Subglacial drainage systems

As water flows down the potential gradient, energy is lost. This is converted into heat, which has two effects:

① Keep water at melting point

② Melt more ice away

Amount of energy released by loss of hydraulic potential is

$$Q \frac{d\phi}{dx} \Delta x$$

Q: Volume flux rate of water
x: along water flow path

As ice gets thinner downstream, pressure decreases and pressure melting point increases and so water temperature must increase to keep water from freezing. The amount of heat needed to do this is

$$\rho_w C_w \beta \left[Q \rho_i g \frac{\partial (H-z)}{\partial z} \Delta x \right]$$

C_w: heat capacity of water
β: gradient of melting point line (again!)

$\gamma = \rho_w C_w \beta = 0.41$ dimensionless quant

Or, simplified: $\gamma Q \Delta x \left(\frac{\partial \phi}{\partial x} - \rho_w g \frac{\partial z}{\partial x} \right)$

The rest of the heat from the loss of hydraulic potential goes to melting ice, where

$$\dot{m} \Delta x (2\pi r) \rho_i L_f$$

$\dot{m}$: melt rate of channel radius
$2\pi r$: circumference of a possible channel

Thus, we have:
$$Q \frac{\partial \phi}{\partial x} \Delta x = \dot{m} \Delta x (2\pi r) \rho_i L_f + \gamma Q \Delta x \left(\frac{\partial \phi}{\partial x} - \rho_w g \frac{\partial z}{\partial x} \right)$$

Solving for $\dot{m}$, we have

$$\dot{m} = \frac{Q}{2\pi r \rho_i L_f}\left[(1-\gamma)\frac{\partial \phi}{\partial x} + \gamma \rho_w g \frac{\partial z}{\partial x}\right]$$

Since a pipe-like conduit will result in the fastest Q of water flow, which causes more melting, this melt rate will want to preferentially create pipe-like conduits to accomodate subglacial water flow

Gaukler-Manning-Strickler empirical relation for water flow through a pipe:

$$Q = \frac{\pi r^2}{2^{2/3} n_m} r^{2/3} \left(\frac{1}{\rho_w g}\frac{\partial \phi}{\partial x}\right)^{\frac{1}{2}}$$

n_m is an empirical param derived from lab measurements - depends on conduit roughness

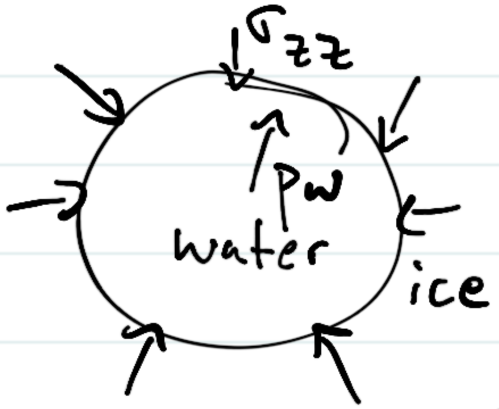
$n_m \sim 5 \times 10^{-3}\ m^{-\frac{1}{3}} s$ for an actual smooth pipe

$n_m \sim 10^{-2} - 10^{0}\ m^{-\frac{1}{3}} s$ for subglacial channels

If water is being supplied from a warm source, there is extra heat to use for melting:

$$\dot{m} = \frac{c_w Q \rho_w}{2\pi r \rho_i L_f} \frac{\partial T}{\partial x}$$

