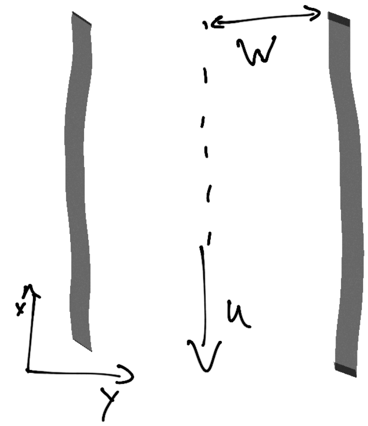
Ice Stream Velocity

GT EAS 4803/8803

Let us consider an ice stream of half-width W, which is primarily flowing in one direction.



First, we can go back to the high-order ice flow approximation for ice flow along the x-direction:

$$4\frac{\partial}{\partial x}\left(\eta\frac{\partial u}{\partial x}\right)+2\frac{\partial}{\partial x}\left(\eta\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial y}\left(\frac{\eta}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)+\frac{\partial}{\partial z}\left(\eta\frac{\partial u}{\partial z}\right)=\rho g\frac{\partial h}{\partial x} \tag{1}$$

Let us make two key assumptions: (1) flow is primarily along the x-direction ($u >> v$) and the changes in velocity along flow are negligible ($\frac{\partial u}{\partial x}$ is small). This leaves

$$\frac{\partial}{\partial y}\left(\frac{\eta}{2}\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\eta\frac{\partial u}{\partial z}\right)=\rho g\frac{\partial h}{\partial x} \tag{2}$$

Reminding ourselves that $\eta\frac{\partial u}{\partial z}=\tau_{xz}$ and $\tau_d=\rho g\frac{\partial h}{\partial x}$. We can then integrate this equation with respect to depth (z), producing

$$h\frac{\partial}{\partial y}\left(\frac{\eta}{2}\frac{\partial u}{\partial y}\right)+\left(\tau_{xz}|_{z=h}-\tau_{xz}|_{z=0}\right)=\tau_d \tag{3}$$

Shear stress at the ice sheet surface should be negligible ($\tau_{xz}|_{z=h}=0$), and $\tau_{xz}|_{z=0}$ is simply the basal shear stress, τ_b, leaving

$$h\frac{\partial}{\partial y}\left(\frac{\eta}{2}\frac{\partial u}{\partial y}\right)=\tau_d-\tau_b \tag{4}$$

We can now integrate along the y-direction, starting from the center of the ice stream where τ_{xy} will be small to the ice stream edge (i.e. the "shear margin")

$$h\int_0^y\frac{\partial}{\partial y}\left(\frac{\eta}{2}\frac{\partial u}{\partial y}\right)dy=\int_0^y\left(\tau_d-\tau_b\right)dy \tag{5}$$

Giving

$$\frac{\eta}{2}\frac{\partial u}{\partial y} = \frac{y}{h}\left(\tau_d - \tau_b\right) \tag{6}$$

It is now useful to re-examine the effective viscosity of ice in the higher-order approximation

$$\eta = A^{-1/n}\left(\frac{1}{2}\left(\dot{\epsilon}_{xx}^2 + \dot{\epsilon}_{yy}^2 + \dot{\epsilon}_{zz}^2\right) + \dot{\epsilon}_{xz}^2 + \dot{\epsilon}_{xy}^2\right)^{\frac{1-n}{2n}} \tag{7}$$

Vertical strain rates and along-flow strain rates are all negligible compared to the cross-flow strain rates, causing all these terms to drop out except the last one

$$\eta = A^{-1/n}\left(\frac{1}{2}\dot{\epsilon}_{xy}\right)^{\frac{1-n}{n}} \tag{8}$$

Inserting this back into the momentum balance above (noting that $\dot{\epsilon}_{xy} = \frac{\partial u}{\partial y}$), we have

$$\frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^{1/n} = \frac{y}{h}\left(\tau_d - \tau_b\right) \tag{9}$$

This equation can be integrated one final time in the y-direction

$$\int_0^y \frac{\partial u}{\partial y} dy = \int_0^y \frac{2^n A y^n}{h^n}\left(\tau_d - \tau_b\right)^n dy \tag{10}$$

Leading to (with some reorganization)

$$u(y) = U_d\left[1 - \left(\frac{y}{W}\right)^{n+1}\right] W^{n+1}\left(\tau_d - \tau_b\right)^n \tag{11}$$

where $U_d = \frac{2^n A}{h^n (n+1)}$.

A plot of the shape of velocity in an ice stream profile can be seen below.