# An Air Cannon, Laser-Gate System to Determine the Coefficient Of Restitution of a Baseball

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***Abstract-*** An air cannon, optical laser gate, collision tube assembly was created to determine the coefficient of restitution (COR) between a baseball and a piece of wood as a function of the relative velocity between the two bodies. A PVC compressed air cannon rated for 120 psi accelerated a baseball to speeds nearing 70 mph, and an optical laser gate captured the speed of the baseball before and after a collision with a piece of wood (ash or pine). The collision was facilitated by a collision tube racked at the end of the cannon barrel. Values for COR from baseball to baseball were consistent to within plus or minus 0.4 at various inbound velocities between 20 and 70 mph. The COR appeared to decrease as the inbound velocity increased. However, not enough data were collected to confidently apply a mathematical model to relate the two quantities. The coefficient of determination for two attempted linear fits were 0.59 and 0.78.

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## I. Project Description

The coefficient of restitution (COR) is a ratio between the energy of a system before and after a collision. In a two body collision, if one body is fixed throughout the interaction and energy in the form of spin can be ignored, the COR can be expressed as the square root of the ratio of the final velocity over the initial velocity of the moving body. When a moving ball collides with a stationary surface, the ball will rebound with a fraction of the energy that it started with. The central task of this project was to design and construct a system to reliably and accurately determine the COR of a baseball with a piece of wood as a function of the relative velocity between the two bodies.

## II. Presentation of Final Design

It is often useful to present the final design first, allowing for specific and direct reference to the actual system when exploring design alternatives and problem evaluation. This section will detail the final implemented design of the system as well as describe how the subsystems work and interact with each other. Below, in Fig. 1, is an image of the system in its entirety.

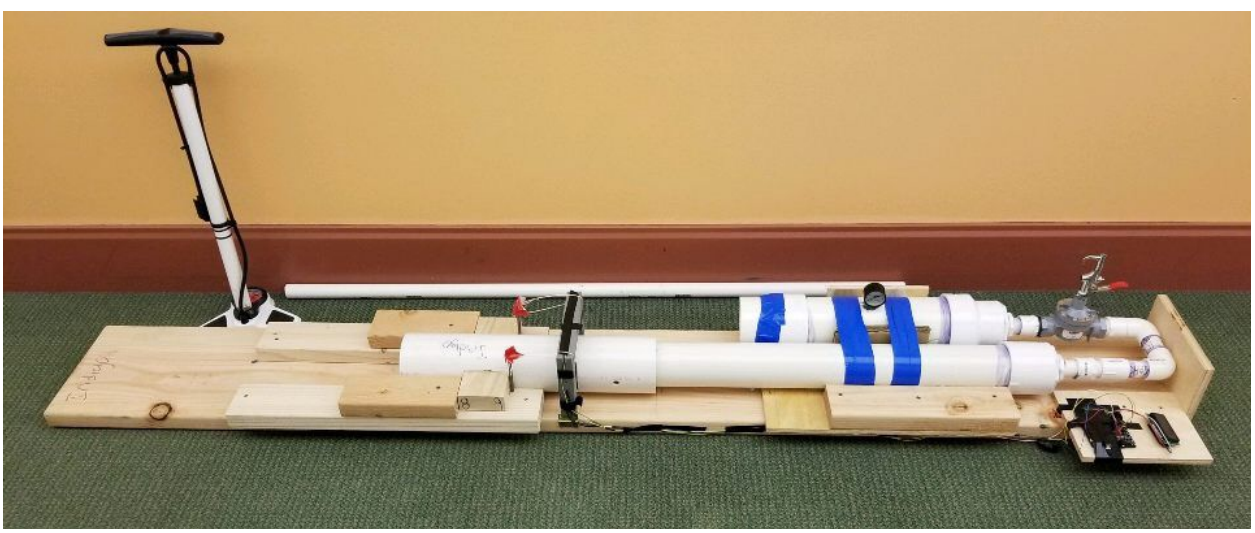

**Figure 1**. Final implemented design.

As seen above in Fig. 1, there are four main pieces to our system. There is the PVC air cannon, the laser-gate system, the collision tube and rack, and the overall frame to the design. These sections will be discussed separately, although none of them were designed or constructed independently, as the elegant organization of our system seen in Fig. 1 suggests.

The pvc air cannon acts as the agent of acceleration for the baseball to establish a relative velocity between the baseball and the piece of wood (which is housed in the collision tube). Below in Fig. 2 is a close up image of the air cannon.

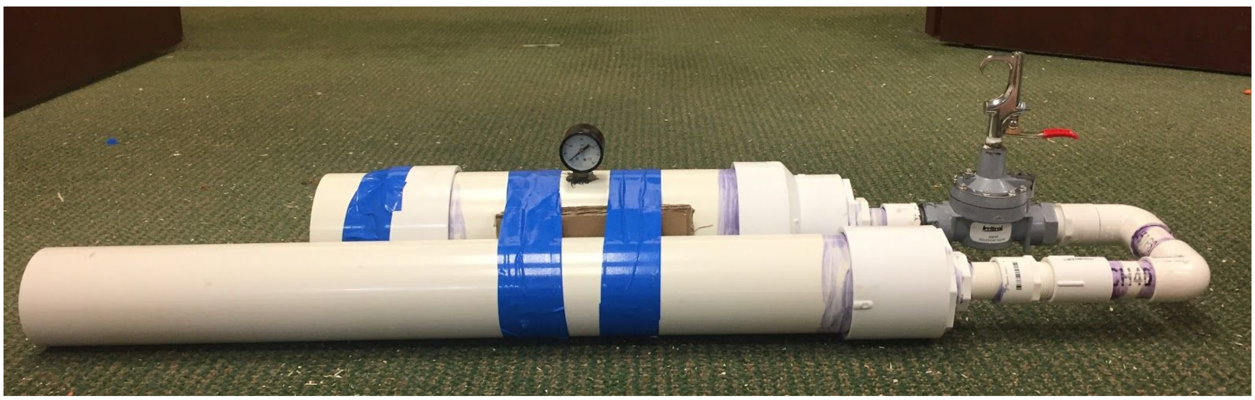

**Figure 2**. The pvc air cannon. Consists of three sections: the pressure chamber, the quick release trigger, and the barrel.

The air cannon works in a very simple manner. The pressure chamber is pumped with air to high pressures, and upon pulling the quick release valve, the large differential pressure between the chamber and the barrel forces a wave of fast moving air to force the baseball out of the barrel at high speeds. The compressed air provides an impulse to the baseball as it propagates down the length of the barrel. The highest delivered speed we measured for the air cannon was 68 mph at a chamber pressure of 100 psi. We did not find consistent speeds for psi values lower than 40, but at 40 psi the delivered speed ranged between 20-25 mph.

The barrel is quite simple, and does not require much additional explanation. The barrel is 3 inches in diameter, which fits a baseball (with a diameter of about 2.94 inches) tight enough for consistent speeds when firing. It is also worth mentioning that the barrel is mounted to the pipe bend seen in Fig. 2 with a threaded connection. This threaded connection makes it very convenient to remove and exchange the barrel with other barrels, allowing this cannon to be used for projectiles of different calibers and also with barrels of different lengths.

The bend itself in the air cannon also serves two purposes as well that enhance the overall design of the cannon. The bend reduced the overall length profile of the cannon, making it easier to move and handle. Additionally, the bend and the introduction of the small slits of cardboard between the chamber and the barrel seen in Fig. 2 ensure that the quick release valve and all connections between the chamber and barrel do not experience any mechanical stress when lifting or moving the cannon. This feature of the cannon is critical for ensuring its longevity and reliability. All of the seals under high pressure are ensured using high strength JB Weld epoxy. This epoxy is excellent for withstanding high static pressures, but the epoxy fails very easily if a shear or bending stress is applied to it. The bend and cardboard slits in concert with the taped connection ensure a comfortable seating of the chamber to the barrel, allowing for stress-free movement of the cannon. We utilize the ease of moving the cannon around as well as its reduced length later on in our design when designing our mounting system.

The pressure chamber is a length of 4 inch diameter schedule 40 PVC pipe fitted with an endcap, a Schrader valve, a pressure gauge, and a series of connections to fit to a one inch diameter PVC threaded connection. Seen below in Fig. 3 is a close up of the Schrader valve connection as well as a portrait profile along the length of the cannon.

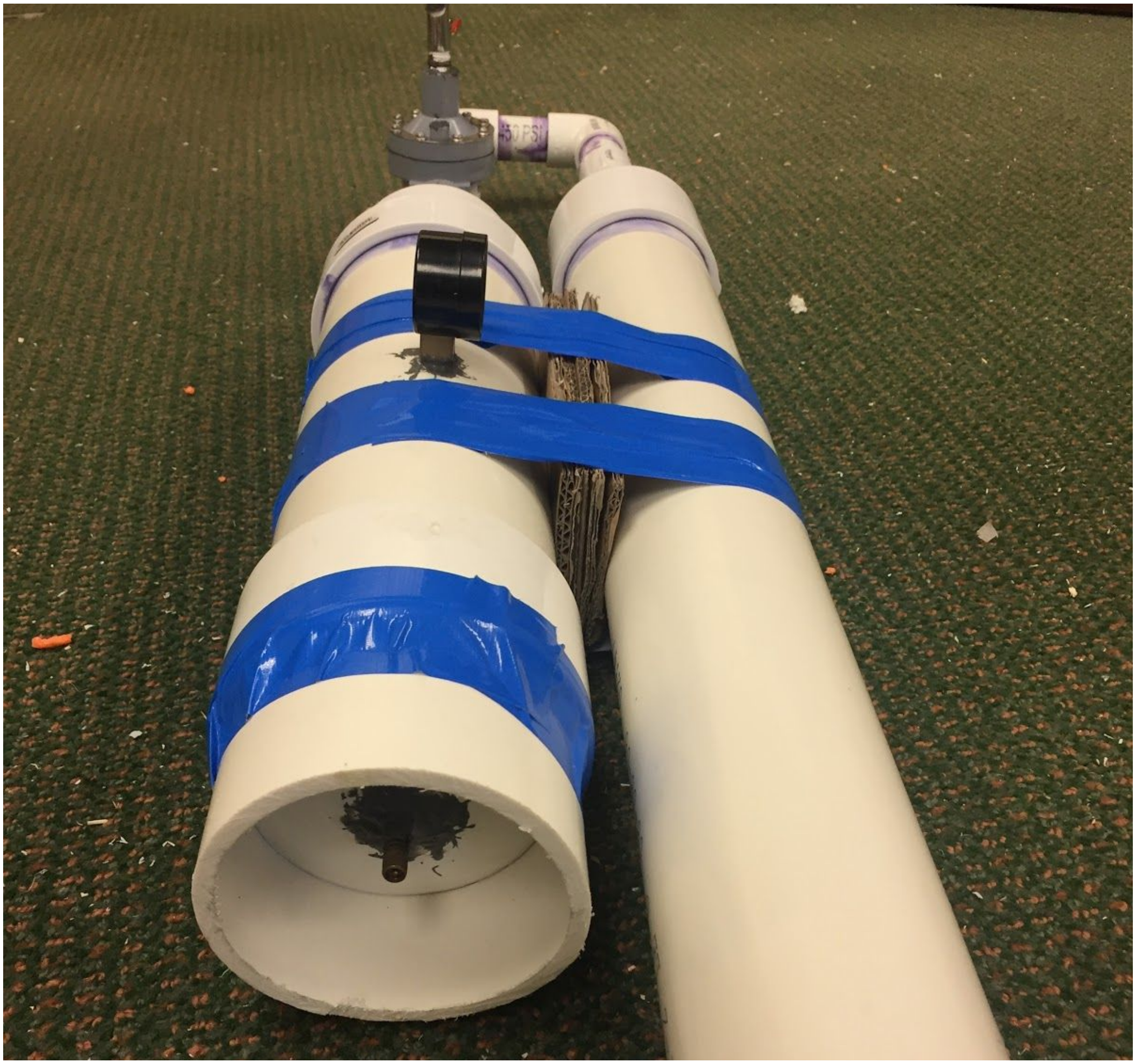

**Figure 3**. Portrait profile of the cannon. The Schrader valve connection can be seen in detail towards the bottom left.

As seen above in Fig 3. A Schrader valve connection, the typical connection used for bike and car tires, was fitted to the endcap through a drilled hole and sealed using JB Weld epoxy. Another section of 4 inch diameter PVC pipe was glued and taped onto the end to protect the valve from experiencing any mechanical stress from bumping into the ground or wall when being moved around. Fig. 3 also demonstrates the effectiveness of the cardboard slits and connecting tape to seat the chamber and barrel to one another. The 4 inch PVC pipe was rated for 120 psi, while all the JB Weld connections were rated for 200+ psi.

To create the quick release valve, we modified a sprinkler solenoid valve. A close up of the valve can be seen below in Fig. 4.

**Figure 4**. Close up of quick release valve. Modified from an Irritrol 205TF 1” Glove valve and a Neiko 31112 Air Blow Gun Oversized Trigger.

To explain how the quick release valve works, we will refer to the diagram below in Fig. 5.

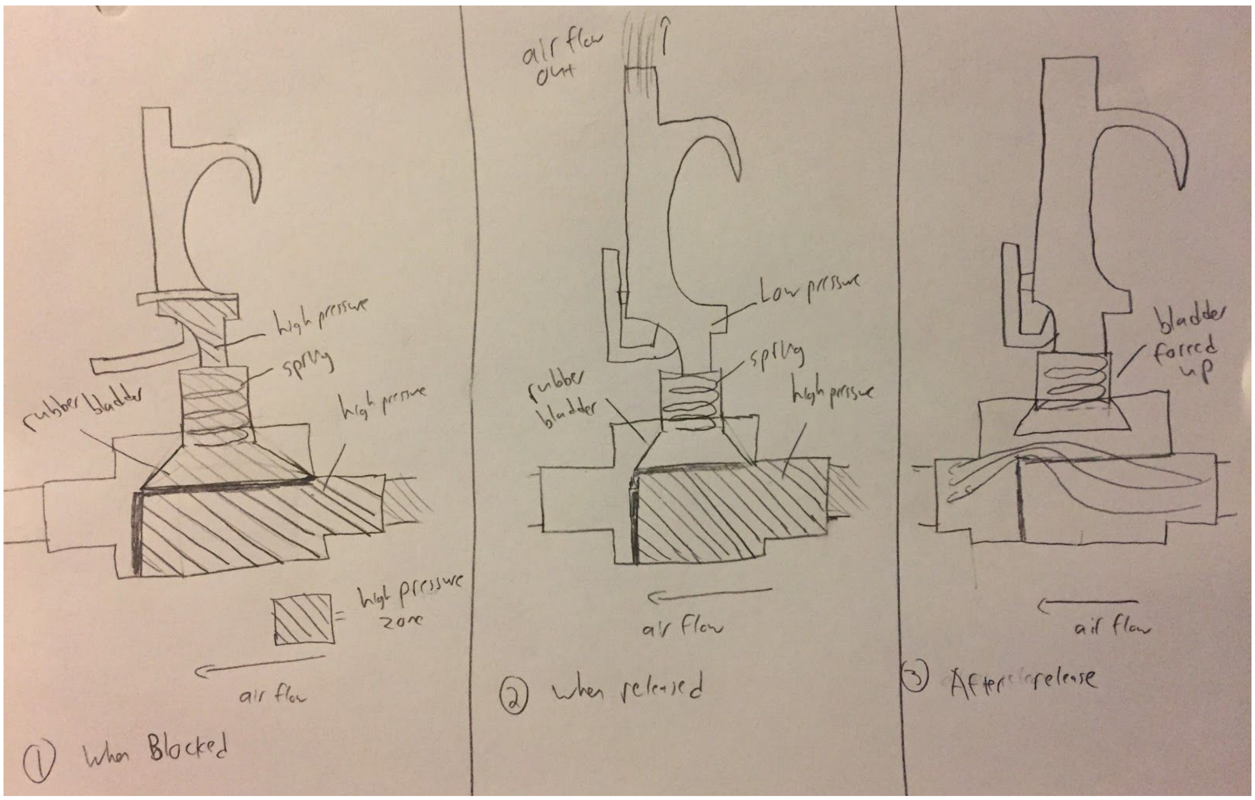


**Figure 5**. Three-phase diagram explaining the quick release mechanism.

The modified sprinkler valve has a compressed spring that pushes down on a rubber bladder that forces a seal between the high pressure zone upstream (denoted by areas with diagonal lines) and the low pressure zone downstream. At the top of this rubber bladder is a small hole that allows the volume directly above the bladder to also pressurize to the same pressure as inside the pressure chamber. The way the blow gun trigger works is that in the down position, there is a small piece that blocks airflow to the atmosphere out of the top of the trigger. When in the up position, this small piece moves, and allows air to flow through the trigger piece to the atmosphere. As seen in Fig. 5, at stage 1, both the volume above and below the rubber bladder are pressurized at the chamber pressure. In stage 2, once the blow gun trigger is pulled to the up position, the small amount of air at high pressure within the trigger quickly flows out of the top of the trigger piece. Then in stage 3 there is a very large pressure differential above and below the rubber bladder. With a very high pressure below the bladder and a low pressure above the bladder, the bladder is forced up, breaking the seal, allowing the compressed air from the chamber to flow through to the barrel very quickly. This quick flow of compressed air is what provides the impulse to the baseball and accelerates it. The sprinkler valve was originally fitted with a solenoid switch, but we found that the switch did not work well or quickly enough to provide the desired impulse to the baseball at higher pressures.

