Ice thermodynamics

Glacier and Ice Sheet Dynamics
EAS 4403/8803

1. Why do we study ice temperature?
2. Thermal structures of glaciers
3. Surface energy balance
4. Heat equation
5. Ice thermodynamical parameters
6. Modeling ice temperature
7. Effects of vertical and horizontal advection
8. Measured temperature profiles in ice sheets
9. Important current research topics
10. Analysis of recent research

1) Why do we study ice temperature?

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- Ice deformation rates depend on ice temperature
- Basal melting, basal sliding, and basal erosion only happen if the ice base is temperate
- Ice temperature preserves memory of past climatic changes
- The rate of densification from firn into ice is sensitive to temperature
- The speed of electromagnetic waves depends on ice temperature

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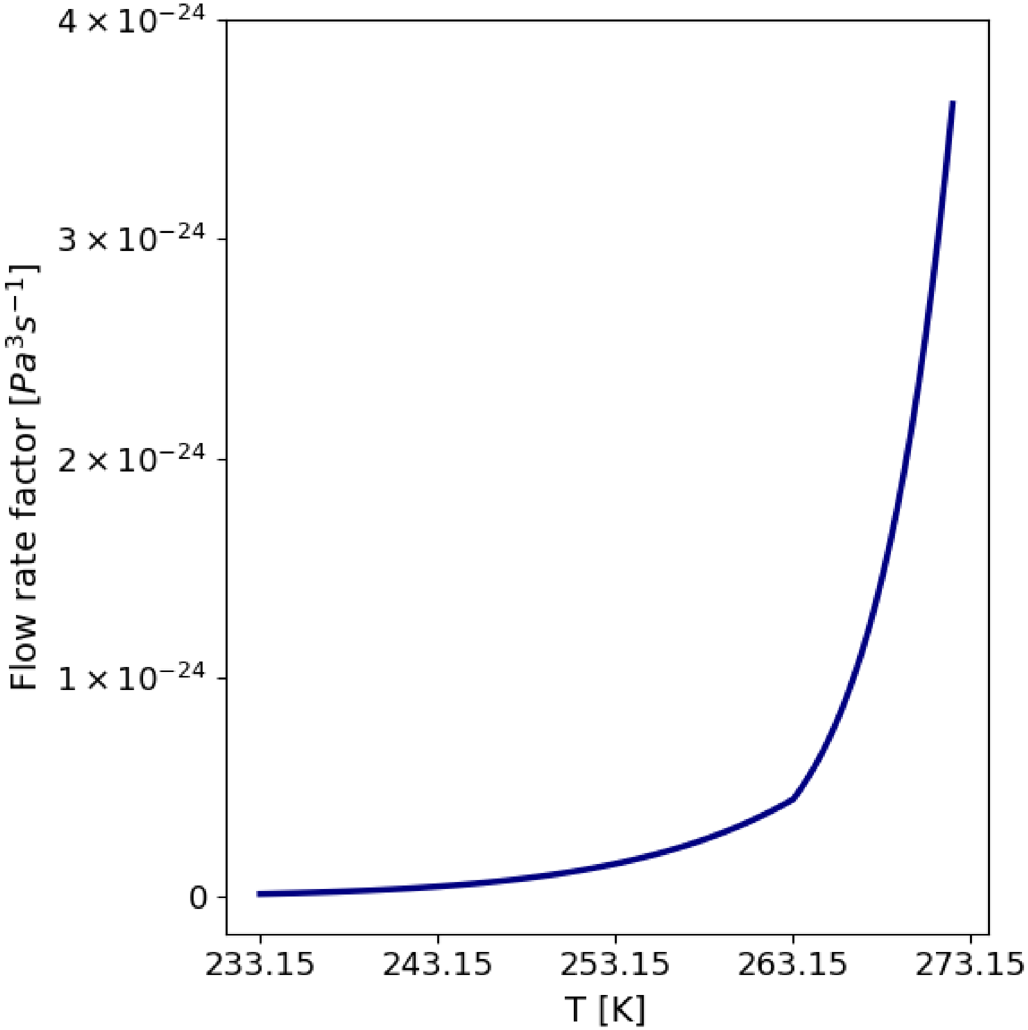

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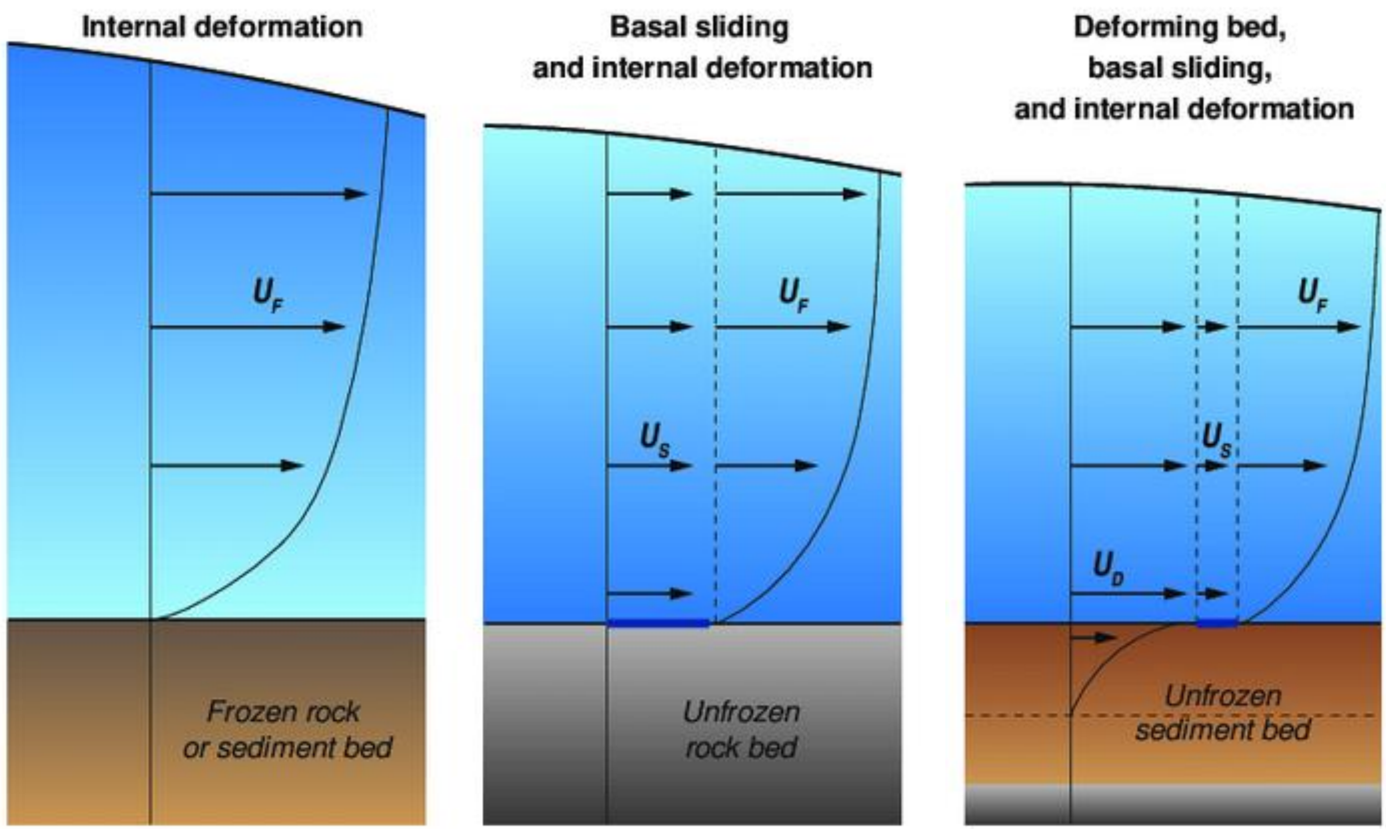
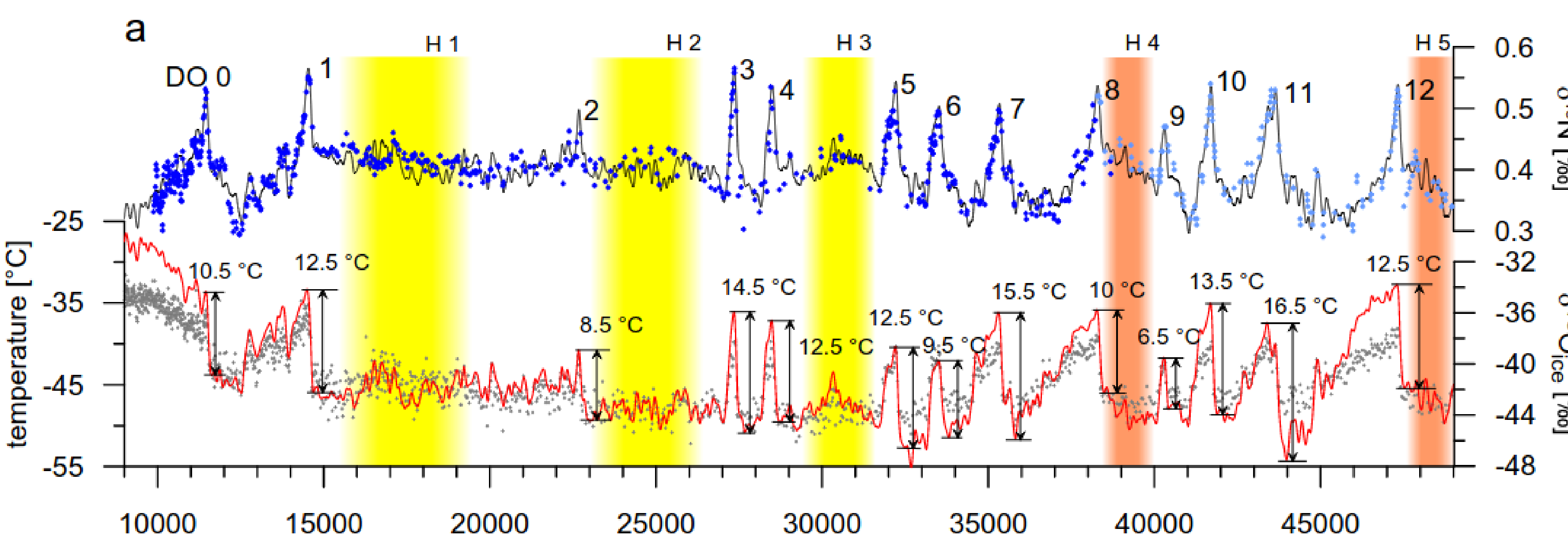


Becher et al. (2021)

1) Why do we study ice temperature?