The closure of the conduit due to creep deformation in the presence of a stress inside the conduit is



$$\dot{\varepsilon}_{rr} = A \tau_{RR}^{\,n}$$

$$\tau_{RR} = \frac{N}{n} = \frac{\rho g H - P_w}{n}$$

$$\dot{\varepsilon}_{rr} = \frac{\dot{r}}{r} = A \left(\frac{N}{n}\right)^n$$

$$\boxed{\dot{r} = A r \left(\frac{N}{n}\right)^n}$$

At a steady state with conduits

$$\dot{m} = \dot{r}$$

which can be rearranged to show that

$$Q^{\alpha} \propto N$$

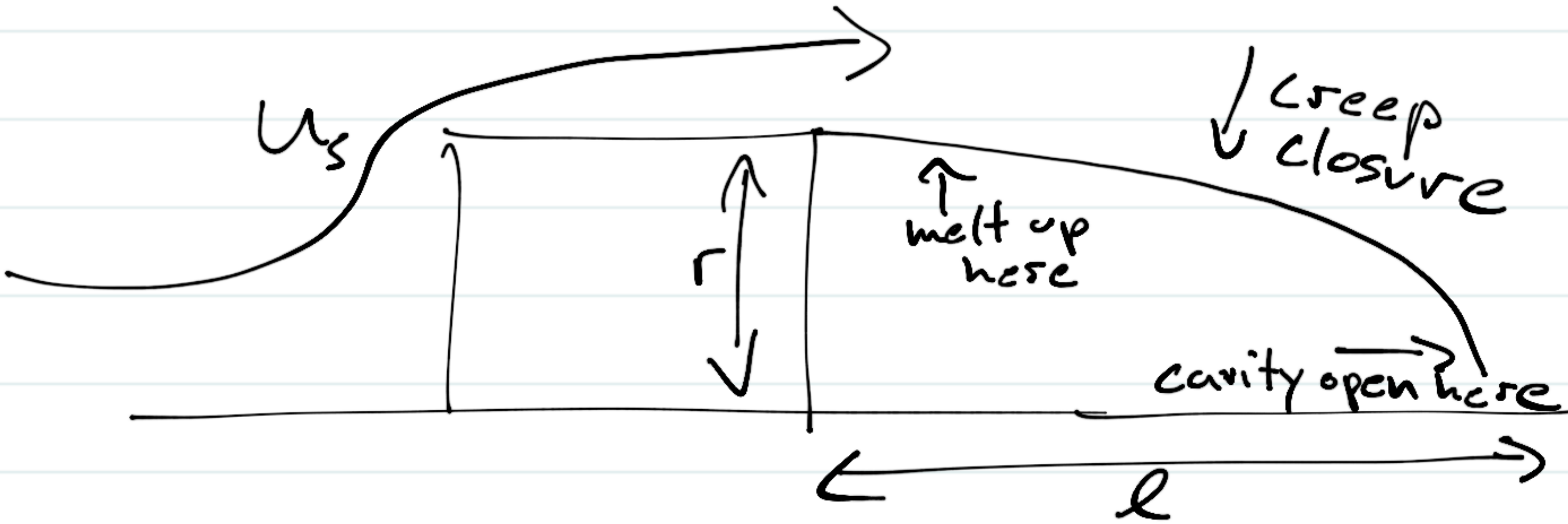
or in other words

$$P_w \propto \rho g H - Q^{\alpha}$$

As water supply Q increases, water pressure decreases.

Cavity opening

Conversely, we might consider a system of linked cavities where the rate of cavity opening is set by the glacier sliding speed:



As for conduits the melt rate of cavity walls goes like

$$\dot{m}_{cav} = \frac{1}{\rho_i L_f \ell} Q \frac{d\phi}{dx}$$

The net rate at which the cavity length changes

$$\dot{r}_{creep} - \dot{m}_{cav} = \frac{r}{\ell} u_s$$

For these types of cavities:

$$Q = (\dots) \frac{r^{5/3} \ell}{n_m} \left(\frac{d\phi}{dx}\right)^{\frac{1}{2}}$$

Subbing for N:

$$N^n = (\ldots)\left[\frac{r^{5/3}\, u_s \left(\frac{d\phi}{dx}\right)^{\frac{1}{2}}}{Q} + (\ldots)\, r^{2/3} \left(\frac{d\phi}{dx}\right)^{3/2}\right]$$

In contrast to conduits, for cavities N decreases (P_w increases) as Q increases.

More water supplied $\longrightarrow$ Higher water pressure at bed