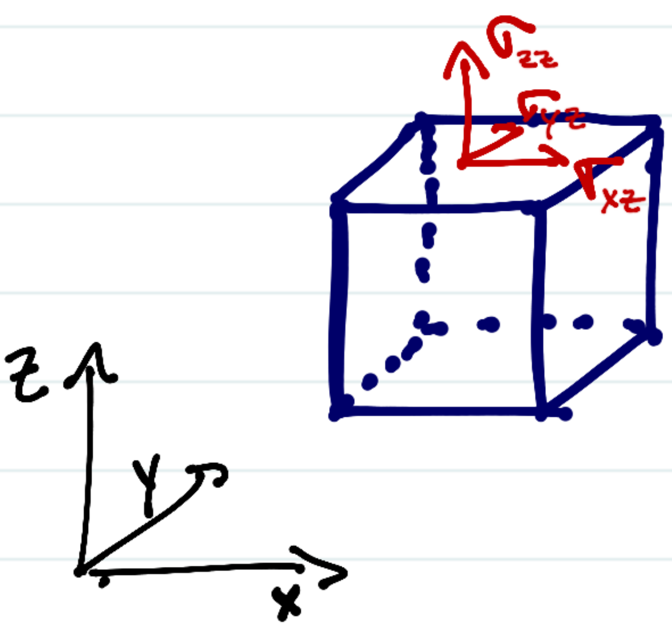
Concept review

What is the difference between?

- Scalar
- Vector
- Tensor

Continuum Mechanics

Lets consider a cube of some material that can deform (e.g. ice)



For each face of the cube, there are 3 possible stress directions

- Normal stress (into/out of face)
- Shear stress (along face in two perpendicular dir)

Question: for all the faces of a cube, how many unique numbers are needed to completely describe the stress state?

Answer: 9

These can be written in the 3x3 Cauchy (second-order) stress tensor:

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

Where σ_{ij} $\quad i \rightarrow$ the stress direction

$j \rightarrow$ the face normal dir

Normal stresses:
$i = j$

Shear stresses:
$i \neq j$

"The i-directed stress acting on the j-normal face"

Surface stress vs. body force

For our cube of material

→ Surface stresses act at the cube faces and <u>not</u> in the interior

→ Body forces act on every point in the volume of the cube equally

Among surface stresses there are:

① Normal stresses

② Shear stresses

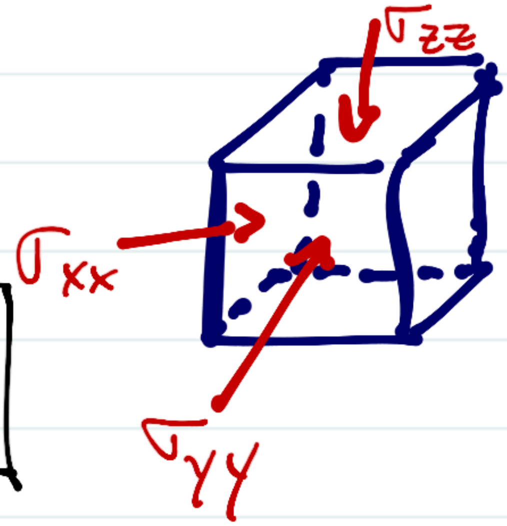
(see explanations above)

The hydrostatic stress (P)

The mean of all normal stresses

$$P = \frac{1}{3}\sum_{i=1}^{3} \sigma_{ii}$$

$$\boxed{P = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})}$$



The deviatoric stress tensor (τ_{ij})

The shearing deviation from the mean compressive stress

$$\tau_{ij} = \sigma_{ij} - \delta_{ij} P \quad \text{where } \delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$$\tau_{ij} = \begin{pmatrix} \sigma_{xx} - P & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} - P & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} - P \end{pmatrix}$$