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BS18-02

Homework 2

Variant: C

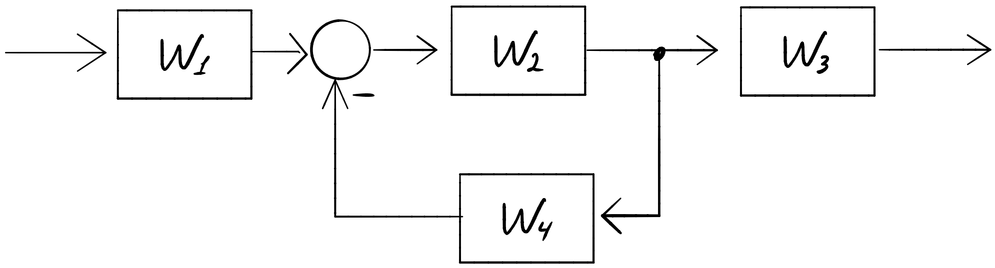
Question 2

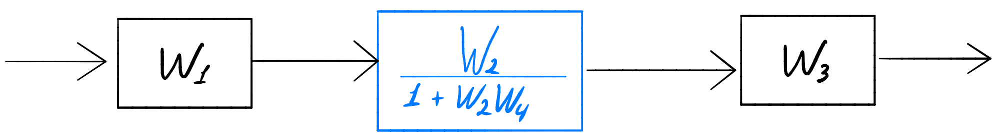
Step-by-step calculations are present on the following page

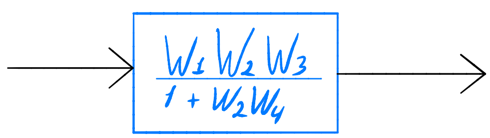
1. The total transfer function is

$$\frac{100s^2 + 120s + 20}{150s^5 + 325s^4 - 16s^3 - 220s^2 - 197s - 42}$$

Ex 2.1 Calculations







(c) $W_1 = \frac{2}{s^2+s-2}$, $W_2 = \frac{1}{3s+2}$, $W_3 = \frac{s+1}{s+0.3}$, $W_4 = \frac{1}{s+0.2}$

$$\frac{2(s+1)}{(s^2+s-2)(3s+2)(s+0{,}3)} \cdot \frac{1}{1+\frac{1}{(3s+2)(s+0{,}2)}}$$

$$\frac{100s^2+120s+20}{150s^5+325s^4-16s^3-220s^2-197s-42}$$

2. On the figure 1 a scheme to compare input signal, simplified and original (shown at fig. 2) systems' output is shown.

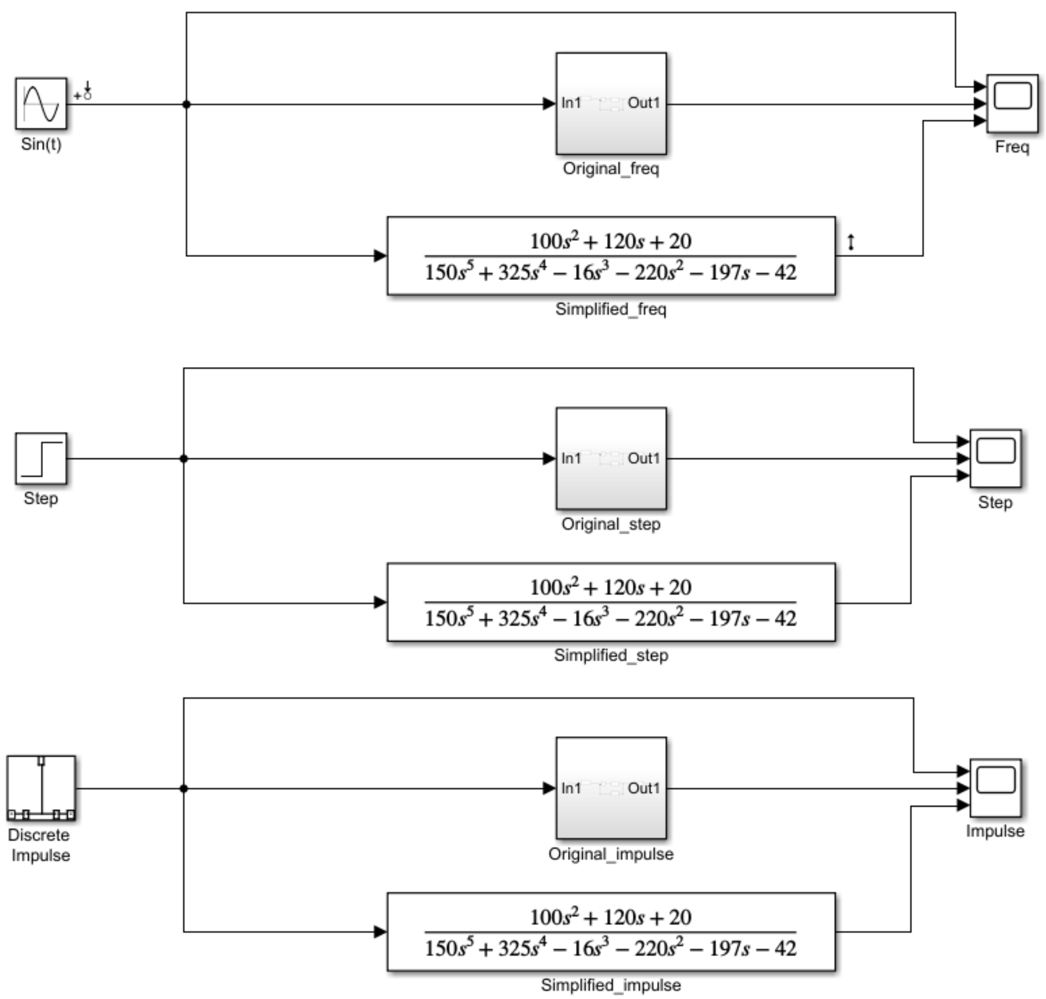


Figure 1: A schema for comparison original and simplified schema on different inputs