This quick release mechanism proved to be very reliable and very safe as well. The quick release trigger was rated to more than 300 psi, and the JB Weld in concert with teflon tape created very sturdy high pressure seals for confident operation. This simple trigger allows for very easy use of the gun, and does not require nearly any physical exertion on the part of the user. This design allows for very straightforward point and shoot operation. There was however one notable drawback to this trigger mechanism. To the lift the bladder and allow the compressed air to flow from the chamber to the barrel, we found that a minimum of a 12 psi pressure difference was required. As a result of this requirement, after firing the cannon, around 12 psi would always remain in the chamber. This means that not all of the air from the chamber was delivered to the baseball upon pulling the trigger.. As the ball propagates down the barrel, the pressurized air behind it is expanding, losing its pressure as it grows to fill more volume. With less total air being pushed behind the ball, that pressure will drop quicker, resulting in a smaller total delivered impulse to the baseball, and thus a smaller exit velocity. This tradeoff in speed however was compensated for with the increased ease of use and safety the trigger mechanism provided.

The next major sub system to our final design was the laser-gate system. We used a laser-gate in concert with an DFRobot Romeo to measure the speed of the ball both before and after the collision. Pictured below is our laser system implemented in our final design.

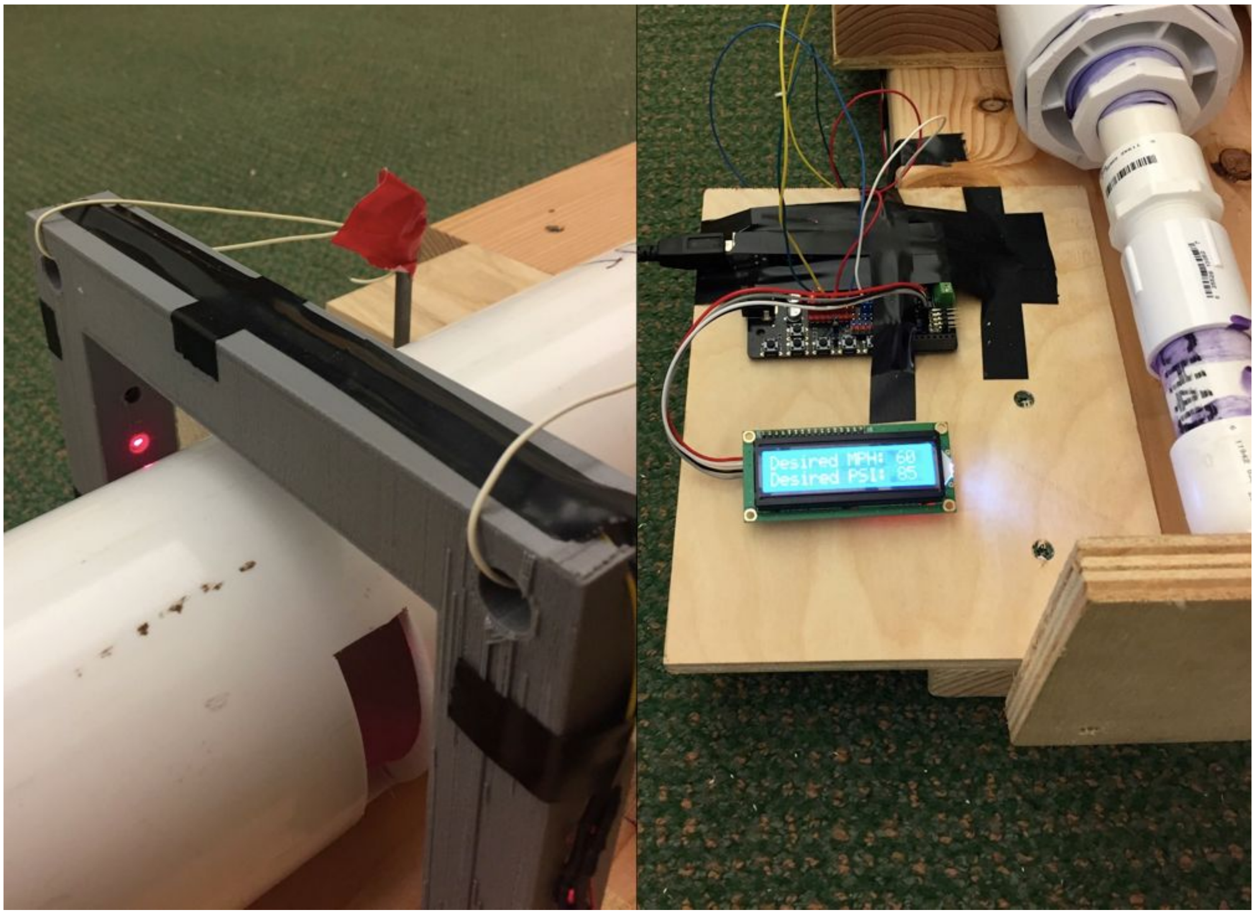


**Figure 6**. Implemented laser gate and microcontroller/lcd system. Used to measure the pre and post-collision speeds and calculate the COR.

As seen in Fig. 6, the laser-gate system was mounted to peer through a slit in the collision tube, and the microcontroller/lcd system was mounted on a platform next to the release trigger. Our laser-gate system consisted of three laser diodes incident on three photoresistors wired in series with one another. These three lasers were mounted inside of a 3D-printed rectangular gate, printed to form a perfect plane for the baseball to pass through. Using a plane of lasers instead of a single laser ensures we capture the entire diameter of the ball as it passes through our gate. Using one laser introduces the risk of only capturing a fraction of the baseball's diameter as it breaks the laser, since the cross-sectional thickness of a sphere is smaller as you move toward the top and bottom of the object. The signal output of the three photoresistors was routed to an Arduino Romeo microcontroller. Upon the lasers being broken, the voltage output from the photoresistors increases significantly. The microcontroller records when these large steps occur. The time difference between these drastic steps is then compared to the diameter of the object to calculate the speed of the object. This method relies on a consistent value for the diameter of a baseball. The diameter of a baseball is listed as varying from 2.86 in to 2.94 in, which is a spread of plus or minus 1.4 % from the average value of 2.9 in. Due to this uncertainty, we can expect our values for inbound and outbound speed to have *at least* an uncertainty of 1.4%. Due to the capture rate of the microcontroller, the actual percentage of uncertainty is likely to be larger. After collecting the speeds before and after the collision, the microcontroller calculates the COR and displays the speed and COR information on the LCD for the user.

The next major piece of the design to explore, and perhaps the most important and most clever, is the collision tube and rack. The collision tube in our system does many things at once, and makes the system very easy and safe to use. Below is a picture of the unmounted, unracked collision tube.

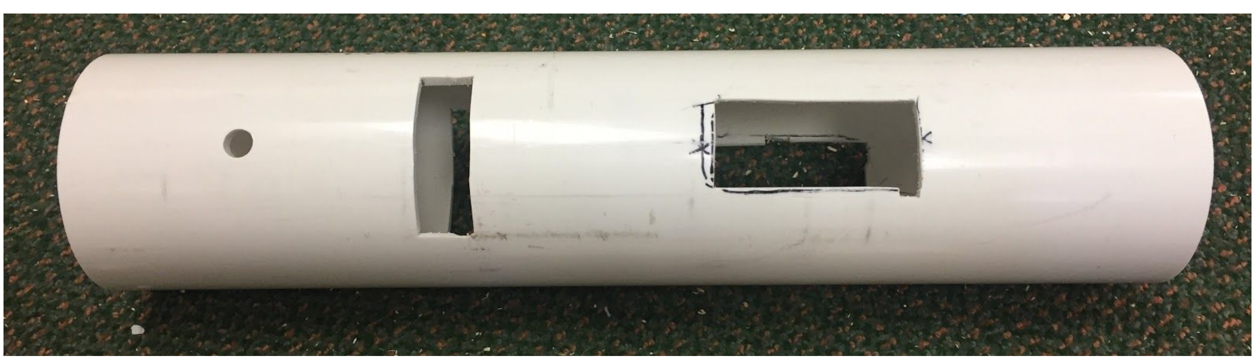

**Figure 7**. Collision tube. Holes described from right to left: 2"x4" wood sample slot, laser-gate slot, optional secondary laser hole.

The collision tube is a piece of 4 inch PVC pipe cut with three slots that serve different and very important purposes. There is a slot for inserting the piece of wood for the collision, a slot to allow the laser-gate to peer through, and an additional third slot for another laser if we wanted to implemented another speed measurement system that utilized a fixed distance step. The essential other piece to the collision tube is the rack that it fits into. The collision tube rack is shown below in Fig. 8.

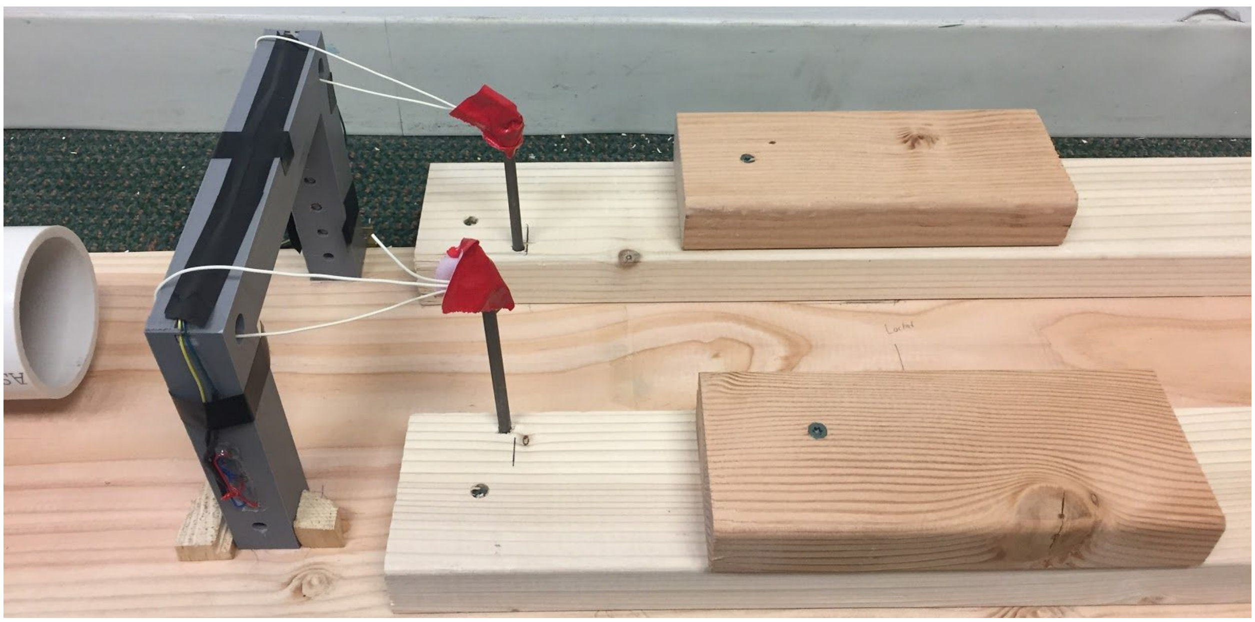

**Figure 8**. Collision tube rack.

The collision tube rack is responsible for holding the collision tube in place during testing. The openness of the collision tube rack design allows for complete freedom of the collision tube, making reloading very easy, safe, and convenient. The way the collision tube and rack work in concert is very simple but also very effective. First, a standard 2”x4” piece of wood is inserted into the horizontal slot of the collision tube. The user then rests this piece across the upper two pieces of wood while pointing the collision tube through the rectangular laser-gate. Fig. 9 below shows step one.

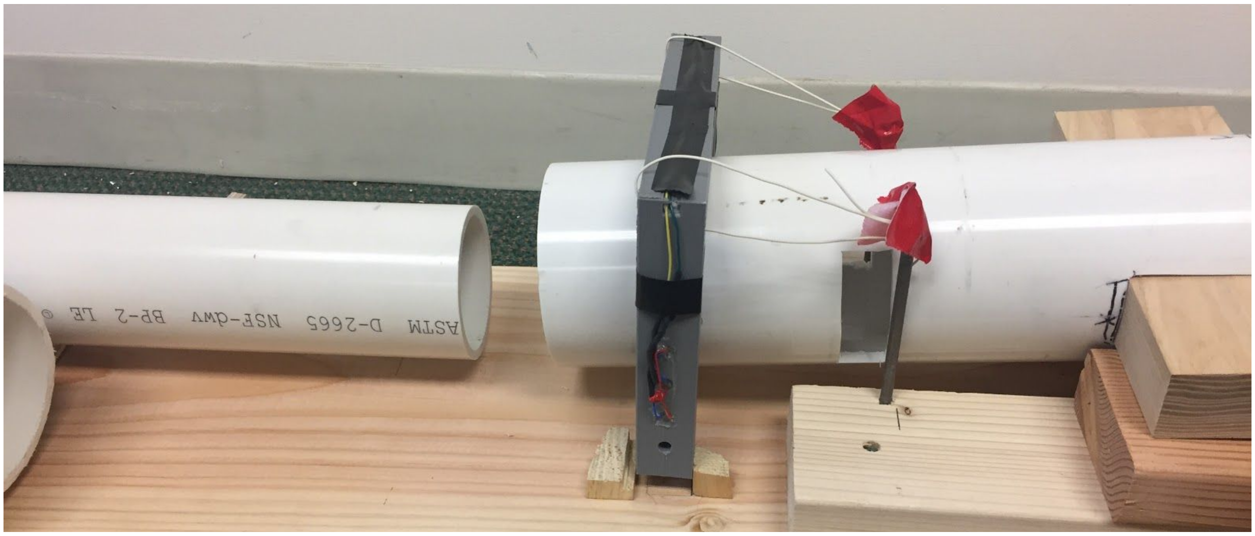

**Figure 9**. Step one of racking the collision tube into place.

Then just by simply pushing the whole collision tube slightly forward, the piece of wood used for the collision racks down and back. The collision tube than is firmly seated between the metal pegs and the horizontal pieces of wood seen below in Fig. 10.