- Ice temperature preserves memory of past climatic changes

Kindler et al. (2014)



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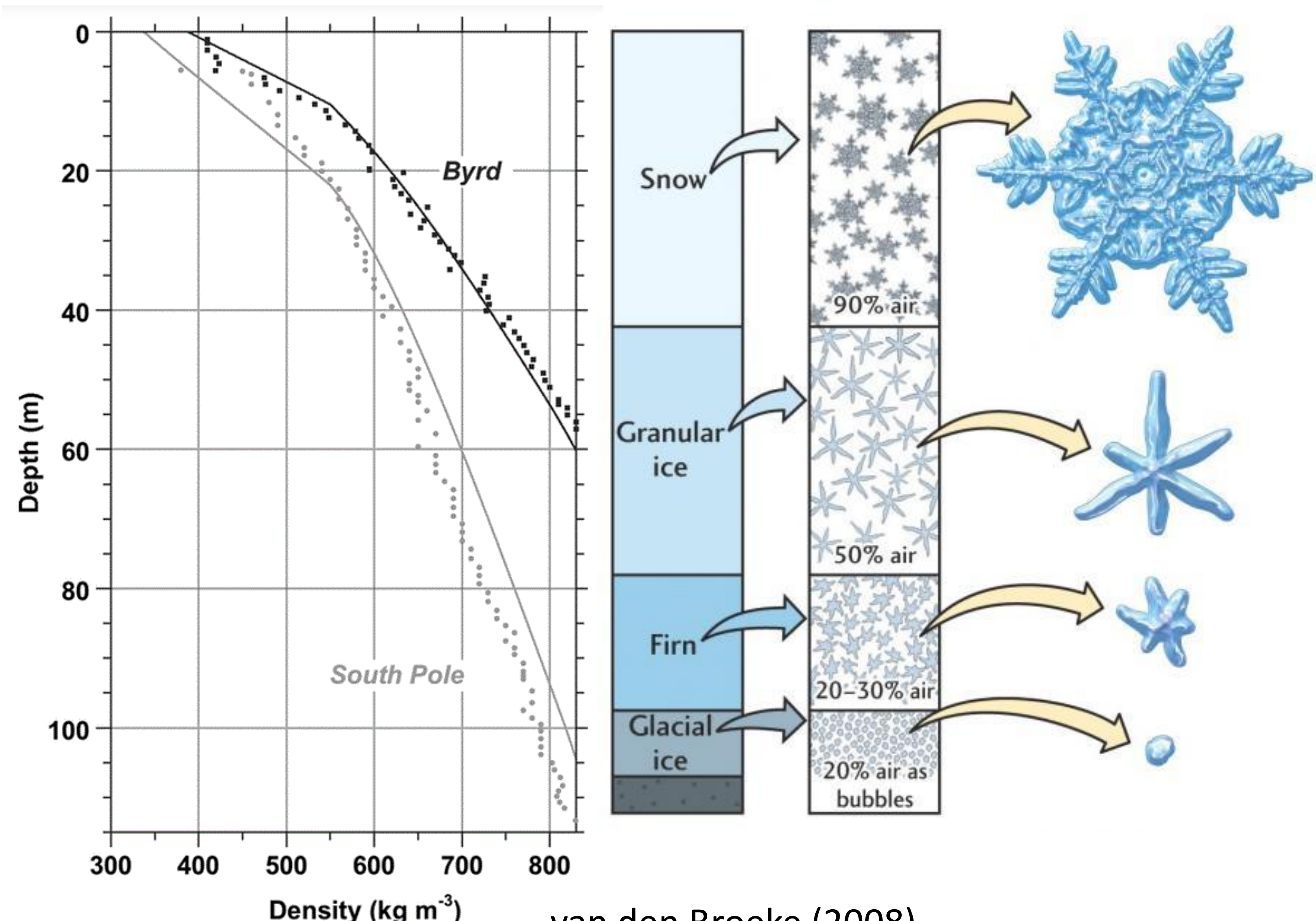
- The rate of densification from firn into ice is sensitive to temperature



van den Broeke (2008)

1) Why do we study ice temperature?

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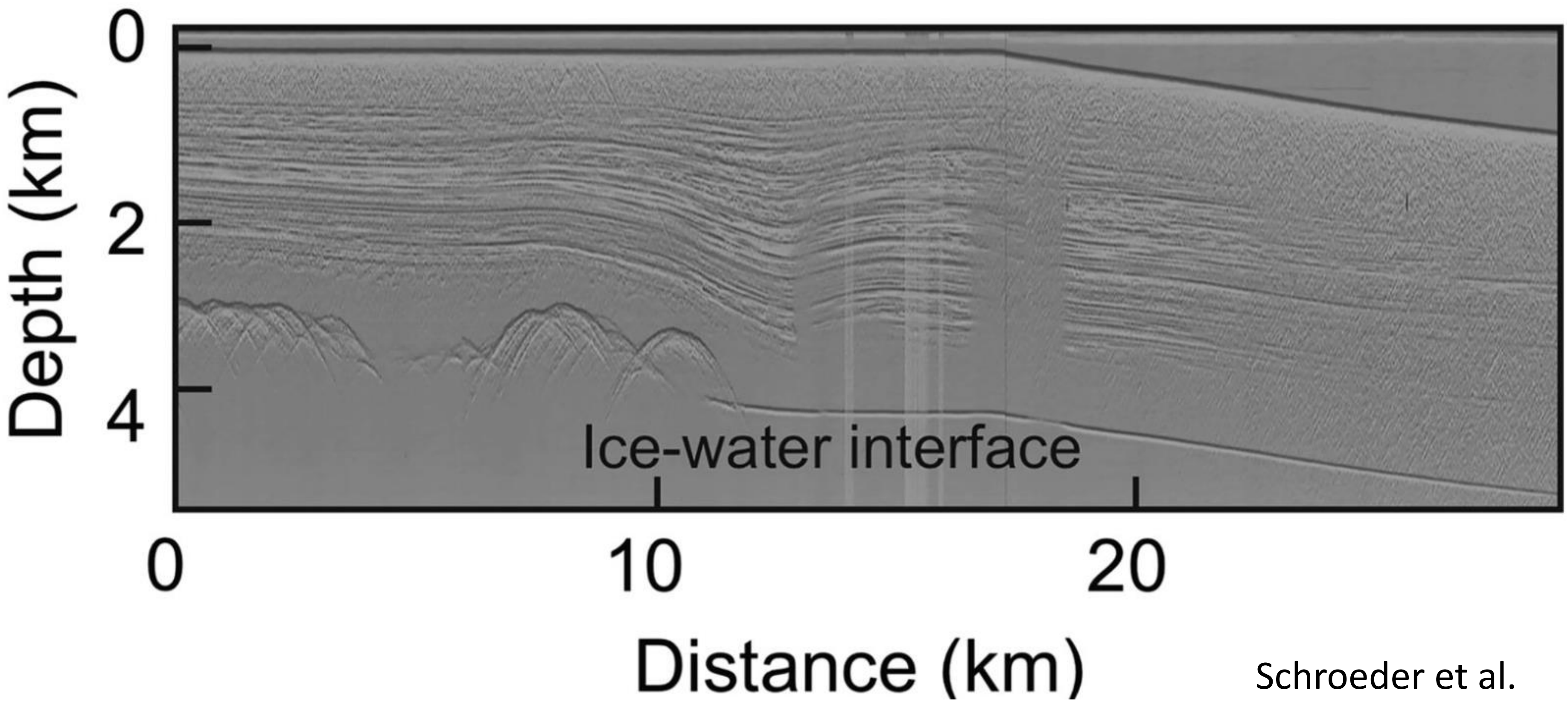
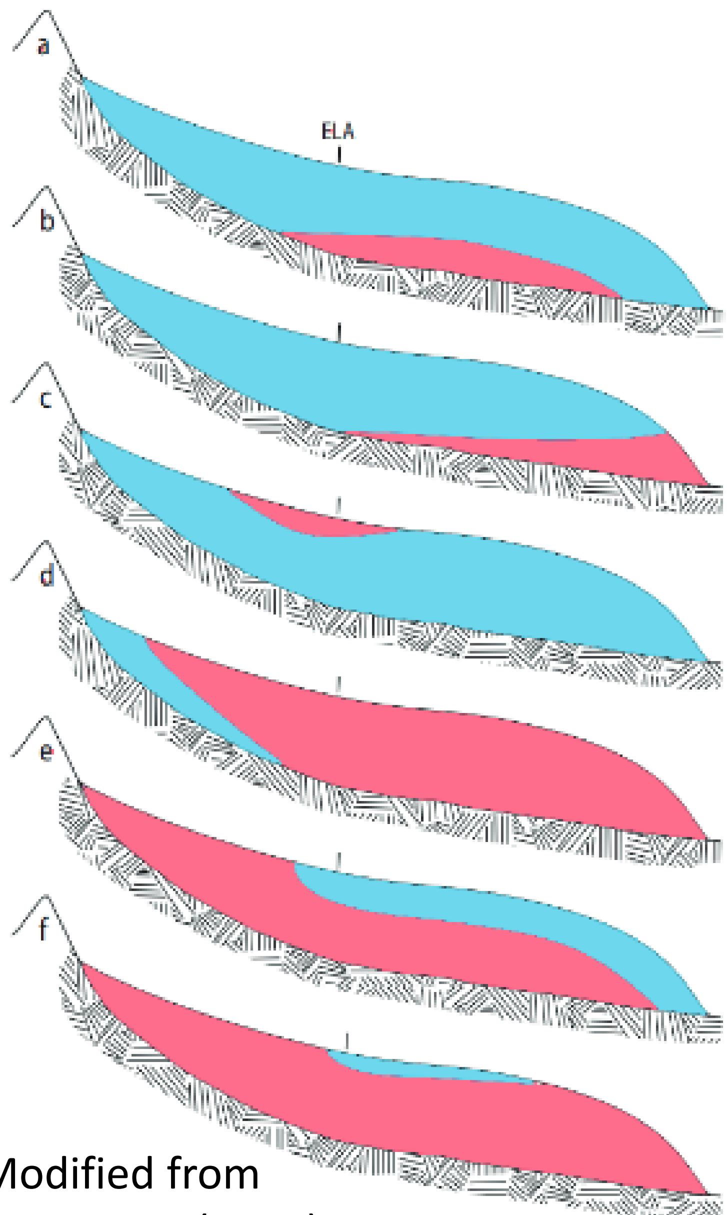


Schroeder et al. (2019)

2) Thermal structures of glaciers

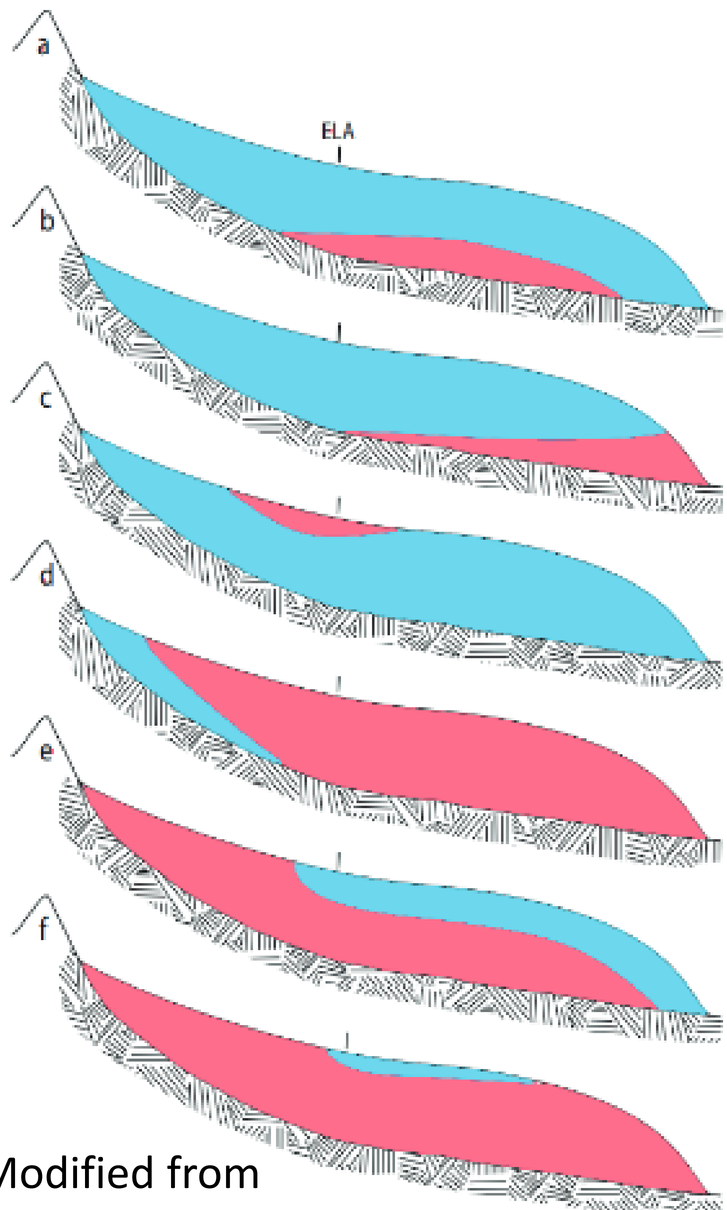


Modified from Pettersson (2004)

- Glaciers can be classified according to their thermal regime. Here red shows temperate ice (at melting point) and blue shows cold ice (sub-freezing conditions)