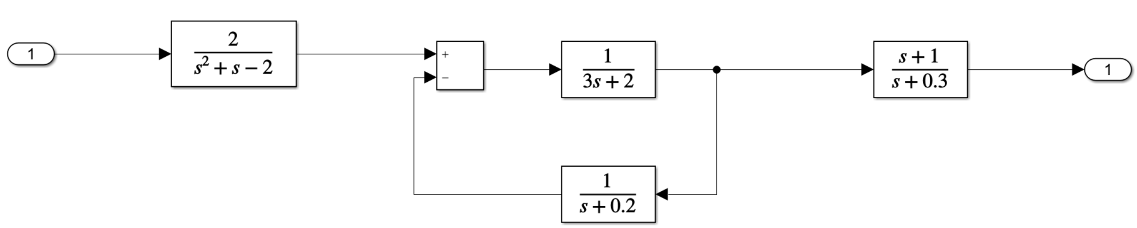


Figure 2: A sub-module containing the original schema

The graphs showing system's behaviour on different inputs are depicted on figures:

- 3 for frequency input;
- 4 for step input;
- 5 for impulse.

For all of 3 given inputs the system is unstable (will be formally proven further).

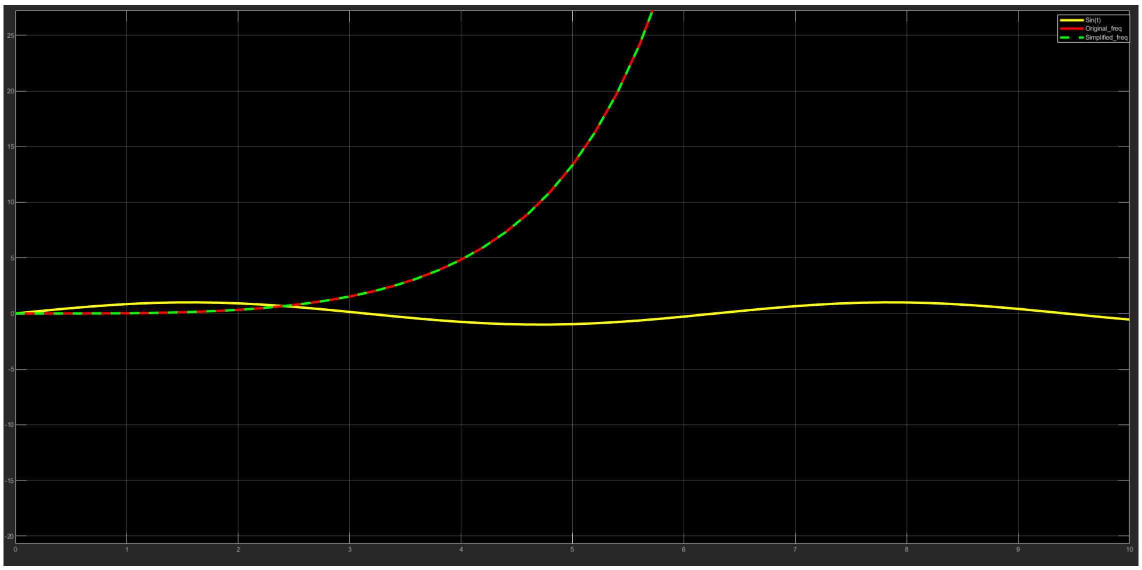


Figure 3: A response for frequency input.

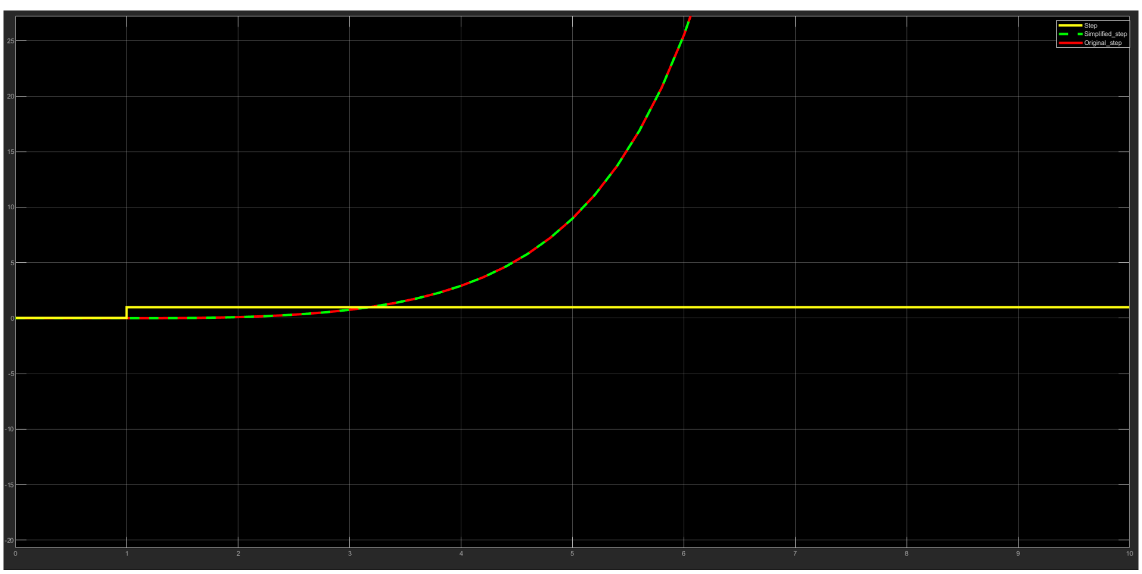


Figure 4: A response for step input.