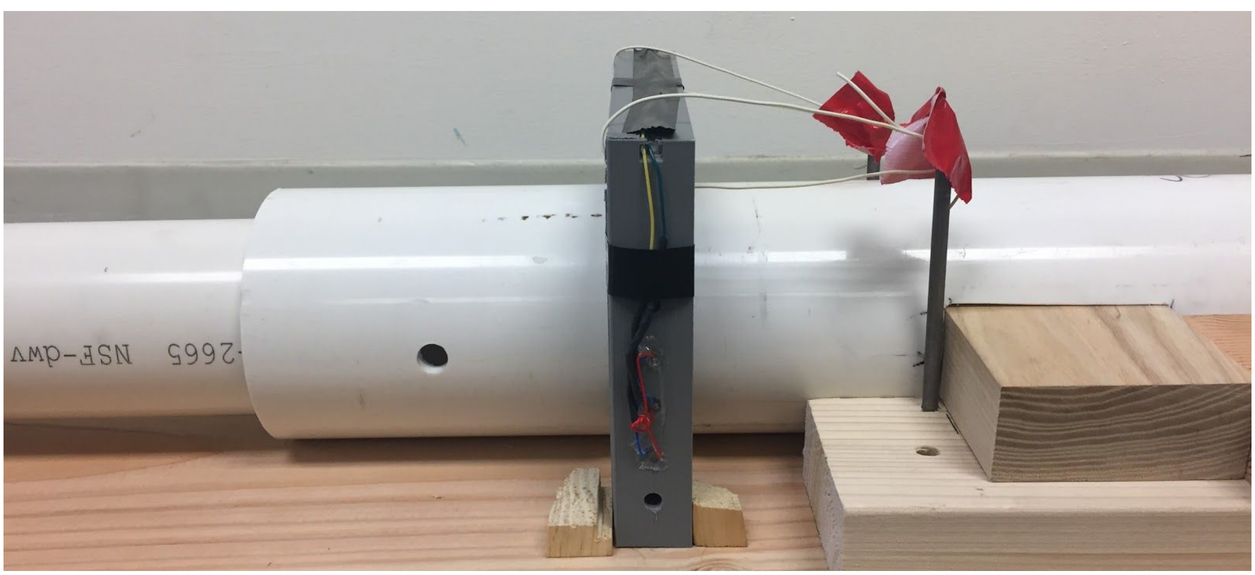

**Figure 10**. Collision tube in the fully racked position.

In this fully racked position, the collision tube mates perfectly with laser gate and the barrel of the gun. The gun barrel lies nearly perfectly coradial with the collision tube in this position, ensuring a nearly head perpendicular collision with the piece of wood. Fig. 11 below illustrates how the laser-gate peers perfectly through the center vertical slot of the collision tube when fully racked.

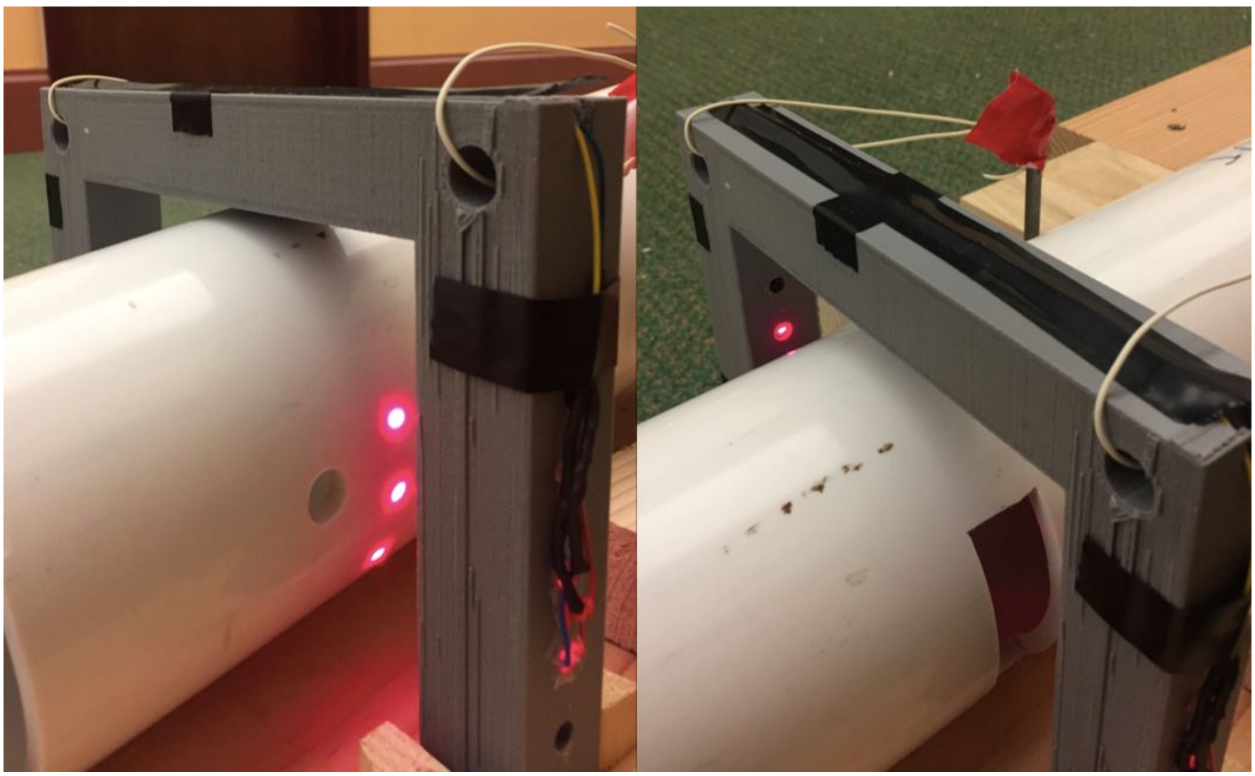

**Figure 11.** Demonstration of how racking mechanism and slot in collision tube enables the laser gate to function properly when in fully racked position.

The collision tube and its racking system also creates a perfectly safe environment for testing. When fully racked, the barrel end is covered by the collision tube, and the collision tube exit is completely blocked by the piece of wood, but still allows for air to easily flow around it. There is no physical way for the ball to escape the system when firing, making it exceedingly safe to use.

Additionally, as the collision tube is completely removable, the gun can be reloaded easily without having to move the gun. Fig. 12 below illustrates how the collision tube blocks the exit of the ball, but allows for excellent airflow through the barrel of the gun, and how the system is reloaded.

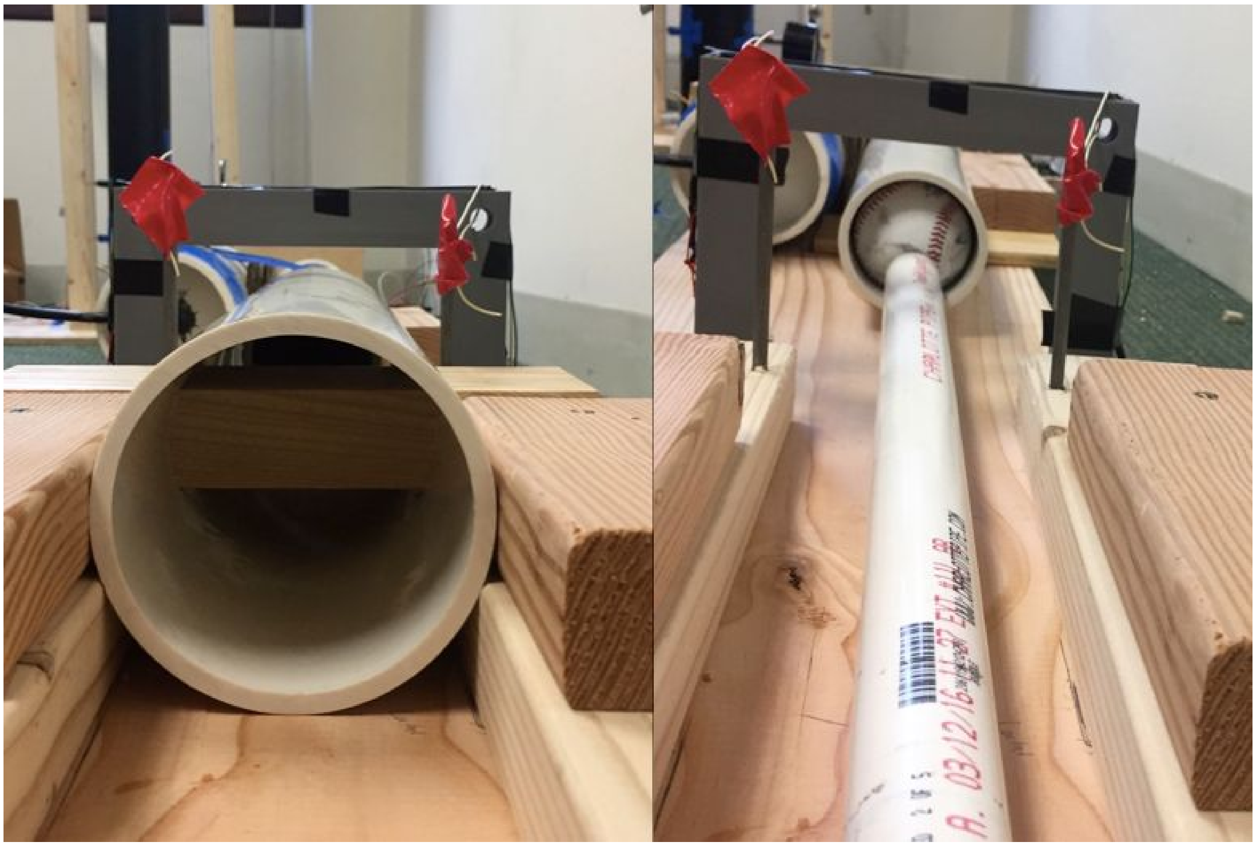

**Figure 12**. Back of collision tube when racked, and reloading process.

The collision tube and rack are most likely the two most unique elements to our design. Together they simplify the design immensely and allow for easier implementation of other pieces of the system, such as measuring the energy of the system after the collision. The collision tube allows for enough space for convenient airflow and reloading, while also keeping the collision confined so as to only require the use of one laser-gate. The collision tube is also quite small in length, which minimizes the impact gravity has on the path of the ball. This system also allows us to conveniently test different types of wood for very low cost. The typical 2”x4” is manufactured in nearly every type of wood as the standard size used for building, so viable test pieces are very easy to obtain. The racking system also removes much of the possibility for user error. Our system does not require the user to line anything up or be careful with any aspect of the setup. Everything simply fits together as it should, and all the user has to do is pump up the chamber and pull the trigger.

The final piece of the design to discuss is the mounting of all of the subsystems into one contained unit. The mounting process for the gun and rack was performed at the same time, and the mounting process for the laser gate and the microcontroller was performed somewhat iteratively afterwards. As seen in previous figures, the entire system was mounted on a single 10 ft piece of pine. Below in Fig. 13 is the mounting section for the PVC air cannon.

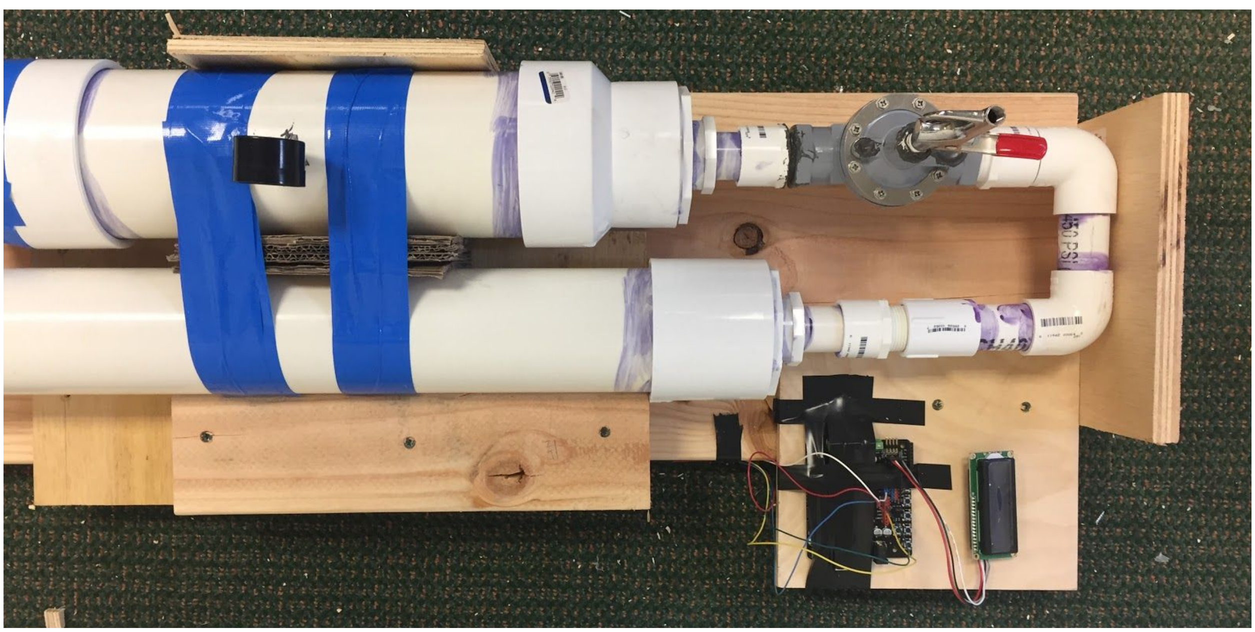

**Figure 13**. Mounting section for the PVC air cannon and microcontroller/LCD.

There are five features to notice from Fig. 13. The first and least important/impressive feature is the mount for the microcontroller and the LCD. This platform provides a sturdy location for the user to interact with the laser gate and view the outputs of the electrical system. The second aspect to notice is the piece of plywood on the end of the base. This piece of wood racks the gun lengthwise along the base, ensuring that the kickback from firing the air cannon does not displace the cannon in the backwards direction for the next trial. The third important aspect to discuss is the piece of plywood attached along the the top of the base. This piece of plywood racks the cannon widthwise along the base by making contact along the length of the pressure chamber. The fourth aspect to notice is the 2x4 piece of pine that lies incident along the barrel. This piece of wood serves two purposes. The first is it racks the cannon widthwise along the base, but it also mates perfectly with the 3 inch transition piece at the base of the barrel. This small detail racks the cannon lengthwise along the cannon in the forward direction. This last piece of wood prevents the cannon from moving in the positive and negative x and y directions, while still allowing the gun to be picked up and removed from the base at any time. The final feature to notice from Fig. 13 is the piece of plywood beneath the pine 2x4. A close up of this feature is given below in Fig. 14.