- 'Cold glaciers': temperature everywhere below the pressure melting point.
- 'Temperate glaciers': temperature at the pressure melting point (except for thin surface layer)
- 'Polythermal glaciers': composed of both cold and temperate ice

2) Thermal structures of glaciers



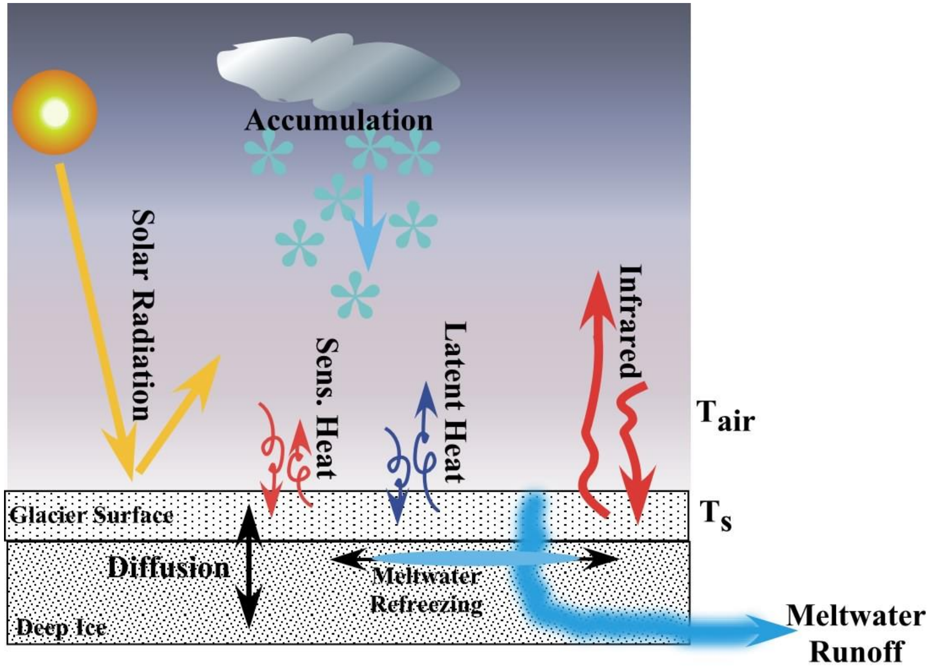
Modified from Pettersson (2004)

- **a,b**: Cold climate, low to zero melt in the accumulation area, strain heating at the bed raises ice to the pressure melting point
- **c**: Latent heat release from meltwater refreezing in the lower accumulation area heats upper ice
- **d**: Process of **c** is further enhanced, but some cold ice is still advected from the uppermost accumulation zone
- **e**: Bare solid ice in the ablation area prevents refreezing, and latent heat release occurs only in accumulation area. Cold surface layer of the ablation area is due to sufficient winter cooling.
- **f**: Same process as in e, but surface melt at lower elevations is high enough to remove the cold ice layer.

3) Surface energy balance (overview)

$$E_{net} = (1 - \alpha)E_s + E_{l,in} - E_{l,out} + E_g + E_h + E_k + E_p$$

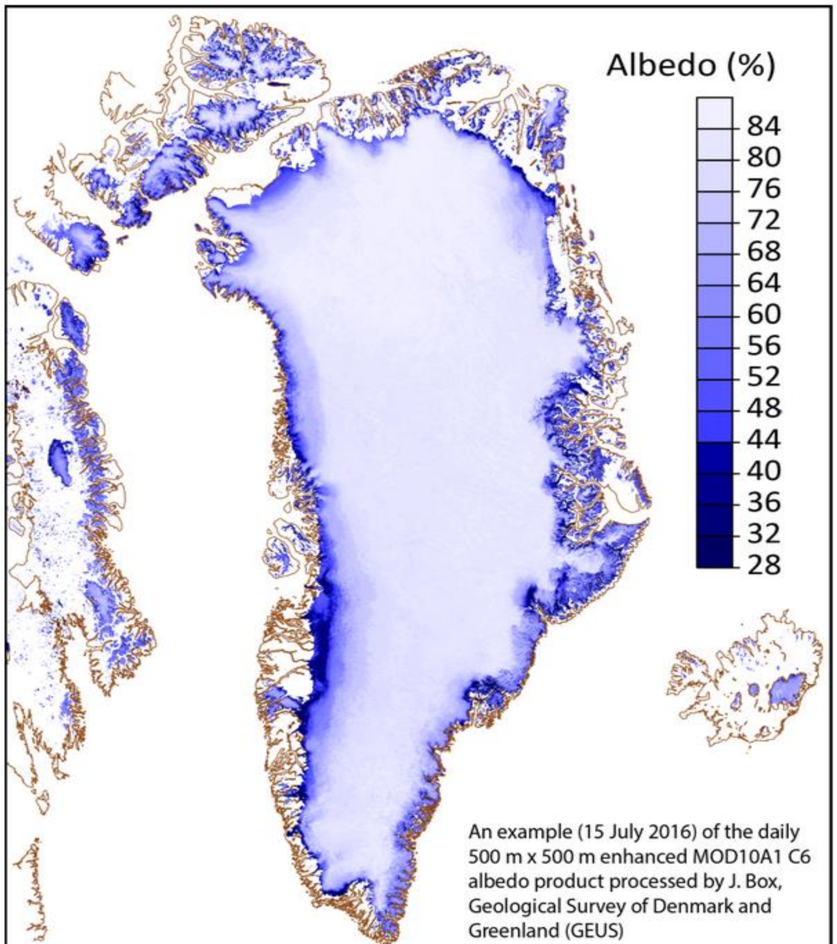
E_s : shortwave radiation, depends on incidence angle, shading, clouds, etc.
α : albedo
$E_{l,in}$: depends on vertical profile of temperature and humidity in the atmosphere, $E_{l,in} \approx \sigma T_{2m}^4$
$E_{l,out}$: infrared energy radiated from the surface, $E_{l,out} \approx \sigma T_S^4$
E_g : conduction of heat into/from underlying ice
E_h : turbulent flux of latent heat, driven by humidity difference between surface and atmosphere
E_k : turbulent flux of sensible heat, driven by temperature difference between surface and atmosphere
E_p : heat flux from precipitation (typically small)



3) Surface energy balance (overview)

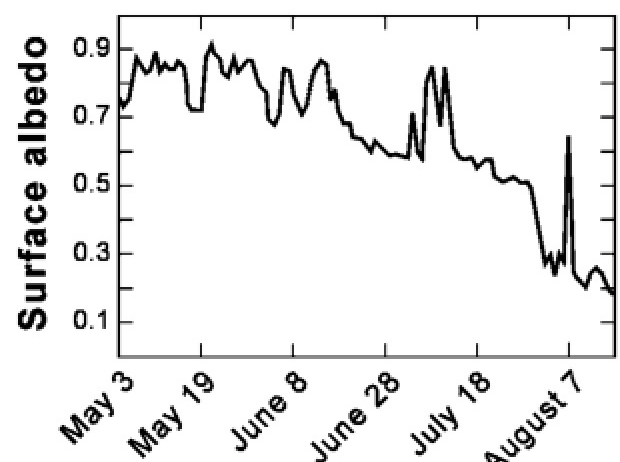
Albedo quantifies the reflectivity of the surface. It depends strongly on the properties of the snow/firn/ice surface layer. It can range between 0.1 to 0.9, and has a very strong impact on the amount of surface melting. Yet, it is challenging to incorporate into model simulations.

Albedo on July 15 2016, from MODIS satellite



Box et al. (2017)

Surface type	Recommended	Minimum	Maximum
Fresh dry snow	0.85	0.75	0.98
Old clean dry snow	0.80	0.70	0.85
Old clean wet snow	0.60	0.46	0.70
Old debris-rich dry snow	0.50	0.30	0.60
Old debris-rich wet snow	0.40	0.30	0.50
Clean firn	0.55	0.50	0.65
Debris-rich firn	0.30	0.15	0.40
Superimposed ice	0.65	0.63	0.66
Blue ice	0.64	0.60	0.65
Clean ice	0.35	0.30	0.46
Debris-rich ice	0.20	0.06	0.30

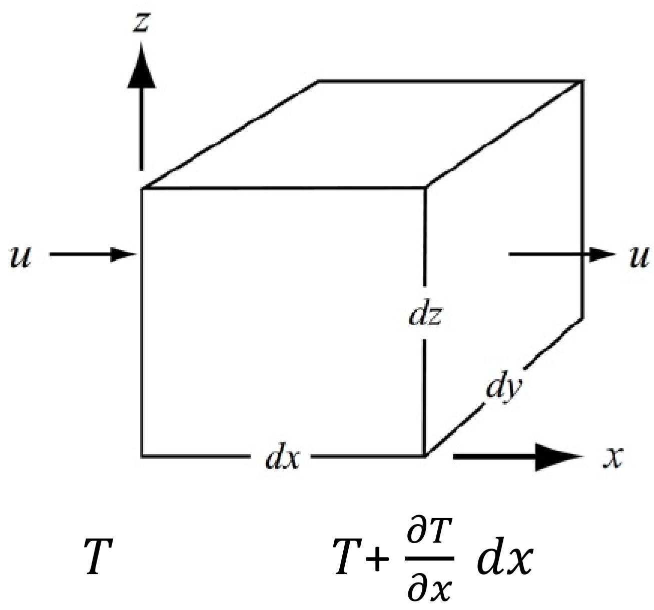


Cuffey and Paterson (2010)

4) Heat equation

Conservation of energy implies that the temperature evolution of ice depends on transfers of heat within the ice and heat source terms

Advection



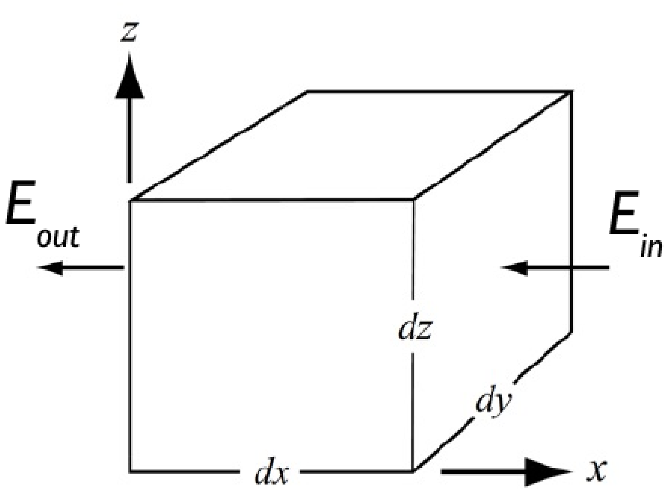
Heat is transferred due to the bulk motion of ice. Through ice flow, the temperature characteristics of ice is carried along the flow.

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x}$$

4) Heat equation

Conservation of energy implies that the temperature evolution of ice depends on transfers of heat within the ice and heat source terms

Conduction



$\frac{\partial T}{\partial x}$ $\qquad$ $\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) dx$

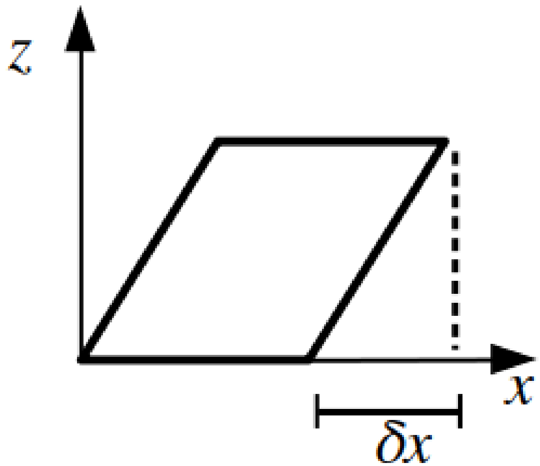
Heat spontaneously flows along a temperature gradient. As such, the amount of heat transferred is proportional to $\frac{\partial T}{\partial x}$. The difference in conductive heat flux at different points is thus proportional to $\frac{\partial^2 T}{\partial x^2}$.

