Homework 7

Problems to turn in individually

1. SGI (Soylent Green Industries) will make an annual profit of $P(x, y)$ (in millions of dollars), where x is the cost (in dollars) per pound of obtaining the raw materials to be made into their product and y is the average salary (in thousands of dollars) paid to employees. This year the value of x is 100 and the value of y is 50. Next year the company expects x to go up to 103, and they are thinking of reducing y to 48.

   (a) SGI scientists have determined that, near $(x, y) = (100, 50)$,

   $$P(x, y) \approx ye^{-\frac{xy}{10,000}}.$$

   Determine two approximations of $P(103, 48)$ by using a first order Taylor polynomial approximation and then a second order Taylor polynomial approximation centered at $(x, y) = (100, 50)$. (Note, a first order approximation uses up to first derivatives (that is, no second, third, etc. derivatives). A second order approximation uses up to second derivatives.)

   (b) If the SGI scientists' formula ended up being valid at $(x, y) = (103, 48)$, as well as at $(x, y) = (100, 50)$, how close were your approximations in part (a) to the actual value for $P(103, 48)$?

2. You enter into a short crude oil futures contract at \$43 per barrel. The initial margin is \$3,375 and the maintenence margin is \$2,500. One contract is for 1,000 barrels of oil. By how much do oil prices have to change before you receive a margin call?

3. The forward price of wheat for delivery in three months is \$3.90 per bushel, while the spot price is \$3.60. The three-month interest rate in continuously compounded terms is 8% per annum. Is there an arbitrage opportunity in this market if wheat may be stored costlessly?

4. A stock will pay a dividend of \$1 in one month and \$2 in four months. The risk-free rate of interest for all maturities is 12%. The current price of the stock is \$90.

(a) Calculate the arbitrage-free price of (i) a three-month forward contract on the stock and (ii) a six-month forward contract on the stock.

(b) Suppose the six-month forward contract is quoted at 100. Identify the arbitrage opportunities, if any, that exist, and explain how to exploit them.

5. A three-month forward contract on a non-dividend-paying asset is trading at 90, while the spot price is 84.

   (a) Calculate the implied repo rate.

   (b) Suppose it is possible for you to borrow at 8% for three months. Does this give rise to any arbitrage opportunities? Why or why not?

Problems to turn in as a group

1. You have a position in 200 shares of a technology stock with an annualized standard deviation of changes in the price of the stock being 30. Say that you want to hedge this position over a one-year horizon with a technology stock index. Suppose that the index value has an annual standard deviation of 20. The correlation between the two annual changes is 0.8. How many units of the index should you hold to have the best hedge?

2. You manage a portfolio of GM bonds and run a regression of your bond's price changes on the changes in the S&P 500 index futures and changes in the ten-year Treasury note futures. The regression result is as follows:

$$\delta_P = 0.02 - 0.2\,\delta_{S\&P} + 0.5\,\delta_{TRY}, \quad R^2 = 0.7$$

   where the regression above is in changes in index values for all the right-hand side variables. What positions in the two index futures will you take? What proportion of the risk remains unhedged? What implicit assumption might you be making in this case?

3. You are asked to hedge price changes in a security S over a maturity T. The correlations of price changes in S, and price changes in futures contracts F_1, F_2 are given by the following correlation matrix:

	S	F_1	F_2
S	1.00000	0.98757	0.82923
F_1	0.98757	1.00000	0.84939
F_2	0.82923	0.84939	1.00000

If the standard deviations of the price changes on the three assets are given by

$$\begin{aligned} \sigma(\Delta_S) &= 0.30 \\ \sigma(\Delta_{F_1}) &= 0.25 \\ \sigma(\Delta_{F_2}) &= 0.15 \end{aligned}$$

then, find the minimum-variance hedge for S using *both* futures contracts F_1 and F_2. Express your solution in terms of the number of units in the futures contracts F_1 and F_2 to hedge a 1 unit position in S. What can you say about the solution(s) you have arrived at?