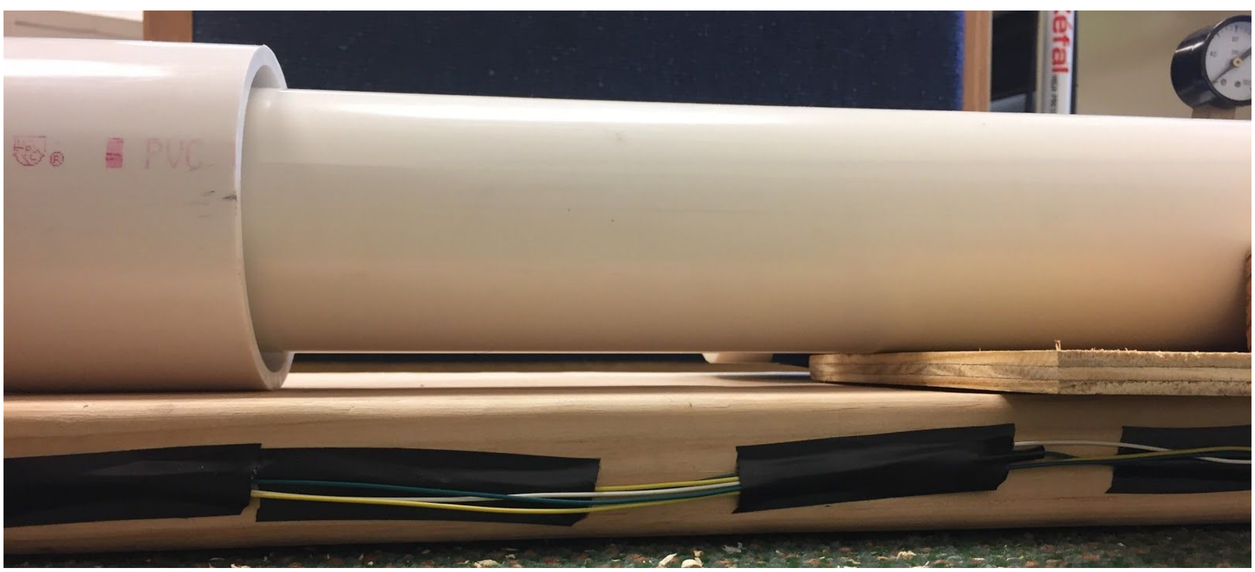


**Figure 14**. Plywood piece on base that mates the barrel to the collision tube at proper height.

As seen above in Fig. 14, this last piece sets the height of the barrel to be nearly perfectly coradial with the collision tube, ensuring that upon entry, the baseball will not hit any of the walls of the collision tube and introduce inaccuracies into our COR or speed measurements. After completing the mounting of the air cannon, the mounting for the racking system was implemented to make the collision tube coradial with the barrel of the gun, as I discussed when explaining the collision tube and rack system.

The mounting system for the laser gate was adjusted slightly as we tested. The primary mount was the 3D-printed frame as we discussed previously. However, other important aspects we neglected to mention were the introduction of the hinge, small wood racks, and tension wires. Below in Fig. 15, we present the opposite side of the laser gate frame, showing the hinge.

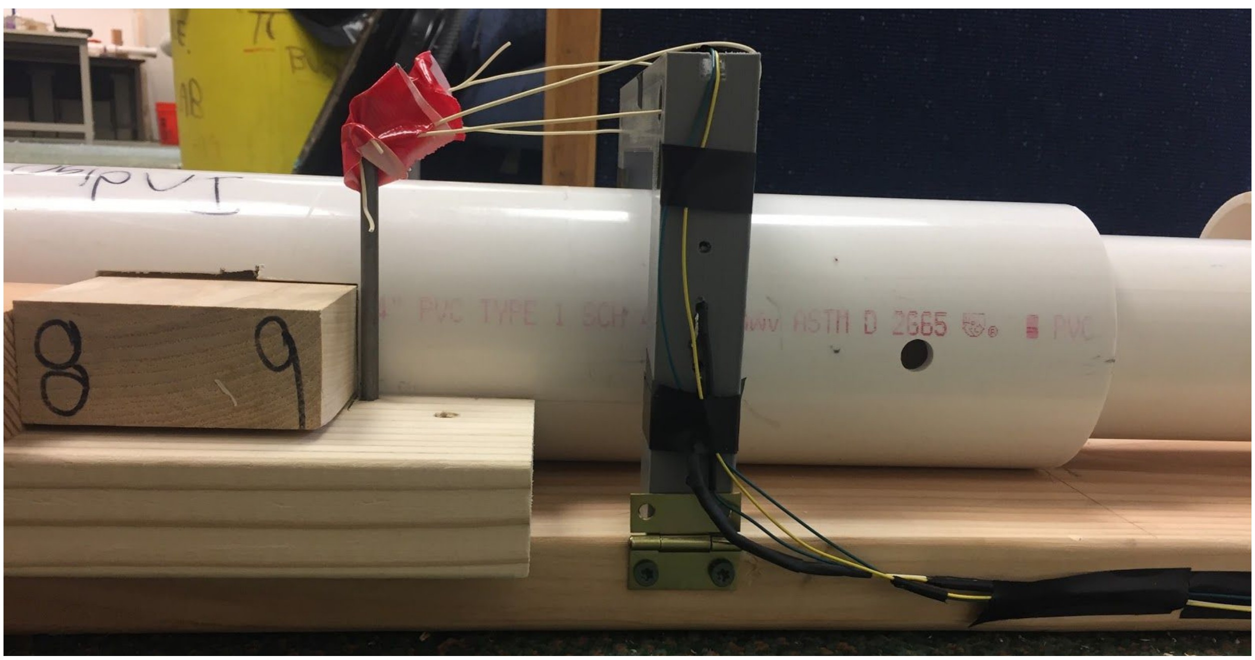


**Figure 15**. Laser gate frame hinge allowing for smooth removal and racking of the collision tube.

We introduced the hinge because we found when removing and racking in the collision tube, the laser gate frame was slightly too short to allow for smooth and natural movement. As a result of this discovery, we needed to have the frame free to move, even if only slightly, in the z-direction (up from the base). The hinge was the perfect solution to this problem, and also provided a solid way to connect the gate to the base. The hinge in concert with the small wood racks and the tension wires presented a perfect solution to our laser gate mounting problem.

The small wood racks and tension wires can be seen in Fig. 10. These small pieces serve to ensure the laser gate is perpendicular to the collision tube with every trial. They remove the need for the user to align the lasers with each trial and help to keep the lasers steady after a collision. The tension wires prevent the 3D-printed frame from recoiling after a collision occurred inside the tube. Especially at higher impact speeds (greater than 50 mph) and impacts with harder woods (ash compared to pine), the entire system experiences a significant jolt after impact. This jolt frequently caused the lasers to be broken by the collision tube itself since the frame would displace so violently after the collision. The introduction of these tension wires solved this problem beautifully without needing to introduce any complicated new features, and still allowed for enough freedom of motion for the frame to accomplish smooth raking and removal of the collision tube.

Our design was successful in delivering consistent COR values for two different types of wood across a range of different speeds. Although the end product we would consider a success, there are many ways in which the system could have failed and in fact did fall short. We will explore these scenarios and instances now in our failure analysis.

**III. Failure Analysis and Assignment of Uncertainty**

When performing failure analysis, it is important to remember exactly what the goals of the system are. Referring back to our problem statement, we needed to deliver a "system to reliably and accurately determine the COR of a baseball with a piece of wood as a function of the relative velocity between the two bodies." We can therefore define failure as anything that prevents us from delivering such a system. In our minds there are two ways in which we can fail. There is failure by catastrophe, where our system is either not safe or can not consistently yield values for the COR, and there is failure by inaccuracy, where our presented values for the COR are riddled with systematic error. We will first explore the possibility for catastrophic failure in our system, and then accuracy driven failure.

We have collected the following instances to analyze that in our minds constitute catastrophic failure: system self-destruction, seal failure, and data-collection failure. The first thing we acknowledged was that we would have compressed air under high pressure within our system. Amongst all the pieces that would need to sustain prolonged high pressure, the pressure rating on the 4 inch PVC pipe was the lowest at 120 psi. We never enacted a block in the system (either mechanical with a relief valve or electrical by cutting current to a pump) that could prevent the user from pressurizing the chamber above this rating, but we figured that simply telling the user that he or she should not exceed 100-110 psi would suffice for prototype purposes.

Seal failure was a more pervasive problem we needed to account for. A failure in a seal would prevent the air cannon from achieving high enough chamber pressures to propel the ball at meaningful speeds (greater than 20 mph) or even propel the ball at all (due to the rubber bladder needing at least 12 psi pressure differential to release). As we mentioned in our design presentation section, the primary agent for ensuring our most important seals (around the Schrader valve, around the pressure gauge, and the ports of the modified sprinkler valve) was JB Weld. JB Weld fails quickly under shear or bending loads. To prevent failure at the fill valve and the quick release valve, we introduced the end cap cover and the bend as we have discussed prior. However, the pressure gauge remained exposed even in our final design, and if it were to be bumped or the cannon were dropped on the gauge, the JB Weld seal likely would have broken. Although a repair on the seal is simple, high strength epoxy takes 24 hours to set, so a seal break on performance day would indeed constitute complete catastrophic failure. As the cannon would be set up in its racked position as seen in Fig. 13, we assumed that the pressure gauge would be safe from violent bumps or drops come performance time. However, a better design would include a more protected pressure gauge.

The final possibility for catastrophic failure we identified was data-collection failure. This groups together many potential electrical and mechanical problems. We did our best to combat electrical failure by ensuring our component input and output voltage ranges were correct, and any and all electrical pieces, like the wires, microcontroller, and LCD, were all fastened down in a mechanically stable way. The most likely failure possibilities we envisioned with data collection had to do with aiming accuracy and object interference. The introduction of the collision tube and our racking system essentially prevented any potential failure with the ball not passing through the laser-gate plane, so aiming accuracy was of negligible concern come demo day. However, object interference, as in something other than the baseball tripping our laser gate, proved to be a more pervasive failure possibility we needed to account for. Due to the jolt caused by the collision between the baseball and the piece of wood, there were two things that could cause object interference. The first is that the collision tube itself would rebound after the collision and move as far as to rest in front of the lasers, preventing any sort of accurate measurement. The second is the frame holding the laser gate would jolt violently towards the air cannon barrel, as its inertia wants to keep it from accelerating. To combat these two problems, we iteratively made one modification to the collision tube and introduced two features to the system. First, we made the viewing slit on the collision tube much wider, allowing for more wiggle room for the lasers to peer through to their respective photoresistors. Secondly, we introduced the metal pegs in the wood to prevent the collision tube from sliding forward after the collision. Thirdly, we introduced the small wooden racks for the frame and the tension wires to prevent the frame from jolting so severely. With the introduction of these features, we experienced excellent data collection consistency for collisions between speeds of 20 mph and 65 mph, our target range.

Accuracy driven failure analysis is a much more rigorous and thought intensive process than catastrophic failure analysis. We will begin our accuracy failure analysis with a discussion of our assumptions. To deliver a perfectly accurate value of COR, our system assumes the following:

1. Every baseball has a diameter of 73.7 mm (2.90 in)
2. Our energy balance boundary conditions:
    a. Kinetic energy of piece of wood is 0 before and after collision
    b. Gravitational potential energy of all bodies are the same before and after collision
    c. Rotational energy of the ball (spin) is 0 before and after collision
3. The baseball passes through the laser gate plane in the same orientation before and after the collision (related to the no spin assumption)
4. The plane of the laser gate is perfectly normal to the path of the ball before and after the collision
5. Air resistance is negligible over the course of the collision measurement times
6. Infinitely fast sampling rate

These assumptions are presented in order of difficulty of analysis, and will be analyzed in the order presented. While all of these assumptions are incorrect, we will analyze how much effect they could potentially have on our measurement outcomes.

As we have mentioned, baseballs have listed diameter ranges of 2.86-2.94 in, and are not uniform spheres. This assumption will automatically insert an uncertainty of +/- 1.4% into our velocity measurements, and +/- 2% into our COR value due to the combination of uncertainty of pre and post collision measurements.

The second assumption is equally easy to determine incorrect but much more difficult to quantify. The way we measure the COR is by comparing the velocity of the ball after the collision to the velocity before the collision. This is only viable when making the boundary condition assumptions listed for number 2 above. The conditions spelled out in 2a are the easiest to analyze. The piece of wood is at rest prior to the collision, but after the collision the wood has a few millimeters of space to move before it is stopped by the metal pegs seen in Fig. 15. During testing, we observed the block of wood and the whole collision tube actually move these few millimeters in response to the collision. As a result we cannot assume the kinetic energy of the block after the collision is 0. However, as we have no way to quantify this amount of energy, we cannot provide a numerical analysis and only a heuristic application of uncertainty. We will assign an additional 2% of uncertainty to our COR measurements due to this rebounding effect.

The conditions given by 2c also cannot be quantified consistently. In our system, the baseball is not stuffed into the barrel with any sort of wrapping material. As a result, it is likely rolling or touching the surface of the barrel at some point along its journey out of the barrel. Similarly, inside the collision tube, both before and after the collision, if the ball were to touch the sides of the tube, additional rotational impulses would be applied to the baseball by friction. Also, the collision itself could induce spin on the ball. As a result of these realities, the assumption that the ball has no spin before and after the collision is almost certainly false. However, when observing high speed video of baseballs being shot out of air cannons, similarly without any wrapping material, there is very little spin on the ball (1). Additionally, perpendicular collisions (or nearly perpendicular collisions) with static objects do not tend to add significant spin energy to baseballs. In fact, they tend to reduce overall spin significantly (2). To attempt to apply any sort of numerical uncertainty, let us look into some aspects of spin with baseballs. According to resource 3, the spin rate on fastballs appears to reduce somewhat linearly with speed of the ball. For one observed pitcher, an 80 mph fastball had a 2000 RPM spin rate, where a 60 mph fastball had a 1500 RPM spin rate. Since our air cannon can shoot 60 mph, and we observed that spin out of an air cannon is typically very small, let's analyze the following "worst case" situation for our no spin assumption:

1. KE for the block of wood is 0 before and after collision
2. No changes in PE for any body involved in collision
3. Ball inbound velocity is 60 mph with 750 RPM spin rate (half that of a pitched ball)
4. Ball post-collision velocity is 30 mph with 0 RPM spin rate
5. Approximate the ball as a solid sphere, with a radius of 1.45 in

Performing the energy balance for the spin and no spin cases yields the following relationships for COR.

$$COR = \sqrt{\frac{E_{final}}{E_{initial}}}$$
$$E_{initial} = \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2$$
$$E_{initial} = \frac{1}{2}m(v_i^2 + \frac{2}{5}r^2\omega_i^2)$$
$$E_{final} = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$
$$E_{final} = \frac{1}{2}mv_f^2$$
$$COR(spin) = \sqrt{\frac{v_f^2}{v_i^2 + \frac{2}{5}r^2\omega_i^2}}$$
$$COR(nospin) = \frac{v_f}{v_i}$$

We first must centralize our units:

$$r = 1.45in = .1208ft$$
$$\omega = 750RPM = 78.538\frac{rad}{s}$$
$$v_i = 60mph = 88\frac{ft}{s}$$
$$v_f = 30mph = 44\frac{ft}{s}$$

After performing calculations, the COR with spin, or our "worst case", is 0.499. The COR for the no spin case, or what we assume in our system, is 0.50. So even in this absolute worst case scenario, the COR is only affected by 1%. This percentage is significant, but not large enough to constitute an assignment of failure, even in the worst possible case. Moving forward, we will assume the worst, and apply a 1% uncertainty due to spin considerations for values of COR.

Now to consider the assumptions given by 2b, where the gravitational potential energy of all bodies are the same before and after a collision. Let us again look at a worst case scenario for this assumption. The slowest speed that we have reliable tests with is around 20 mph. The maximum possible change in height of the ball from before the collision to afterwards refers to the scenario where initially the ball is touching the top of the collision tube, and afterwards the ball is touching the bottom of the collision tube. For a 4 inch diameter collision tube and a 2.9 inch diameter baseball, this means the maximum change in height is 1.1 inch. For this worst case scenario, let us also assume the ball never spins, and that the rebound velocity is 10 mph.

$$E_{initial} = \frac{1}{2}mv_i^2 + mgh$$
$$E_{final} = \frac{1}{2}mv_f^2$$
$$COR = \sqrt{\frac{v_f^2}{v_i^2 + 2gh}}$$
$$h = 1.1in = 0.0912ft$$
$$g = 32.2\frac{ft^2}{s}$$
$$v_f = 10mph = 14.7\frac{ft}{s}$$
$$v_i = 20mph = 29.3\frac{ft}{s}$$

For our worst case, with the height change, the COR is 0.499999, as compared to 0.50 when ignoring height changes. As this calculation demonstrates, changes in height have a negligible effect on the outcome COR.