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$$

4) Heat equation

Conservation of energy implies that the temperature evolution of ice depends on transfers of heat within the ice and heat source terms

Strain heating (source term)



$W = F\ \delta x$
(heat energy)

Ice deformation implies that mechanical energy is converted into heat. The amount of heat released per unit time depends on the rate of ice deformation and the pressure applied.

$$\frac{\partial T}{\partial t} = \frac{2}{\rho\ c_p} \dot{\varepsilon}_{zx}\ \sigma_{zx}$$

4) Heat equation

Conservation of energy implies that the temperature evolution of ice depends on transfers of heat within the ice and heat source terms

(3-D heat equation on the whiteboard)
simplifies approximately to:

$$\boxed{\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} - \mathbf{v} \bullet \nabla T + \frac{2}{\rho\, c_p} \dot{\varepsilon}_{zx}\, \sigma_{zx}}$$

5) Ice thermodynamical parameters

- **Thermal conductivity** (k): ability of ice to conduct heat under a given temperature gradient, higher at low temperatures (strong intermolecular forces).
- **Specific heat capacity** (c_p): energy required to raise 1 kg of ice by 1 K, higher at high temperatures (energy used to increase rotation of molecules).
- **Latent heat of fusion** (L_f): amount of energy released when liquid water freezes, equivalent to amount of energy required to melt ice, particularly high for water: energy need to melt 1 kg of ice is equal to the energy need to raise 1 kg of water from 0 to 79.8 °C.
- **Density** (ρ): after firn densification, ice density is approximately constant 917 kg/m^3
- **Thermal diffusivity** (κ): $\kappa = \frac{k}{\rho\, c_p}$, quantifies the thermal inertia: the relative difficulty to increase temperature using a given heat source.
- **Temperature at the pressure melting point** (T_{pmp}): melting point temperature adjusted for the pressure exerted by the weight of the ice column (T_{pmp} is lower at high pressure).

6) Modeling ice temperature

In general, the heat equation must be solved numerically instead of analytically because:

- Boundary conditions vary in space and time
- The ice domain changes in three dimensions over time
- Longitudinal heat transport through advection is not well handled analytically
- Ice flow depends on temperature and vice-versa
- Thermodynamical parameters depend on temperature and vice-versa

Still, analytical solutions can provide relatively good solutions in particular cases. They also provide a first-order understanding of the different mechanisms influencing a steady-state temperature profile, and their respective effects on the temperature profile. We will now analyze the Robin analytical solution (Robin, 1955).

6) Modeling ice temperature

Robin model assumptions:

- Temperature profile and ice thickness are in steady-state.
- Basal temperature is below the pressure melting point (cold-based).
- No horizontal heat advection.
- All ice thermodynamical parameters are constant with respect to temperature.

Robin solution (see derivation on whiteboard):

$$T(z) = T_S + \frac{\sqrt{\pi}}{2}\left(\frac{dT}{dz}\right)_{z=0}\left[\operatorname{erf}\left(\frac{z}{q}\right) - \operatorname{erf}\left(\frac{H}{q}\right)\right] q$$

with $q = \sqrt{\frac{2\,\kappa\, H}{\dot{b}}}$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2)\ du$$

For analyzing the Robin solution and doing some sensitivity experiments, go to Jupyter Notebook:
github.com/wc2421/Glacier-and-Ice-Sheet-Dynamics-Labs/tree/master/IceThermodynamics

7) Effects of vertical and horizontal advection

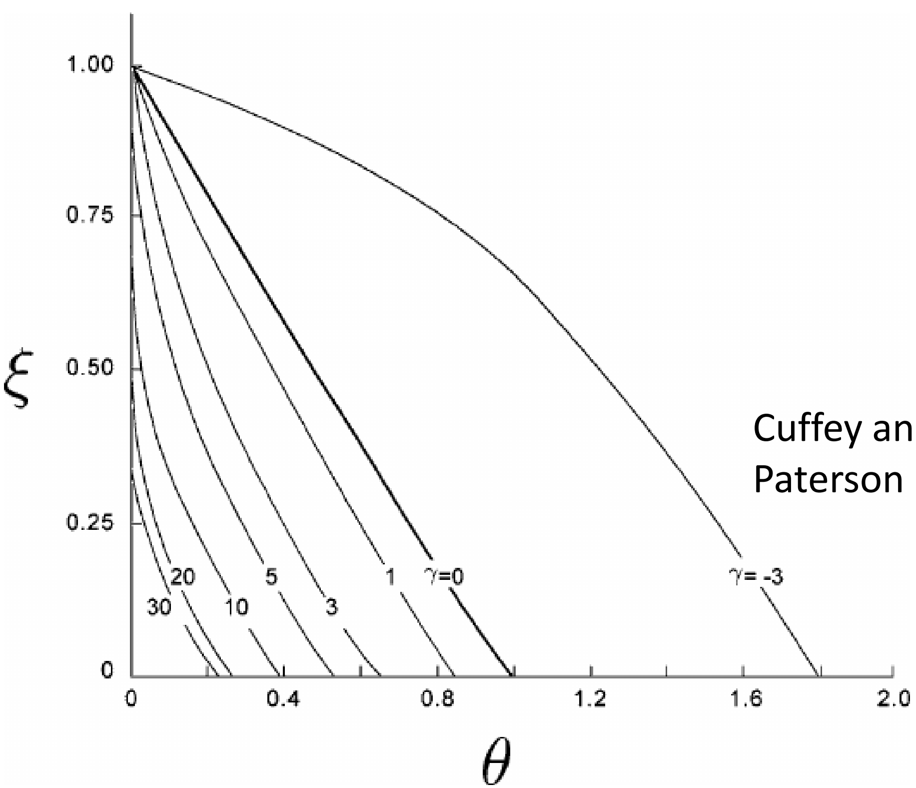
The Robin solution is instructive to understand the impact of vertical advection on the temperature profile: for *higher* vertical advection rates, the surface temperature signal penetrates *deeper* into the ice column. If the advection rate is low, heat diffusion dominates.

The relative influence of advection versus diffusion can be quantified by the Péclet number:

$$\gamma = \frac{\dot{b}\,H}{\kappa}$$

here: $\xi = \frac{z}{H}$ and $\theta = \frac{k\,(T-T_S)}{G\,H}$

Note that γ<0 corresponds to the ablation zone. In this case, ice is advected upwards towards the surface.



Cuffey and Paterson (2010)

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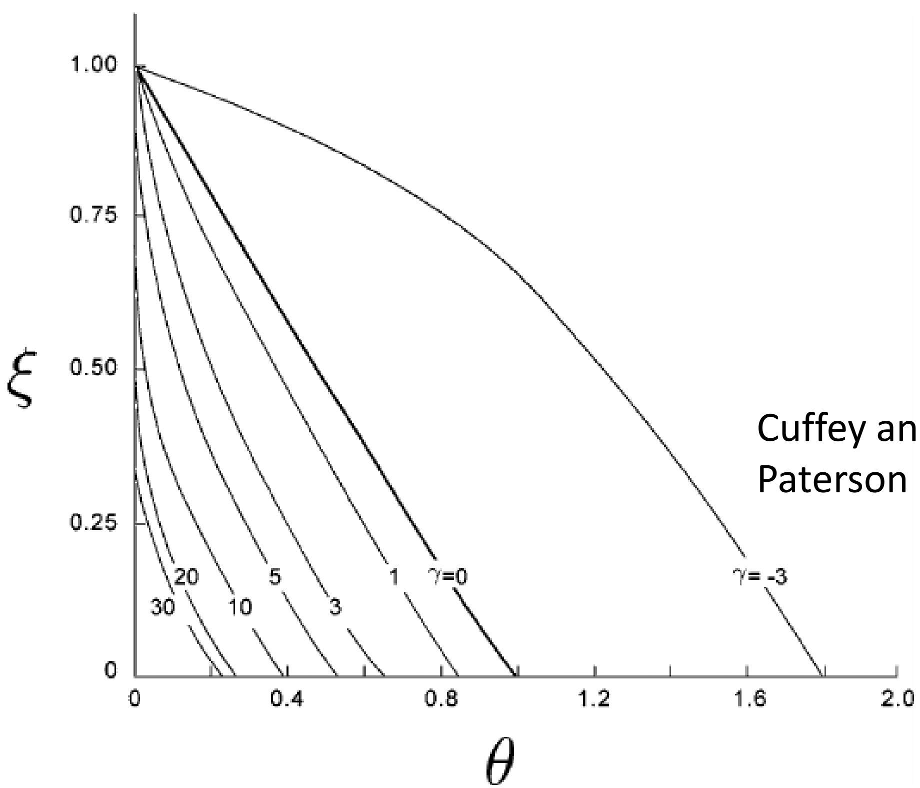
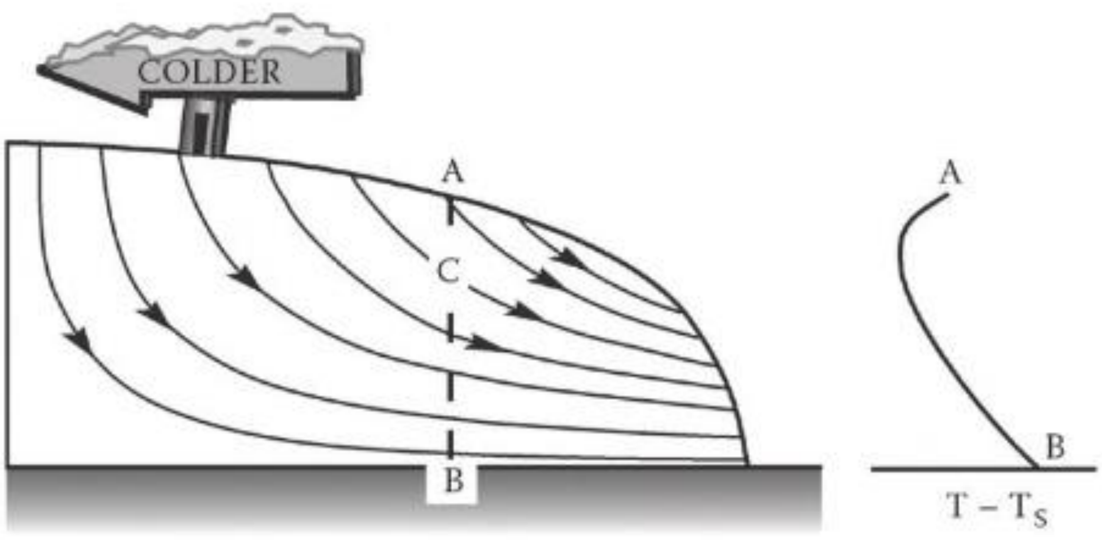
Note that γ <0 corresponds to the ablation zone. In this case, ice is advected upwards towards the surface.
<u>For which case the Robin profile is unrealistic, as the surface should be close to the melting point.</u>



Cuffey and
Paterson (2010)