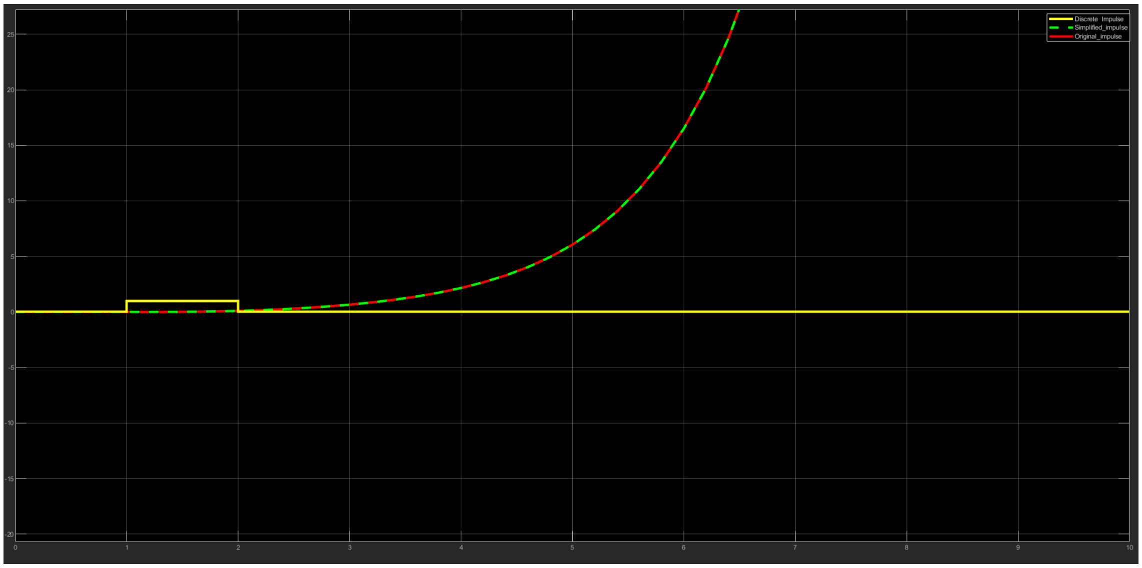


Figure 5: A response for impulse input.

3. Figures 6 and 7 show pole-zero map for frequency input.

Zeroes:

$$\begin{cases} s = -1 \\ s = -\frac{1}{5} \end{cases}$$

Poles:

$$\begin{cases} s = -2 \\ s = -\frac{3}{10} \\ s = 1 \\ s = \frac{1}{30}(-13 \pm i\sqrt{251}) \end{cases}$$

There is a pole at (1; 0), which makes the system unstable (Re $>$ 0).

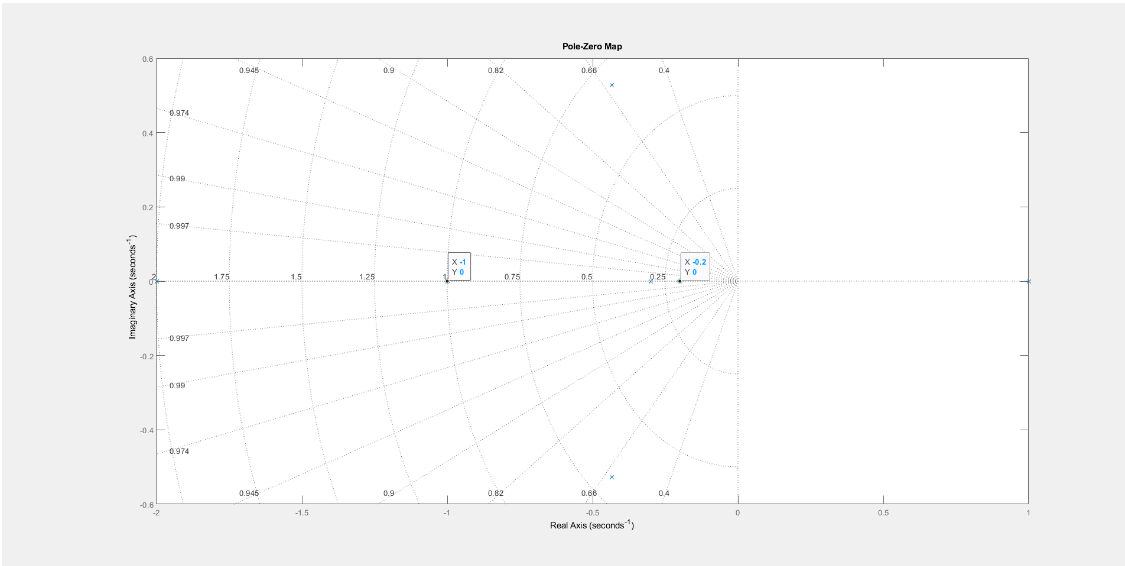


Figure 6: Pole-zero map with zeroes marked.