Assumption three is related to the idea of spin and the type of baseball being used. A flat seam baseball has seams raised 0.031 inches from the spherical surface of the ball. High seam baseballs are the more traditional kind, and their seams are raised 0.048 inches from the surface of the baseball, giving it a larger radius at certain locations (4). For this analysis, let us assume we are using a high seam baseball. In the scenario where the baseball passes the laser gate along its largest width before the collision, and passes the laser gate along its thinnest width after the collision, let us calculate how much effect this will have on the COR assuming all other assumptions are maintained. Let us use 60 mph for initial velocity and 30 mph for the final velocity, and that the ball's true diameter is 2.9 inches. The initial velocity measurement is therefore overestimated, since we compared a time step to a distance of 2.9 inches as opposed to 2.996 inches (height of seams added to either side). The true initial velocity is 58.1 mph. The post collision velocity is true, however, since it crossed the laser gate at its true diameter of 2.9 in. So the real values COR is 0.517, compared to the yielded value of 0.5. This is a deviation of roughly 3.3% from the truth value. This is significant, but again, the worst case scenario is unlikely, and high seamed baseballs are only used in certain circumstances, mostly for beginners in little league. In the mlb, they use rolled-seam balls, which have a much lower raised height of the seams (5). As a result of these observations, we will apply a 2% random uncertainty to compensate for assumption three.

The fourth assumption is the most geometrically interesting to analyze. An assumption like this can overlook errors because the ball might pass through the laser gate at an angle, breaking the lasers for longer than they would be broken given a crossing normal to the plane of the lasers. Picking out the worst case scenario for this assumption takes a bit of thought. Let us again assume a 60 mph initial velocity measurement and a 30 mph initial velocity measurement. Assume that the initial path of the ball is perfectly normal to the plane of the laser gate. But for the crossing on the way back, humor a situation depicted below in Fig. 16.

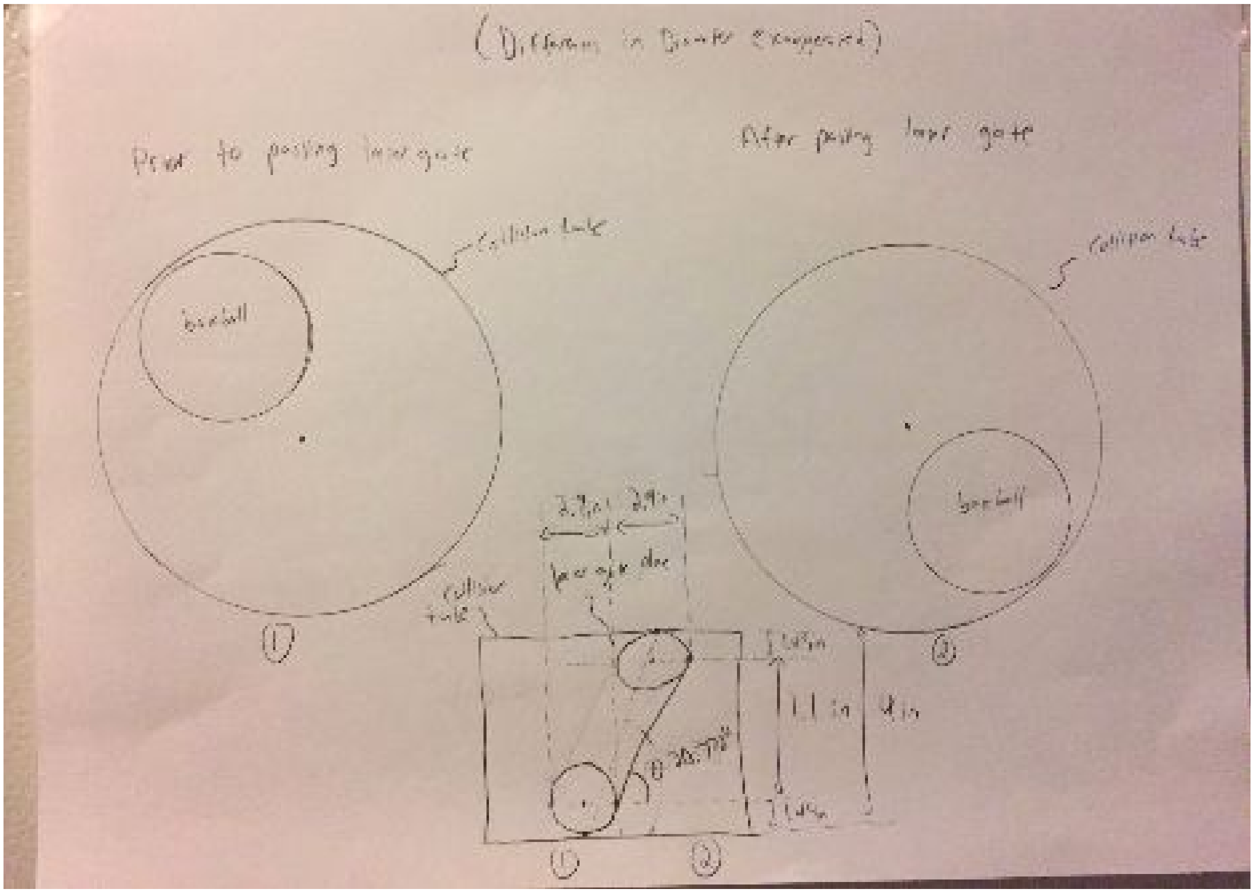


**Figure 16**. Depiction of worst case measurement scenario for assumption 4.

Although it is extremely unlikely that this physical depiction would ever actually occur, it does represent the absolute worst case scenario where our laser system would actually record a measurement for velocity. For a situation like this to occur, the ball would have to pass through the beam plane at an angle of 20.772 degrees. To achieve an angle like this, a serious random deflection is required off of the piece of wood, which would surely result in very inaccurate COR readings for reasons other than the one currently discussed. Nonetheless, when accounting for this angle of incidence with the plane, the effective diameter of the baseball as far as the laser gate is concerned changed from 2.9 in (the default radius) to 3.1 in, resulting in a true final velocity of 32.1 mph. After making this adjustment, the true COR is 0.535. The outputted value would have been 0.50, which is a deviation of 6.5% from the truth value. This percentage is significant, but again, this represents a nearly impossible fringe scenario. Angles closer to 5 degrees are much more reasonable to assume, which would result in a deviation of 0.4% from truth. We will assign an uncertainty of 1% to our values of COR due to the implications of assumption number four.

To analyze the implications of assumption five, we must look at our physical system. For air resistance to significantly affect the value of the COR, the air drag must take a significant amount of energy away from the ball from the time it first breaks the laser gate to when it passes the gate completely after the collision. The laser gate is close to 4 inches from the face of the piece of wood. The ball will experience air drag for the entire 4 inches on its way to collide with the wood. After the collision, the opposite side of the ball with experience drag, and it will need to travel 4 inches again until it completely breaks the laser gate. So the ball will experience a drag force for a total of 8 inches of distance, 4 inches at the incoming velocity, and 4 inches at the rebound velocity. For these short 4 inch distance, let us assume the velocity is constant (60 mph inbound, 30 mph rebound). Let us perform an impulse estimate to see of a significant amount of energy is lost due to air drag over this short journey. At 60 mph, the ball traverses 4 inches in roughly 0.004 seconds. At 30 mph, the ball traverses 4 inches in roughly 0.008 seconds. We will estimate the drag coefficient of a baseball at C=0.4. Performing an impulse estimation:

$$F_D = \frac{1}{2} C \rho A v^2$$

$$J = F\Delta t = m\Delta v$$

$$m = 0.01 slug$$

$$\rho = 0.0023769 \text{ slug/ft3}$$

$$C = 0.4$$

$$A = 0.046 ft^2$$

$$J_{in} = \frac{1}{2} * 0.4 * 0.0023769 \frac{slug}{ft^3} * 0.046 ft^2 * (88 \frac{ft}{s})^2 * 0.004s = 6.774 * 10^{-4} N * s$$

$$J_{out} = \frac{1}{2} * 0.4 * 0.0023769 \frac{slug}{ft^3} * 0.046 ft^2 * (44 \frac{ft}{s})^2 * 0.008s = 3.387 * 10^{-4} N * s$$

$$\Delta v_{rebound} = (J_{in} + J_{out}) / m = 0.102 \frac{ft}{s} = 0.07 mph$$

We see that the rebound velocity would only be reduced by about 0.07 mph. So the true measurement speed given no drag would be 30.07 mph, resulting in a true COR of 0.501. This is a deviation of 0.23% from the measured COR value in this scenario. So as a result, it is fair to say that air drag can be ignored.

The final assumption to analyze is the assumption of infinite sampling rate of the microcontroller. In reality, the sampling rate of the Arduino Romeo is 9600 samples a second, meaning that information is collected from its ports 9600 times every second. This reality most certainly will add some uncertainty to our COR output. Let us again consider the worst case scenario with collection at the highest recorded speed we measured, 68 mph. At 68 mph, the ball translated 2.9 inches in 0.0024 seconds. With a sampling rate of 9600 samples per second, the Romeo board is collecting a sample every 0.000104 seconds. That means over the course of the baseball breaking the lasers, the sample period will occur 23.27 times. But since samples are discrete, the board can only take 23 samples, which means the board will assign a traversal time of 23*0.000104 seconds = 0.002392 seconds compared to the real value of 0.0024 seconds. If we imagine the worst case scenario, where the ball breaks the laser plane the instant after a sample and leaves the plane the instant before another sample, the board will assign 22 sampling periods to the ball traversal time. This sampling error corresponds to a maximum of +4.7% error in the inbound velocity values. However, this is not the whole story, because on the way out, after the rebound, the ball is just as likely to get caught in a sampling "worst case" scenario. Due to the discrete nature of sampling, both the inbound and rebound velocities will almost always be overestimated because the board assigns a shorter time for a traversal than what really happened. However, this overestimation is fractionally larger the faster the ball is moving because there are fewer sampling periods over the period of traversal. Due to the mirroring of the speed measurement inaccuracies, the COR will be affected less than the individual speed measurements. If we are to expect a rebound velocity of 34 mph for an inbound velocity of 68 mph, and we assume the same sample clipping effect for the rebound velocity measurement, the deviation of the COR from the truth value is 2%. As this effect will scale with velocity, and there is usually only a single period clipping in the measurement (as opposed to two periods for the worst case scenario), we will assign a 1% uncertainty to all COR values to compensate for assumption number six.

Now having parsed through all of our assumptions and attempted to numerically assign a fractional error to all them, we will collect our terms and see what our overall error estimate come to. If we assume all sources of uncertainty are random, we combine them with the least squares method.

**Table 1**. Summary of assignment of uncertainty due to each assumption

| Assumption | Number | Assigned Uncertainty |
|---|---|---|
| Universal Diameter | 1 | 2% |
| Wood block never moves | 2a | 2% |
| No change in gravitational potential energy | 2b | 0% |
| The baseball never spins | 2c | 1% |
| Identical orientations for crossing laser gate | 3 | 2% |
| Path of ball is always normal to plane of laser gate | 4 | 1% |
| Negligible Air Drag | 5 | 0% |
| Infinite Sampling Rate | 6 | 1% |
| Least Squares Total | - | 3.91% => 4% |

So as seen from Table 1, the uncertainty we can attempt to quantify from our system leads to an assignment of 4% uncertainty. There are other potential sources of uncertainty that have nothing to do with our system, such as the inconsistency of the material properties of a baseball. Collisions are very random events and can rarely be predicted with perfect accuracy, even in such a controlled setting.

**IV. The Design Story**

The first step in designing an engineering solution is to identify the problem. Our task was to develop a method of measuring the Coefficient of Restitution of a baseball against a wood surface as a function of the relative velocity between them. With the problem defined, we moved onto problem evaluation.

***Problem Evaluation***

A practical use of this information would be for the impact of the ball of a baseball bat after a pitch. The average fastball speed in the MLB is 90.9mph with a range of 50-100mph for all pitches. The average baseball bat is made out of hardwoods like ash, hickory, and maple and 2.61in thick at the thickest point. Baseball dimensions fluctuate between 9.00-9.25in in circumference, 2.86-2.94in in diameter, and 5.00-5.25 ounces in weight. The MLB does not release the COR measurements for their baseballs, but Rodney Cross in "Physics of Baseball and Softball" writes: "Elastic behaviour in the vertical direction is described by a number called the coefficient of restitution (COR) which is about 0.5 for a baseball, depending on ball speed." This is verified by the UMass-Lowell Baseball Research Center which found the COR to vary between .514 and .578 at at 70℉ and 50% relative humidity within ±5% in relative humidity and within ±5℉ in temperature, using the ASTM wall test. The ASTM wall test uses an air cannon to propel a baseball into a stationary piece of ash. With this information, we knew that we wanted to design our system to establish a relative velocity between 50-100mph between the wood and the ball, impacting against a hardwood (preferably ash) about 2.5in thick.

We also wanted the system to be easy to use, repeatable, adjustable, inexpensive, and safe. We decided that a self contained system that displayed a high level of information and could be controlled with minimal effort would best achieve these standards. The wood surface should be flat (perpendicular to the direction of motion) and stationary to avoid complication with movement or angles. The method of propulsion should be adjustable in power and easy to set up. The methods of measurement should be non interfering.

***Considering Alternatives***

We first ruled out most methods of mechanically measuring the inbound and outbound speeds or displacement. Some systems we considered were a spring catching mechanism or pendulum catching mechanism. With our limited resources, it would be difficult to create a mechanical device that could accurately and reliably collect information on the system's energy while also maintaining good usability and minimizing complexity. This lead us to the decision that the system of propulsion should be decided upon first, and then iterated upon once designing the measurement system. This was because modifying an electrical system of measurement was much simpler than modifying a propulsion system, because of the small and stationary nature of sensors.

There were three main methods of propulsion explored: elastic, gravitational, and pressure. Combustion and magnetic propulsion methods were also suggested, but abandoned quickly with concerns of safety, cost, and feasibility. Elastic propulsion would involve a series of springs or bands to propel the ball. A ratcheting mechanism would be required to load the ball into any system with enough power to propel the ball at the desired speeds. This idea was abandoned after considering the cost of springs or bands with a large enough spring coefficient to propel a baseball to our desired speeds. Additionally, we envisioned the mechanical complexity of a ratcheting mechanism would compromise safety and simplicity to a fault. Gravitational propulsion was rejected because the height required to reach 60 mph would be about 120 ft high, or approximately 12 stories up. It would be exceedingly difficult to maintain any sort of accuracy or control of the system when dropping from such heights.

With all these methods considered, air pressure seemed like the best propellant. It was easy to contain at high pressures, powerful, accessible, and low cost. Some quick calculations showed that a system of 80 psi could propel a baseball ball well over 70 mph. The standard bike pump is rated for up to 130 psi, and the 4-in schedule 40 PVC pipe in the Trinity Engineering Woodshop is rated for operation at 133 psi. Barrel length for the air cannon could be varied to control for accuracy and easy loading, and a collision chamber could be mounted on the end to contain the ball and collision at launch. The team agreed that this was the most effective method of propulsion for this project. Below is the quick impulse-momentum calculation we performed to confirm the feasibility of an air cannon propelling a baseball in our desired speed range.

$$
\begin{aligned}
&J = \Delta P \\
&F\Delta t = m\Delta v \\
&F = P * A \\
&P_{initial} = 80\,psi
\end{aligned}
$$

Given an initial pressure of 80 psi, we conservatively assumed that the cannon would deliver an average pressure of 10 psi to the rear of the baseball on its way through the barrel. We selected one eighth the input pressure because the expansion of air in the barrel and also air escaping around the sides of the barrel would drastically reduce the pressure behind the ball as it propagated. For an effective barrel length of 2 ft, and ball moving at 60 mph, the time in the barrel is around 0.02 seconds.

$$
\begin{aligned}
&P_{average} = 10\frac{lb}{in^2} * \frac{144in^2}{1ft^2} = 1440\frac{lb}{ft^2} \\
&A = \pi / 4 * (2.9in)^2 = 6.6052in^2 = 0.0459ft^2 \\
&m = 0.145kg * \frac{1slug}{14.539kg} = 0.01slug \\
&\Delta t = 0.02s \\
&\Delta v = \frac{F\Delta t}{m} = \frac{1440\frac{lb}{ft^2} * 0.0459ft^2 * 0.02s}{0.01slug} = 132.2\frac{ft}{s} = 90.1mph
\end{aligned}
$$

So at 80 psi, our most conservative estimate landed the baseball at an exit speed of 90.1 mph. Well above our target range.