7) Effects of vertical and horizontal advection

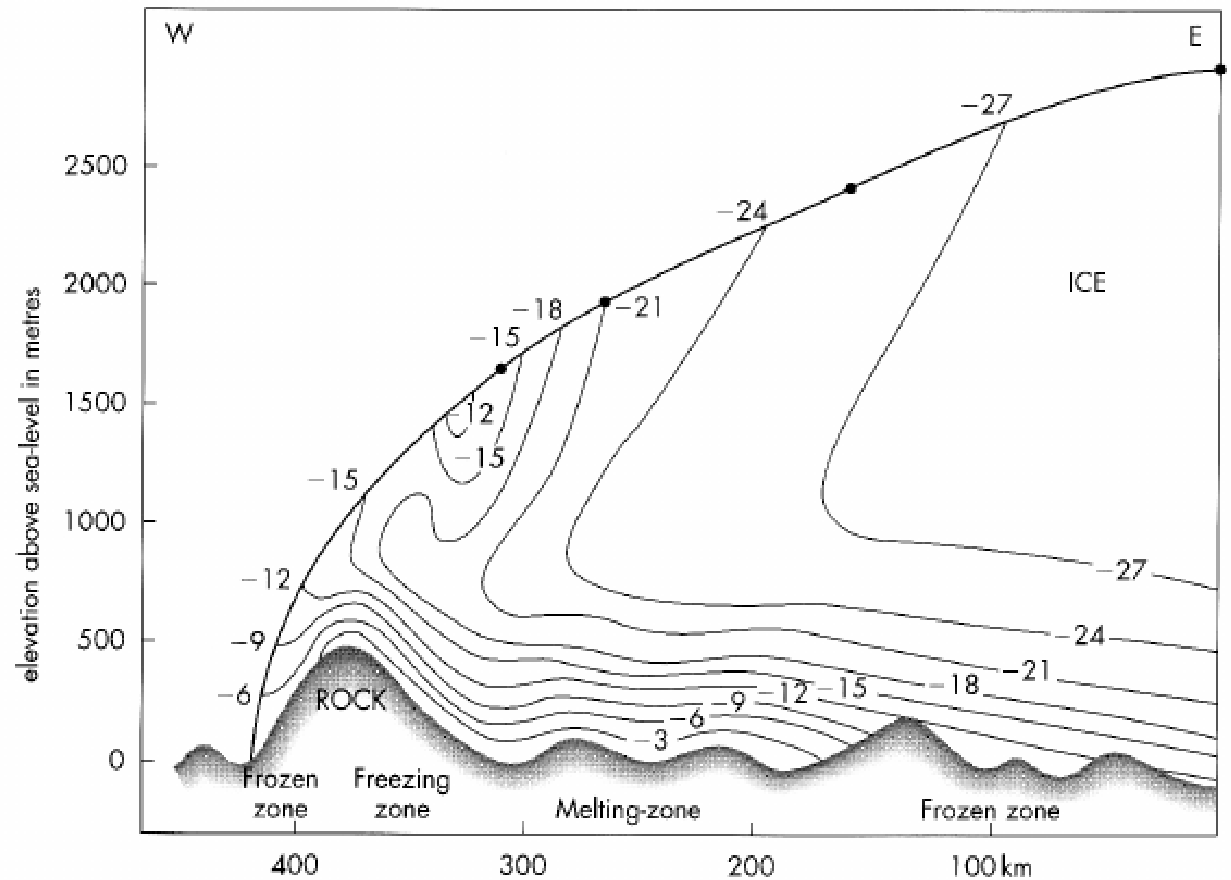
The horizontal inflow of upstream ice affects temperature. Typically, ice from higher elevations was deposited under colder conditions, and carries this cold signal.
This can cause an inversion in the temperature gradient in ice columns at lower elevations.
The magnitude of this inversion depends on (1) the gradient in surface temperatures along the flow line, and (2) the ice flow velocity.
The deeper in the ice column, the higher the ice is originating from. But this is compensated by longer exposure to the heat sources close to the base.



van der Veen (2013)

7) Effects of vertical and horizontal advection

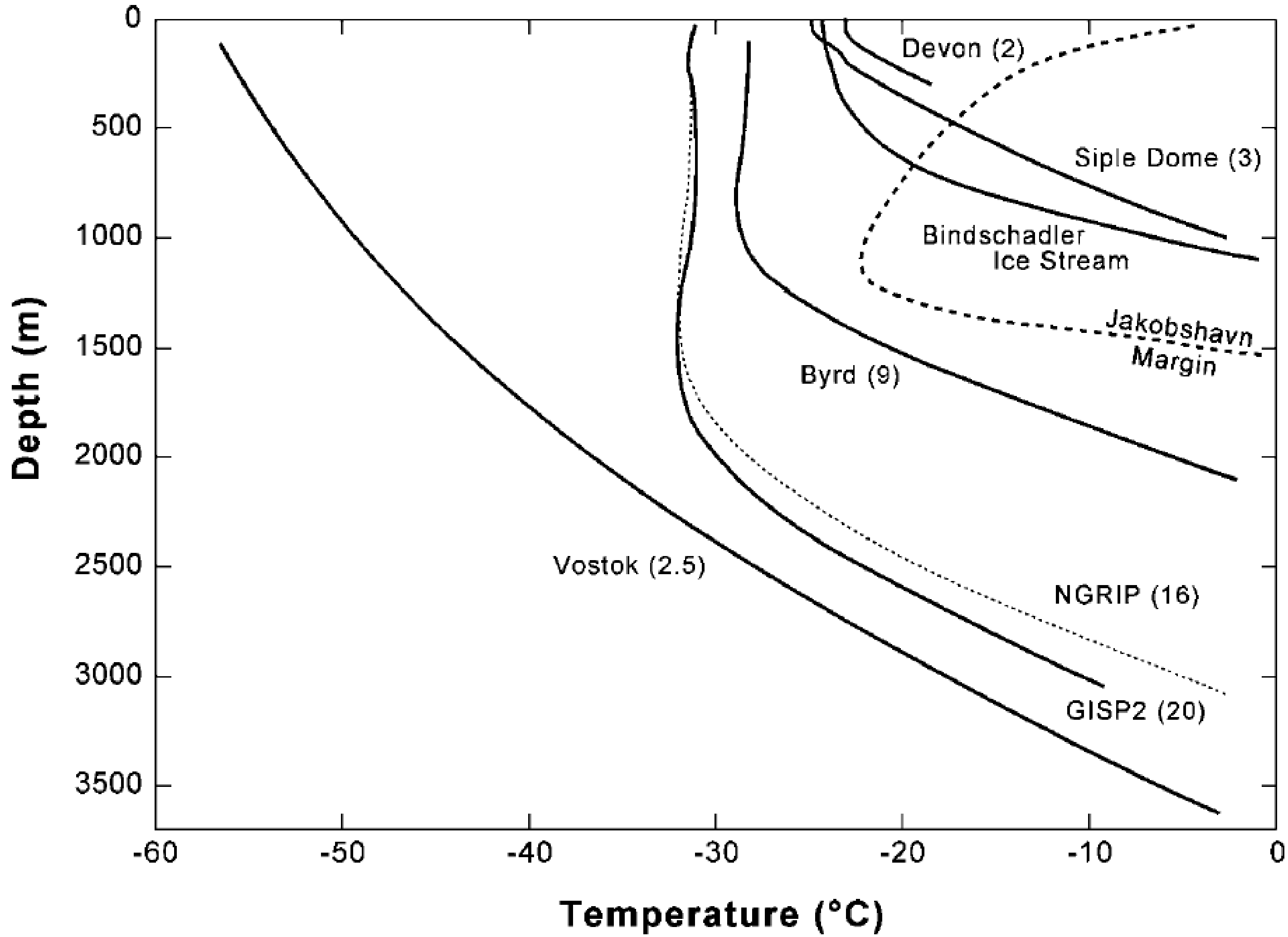
The effect of horizontal advection can be perceived in ice sheets. For example, this figure shows isotherms within a cross section of the Greenland ice sheet at 70°N.



Chapman et al. (2005)

8) Measured temperature profiles in ice sheets

This figure shows measured temperature profiles at different locations of the Greenland and Antarctic ice sheets. How can we relate these profiles to the different processes that we studied in this chapter?

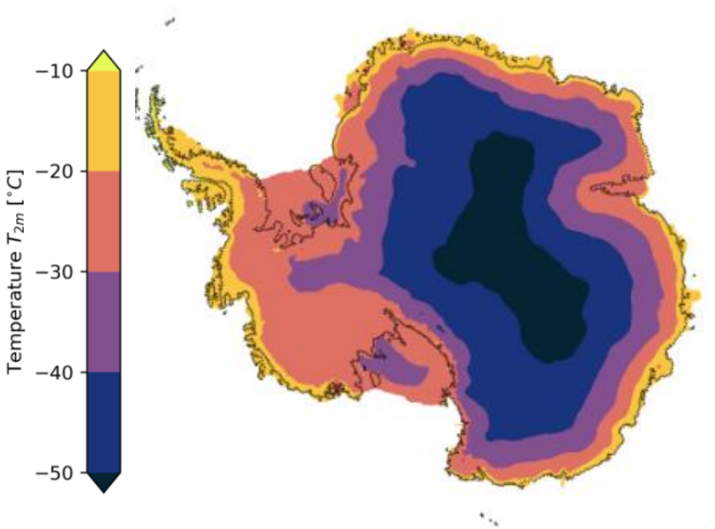


Cuffey and Paterson (2010)

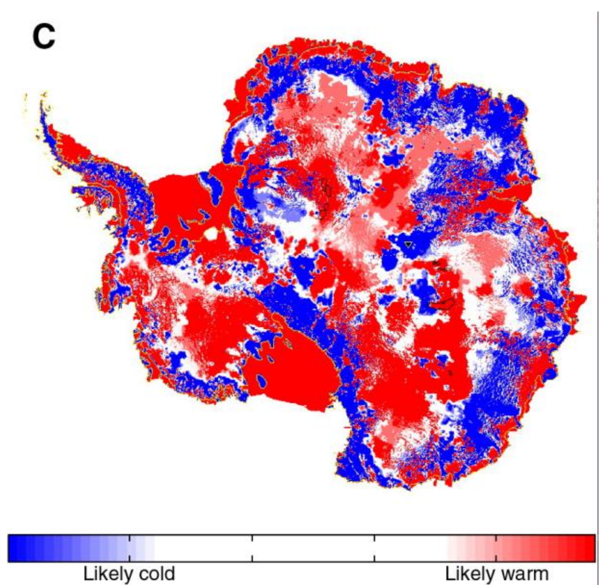
9) Important current research topics: basal thermal state

Basal thermal state is important, as it governs basal melting, basal sliding, bedrock erosion, possible presence of subglacial lakes, etc. But it is difficult to determine!

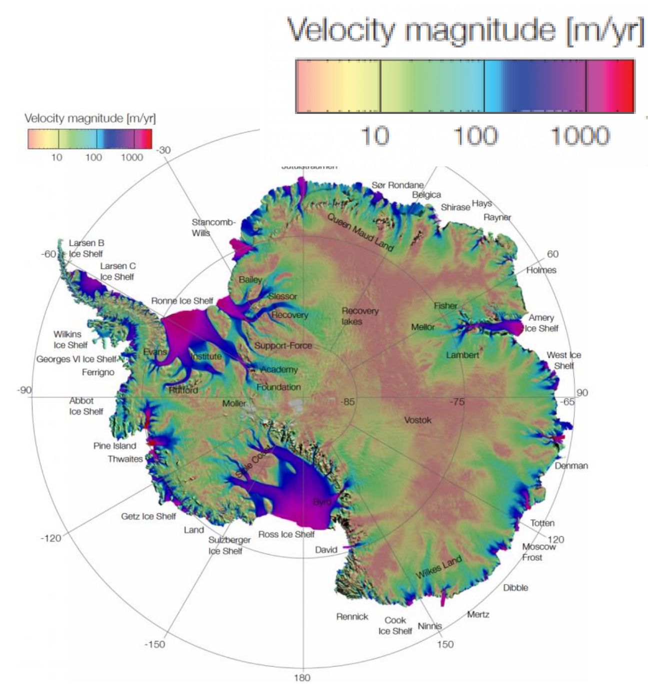
- Basal thermal state is anti-correlated with surface temperature. Thicker ice insulates the basal heat flux better, even though this means colder surface conditions.
- Thick ice at the slow-moving ice divides favors warmer basal conditions. On the other hand, warmer basal conditions at the periphery promotes fast moving ice.



