Subglacial hydraulic potential

Bernoulli's equation tells us that work done by the motion of a fluid is constant along the path of the fluid:

$$\underbrace{P_w}_{\text{pressure energy}} + \underbrace{\rho_w g z}_{\text{potential energy (gravity)}} + \underbrace{\tfrac{1}{2}\rho_w g v^2}_{\text{kinetic energy due to flow}} = \underbrace{\phi}_{\text{constant hydraulic potential}}$$

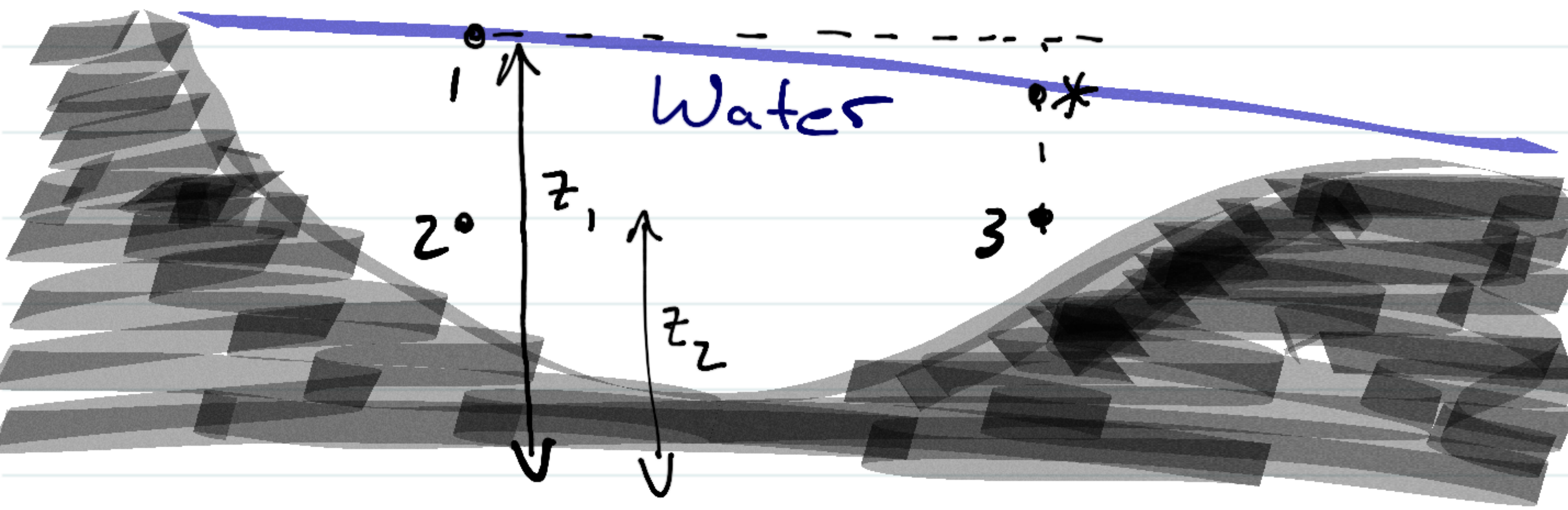
Subglacial fluid flow tends to be slow, so we assume $v = 0$

$$\underbrace{H_h}_{\text{Hydraulic Head}} = \frac{\phi}{\rho_w g} = \frac{P_w}{\rho_w g} + z$$

Hydraulic Head → If you drilled a borehole to the bed of the glacier, water would rise to this height in the borehole

Differences in H (or ϕ) drive water flow.

Consider a lake which feeds a river



$$\phi_1 = \rho_w g z$$

$$\phi_2 = \underbrace{\rho_w g (z_1 - z_2)}_{P_w} + \rho_w g z_2$$

$$\phi_2 = \rho_w g z_1$$

No flow from 1 to 2

$$\phi_3 = \rho_w g (z_* - z_3) + \rho_w g z_3$$

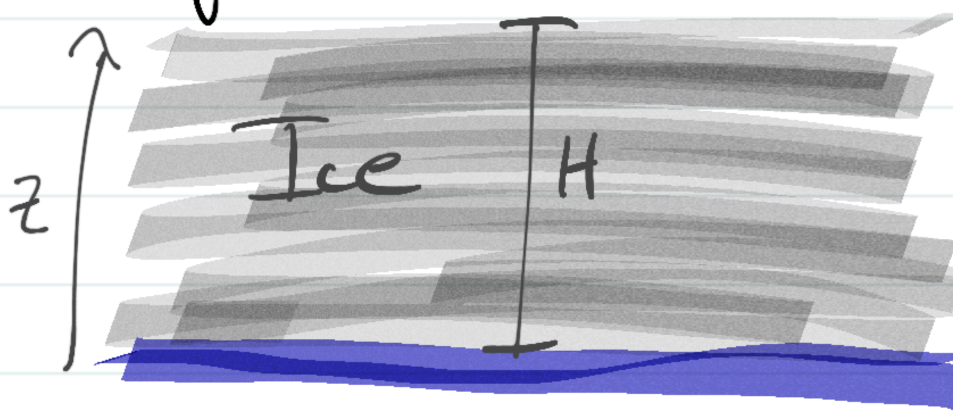
$$\phi_3 = \rho_w g z_* \leftarrow \quad z_* < z_1$$

$\rightarrow$ Flow from $2 \rightarrow 3$

Underneath a glacier, water pressure may be assumed to be at the glaciostatic pressure $\rho_i g(H_i - z)$ (unless there is a drainage system)

$$\phi = \rho_i g (H - z) + \rho_w g z$$

The hydraulic gradient in the horizontal is then



$$\frac{d\phi}{dx} = \rho_i g \left(\frac{dH}{dx} - \frac{dz}{dx} \right) + \rho_w g \frac{dz}{dx}$$

$$\frac{d\phi}{dx} = \rho_i g \underbrace{\frac{dH}{dx}}_{\text{Ice sfc slope}} + g(\rho_w - \rho_i) \underbrace{\frac{dz}{dx}}_{\text{Bed slope}}$$

$\rho_i \approx 10(\rho_w - \rho_i) \rightarrow$ so ice sfc slope tends to control the direction of water flow along the bed.