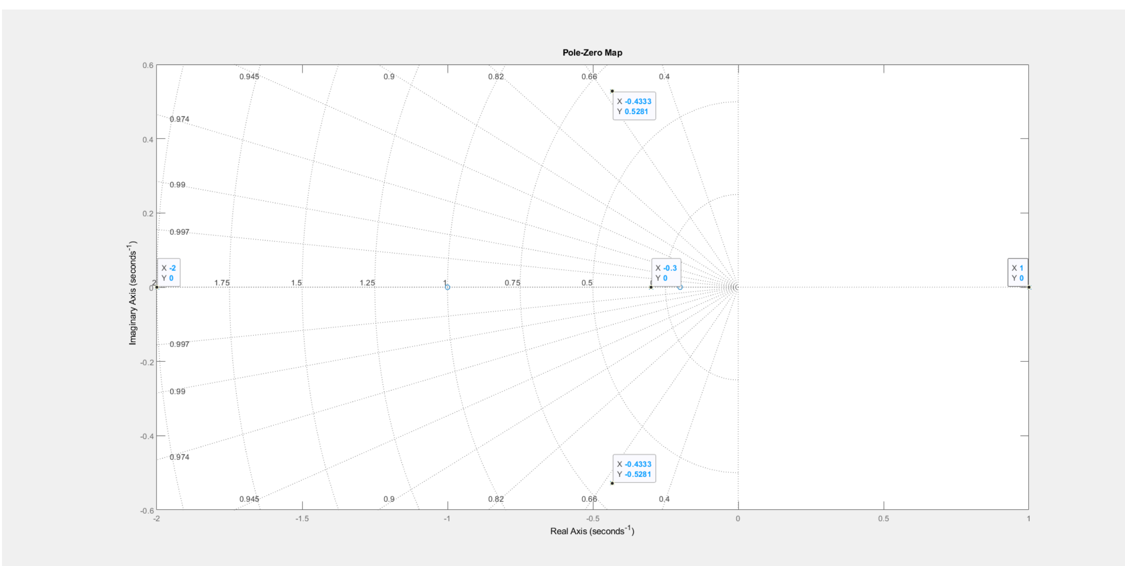


Figure 7: Pole-zero map with poles marked.

4. Figure 8 shows the Bode plot.
   The phase graph intersects -180 at frequency $= 0.322\ rad/sec$, value of magnitude at this point is a gain margin $\approx 4.17\ dB$. The magnitude graph never intersect 0, thus the phase margin is ∞.

   Let's calculate asymptotes:

(a)

$$\frac{100s^2+120s+20}{150s^5+325s^4-16s^3-220s^2-197s-42} = -\frac{20}{42}\frac{5s^2+6s+1}{-\frac{150}{42}s^5-\frac{325}{42}s^4+\frac{16}{42}s^3+\frac{220}{42}s^2+\frac{197}{42}s+1}$$

Thus the graph starts at magnitude $-20\log_{10}\left(\frac{20}{42}\right) = -6.44dB$ at phase $-180°$ with zero growth rate($\Delta = 0$).

(b) Then we consider a pole -2 which produces a break frequency 10^{-1} rad/sec. The magnitude starts to decrease at rate $\Delta = -20$ dB/dec up to zero -1.

(c) A zero -1 increases grow rate by 20 dB/dec, so overall rate become $\Delta = 0$ dB/dec up to next pole

(d) The next pole is a pair of complex-conjugate roots at $Re(\dots) = -\frac{13}{30}$, that decreases overall rate by 40 dB/dec resulting in $\Delta = -40$ dB/dec

(e) Following critique point is a pole at -0.3 which will reduce growing by 20 dB/dec, thus having $\Delta = -60$ dB/dec

(f) Zero at -2 will increase growth rate resulting in $\Delta = -40$ dB/dec

(g) Finally a pole at 1 will decrease a rate, so after this point $\Delta = -60$ dB/dec

As we can see, the majority of growth rate changes happen between -2 and 1 which is a very small interval compared to logarithmic scale.