Mean surface temperature
(Rodehacke et al., 2021)

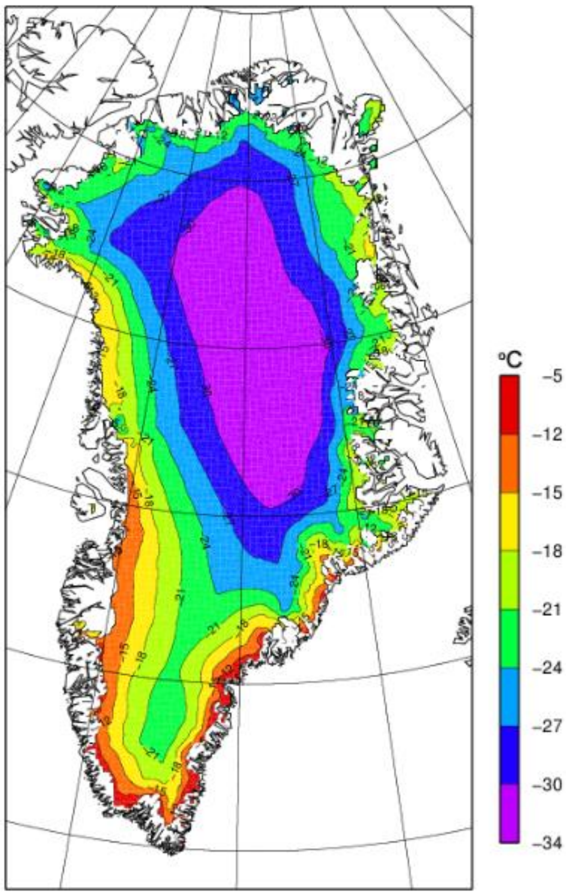


Basal thermal state
(Pattyn, 2010)

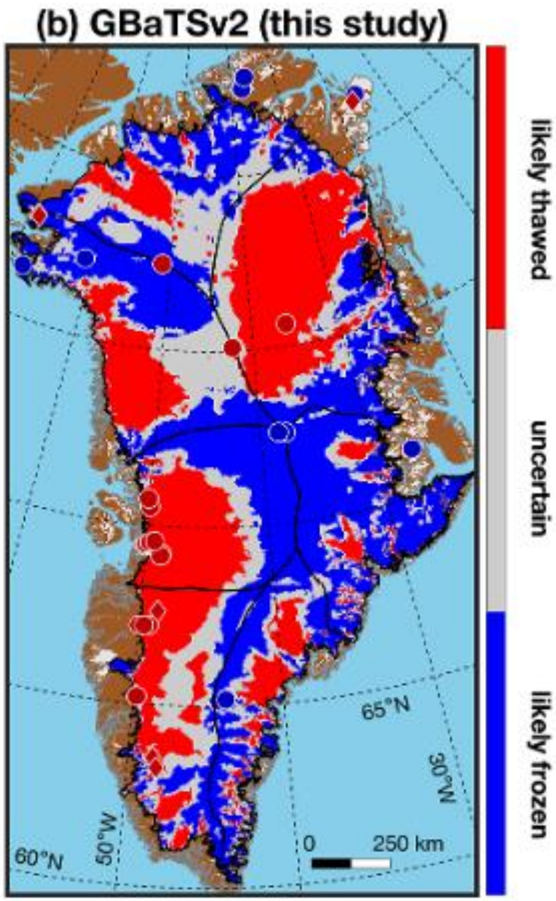


Surface ice velocities
(Rodehacke et al., 2021)

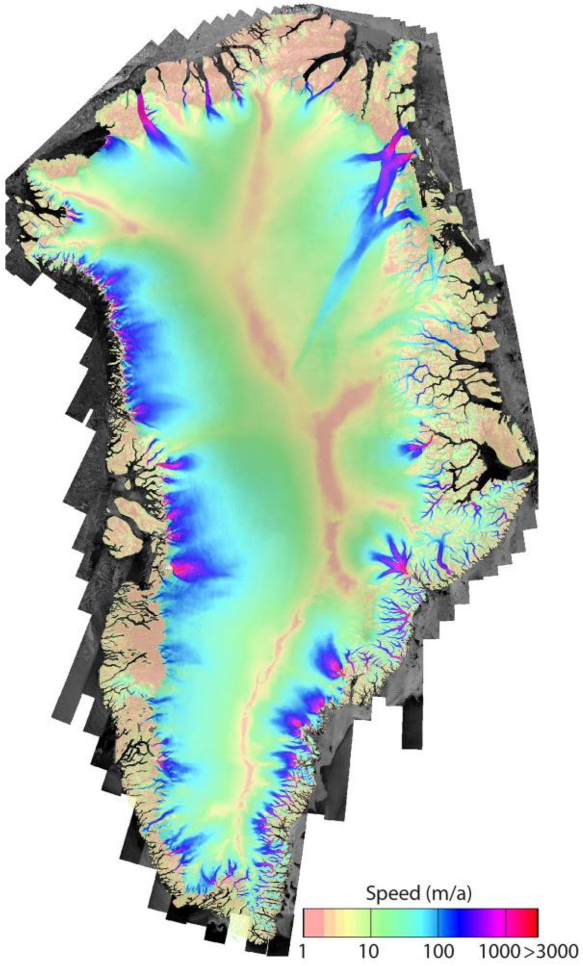
9) Important current research topics: basal thermal state



Mean surface temperature
(Reijmer et al., 2012)



Basal thermal state
(MacGregor et al., 2022)



Surface ice velocities
(Joughin et al., 2017)

9) Important current research topics: geothermal heat flow

An important factor in determining the basal thermal state is the geothermal heat flow (GHF). Transporting heat from the Earth interior, GHF mostly originates from:

1) Primordial heat remaining from the transformation of kinetic energy into heat at the formation of Earth through celestial collisions.
2) Radioactive decay of heat-producing elements (U, Th, K), primarily concentrated in the Earth crust.

GHF is higher on oceanic crust than continental crust (≈14%). This reflects the smaller thickness of oceanic crust, and hot mantle rocks thus being at shallower depths.

Continental GHF varies in response to crustal heat production, age, composition, tectonic history, distance to continental margins, and thickness of crust and mantle.

Constraining GHF beneath ice sheets is challenging. Local measurements calculate the vertical temperature gradient in the bedrock, in the ice sheet, and in unconsolidated sediments. But such measurements reaching the base of ice sheets are rare.

9) Important current research topics: geothermal heat flow

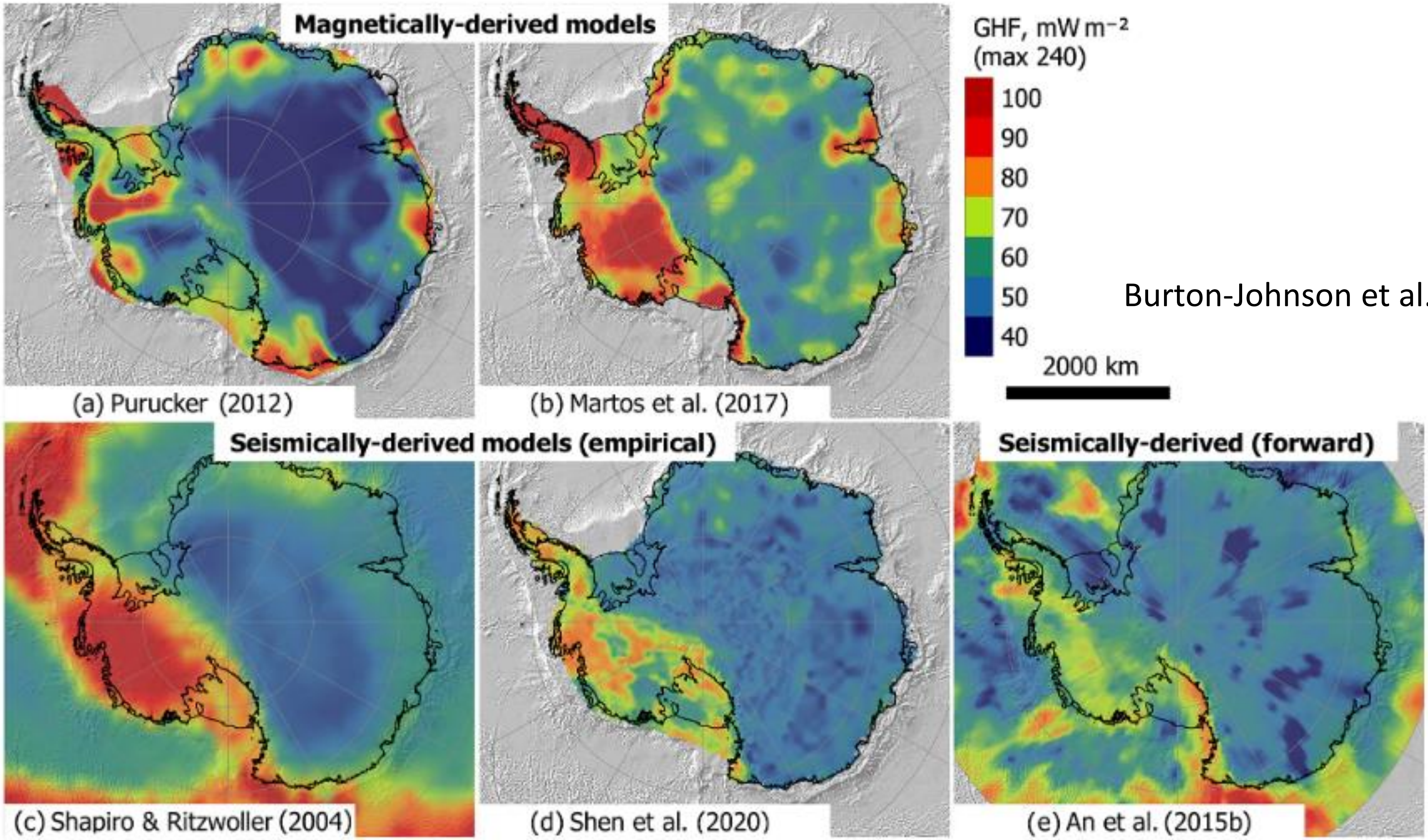
Methods have been developed to estimate GHF, based on the existing observations. Different estimation methods exist:

- Magnetic methods: magnetic survey data is used to determine the depth at which the maximum temperature of ferromagnetic magnetization is exceeded (Curie point depth). Regions with shallower Curie point depths have higher vertical temperature gradients, and thus higher GHF.
- Seismic methods: temperature is the primary control on seismic velocity in the mantle. Furthermore, the lithosphere-asthenosphere boundary can be detected. GHF is then calculated from the estimated mantle heat flow, and the heat production and conductivity characteristics of the lithosphere above this boundary.
- Gravity methods: satellite gravity data is used to estimate the crustal thickness (instead of seismic estimates). With additional constraints, GHF is calculated as in the seismic methods.

