欧拉函数

定义

$$
\begin{aligned}
&\text{定义式：小于}n\text{的正整数与}n\text{互质的数目} \qquad (1)\\
&\varphi(n)=\sum_{d=1}^{d<n}[gcd(d,n)==1]\\
&\qquad\qquad \varphi(1)=1\\
&\text{通式：}n=\prod_{i=1}^{k}p_i^{\alpha_i},\varphi(n)=n*\sum_{i=1}^{k}\left(1-\frac{1}{p_i}\right)=n*\prod_{i=1}^{k}\left[\frac{1}{p_i}*(1-p_i)\right]
\end{aligned}
$$

推论

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\begin{aligned}
& \qquad (2)\\
&\text{当}n=p^k\text{时},\varphi(n)=p^k-p^{k-1}=(p-1)*p^{k-1} \qquad (3)\\
&\text{当}n=p\text{时},\varphi(n)=n-1 \qquad (4)\\
&\varphi(n)\text{是积性函数},gcd(a,b)=1\Rightarrow\varphi(a*b)=\varphi(a)*\varphi(b) \qquad (5)
\end{aligned}
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小结论

$$\tag{6}$$

$$1:\sum_{d=1}^{d<n} d*[gcd(d,n)==1]=\frac{n*\varphi(n)}{2} \tag{7}$$

$$2:x\%p=0\Rightarrow\varphi(x*p)=p*\varphi(x) \tag{8}$$

$$3:n=\sum_{d|n}\varphi(d) \tag{9}$$

$$4:\text{欧拉定理}:gcd(a,n)=1\Rightarrow a^{\varphi(n)}=1\pmod n,\text{特殊情况(费马小定理)}:a^{\varphi(p)}=x^{p-1}=1\pmod n \tag{10}$$

$$5:\text{欧拉降幂}:\begin{cases}a^n\equiv a^{n\%\varphi(m)}\pmod m,gcd(a,m)=1\\a^n\equiv a^n\pmod m,gcd(a,m)\neq1,n<\varphi(m)\\a^n\equiv a^{n\%\varphi(m)+\varphi(m)}\pmod m,gcd(a,m)\neq1,n>\varphi(m)\end{cases} \tag{11}$$

$$6:gcd(x,y)=d\Rightarrow\varphi(x*y)=\frac{\varphi(x)*\varphi(y)}{\varphi(d)} \tag{12}$$

$$7:n>2\text{时},\varphi(n)\in\text{偶数} \tag{13}$$

$$8:\frac{\varphi(n)}{n}=\sum_{d|n}\frac{\mu(d)}{d} \tag{14}$$