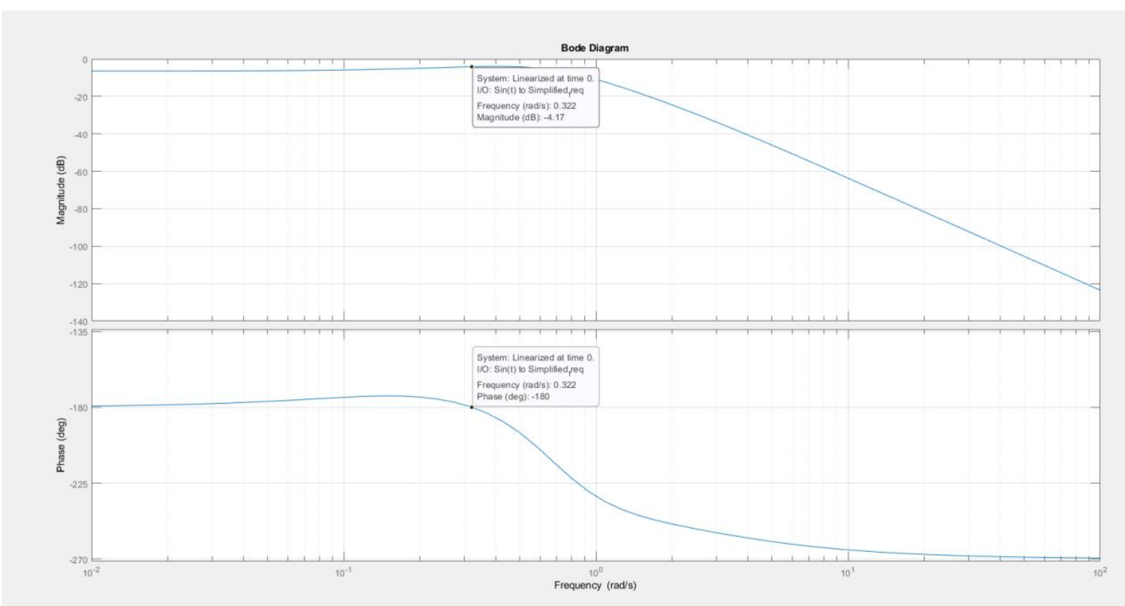


Figure 8: Bode plot.

Question 3

Step-by-step calculations are present on the following page
Given:

$$W(s) = \frac{s+4}{3s+2} \qquad M(s) = \frac{1}{s+1}$$

1. Transfer function for $g(t)$, given $f(t) = 0$:

$$W_{xg} = \frac{W}{1+W} = \frac{s+4}{4s+6}$$

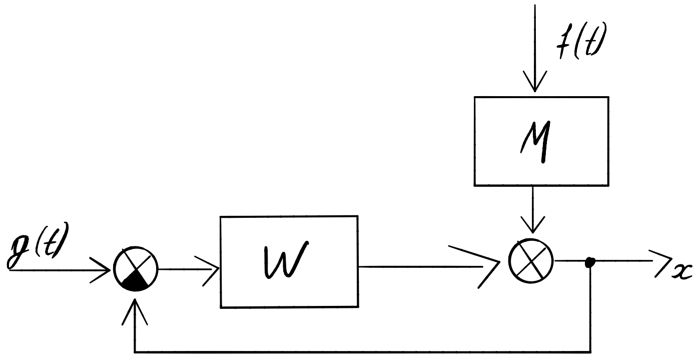
2. Transfer function for $f(t)$, given $g(t) = 0$:

$$W_{xf} = \frac{M}{1+W} = \frac{3s+2}{4s^2+10s+6}$$

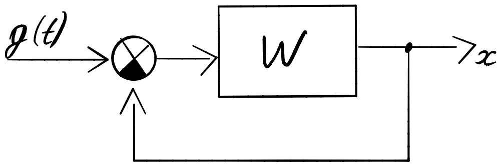
3. Total transfer function:

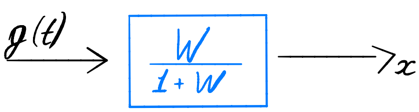
$$\begin{aligned} x &= W_{xg}(s)g(t) + W_{xf}(s)f(t) \\ &= \frac{s+4}{4s+6} * g(t) + \frac{3s+2}{4s^2+10s+6} * f(t) \end{aligned}$$

Ex 3 Calculations

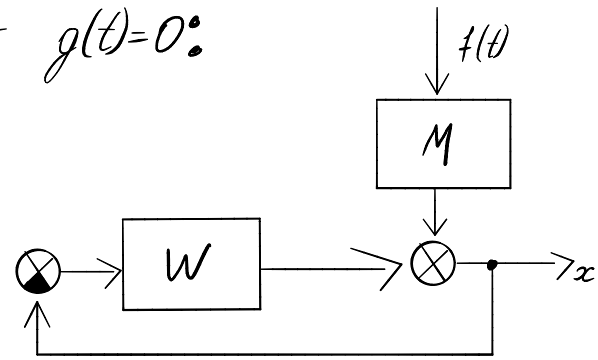


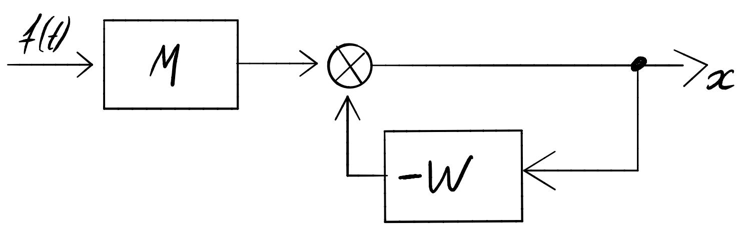
for $f(t)=0$:

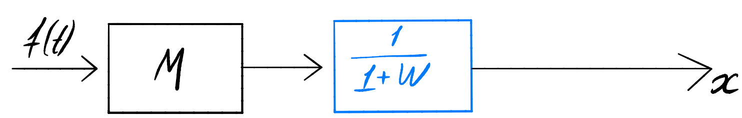


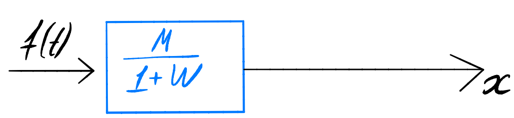


for $g(t)=0$:









Question 4

Find transfer function of the system:

$$A = \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$C = \begin{pmatrix} -2 & 0 \end{pmatrix} \qquad D = \begin{pmatrix} 2 \end{pmatrix}$$

We know, that the transfer function for a SS is: $C(sI - A)^{-1}B + D$.