9) Important current research topics: geothermal heat flow

Geothermal heat flux in Antarctica

- Very high spatial variability
- Uncertainty between different estimates



Burton-Johnson et al., (2020)

9) Important current research topics: geothermal heat flow

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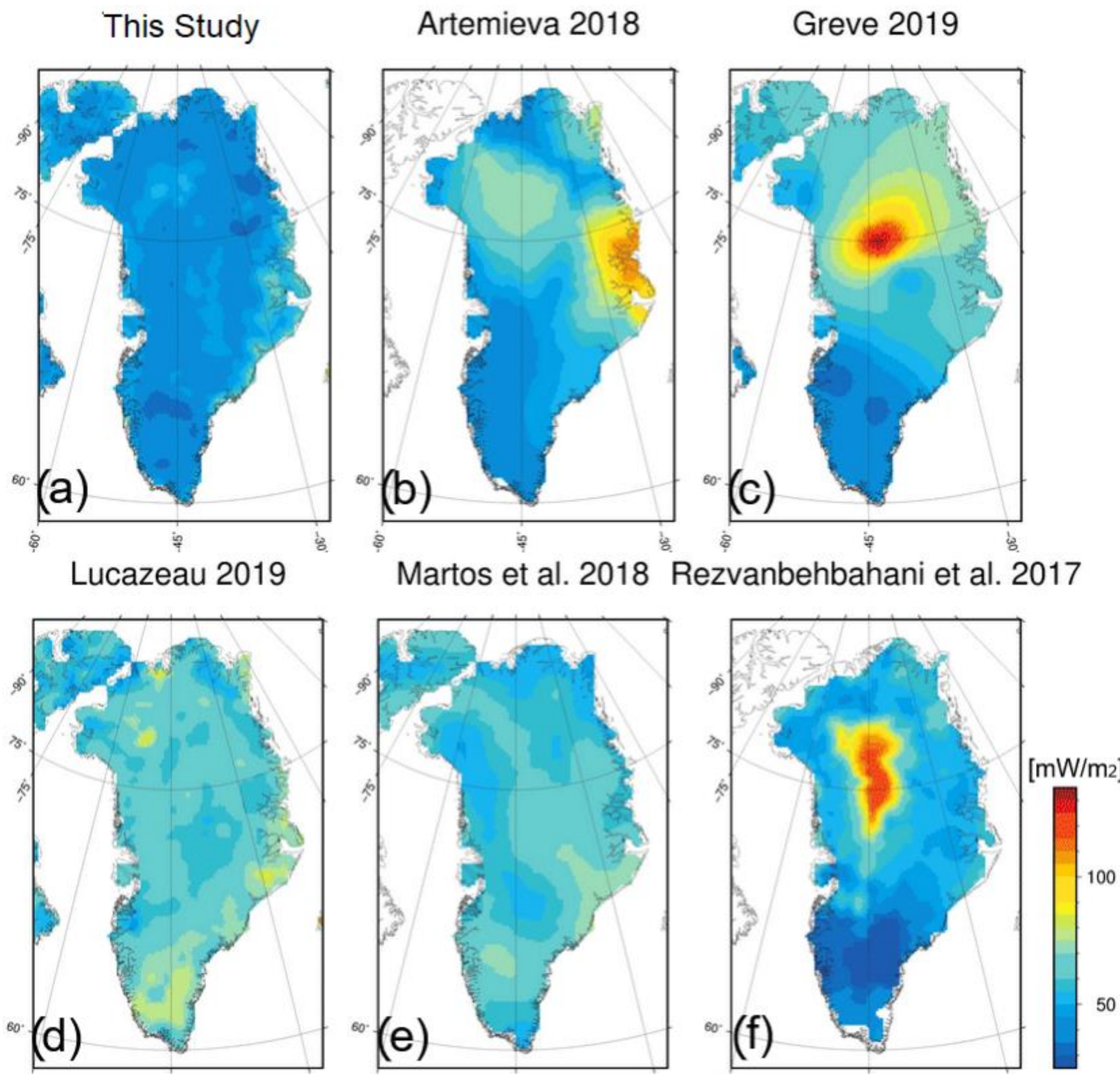

Colgan et al., (2022)

10) Analysis of recent research

Ice thermodynamical modeling is still a crucial research topic. We will now focus on the publication of Van Liefferinge et al. (2018).

- Understand the goals of the study
- Describe their methods and link them to the course material
- Analyze their results

The Cryosphere, 12, 2773–2787, 2018
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The Cryosphere
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Promising Oldest Ice sites in East Antarctica based on thermodynamical modelling

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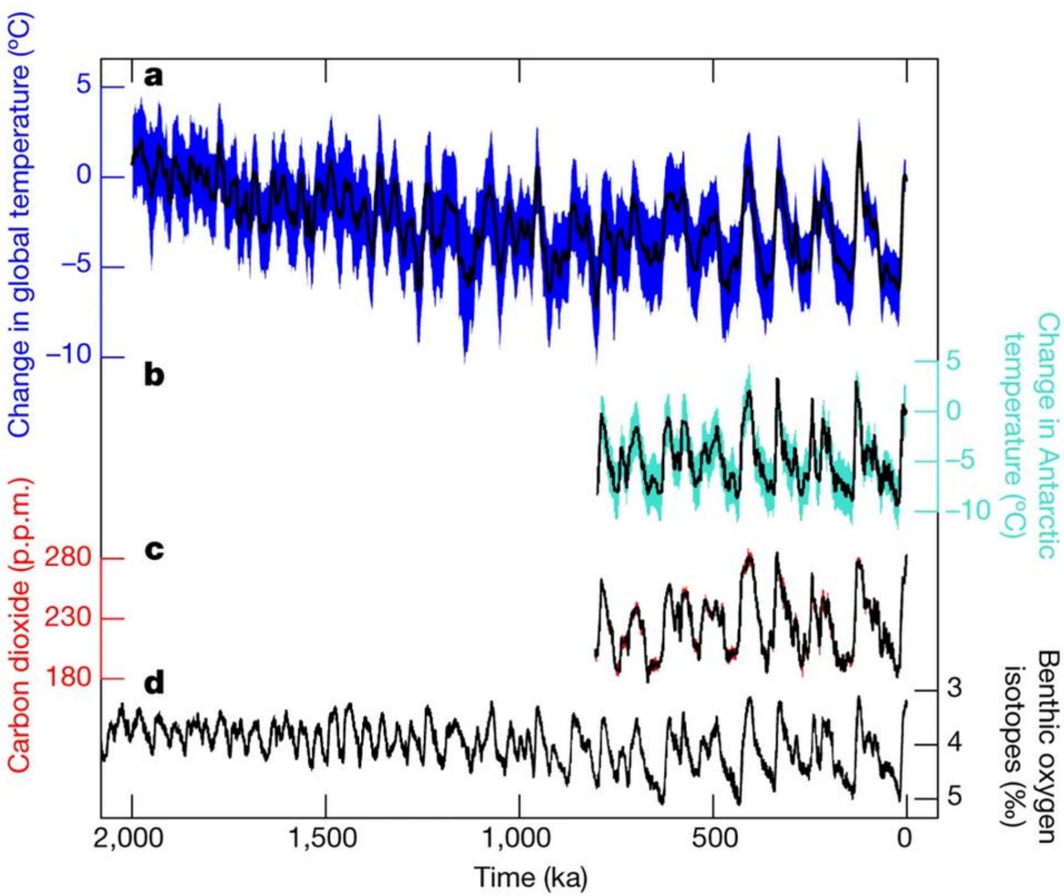
A) Goals of the study

- Current oldest ice cores go back to 800 kyr BP.
- Glaciological and paleo-climate community strive to find >1Myr old ice
- Key to understand the mechanisms explaining the change in climate periodicity at the Mid-Pleistocene Transition (0.9 to 1.2 Myr BP)
- Only 4 ice cores have reached the ice-bed interface in Antarctica, all revealing basal temperatures close to or at the melting point



2006; Elderfield et al., 2012). To resolve the mechanisms behind the major climate reorganisation during the MPT, the recovery of suitable 1.5 million-year-old ice core samples is fundamental. This old ice would provide us with unique and crucial insights into air composition as well as the isotopic composition and dust content of the ice throughout the MPT.

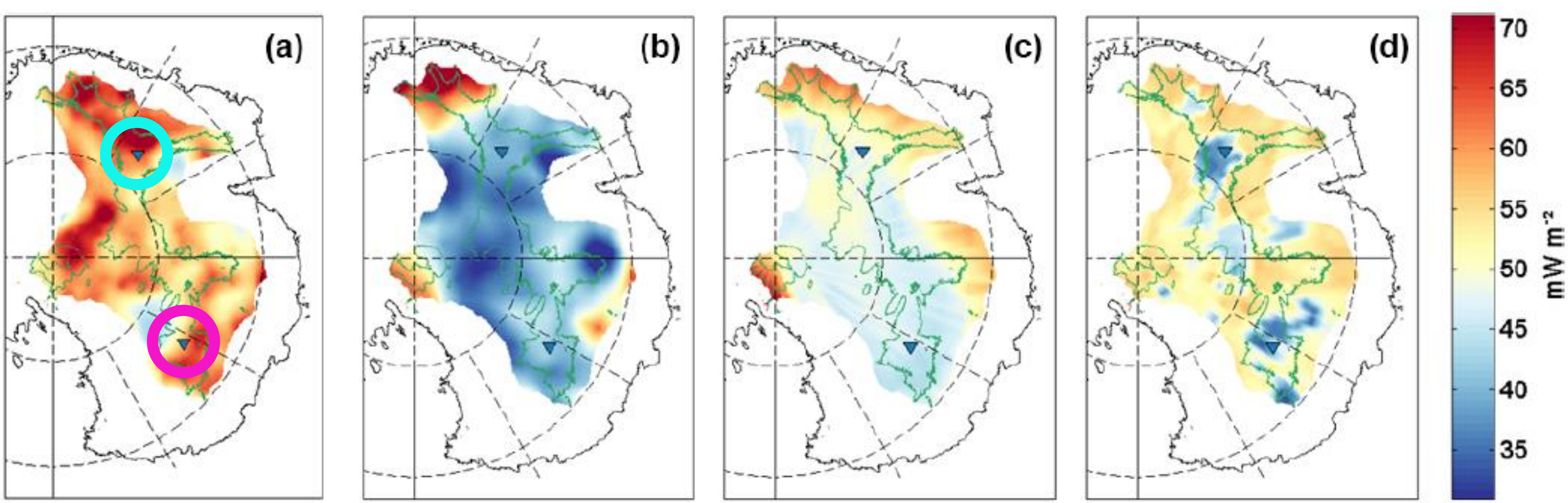
Snyder (2016)

A) Goals of the study

- Where to drill?
- Main focus is on the East Antarctic ice sheet
- 4 criteria:

a) No basal melt, which would remove the oldest ice layers
b) Smooth bed to avoid folding of the ice at the base
c) Low ice velocities, otherwise ice is transported to the ocean
d) High ice thickness, for an optimal vertical resolution of the oldest ice layers

- Major unknown: geothermal heat flow (GHF)



Dome Fuji
Dome C

B) Methods (part 1)

- Use a thermodynamical 1D model at all locations in slow-flowing areas of East-Antarctica
- Constrain the model with (a) reconstruction of past temperature variations, and (b) past ice thickness variations from an ice sheet model
- Repeat multiple simulations with different GHF values
- Find the threshold GHF value that causes basal melting (G_{pmp})

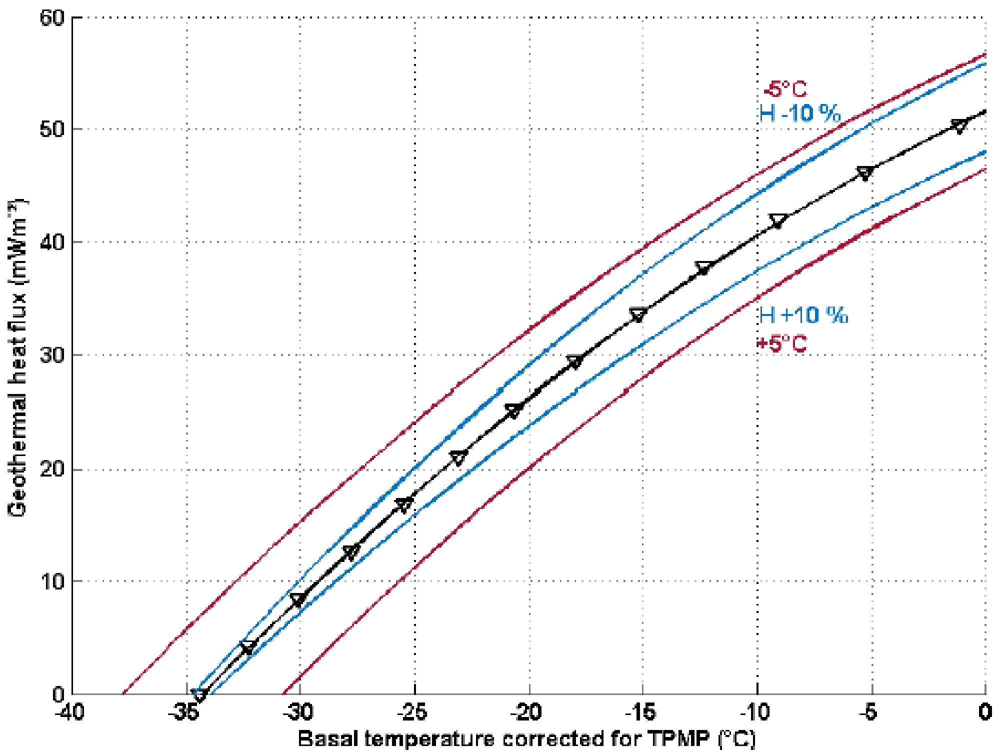


Figure 3. Example model result for a location near Dome C. The

In this paper we solve the vertical temperature profile over time, taking into account vertical diffusion and advection,

$$\frac{\partial T}{\partial t} = \frac{k}{\rho_i c_p} \frac{\partial^2 T}{\partial z^2} - w \frac{\partial T}{\partial z}, \qquad (1)$$

