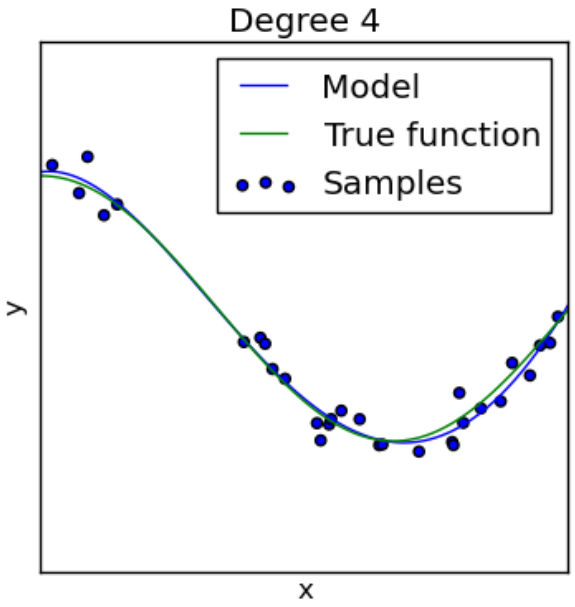
Underfitting



$$J(\beta_0,\beta_1)=\frac{1}{2m}\sum_{i=1}^{m}\left(y_\beta(x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2+\lambda\sum_{j=1}^{k}\beta_j^2$$

J = V. High + Low

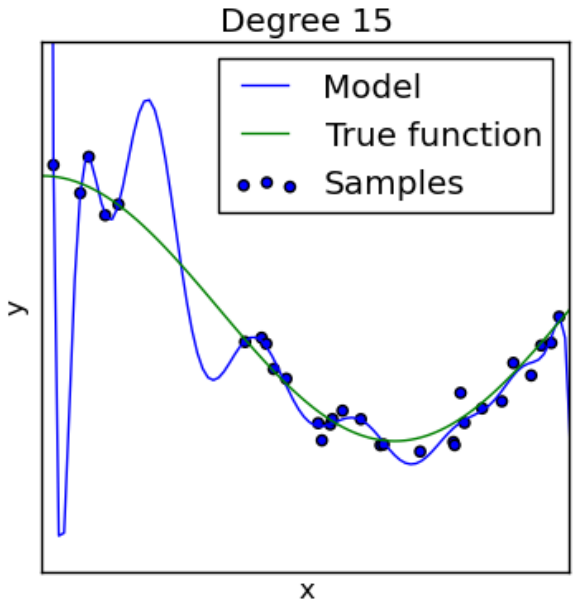
Just Right



$$J(\beta_0,\beta_1)=\frac{1}{2m}\sum_{i=1}^{m}\left(y_\beta(x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2+\lambda\sum_{j=1}^{k}\beta_j^2$$

J = Low + Low

Overfitting



$$J(\beta_0,\beta_1)=\frac{1}{2m}\sum_{i=1}^{m}\left(y_\beta(x_{obs}^{(i)})-y_{obs}^{(i)}\right)^2+\lambda\sum_{j=1}^{k}\beta_j^2$$

J = Low + V. High

Ridge Regression

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Underfitting
J = V. High + Low