$$sI - A = \begin{pmatrix} s-2 & 0 \\ 3 & s-1 \end{pmatrix} \tag{1}$$

Since matrix obtained after step (1) is 2x2, we can find it's inverse according to the following formula:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - cb} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Thus

$$\begin{aligned}
(sI - A)^{-1} &= \frac{1}{s^2 - 3s + 2} \begin{pmatrix} s-1 & 0 \\ -3 & s-2 \end{pmatrix} \\
(sI - A)^{-1}B &= \frac{1}{s^2 - 3s + 2} \begin{pmatrix} s-1 & 0 \\ -3 & s-2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} \\
&= \frac{1}{s^2 - 3s + 2} \begin{pmatrix} 1-s \\ s+1 \end{pmatrix} \\
C(sI - A)^{-1}B &= \frac{1}{s^2 - 3s + 2} \begin{pmatrix} -2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1-s \\ s+1 \end{pmatrix} \\
&= \frac{2s - 2}{s^2 - 3s + 2} \\
C(sI - A)^{-1}B + D &= \frac{2s - 2}{s^2 - 3s + 2} + 2 \\
&= \frac{2s^2 - 4s + 2}{s^2 - 3s + 2}
\end{aligned}$$

Answer:

$$W = \frac{2s^2 - 4s + 2}{s^2 - 3s + 2}$$

Question 5

Find transfer function of the system:

$$A = \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 3 \end{pmatrix} \qquad D = \begin{pmatrix} 1 & 2 \end{pmatrix}$$

We know, that the transfer function for a SS is: $C(sI - A)^{-1}B + D$.

$$sI - A = \begin{pmatrix} s-4 & -1 \\ 2 & s-1 \end{pmatrix} \tag{2}$$

Since matrix obtained after step (2) is 2x2, we can find it's inverse according to the following formula:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - cb} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Thus

$$\begin{aligned}
(sI - A)^{-1} &= \frac{1}{s^2 - 5s + 6} \begin{pmatrix} s-1 & 1 \\ -2 & s-4 \end{pmatrix} \\
(sI - A)^{-1}B &= \frac{1}{s^2 - 5s + 6} \begin{pmatrix} s-1 & 1 \\ -2 & s-4 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix} \\
&= \frac{1}{s^2 - 5s + 6} \begin{pmatrix} 2s+1 & s-1 \\ 3s-16 & -2 \end{pmatrix} \\
C(sI - A)^{-1}B &= \frac{1}{s^2 - 5s + 6} \begin{pmatrix} 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2s+1 & s-1 \\ 3s-16 & -2 \end{pmatrix} \\
&= \frac{1}{s^2 - 5s + 6} \begin{pmatrix} 11s-47 & s-7 \end{pmatrix} \\
C(sI - A)^{-1}B + D &= \frac{1}{s^2 - 5s + 6} \begin{pmatrix} 11s-47 & s-7 \end{pmatrix} + \begin{pmatrix} 1 & 2 \end{pmatrix} \\
&= \frac{1}{s^2 - 5s + 6} \begin{pmatrix} 11s-46 & s-5 \end{pmatrix}
\end{aligned}$$

Answer:

$$\begin{cases} W_1 = \frac{11s-46}{s^2-5s+6} \\ W_2 = \frac{s-5}{s^2-5s+6} \end{cases}$$

Question 6

Step-by-step calculations are present on the following page

1. Transfer function for f, given $x = 0$:

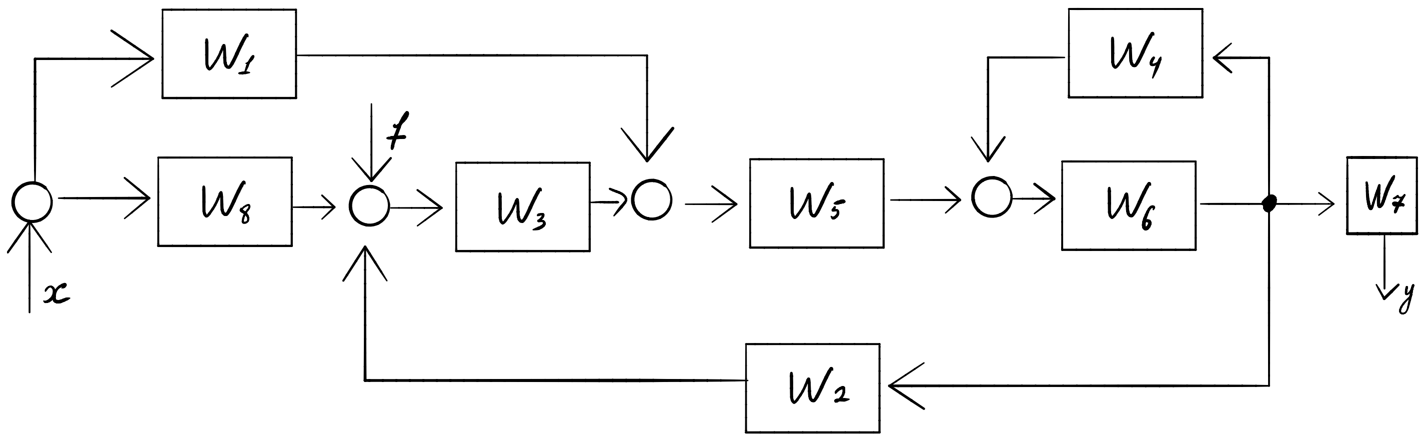
$$W_{yf} = \frac{W_3W_5W_6W_7}{1 - W_4W_6 - W_2W_3W_5W_6}$$

