Homework 5: Solutions

Problems to turn in individually

1. Stock A's (annual) expected return is 10% and its (annual volatility) is 15%.

   (a) What is the stock's expected return over a half-year? (This means the expected value of $\int_0^{.5} \frac{dS}{S}$.)

   (b) What is the stock return's volatility over a half-year? (This means the standard deviation of $\int_0^{.5} \frac{dS}{S}$.)

   ANS:

$$\int_{t_1}^{t_2} \frac{dS}{S} = \underbrace{\mu(t_2 - t_1)}_{\text{mean}} + \underbrace{\sigma\sqrt{t_2 - t_1}}_{\text{std. dev}} Z \qquad \mu = .1, \sigma = .15$$

   (a)

$$\mu(t_2 - t_1) = .1(.5) = \boxed{.05}$$

   (b)

$$\sigma\sqrt{t_2 - t_1} = .15\sqrt{.5} = \boxed{.106}$$

2. An $n \times n$ matrix P is called "positive semi-definite" if, for any vector x, we have that

$$x^T P x \geq 0.$$

   For example, the identity matrix I is clearly positive semi-definite. In class we defined the matrix Σ to equal $\sigma\sigma^T$, where σ is the volatility matrix.

   (a) Show that Σ is positive semi-definite.

   (b) Show that Σ is symmetric. That is, show that $\Sigma^T = \Sigma$.

ANS:

We know that $x^T x \geq 0$ for any x.

(a) Since $x^T \sum x = x^T \sigma\sigma^T x = w^T w \geq 0 \quad \left(\text{where } w = \sigma^T x\right)$, we have that $\sum$ is positive semi-definite.

(b) $\sum^T = \left(\sigma\sigma^T\right)^T = \left(\sigma^T\right)^T \sigma^T = \sum$

3. In class you learned that diversification comes quickly when we invest equally in n stocks in a portfolio. What if we do not invest equally? Say $n = 100$, and we invest a certain amount in the first stock. We invest twice as much in the second stock, and thrice as much in the third stock and so on. Do we still get adequate diversification from this lopsided allocation scheme?

   ANS: Yes. Say we invest \$1 in the first stock, \$2 in the second, etc. Then the total amount invested in the stocks is $1 + 2 + ... + 100 = 5050$. The amount invested in stock 100, is \$100, and this is less than 2% of the total portfolio of 5050. So, yes, we still are plenty well diversified!

Problems to turn in as a group

1. Navigate to the Fama-French data web site and download the weekly return series for the risk free rate, the excess stock return, SMB and HML for the past 50 years. Use this data to compute the following results:

   (a) Compute the covariance matrix of returns for each of the five 10-year blocks of data. Do you see a trend in the variances? How about the covariances?

   (b) What is the empirical relation between the variance values in each 10-year block and the covariances?

   ANS: No fixed answer, depends on the data.

2. Using the WRDS data web site, download the returns on a bond index and a stock index of your choosing. Do so for the past 50 years (daily).

   (a) Compute the mean and standard deviation of returns of the two indexes.

(b) Suppose you invested \$1 in each index 50 years ago and every day re-invested the returns. Plot the value of the \$1 in each index over the 50-year period. How much does each \$1 become after 50 years?

(c) Now suppose you were clairvoyant, and knew in advance which index would perform better each day, and invested all your cumulated money each day in the better index. How much would your \$1 be worth after 50 years? Compare your answer to the ones in the previous part and comment.

ANS: No fixed answer, depends on the data.