C) Results of G_{pmp}

- If G_{pmp} is high: high GHF can be accommodated while preserving ice frozen to the bed
- If G_{pmp} is low: even a low GHF would cause melting at the ice base

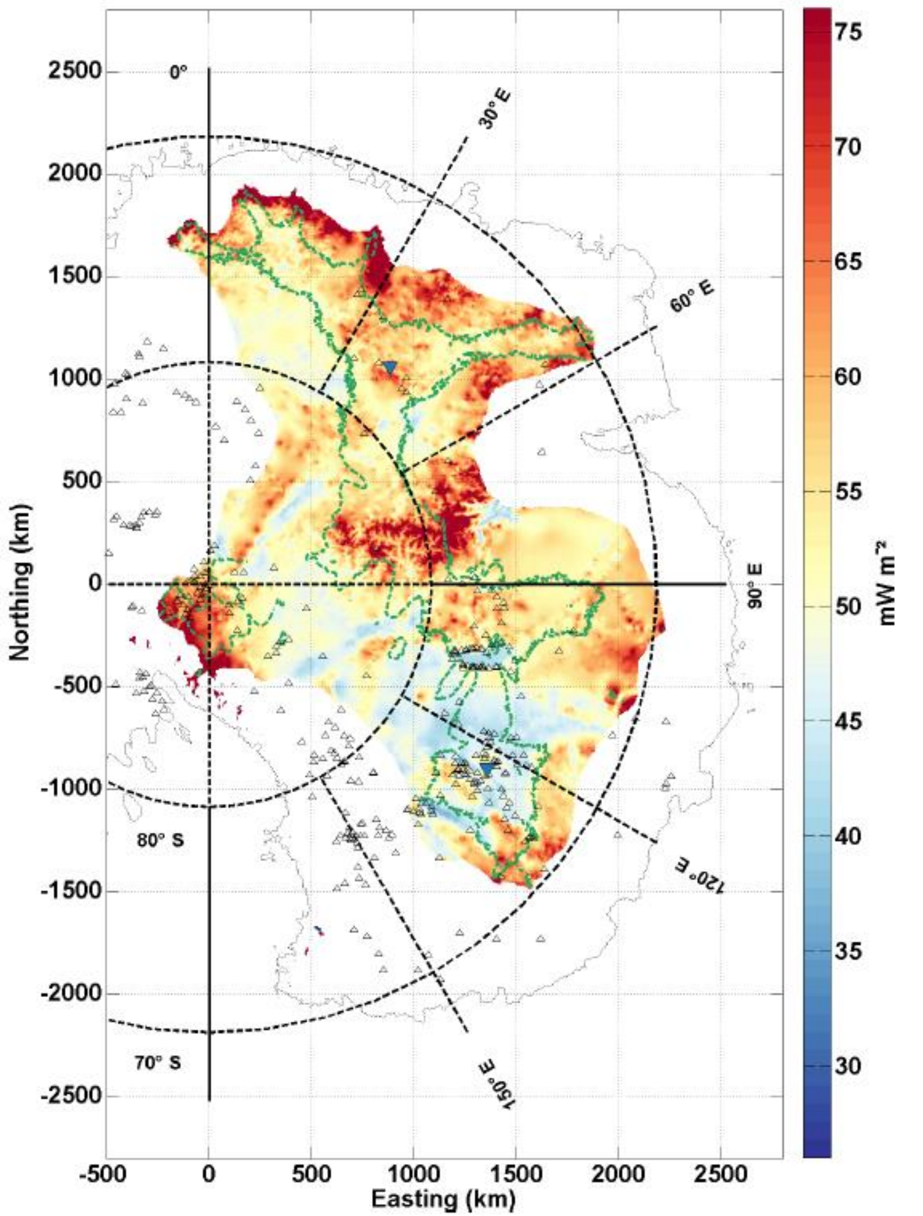


Figure 4. Map of G_{pmp}, i.e. the maximum GHF needed to keep a frozen base over 1.5 Ma. Colours represent the magnitude of the GHF (mW m^{-2}). The colour scale's central GHF value, in yellow,

C) Results of G_{pmp}

The authors compare their G_{pmp} values with the GHF of the datasets, and the respective uncertainties in GHF → Probability of G_{pmp} exceeding GHF

This is not the end-result, as there are multiple criteria for the most promising drilling site!

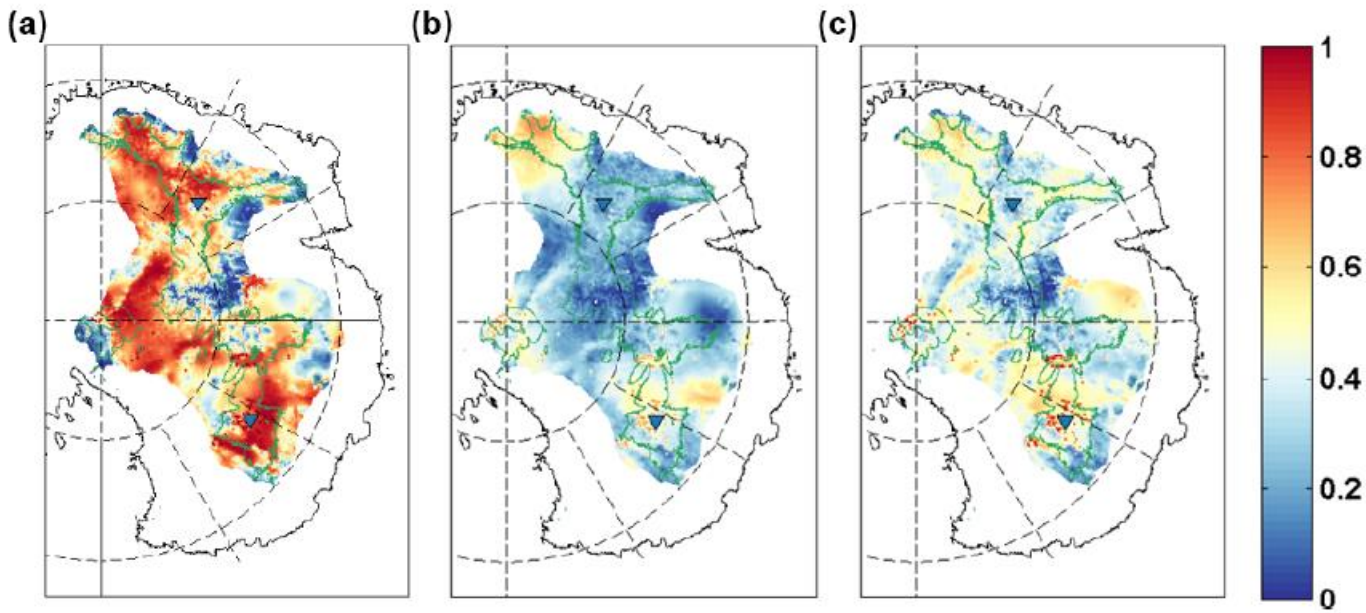
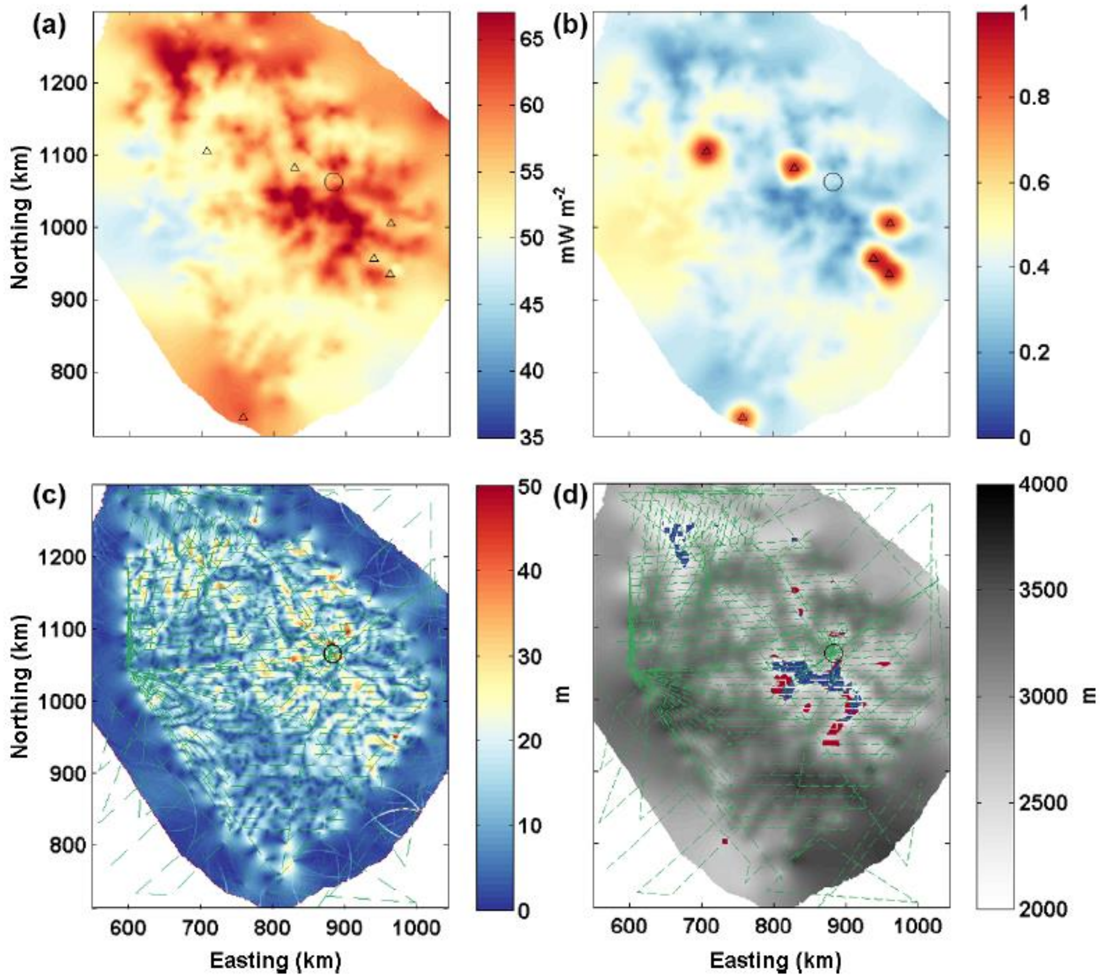


Figure 6. Probability that ice reached the pressure melting point over the last 1.5 Ma according to the GHF data sets from Martos et al. (2017), Purucker (2013) and Shapiro and Ritzwoller (2004) from left to right.

C) Results of most promising sites (around Dome Fuji)

G_{pmp}

Bedrock roughness



Probability of G_{pmp}< GHF

1st and 2nd choices for most promising sites

Figure 7. **(a)** Map of G_{pmp} results for the Dome Fuji area with the 1-D model calculated on a 500 m × 500 m grid, given in a WGS 84 northing–easting coordinate system (km). **(b)** Probability that ice reached the pressure melting point over the last 1.5 Ma according to Shapiro and Ritzwoller (2004). The small black triangles locate subglacial lakes and the circle locates the Dome Fuji ice-core site. **(c)** Standard deviation of bedrock variability; **(d)** ice thickness from Karlsson et al. (2018). In blue are potential locations of Oldest Ice with $H > 2000$ m, $\sigma_b < 20$ m, a probability that ice reached the pressure melting point < 0.4, surface velocity $< 2\,\mathrm{m\,a^{-1}}$, distance to radar lines < 4 km. In red are potential locations with $H > 2500$ m and a surface velocity $< 1\,\mathrm{m\,a^{-1}}$. The green dashed lines outline the new radar survey (Karlsson et al., 2018).