

# 积性函数

## 定义

$$\gcd(x, y) = 1, f(x * y) = f(x) * f(y) \Leftrightarrow f(n) \text{ 为积性函数} \quad (1)$$

## 性质

$$f(x), g(x) \text{ 为积性函数, 则: } f(x^p), f^p(x), f(x) * g(x), \sum_{d|n} f(d) * g\left(\frac{n}{d}\right) \text{ 都是积性函数} \quad (2)$$

## 常见的积性函数

$$\text{约数个数函数} \quad d(n) = \sum_{d=1}^n [d|n] = \sum_{d|n} 1 \quad (3)$$

$$\text{约数和函数} \quad \sigma(n) = \sum_{d=1}^n d * [d|n] = \sum_{d|n} d \quad (4)$$

$$\text{约数} k \text{ 次幂函数} \quad \sigma_k(n) = \sum_{d=1}^n d^k * [d|n] = \sum_{d|n} d^k \quad (5)$$

$$\text{欧拉函数} \quad \varphi(n) = \sum_{i=1}^n [\gcd(i, n) = 1] \quad (6)$$

$$\text{莫比乌斯函数} \quad \mu(n) = \prod_{i=1}^k [p_i \nmid n] * (-1)^{[p_i|n]} \quad (7)$$

$$\text{单位元函数} \quad \epsilon(n) = [n = 1] \quad (8)$$