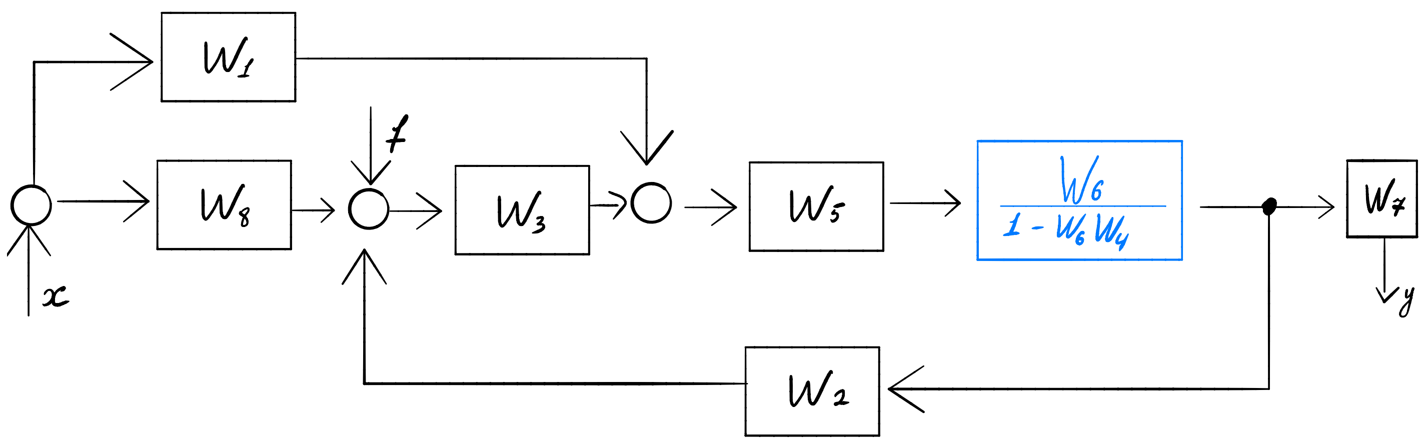
2. Transfer function for x, given $f = 0$:

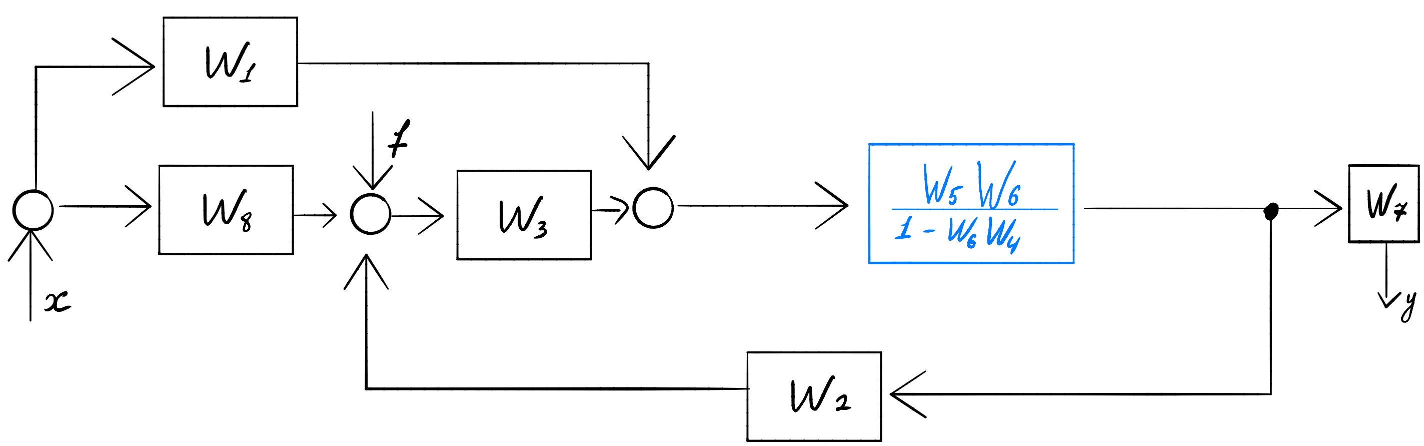
$$W_{yx} = \frac{W_1 + W_3W_8}{W_3} * \frac{W_3W_5W_6W_7}{1 - W_4W_6 - W_2W_3W_5W_6}$$

