

dp优化-矩阵快速幂

假设转移方程： $f_i = \sum_{j=1}^n w_j f_{i-j}$, 则：

$$f_{(0,i)} = \sum_{j=1}^n w_{(0,j)} f_{(j,i)}, \text{ 即矩阵乘法形式}$$

不难构造出以下两种矩阵相乘的形式：

$B = B * A$ 形式

$$\begin{vmatrix} f_{i-1} & f_{i-2} & f_{i-3} & \cdots & f_{i-n} \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{vmatrix} * \begin{vmatrix} w_1 & 1 & 0 & 0 & \cdots & 0 \\ w_2 & 0 & 1 & 0 & \cdots & 0 \\ w_3 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ w_n & 0 & 0 & 0 & \cdots & 1 \end{vmatrix} = \begin{vmatrix} f_i & f_{i-1} & f_{i-2} & \cdots & f_{i-n+1} \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{vmatrix}$$

$B = A * B$ 形式

$$\begin{vmatrix} w_1 & w_2 & \cdots & w_n \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{vmatrix} * \begin{vmatrix} f_{i-1} & 0 & \cdots & 0 \\ f_{i-2} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ f_{i-n} & 0 & \cdots & 0 \end{vmatrix} = \begin{vmatrix} f_i & 0 & \cdots & 0 \\ f_{i-1} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ f_{i-n+1} & 0 & \cdots & 0 \end{vmatrix}$$