***Design Iteration and Construction***

The PVC was bought at Home Depot and the epoxy used for securing all the connections was provided by the Trinity Engineering Department. The standard tire valve, called a schrader valve, was found by visiting a series of bike shops and asking for any busted tires. Eventually, a shop cut the valve off of an old tire tubing and it was mounted on the end of the compression chamber.

The air in the chamber needed to be released quickly and consistently in order to propel the ball at the desired speeds. Butterfly valves were cheap and easily accessible, but difficult to open and put a lot of strain on the system's connections. We wanted something easy to use and consistently fast. Prebuilt solenoid quick releases were expensive, so after some research, we were able to build our own from a sprinkler valve purchased on Amazon.com. This valve prevented 15-20 psi from escaping, but we decided that the ease of use trade off was worth the speed loss.

Implementation of the electrical design required a method of sensing the ball, a controller to count the allotted time the ball was in the plane, and a way of mounting the sensors to the collision tube. Other factors to consider are time and budget constraints. A large portion of our design was contingent around these two factors.  In an effort to compensate for unexpected debugging or complications, parts were chosen in order to begin development and testing quickly. This planning proved beneficial, as the completion of the system early allowed us to correct for several inconsistencies and deliver an accurate prototype on demonstration day.

A series of laser diodes and photoresistors were chosen as the sensors because of these later factors. Other alternatives were considered, such as ultrasonic sensors, infrared emitters and receivers, and photodiodes. Ultrasonic sensors were dismissed quickly, as they are not very precise with round surfaces and each sensor must be individually calibrated for different distances and materials (texture and density). Also, with the small collision tube system desired, the base detecting distance would require the sensor to be farther away from the action than required.

IR emitters were considered because of their conic output, but dismissed because of our decision to mount the sensors outside of the collision tube to avoid damage to the electrical system. This exposed the sensors to other interfering light sources. A housing may have been developed to minimize interference, but would then interfere with the loading and racking mechanisms. The IR emitters were dismissed in lieu of other alternatives. Lasers provide a more intense, stable, and targeted light source than IR emitters, and were used instead.

Photodiodes have a faster response time and are more sensitive than photoresistors, which would make them better for this kind of application. However, they were not used because of time, cost, and familiarity. The photoresistors were available from the engineering department and cost slightly less than the photodiodes. For the estimated speeds of our mechanical system, the ~10ms sensitivity of the photoresistor was thought to be sufficient. From testing, we found this to be correct, although after analysis, we realize that this introduces a significant amount of uncertainty. The members of the team had worked with photoresistors before, and had little experience with photodiodes, and therefore decided to use the more familiar component. This was a logical error, and will be accounted for in later iterations.

Cheap laser diodes were purchased from Amazon.com (with 2-day shipping) and photoresistors were acquired through the engineering department. This allowed us to begin building and testing our system quickly, giving us the ability to debug well in advance of demonstration.

There are two schools of thought on laser speed gates; single point or dual point systems. A dual point system uses two lasers or laser grids a fixed distance apart and measures the time difference between the two points. The advantage of this system is that it can measure any object of any size. This means that this system would remove the uncertainty of the baseball’s diameter. Despite this advantage, we chose to use a single point system for our prototype. That is because this method was easier to accurately implement with our limited resources.

The dual point system provides several challenges that could not be corrected with our limited resources. First, the measurement of distance between the two points must be extremely precise. Even if the original measurement is precise, force from the impact of the ball might shake the system, causing the distance to vary from test to test. Second, the points must be perfectly parallel to each other. If not perfectly parallel, the orientation of the ball as it passes through the points will cause inconsistency of measurement. Finally, the points need to be close so that there is little space for interference from air resistance, gravity, or other external forces. All of these challenges are solved by using a single point system of measurement.

The alignment of the lasers matter significantly less with a single point system, and are much easier to implement. By relying on the diameter of the ball, the system may shake or become slightly misaligned, but still give an accurate reading for the same ball. Because the system was designed specifically for testing baseballs, the inability to test other objects was not considered an issue. In order to make sure the full diameter of the ball was measured every time, a laser grid system was developed.

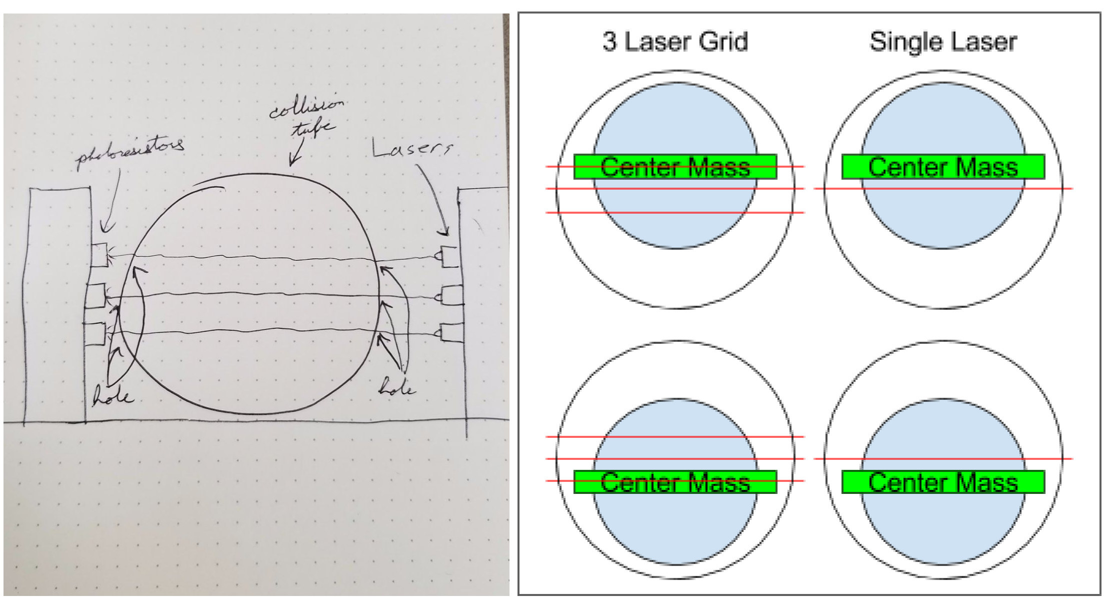


**Figure 17**. Early drafts of the speedgate system and single laser vs laser grid accuracy

The difficulty in mounting this system is that all of the lasers and resistors must be aligned accurately with enough stability to not become misaligned. Any angles with any of the lasers would cause inaccuracy. Although a drill press might be used to align holes, this became difficult when drilling into a round surface at angles, as the tube or drill would slip and become misaligned. The solution was to CAD model and 3D print a speedgate housing. Using one of the Makerbot Replicator 2s found in the Trinity College Library, we were able to print a housing accurate to .1mm.

The DFRobot Romeo board was selected almost purely out of convenience. A microcontroller board was the best way to develop and test the system at low cost. Other alternatives, such as building a digital logic circuit, might be cheaper and more precise, but also significantly more difficult to design and debug. The engineering department had several Romeo boards, as well as LCDs to work with them.

One concern we had for Arduino based boards is the limited sample rate. For a 16 MHz Arduino the ADC clock is set to 16 MHz/128 = 125 KHz. Each conversion takes 13 analog-to-digital clocks so 125 KHz /13 = 9615 Hz. Therefore, by using an arduino based microcontroller such as the Romeo board, we were limited to a 9615Hz sample rate. This would provide us with about 16 samples at any single point for a baseball of 2.9in in diameter moving at 100mph, a speed we never hoped to reach. This showed that the board would not be a bottleneck. The second concern was the cost. The Romeo board costs $29.99 for many features not required for this project. Buying a generic "Arduino" Nano board on ebay (~$2) would accomplish the same tasks, but the Romeo board was more convenient for prototyping.

While the parts were being gathered, the circuit was designed. The initial design was to to run the laser diodes out of a voltage regulator to avoid burnouts, because the lasers were low quality. Then each photoresistor would connect to a transistor that would then connect to an or-gate, and finally to a digital pin of the Romeo board. However, we realized a much simpler circuit could designed with only the resistors and laser diodes.

By running the photoresistors in series, we could avoid the need for an or gate. This works because the resistance increases by several orders of magnitude when not exposed to light. Then by using the photoresistors like the emitter of a transistor, an output voltage can be read from the difference between a reference resistance and the photoresistors. This removed the need for transistors. The Romeo board is equipped with a series of analog ports that can be used as the input for this signal. Finally the voltage regulator can be avoided by undervolting the laser diodes in parallel, which will slightly diminish the intensity of the laser, but also avoid blowouts or fluctuations from the cheap diodes. The difference in intensity turned out to be negligible for the photoresistors used. There is a 3.3V pin on the Romeo board, so we did not need to make a voltage divider. This gave us a final circuit design with minimal cost and a more precise and efficient output than originally conceived.

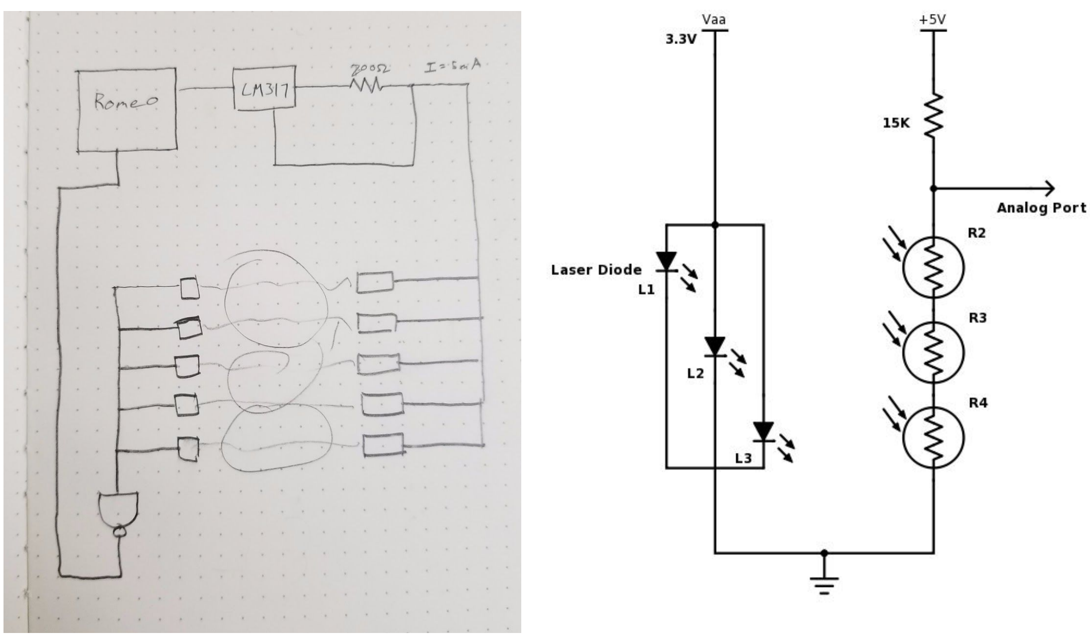


**Figure 18**. The first draft of the speedgate system vs a final circuit design

To prove the functionality of the system, the system was first developed on a breadboard and powered off of a laptop. The code was written in the Arduino IDE. This allowed for a member to code and debug while the mechanical system was being finalized by the rest of the team.

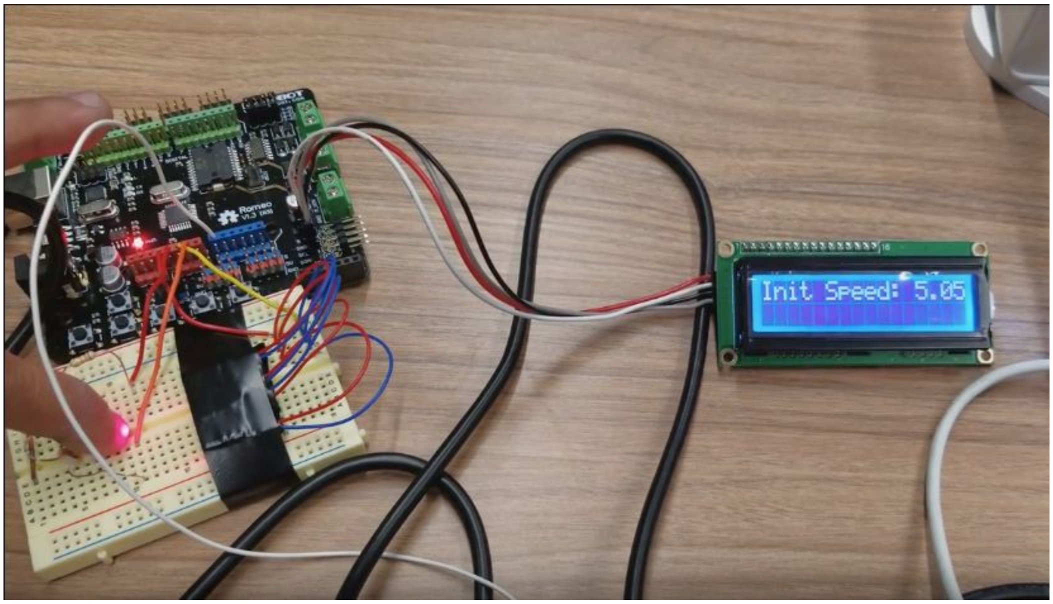


**Figure 19**. Test bench for LCD and Speedgate. The speed was found by breaking the laser with a finger.

The romeo board could be powered from any USB power supply that can output 6-23V. For testing, the system was connected to a laptop, but can be connected to a cheap cellphone charger as we did for demonstration. This increases portability, lowers cost and is more than enough to power the board, lasers, and LCD.

The final issue was debugging the code. At speeds below 50mph, the system had no problem with measurements. Both the inbound and outbound speeds and COR were consistent. Above 50mph, some of the tests would result in extremely high readings upwards of 2000mph, if any reading at all. This was because the code was originally designed to output the inbound speed before measuring the outbound speed, meaning that the microcontroller had to run the calculations for the first speed before taking the next reading. By moving all calculations to the end of the program, the system was able to accurately test up to 68.10mph. This could probably be made even more precise by utilizing photodiodes instead of photoresistors.

**V. Data**

As a user interface feature, we introduced the ability for the user to ask for a desired inbound speed of the baseball. To achieve this sort of feature, we needed to empirically determine what chamber pressures led to what inbound velocities. At the request of the user, the microcontroller could then output a recommended pressure for the chamber. Below in Fig. 19 are three empirical curves we used to implement this speed selection feature.

