HW 2

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1) $A(n) = 1+1+1+A(n-1)+A(n-1)+A(n-1)$

$= 3(1 + A(n-1)), \quad n \geq 0$

$A(0) = 0$

$A(1) = 3(1+0) = 3$

$A(2) = 3(1+3) = 12$

$A(3) = 3(1+12) = 39$

$A(4) = 3(39+1) = 120$

$A(n) = a(3)^n + b$

$A(0) = 0 = a + b \qquad a = -b$

$A(1) = 3 = 3a + b \qquad 3 = -3b + b = -2b$

$b = -\frac{3}{2}$

$3 = 3a - \frac{3}{2} \qquad 3a = 4.5 \qquad a = \frac{3}{2}$

$$\boxed{A(n) = \frac{3}{2}(3)^n - \frac{3}{2}}$$

2) Comparisons

$C(1) = 0$

$C(n) = n-1 + C(n-1) \qquad n > 1$

$C(2) = 2-1+0 = 1$

$C(3) = 3-1+1 = 3$

$C(4) = 4-1+3 = 6$

$$C(n) = \frac{(n)(n-1)}{2}$$

$$C(2) = \frac{(2)(1)}{2} = 1$$

$$n-1 + C(n-1) = n-1 + \frac{(n-1)(n-2)}{2} = \frac{(n-1)n}{2} = C(n)$$

Swap

$$S(n) = S(n-1) + n-1, \quad n > 1$$

$$S(n) = \frac{n(n-1)}{2}$$

$$C(n) = S(n)$$

3) $P(1) = 2$

$P(n) = 3 + 2P(\frac{n}{5})$ $\quad n = 5^m$ $\quad m = \log_5 n$

$P(5^m) = 3 + 2P(5^{m-1}) =$

$3 + 2(3 + 2P(5^{m-2})) = 3 + 2(3) + 2^2 P(5^{m-2})$

$= 3 + 2(3) + 2^2(3 + 2P(5^{m-3})) =$

$2^0(3) + 2^1(3) + 2^2(3) + 2^3 P(5^{m-3})$

$= \ldots 2^m P(5^{m-m}) \ldots 2^2(3) + 2^1(3) + 2^0(3)$

$= 2^m P(1) = 2^m(2) \ldots 2^2(3) + 2^1(3) + 2^0(3)$

$(2^{m+1})(3)$

$\sum_{i=1}^{m}$ $\quad \sum_{j=0}^{m-1} 3(2)^i + 2^m(2) = 3(2^m - 1) + 2^{m+1}$

$3(2^{\log_5 n} - 1) + 2^{\log_5 n + 1} = (3)(2)^{\log_5 n} - 3 + 2^{\log_5 n + 1}$

$M(n) = 3(2^{\log_5 n} - 1) + 2^{\log_5 n + 1}$