Just Right
J = Low + Low

Overfitting
J = Low + V. High

$\lambda = 1$

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

Underfitting
J = V. High + Low

Just Right
J = Low + Low

Overfitting
J = Low + V. High

$\lambda = 1$

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

≈ 0 ≈ 0

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

Underfitting
J = V. High + Medium

Just Right
J = Low + V High

Overfitting
J = Low + VVVHigh

VERY LARGE
underfit

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

≈0 ≈0 ≈0 ≈0

$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

Underfitting
J = V. High + Tiny

Just Right
J = Low + Tiny

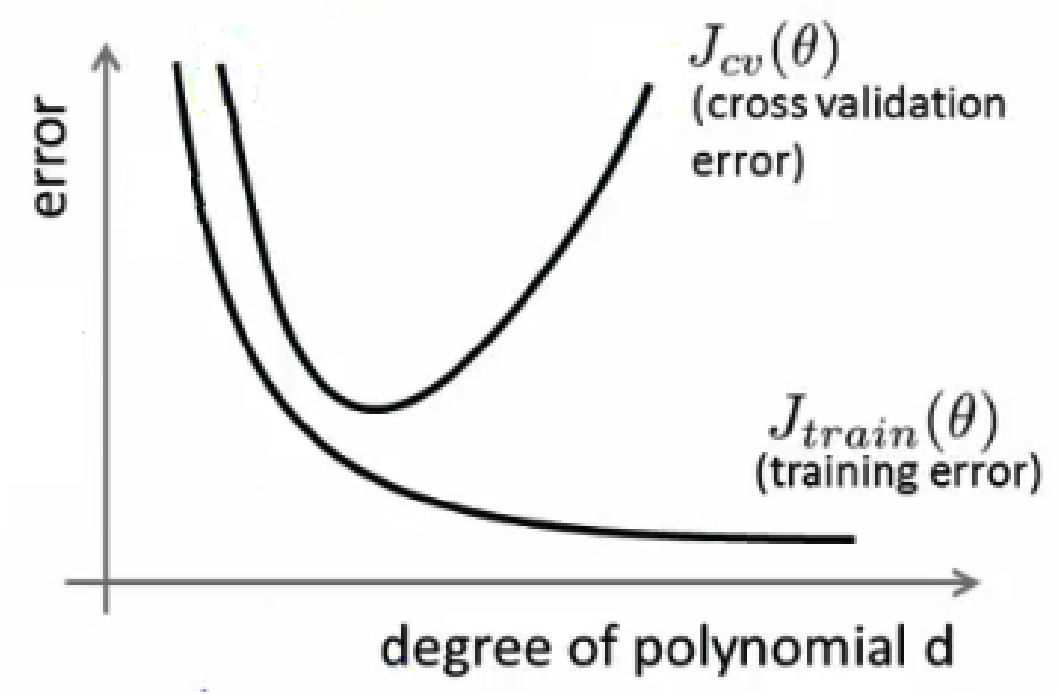
Overfitting
J = Low + Tiny

very small
possible
overfit

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

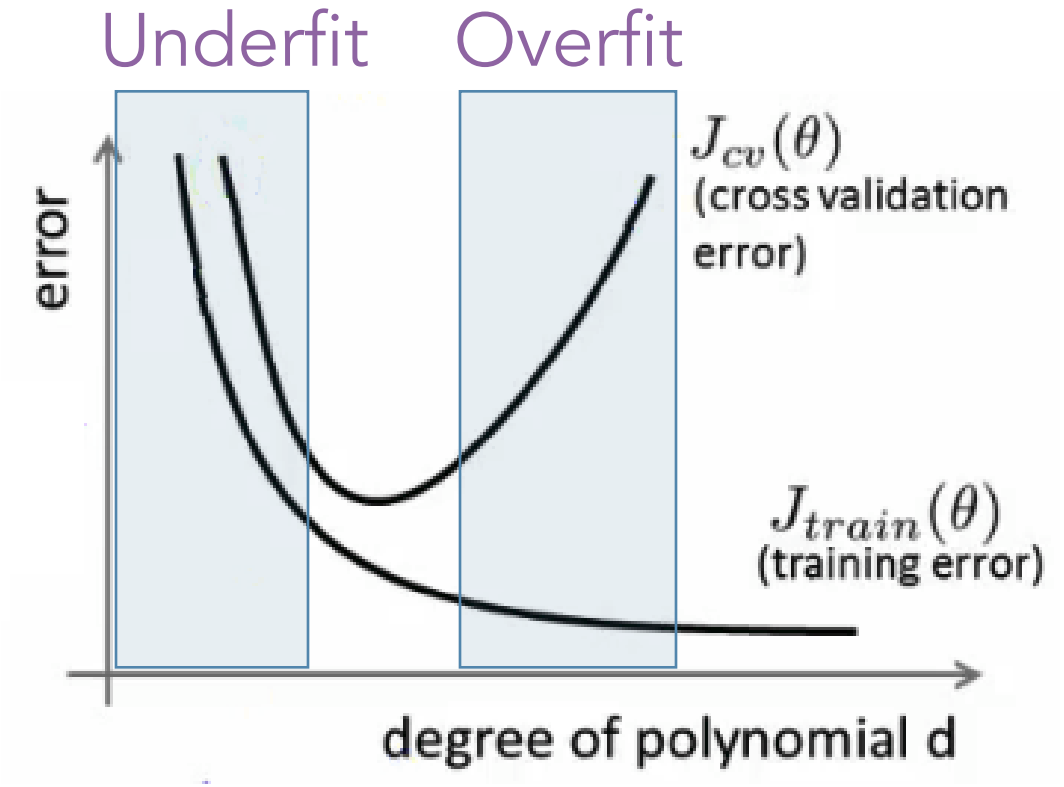
$$y_\beta(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon$$