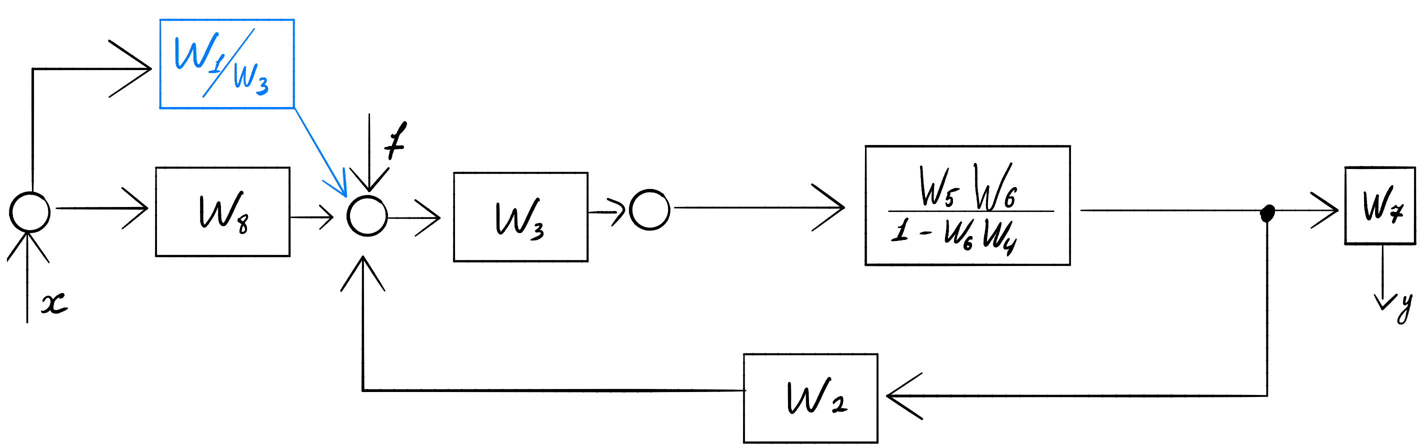
3. Finally:

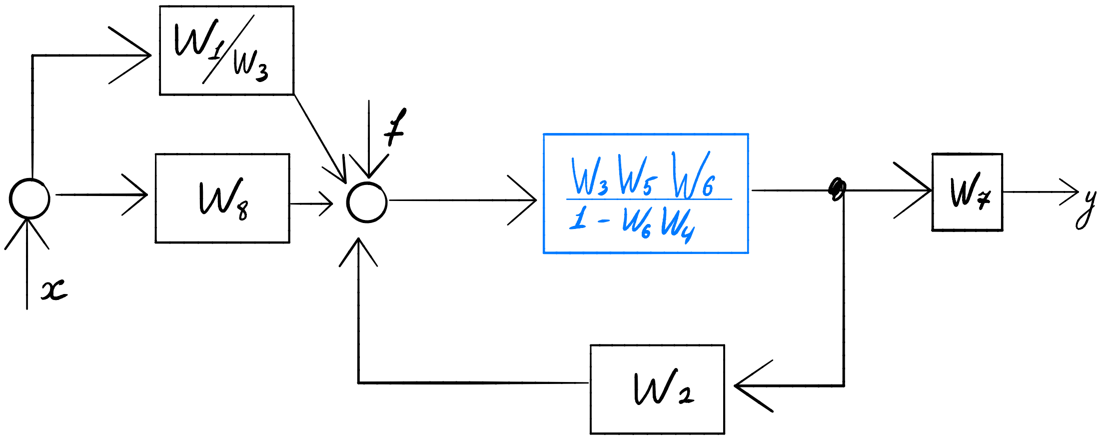
$$\begin{aligned} y &= W_{yf} * f(t) + W_{xf} * x(t) \\ &= \frac{W_3W_5W_6W_7}{1 - W_4W_6 - W_2W_3W_5W_6} * f(t) + \frac{W_1 + W_3W_8}{W_3} * \frac{W_3W_5W_6W_7}{1 - W_4W_6 - W_2W_3W_5W_6} * x(t) \end{aligned}$$

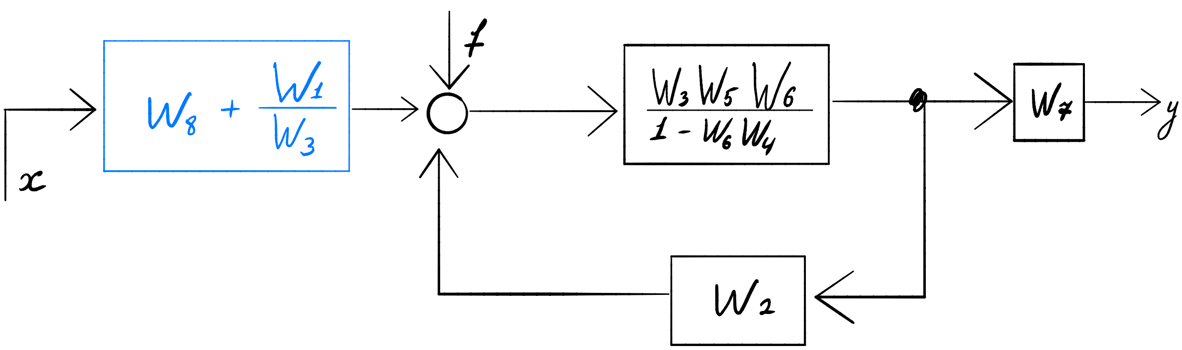

W_1
W_4
f
W_8
W_3
W_5
W_6
W_7
x
y
W_2


W_1
f
W_8
W_3
W_5
$\frac{W_6}{1 - W_6 W_4}$
W_7
x
y
W_2


W_1
f
W_8
W_3
$\frac{W_5 W_6}{1 - W_6 W_4}$
W_7
x
y
W_2


W_1 / W_3
f
W_8
W_3
$\frac{W_5 W_6}{1 - W_6 W_4}$
W_7
x
y
W_2





$$\frac{\dfrac{W_3 W_5 W_6}{1 - W_6 W_4}}{1 - \dfrac{W_2 W_3 W_5 W_6}{1 - W_6 W_4}} = \frac{W_3 W_5 W_6}{1 - W_6 W_4 - W_2 W_3 W_5 W_6}$$