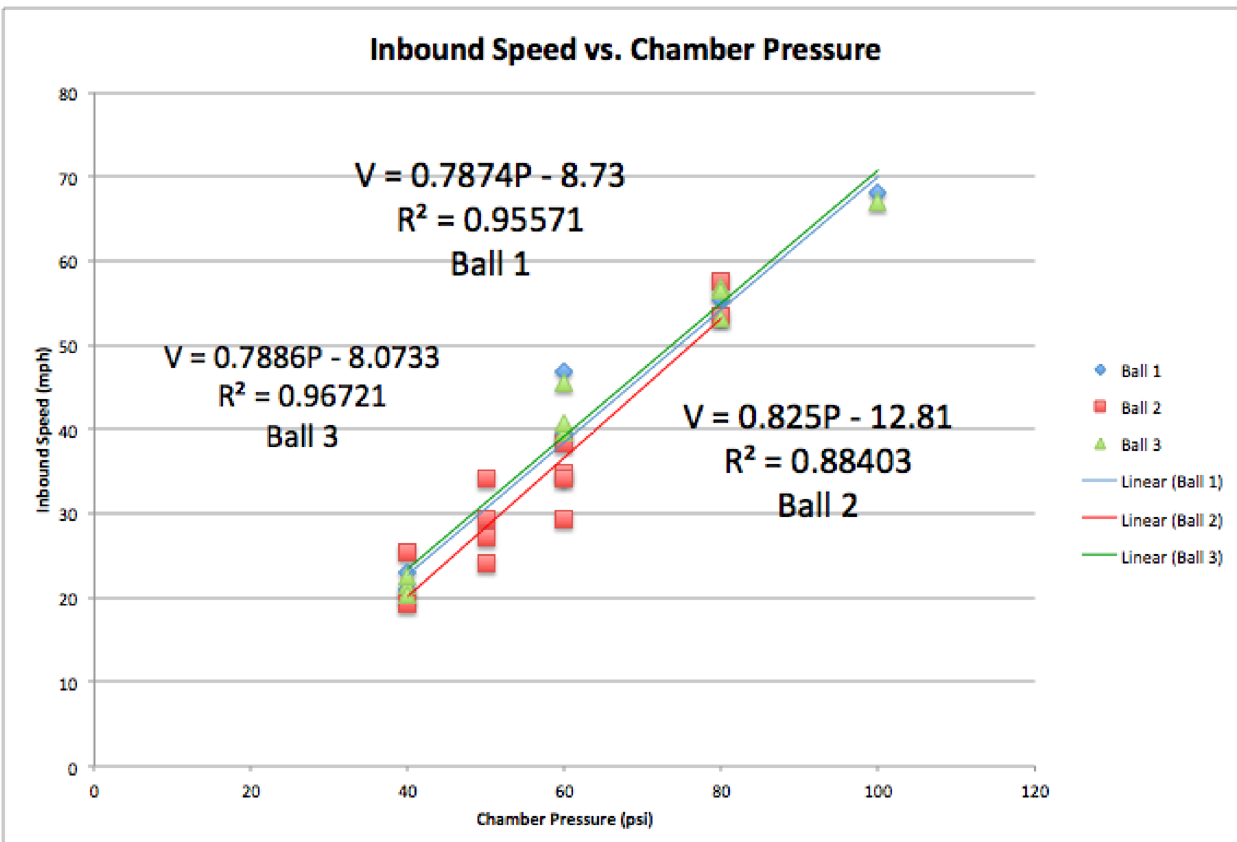


**Figure 20**. Empirical relationship for inbound speed versus chamber pressure for two different baseballs.

Not surprisingly, the three datasets for the two different balls yielded similar inbound speeds given the same chamber pressure. When we implemented the speed selection feature in our microcontroller, we simply inputted velocity, and used the above relations to get three values for chamber pressure, and responded to the user with the average of the three chamber pressures. The user would then pressurize the chamber with either a bike pump or compressor, and fire the cannon.

This next plot provides our answer to the task posed in the project description. Below is our best attempt to quantify how the COR is related to the relative velocity between a baseball and a piece of wood, or in our case, just the inbound velocity of the baseball.

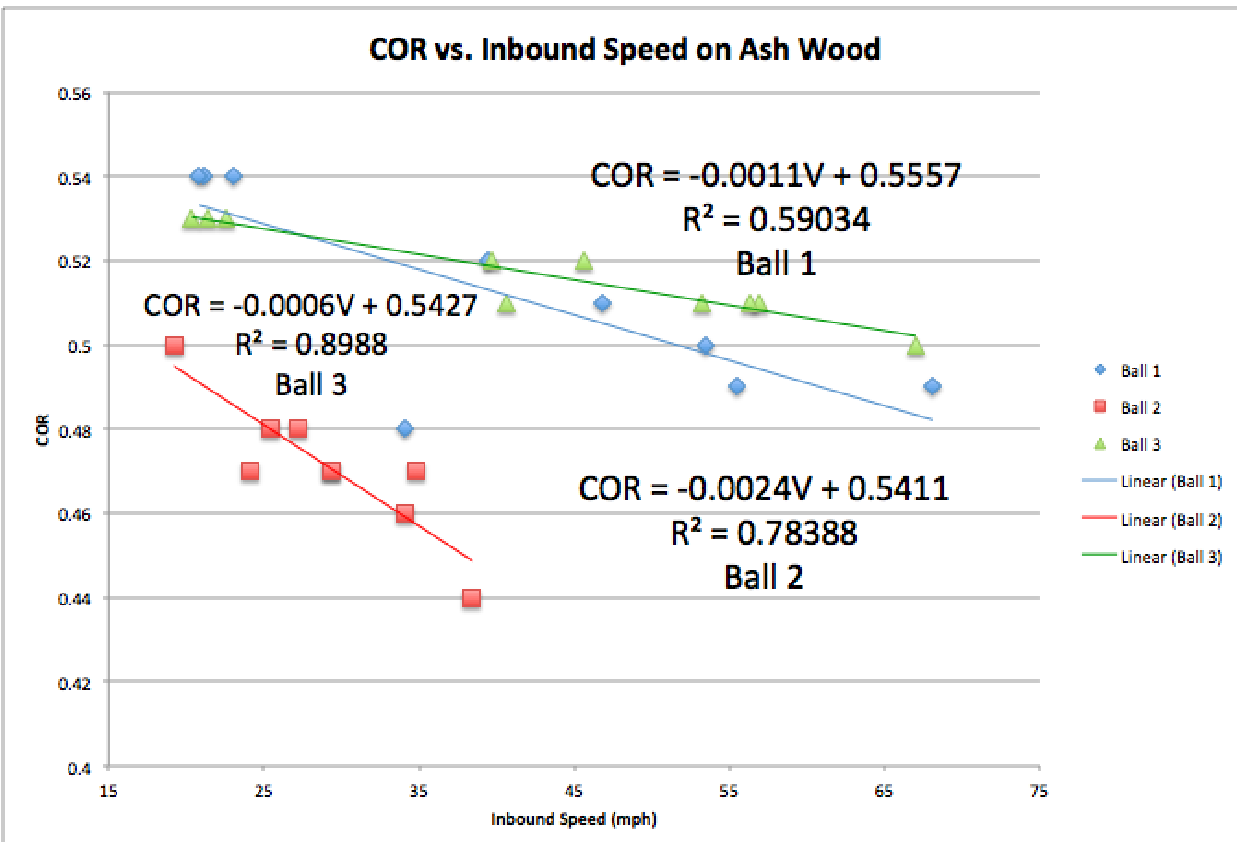


**Figure 21**. COR versus inbound speed. Basic linear fits applied to all sets of data. Uncertainty for values of COR is 4%.

As Fig. 20 above demonstrates, the COR tends to decrease with an increase in inbound velocity. We have attempted to apply linear fits to the data collected from ball 1, ball 2, and ball 3, but none of the fits is very convincing. With more data points spread out across a bigger spread of velocities, applying a mathematical model to the relationship between COR and inbound speed might be possible.

Major League Baseball rule 1.09 states “baseballs must register a rebound 54.6% of the original velocity, plus or minus 3.2 percent” to be considered legal during an “ASTM standard wall test” (6). An ASTM standard wall test constitutes shooting a ball from an air cannon perpendicularly towards a piece of 2.5” thick, 12” wide, 12” tall white ash wood mounted to a steel plate at 60 mph (7). Looking at the data points we have for ball 1 and ball 2 near 60 mph, the values for the COR do not fit within the 0.546 +- 0.032 range constituting a legal ball. This is illustrative, but not exactly damning. Our values for COR at 53.51, 55.49, and 68.10 mph were 0.50, 0.49, and 0.49 respectively for ball 1, and at 56.97, 56.38, and 67.03 mph were 0.51, 0.51, and 0.50 respectively for ball 2. These values are very close to the range identified by the MLB as “legal,” and they are also very consistent with one another. The very close grouping of the COR values demonstrates either a systematic error in the system, induced by one or multiple facets of our system, or perhaps that the balls we used were not legal. MLB baseballs are rolled seam baseballs, and many baseballs fall out of the “legal” category with use due to wearing of leather and stitching (7). The balls we used for testing were certainly not new, and were most likely not of MLB quality or specifications. Since the balls we tested with are possibly, and are frankly likely, not legal baseballs by MLB standards, it is not a fair metric to judge the efficacy of our system based on a the tested baseballs falling within the “legal” criteria. To properly apply this judgement metric, we must acquire some confirmed legal MLB baseballs, and run tests with them.

The Umass Lowell Baseball Research Center also performs COR testing using another method other than the ASTM standard wall test. They use a machine called the Baum Hitting Machine. This machine facilitates a very controlled impact between a moving bat swinging at 90 mph and a baseball moving at 70 mph. This test yielded an average COR of 0.506 in 1999 and 0.503 in 2000. These literature COR values are much closer to the values we reported for ball 1, ball 2, and ball 3. Umass Lowell offers an explanation that begins to make sense of our results. When describing the differences in COR between the ASTM standard wall test and the Baum Hitting Machine tests, Umass Lowell states "the wall method impacts the ball against a flat wall, while the hitting machine impacts the ball against a wood bat—a solid cylinder. The geometrical differences between the wall and the cylinder influence the ball deformation. The wall flattens the ball more than the bat does, and the bat allows the ball to wrap around the barrel. Second, the wall test uses a ball speed of approximately 60 mph impacting a stationary wall, while the hitting machine has a ball pitch of 70 mph impacting a bat swinging at 90 mph (the 90 mph is at the tip of the bat and this tip speed corresponds to 70 mph at the point of contact on the bat, which is 6 inches from the tip of the barrel). The ball deformation is a nonlinear phenomenon. Therefore, it is not unexpected that the COR's between the two testing methods differ." Although we cannot argue that the relative impact speed in the ASTM wall test is far more similar to our experimental system than the collision speed facilitated by the Baum Hitting Machine, the geometry of the collision in our system is more similar to the Baum Hitting Machine test. The piece of wood that the baseball collides with in our system is only 1.5" in height. Since the diameter of a baseball is 2.9", during the collision, the baseball will experience a similar wrapping effect around the piece of wood that is observed in the Baum Hitting Machine tests. Understanding this important similarity, and further acknowledging the consistency with which our system calculates COR values, we argue that our system compares more favorably to the Baum Hitting Machine than the ASTM wall test. While we cannot reconcile the significant difference in relative collision speed between the wood and the baseball between the Baum Hitting Machine and our system, the similar geometry of deformation and similar COR values our system yields to Umass Lowell suggests that our system is certainly effective for determining an accurate COR. Additionally, after applying our uncertainty assignment (plus or minus 4%) to our COR values, the spread of possible values completely encapsulates the values presented from the Baum Hitting Machine tests, and comes closer to the values yielded from the ASTM wall test.

The correlative trend in the COR data (as impact velocity increases, COR decreases) also compares favorably to professional publications. Reports from the Umass Lowell Baseball Research Center confirm the same correlative trend, but similarly struggle to apply a mathematical model to the relationship. However, it is evident that the relationship is not linear, but more of a decay to some threshold COR value at speeds in excess of 120 mph (7).

## VI. Potential Future Work/Scalability

There were some features that our team wanted to incorporate, however because of time and money restrictions, we weren't able to. When thinking of scalability and potential future designs, we thought of a many different ways we could improve our system. In order to increase the ease of use of our air cannon, we wanted to remove the human input of pumping the bike pump and simply add an electric compressor to pressurize the chamber quickly and with less effort. Ideally, the desired initial speed would be inputted and the approximate psi value of the chamber would be outputted on the LCD. The system would then automatically start the compressor until the desired psi is attained and would automatically shut off the compressor.

We additionally want to modify the speedgate to be more precise, flexible, and cheaper. With the use of the Trinity Engineering 3D printers, we can print a speedgate of the same width to include two grids of lasers with a difference in width known to 0.1mm. This would guarantee near perfect alignment, with both grids contained by the same rigid structure to avoid interference. This allows for velocities to be measured without assuming the diameter, and solves all the potential disadvantages of the two point system described earlier. The photoresistors with a 10msec response time would be replaced with photodiodes of a 20nsec response time to improve the precision of the gate. This will allow the sensors to be take full advantage of the 9600Hz sample rate on the Arduino board of choice. In this case, the arduino board should be a generic clone of the Arduino Nano, which we have found for as low as $2 from various retailers.

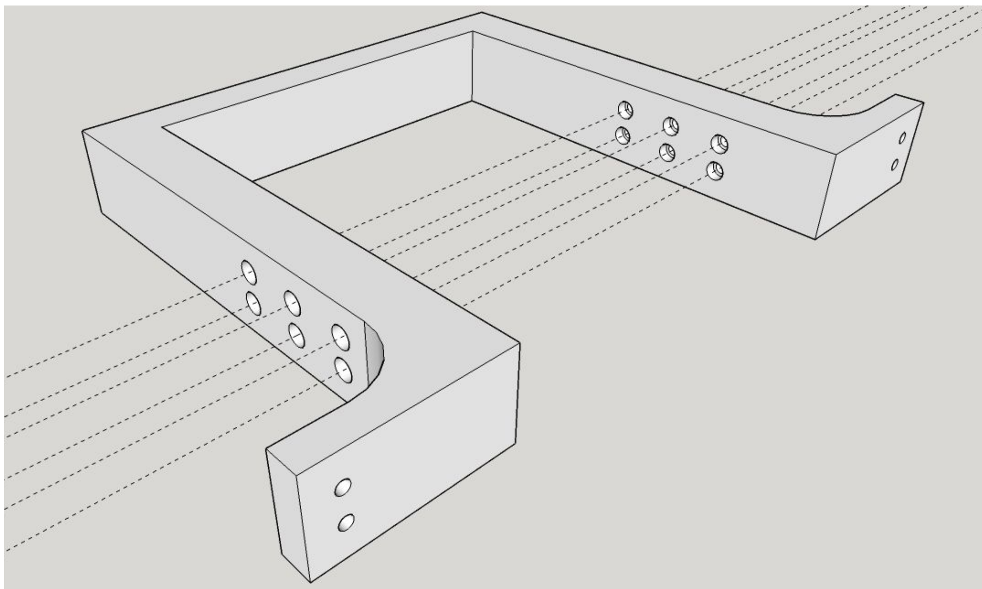

**Figure 22**. An improved speedgate design, tall enough to let the collision tube move freely, with two planes of measurement and added screw holes for permanent mounting to the system.

## VII. Concluding Remarks and Acknowledgements

Let us restate the definition of the central task: "to design and construct a system to reliably and accurately determine the COR of a baseball with a piece of wood as a function of the relative velocity between the two bodies." Per this definition, we conclude that our design was successful. We delivered a reliable, accurate, and easy to use system that resolved COR values for baseballs within the ranges presented by Umass Lowell for the Baum Hitting Machine test. Given more time, money, and resources, we would perform more extensive testing with official MLB quality baseballs to further inquire about our system's efficacy. We also would add more ease of use features to make the system more marketable.

This project was both challenging and an excellent test of collaboration. We attribute the efficacy of our design to considerable thought, planning, and playing to the strengths of our team members.

We would like to acknowledge the YouTube channel Making Random for help in constructing the air cannon. We also would like to acknowledge Andrew Musulin for extensive help with acquiring materials and assembling the system. Finally, we would like to acknowledge Prof. Mertens and Prof. Fixel for guidance along the way.

