欧拉函数

定义

定义式:小于n的正整数与n互质的数目

$$\begin{split} \varphi(n) &= \sum_{d=1}^{d < n} \left[gcd(d,n) == 1 \right] \\ \varphi(1) &= 1 \\ \\$$
 通式: $n = \prod_{i=1}^k p_i^{\alpha_i}, \varphi(n) = n * \sum_{i=1}^k \left(1 - \frac{1}{p_i} \right) = n * \prod_{i=1}^k \left[\frac{1}{p_i} * (1 - p_i) \right] \end{split}$

推论

当
$$n=p^k$$
时, $\varphi(n)=p^k-p^{k-1}=(p-1)*p^{k-1}$
当 $n=p$ 时, $\varphi(n)=n-1$
 $\varphi(n)$ 是积性函数, $gcd(a,b)=1\Rightarrow \varphi(a*b)=\varphi(a)*\varphi(b)$

小结论

$$1:\sum_{d=1}^{d< n} d*[gcd(d,n)==1]=\frac{n*\varphi(n)}{2}$$

$$2: x\%p = 0 \Rightarrow \varphi(x*p) = p*\varphi(x)$$

$$egin{aligned} 2:x\%p &= 0 \Rightarrow arphi(x*p) = p*arphi(x) \ 3:ID(n) &= n = \sum_{d|n}arphi(d) \end{aligned}$$

$$4:$$
欧拉定理: $gcd(a,n)=1\Rightarrow a^{arphi(n)}=1\pmod{n},$ 特殊情况 $($ 费马小定理 $):a^{arphi(p)}=x^{p-1}=1\pmod{n}$

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 欧拉定理: $gcd(a,n)=1\Rightarrow a^{\varphi(n)}=1\pmod{n},$ 特殊情况(费马小定理): $a^{\varphi(p)}=x^{p-1}=1\pmod{n}$ $5:$ 欧拉降幂: $\begin{cases} a^n\equiv a^{n\%\varphi(m)}\pmod{m}, gcd(a,m)=1 \\ a^n\equiv a^n\pmod{m}, gcd(a,m)\neq 1, n<\varphi(m) \\ a^n\equiv a^{n\%\varphi(m)+\varphi(m)}\pmod{m}, gcd(a,m)\neq 1, n>\varphi(m) \end{cases}$

$$6: gcd(x,y) = d \Rightarrow arphi(x st y) = rac{arphi(x) st arphi(y)}{arphi(d)}$$

$$7:n>2$$
时, $arphi(n)\in$ 偶数

$$8:\frac{\varphi(n)}{n}=\sum_{d\mid n}\frac{\mu(d)}{d}$$