莫比乌斯函数

定义

一个关于非平方数的质因子数目的函数(本质容斥定理) (1)

$$\mu(n) = \left\{egin{aligned} 1, n = 1 \ (-1)^k, n = \prod_{i=1}^k p_i \ 0, n = \prod p_i^{lpha_i}, \exists lpha_i > 1 \end{aligned}
ight\} = \prod_{i=1}^k \left[p_i
ight. \mid n
ight] st (-1)^{[p_i|n]}$$

性质

$$1:\mu(n)$$
是个积性函数 (3)

$$2: [n=1] = \sum_{d|n} \mu(d) \tag{4}$$

$$3: \sum_{d|n} \mu(d) = \begin{cases} 1, n = 1 \\ 0, n \neq 1 \end{cases}$$
 (5)

$$4: \sum_{d|n} \frac{\mu(d)}{d} = \frac{\varphi(n)}{n} \tag{6}$$