Error vs. regularization λ



High λ

Low λ


Underfit
Overfit
error
$J_{cv}(\theta)$
(cross validation error)
$J_{train}(\theta)$
(training error)
degree of polynomial d
High λ
Low λ

Ridge Regularization (L2)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Ridge Regularization (L2)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Lasso Regularization (L1)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \left| \beta_j \right|$$

Ridge Regularization (L2)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \beta_j^2$$

Lasso Regularization (L1)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda \sum_{j=1}^{k} \left| \beta_j \right|$$

Elastic Net (L1 + L2)

$$J(\beta_0, \beta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(y_\beta(x_{obs}^{(i)}) - y_{obs}^{(i)} \right)^2 + \lambda_1 \sum_{j=1}^{k} \left| \beta_j \right| + \lambda_2 \sum_{j=1}^{k} \beta_j^2$$

How can I use this?

We were doing:

```
from sklearn.linear_model import
LinearRegression model = LinearRegression()
model.fit(X,Y)
```

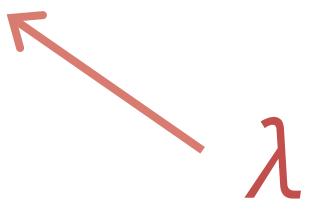
How can I use this?

We were doing:

```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(X,Y)
```

To use Ridge Regularization:

```
from sklearn.linear_model import Ridge
model = Ridge(1.0)
model.fit(X,Y)
```



λ (sklearn Calls It alpha)

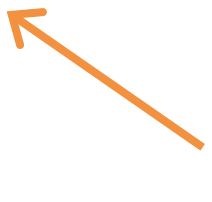
How can I use this?

We were doing:

```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(X,Y)
```

To use Lasso:

```
from sklearn.linear_model import Lasso
model = Lasso(1.0)
model.fit(X,Y)
```



λ (sklearn Calls It alpha)

How can I use this?

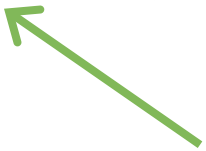
We were doing:

```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
model.fit(X,Y)
```

To use Elastic Net:

```
from sklearn.linear_model import ElasticNet
model = ElasticNet(1.0, l1_ratio = 0.5)
model.fit(X,Y)
```

total weight for the full penalty term



ratio of l1/l2 penalty

My model is not awesome enough.

What do I do?

Try these and check test error (and AIC,BIC,etc.) again:

Use a smaller set of features
Try adding polynomials
Check functional forms for each feature
Try including other features
Use more data (bigger training set)
Regularization

Try these and check test error (and AIC,BIC,etc.) again:

Use a smaller set of features
Try adding polynomials
Check functional forms for each feature
Try including other features
Use more data (bigger training set)
Regularization: Increase/decrease λ