**VIII. Bill of Materials**

**Table 2**. Tabled materials and component list

| **Item** | **Subsystem** | **Price ($)** | **Quantity** | **Total ($)** |
|---|---|---|---|---|
| Irritrol Sprinkler Valve | Air Cannon | 24.89 | 1 | 24.89 |
| Air Gun Valve | Air Cannon | 7.85 | 1 | 7.85 |
| 1” Male PVC Adaptor | Air Cannon | 0.74 | 3 | 2.22 |
| 1” PVC Elbow | Air Cannon | 1.09 | 2 | 2.18 |
| 4”x3” Reducing PVC Couple | Air Cannon | 6.72 | 1 | 6.72 |
| 1”-11/2” Flush Bushing | Air Cannon | 3.70 | 2 | 7.40 |
| 3” PVC Coupling | Air Cannon | 1.43 | 1 | 1.43 |
| Bike Pump | Air Cannon | 19.96 | 1 | 19.96 |
| Ash Wood | Collision Tube | 10.00 | 1 | 10.00 |
| Mini Laser Diode 650nm | Speed Gate | 0.60 | 3 | 1.80 |

**Table 3**. Borrowed or scavenged parts

| **Item** | **Subsystem** | **Est. Price ($)** | **Quantity** | **Total ($)** |
|---|---|---|---|---|
| PVC Components | Cannon | 15.00 | 1 | 15.00 |
| DFRobot Romeo | Speed Gate | 30.00 | 1 | 30.00 |
| 3D printed part | Speed Gate | 5.00 | 1 | 5.00 |
| Photoresistor | Speed Gate | 0.05 | 3 | 0.15 |
| Wires | Speed Gate | 0.10 | 1 | 0.10 |

## IX. References


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4. Fitt, Aaron. "NCAA To Switch to Flat Seamed Balls in 2015." 06 November 2013. http://www.baseballamerica.com/college/ncaa-to-switch-to-flat-seamed-balls-in-2015/#G6uAm6p1zyMpZ2JV.97
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6. Major League Baseball Rules Project. http://mlbrulesproject.blogspot.com/2012/05/rule-109.html
7. Umass Lowell Baseball Research Center. Performance Comparison of the 1999 and 2000 Major League Baseballs. 27 June 2000.

8. Cross, Rodney. Physics of baseball & softball. Springer, 2011.

**X. Appendix**

**Table 4**. Raw Data, Ball 1.

| Pressure (psi) | V1 (mph) | V2 (mph) | COR |
|---|---|---|---|
| 40 | 21.1 | 11.35 | 0.54 |
| 40 | 20.81 | 11.27 | 0.54 |
| 40 | 23.05 | 12.49 | 0.54 |
| 60 | 39.43 | 20.52 | 0.52 |
| 60 | 34.05 | 16.46 | 0.48 |
| 60 | 46.82 | 23.78 | 0.51 |
| 80 | 53.51 | 26.75 | 0.50 |
| 80 | 55.49 | 27.24 | 0.49 |
| 100 | 68.1 | 33.29 | 0.49 |

**Table 5**. Raw Data, Ball 2

| Pressure (psi) | V1 (mph) | V2 (mph) | COR |
|---|---|---|---|
| 40 | 19.21 | 9.67 | 0.50 |
| 40 | 25.39 | 12.28 | 0.48 |
| 50 | 27.24 | 13.03 | 0.48 |
| 50 | 29.38 | 13.75 | 0.47 |
| 50 | 24.17 | 11.44 | 0.47 |
| 50 | 34.05 | 15.61 | 0.46 |
| 60 | 38.42 | 16.83 | 0.44 |

| | | | |
|---|---|---|---|
| 60 | 29.38 | 13.75 | 0.47 |
| 60 | 34.84 | 16.46 | 0.47 |

**Table 6**. Raw Data, Ball 3

| Pressure (psi) | V1 (mph) | V2 (mph) | COR |
|---|---|---|---|
| 40 | 21.42 | 11.44 | 0.53 |
| 40 | 20.37 | 10.76 | 0.53 |
| 40 | 22.61 | 12.07 | 0.53 |
| 60 | 39.66 | 20.51 | 0.52 |
| 60 | 40.70 | 20.92 | 0.51 |
| 60 | 45.65 | 23.58 | 0.52 |
| 80 | 56.97 | 28.83 | 0.51 |
| 80 | 53.21 | 26.99 | 0.51 |
| 80 | 56.38 | 28.54 | 0.51 |
| 100 | 67.03 | 33.42 | 0.50 |

## ***Code***

```
/*
Coefficient of Restitution Test System
Written by Adam Woo

This program was designed to use a laser diode array to identify when a
baseball has entered and left a single plane, giving the velocity of the object
perpendicular to the plane. The program will then analyse the restitution
properties of the object ball by comparing the initial and rebound velocity.
All data will be displayed on a 2x16 character LCD.
*/

/*----------------------------------------------------------------------------
include libraries
*/----------------------------------------------------------------------------
```

```
#include <Wire.h>
#include <LiquidCrystal_I2C.h>
LiquidCrystal_I2C lcd (0x20,16,2);

/*-----------------------------------------------------------------------------
define constants and variables
*/-----------------------------------------------------------------------------
#define baseball_diameter 75 //millimeters
#define milliPerMicro_to_milePreHour 2237.4145431945
int NUM_BUTTON = 3;
int adc_key_val[5] ={30, 150, 360, 535, 760};
int buttonPin = 7;   // choose the input pin (for pushbutton)
int button;
int stage = 0;        // variable for stage of testing
int mph = 0;          // variable for setting MPH
int psi;
int photocellPin = 0;     // the cell and 15K pulldown are connected to a0
int tests;
int buttonPress;
unsigned long SensorOne_timerIn;
unsigned long SensorOne_timerOut;
double timePlaneOne;
double speedIn;
unsigned long SensorTwo_timerIn;
unsigned long SensorTwo_timerOut;
double timePlaneTwo;
double speedOut;
double COR;

void setup(){
/*-----------------------------------------------------------------------------
initialize LCD and photoresistor pin
*/-----------------------------------------------------------------------------
  lcd.init ();
  lcd.backlight ();
  lcd.begin(16,2);
  pinMode(photocellPin, INPUT);      // declare speed gate as analog input
  Serial.begin(9600);
}

void loop(){
  while (stage == 0){
/*-----------------------------------------------------------------------------
push buttons select the desired mph to display the needed PSI, then activates
the speedgate for measurement. Allows for measurement system to ignore loading.
*/-----------------------------------------------------------------------------

    set(button);
    if (button !=0){
      delay(400);
    }
    buttonPress = analogRead(buttonPin);    // read the value from the sensor
    button = get_button(buttonPress);    // convert into key press
  }
```

```
  while (stage == 1){
/*--------------------------------------------------------------------------------
all measurement take place in the "measure" function
*/--------------------------------------------------------------------------------
    measure();
  }

  while (stage == 2){
/*--------------------------------------------------------------------------------a
ll calculations and printing done after all measurements
*/--------------------------------------------------------------------------------
    timePlaneOne = (SensorOne_timerOut-SensorOne_timerIn);
    speedIn = (baseball_diameter/timePlaneOne)*milliPerMicro_to_milePreHour;
    timePlaneTwo = (SensorTwo_timerOut-SensorTwo_timerIn);
    speedOut = (baseball_diameter/timePlaneTwo)*milliPerMicro_to_milePreHour;

    lcd.clear();
    lcd.setCursor(0,0);
    lcd.print("V1:");
    lcd.setCursor(0,1);
    lcd.print(speedIn);
//    Serial.println(timePlaneOne);        test
//    Serial.println(speedIn);

    lcd.setCursor(6,0);
    lcd.print("V2:");
    lcd.setCursor(6,1);
    lcd.print(speedOut);
//    Serial.println(timePlaneTwo);
//    Serial.println(speedOut);


    COR=speedOut/speedIn;
//    Serial.println(COR);
    lcd.setCursor(12,0);
    lcd.print("COR:");
    lcd.setCursor(12,1);
    lcd.print(COR);
    while (COR!=0){}
    }
}


void set(int action){
  if(action==1){
    mph = mph + 10;
     if (mph > 70){      // resets desired mph after 70mph because faster speeds
require dangerous PSI
      mph = 0;
      lcd.clear();
    }
  }
  else if(action==2){
    stage = 1;          // moves to measure stage
```

```
  }

   psi = round((1.2*mph)+13);   // PSI calculations found by data generated
empirical curve

  lcd.setCursor(0,0);
  lcd.print("Desired MPH: ");
  lcd.print(mph);
  lcd.setCursor(0,1);
  lcd.print("Desired PSI: ");
  lcd.print(psi);
}


void measure(){
/*-----------------------------------------------------------------------------
loops if nothing has passed though the speed gate
*/-----------------------------------------------------------------------------
  if (tests == 0){
    lcd.clear();
    lcd.setCursor(0,0);
    lcd.print("Ready for test");
    tests = 1;
  }

  while (analogRead(photocellPin)<60){}  // while grid unbroken do nothing

  if (tests == 1){
/*-----------------------------------------------------------------------------
first measurement
*/-----------------------------------------------------------------------------
     SensorOne_timerIn = micros();   //record the time the ball reaches the gate
first time
    while (analogRead(photocellPin)>60){} // while grid is broken do nothing
    SensorOne_timerOut = micros();

//    Serial.println(SensorOne_timerIn);    testing
//    Serial.println(SensorOne_timerOut);

    tests = 2;      // initiate outbound measurement
  }

  else if (tests == 2){
/*-----------------------------------------------------------------------------
second measurement
*/-----------------------------------------------------------------------------
     SensorTwo_timerIn = micros(); //record the time the ball reaches the first
gate
    while (analogRead(photocellPin)>60){}
    SensorTwo_timerOut = micros();

//    Serial.println(SensorTwo_timerIn);
//    Serial.println(SensorTwo_timerOut);
```

```
    tests = 3;
    stage = 2;
  }
}


int get_button(unsigned int input){
/*----------------------------------------------------------------------------
checks which button has been pressed (not needed if using external buttons)
*/---------------------------------------------------------------------------
  int k;
  for (k = 0; k < NUM_BUTTON; k++){
    if (input < adc_key_val[k]){
      return k;
    }
  }
  if (k >= NUM_BUTTON){
    k = 0;     // No valid key pressed
  }
  return k;
}
```

***Meeting Minutes***

**Meeting 1: 09/07/2017 12:45pm-1:30pm_Recorded by Dana Wensberg**

Meeting:

- Brainstorming: how are we getting the ball moving?
    - dropping it
    - ramp
    - chris sale
- how are we measuring energy prior to impact
    - speed gate
    - potential energy calculation
- how are we measuring after
    - bouncing up and measuring height
    - speed gate on the way out
    - spring system like a pez dispenser
- types of wood, what types, baseball applications
- bike pump solenoid shooting system
- butterfly valve
- getting it going
    - https://www.youtube.com/watch?v=PTh9XtshPIg
    - pvc air cannon, two staged, butterfly valve in between, analog/digital pressure tap in pressure chamber with insulated one way check valve for bike pump filling
    - probably need a custom adaptor for the transition
- massive spring slingshot that catches ball

**Meeting 2: 09/11/2017 6:00pm-8:15pm_Recorded by Mariam Avagyan**

People Present: everyone
Updates: We have not decided on a final design yet.
**Meeting**:
- Recap of previous ideas
- We will use a net to catch the ball after collision
- Research on COR needs to be done to find out about the wood surface that the ball is bouncing off of and whether or not we can allow the board to flex.
- Adam suggested a new idea: we have two speed gates. Fire a ball and block of wood at each other with the same relative momentum, they collide in a PVC pipe. Benefits: this design is closer to the real-life designs.
- We can't put anything on the balls (such as accelerometer) to measure speed since it will change the COR.
- Compressor: [http://www.homedepot.com/p/Husky-120-Volt-Inflator-HY120/202871788](http://www.homedepot.com/p/Husky-120-Volt-Inflator-HY120/202871788)
- Questions to ask to Prof. Mertens: would a bike pump, PVC, and other components in the system count for Ease of Use points because they can be used in everyday life for other applications?
- Consensus: we will use an air cannon to move the ball. We will use a speed gate (we will borrow one or build it). Either way, we will include "How To Build Your Own Speed Gate" in our report.
- We can use a phone camera and use image processing to measure velocity and position of collision. Cons: will make the design bigger, will lower ease of use.
- We can have the ball stationary, everything else would be moving.
- We can put something inside horizontally the PVC pipe to measure the speed (lasers, magnets…)
- Why can we use a speed gate and not a radar gun?
- Wood is hanging, the ball hits it. We measure how much the wood swung back. That's how we measure energy transfer.
- Wood is connected to a spring which is connected to the wall. The ball hits the wood and we measure the spring compression. Cons: realistic applications, reliability.
- We decided to spend most of the budget on things that will help us make better (precise and accurate) measurements.
- Let's use PASCO for a speed gate?
- Another idea: swing a baseball bat down, time it with the cannon. Consistency might be an issue.
- Create a mechanical arm that spins the ball and lets who when it's vertical. Use rpms to find the speed.
- We can have the wood with a less diameter than our PVC pipe. When the ball hits the wood, the air will have room to escape.
- Make a solenoid, railgun?
- Have two strings, pull carts
- [https://www.pasco.com/prodCatalog/ME/ME-9259_laser-switch/](https://www.pasco.com/prodCatalog/ME/ME-9259_laser-switch/)
- [http://archive.iorodeo.com/content/large-physics-photogate](http://archive.iorodeo.com/content/large-physics-photogate)

**Next meeting:** Thursday Sept 14, 4:30-6:30pm
**Assigned Tasks:** Things we need to do before the next meeting:
- Adam will talk to Prof. Mertens
- Mariam will talk to Prof. Fixel and Prof. Huang
- Dana will look into components we need for the air cannon

- Brayan can start the design report outline if he has time but it's not urgent.

**Goals**: by the end of the week we want the design chosen.

**Meeting 3: 09/14/2017 4:30pm-6:00pm_Recorded by Brayan Duarte**

**Updates**: We scavenged for items and found 4" and 3" PVC pipes, one 1" 90 degree bend, and 4" pipe end.

**Meeting:**

-After scavenging, we're finalizing the plan and seeing what else it is that we need

*Things we still need:*

- Bike pump fitting
- 4" - 1" pipe "converter?"
- Quick release, 1" female both sides
- Paste stuff
- Valve
- Drill
- Air blow gun ($7.85 on amazon prime)

**Assigned tasks:**

-Adam, Mariam, and Brayan will go to Home Depot on 09/14/2017

**Meeting 4: 09/15/2017 4:30pm-6:00pm_Recorded by Mariam Avagyan**

**Updates**: We cut the PVC pipe in the woodshop.

**Next Meeting:** 09/16/2017

**Meeting 5: 09/16/2017 10:00am-1:00pm_Recorded by Mariam Avagyan**

**People present**: everyone

**Meeting:** We are going to Home Depot to buy the following items.

- Quick Release mechanism, quick release trigger
- Bike pump, adapter, schrader valve
- 4' to inlet, outlet to 3'
- PVC cement, primer, high pressure rated epoxy
- Teflon tape

**Updates**: - Did not buy anything from Home Depot. Ordered stuff on Amazon.

- Need or order pipe connections on Amazon
- Decided to build a speed gate ourselves.

**Assigned tasks:**

- Adam and Mariam will make the speed gate (3D print, get photoresistors from Andrew)
- Dana and Brayan will find everything needed for the air cannon

***NOTE:** After September 16th, we divided work among ourselves and met numerous times. Meeting minutes were not recorded at these meetings.

**Tasks during this period:**

Dana & Brayan - Assemble Air Cannon

Adam - Model and print speed gate; code, build, and test circuit on breadboard

Mariam & Adam - Assemble speed gate

**Meeting: 9/29/2017 3:00pm-5:00pm**
**People present**: Dana, Adam
**Meeting: -** Dana and Adam built system platform in woodshop
**Updates**: - Electrical System finished and ready for implementation

**Meeting: 9/30/2017 8:00pm-10:00pm**
**People present**: Dana, Adam, Mariam
**Meeting: -** Mounted speedgate
 - Did testing, gathered data.
**Updates**: - Brayan bought a USB power bank to use for demonstration day

**Meeting: 10/1/2017 6:00pm-9:00pm**
**People present**: Dana, Adam
**Meeting: -** Debugging and troubleshooting
 - Did testing, gathered data.