Homework 2: Solutions

Problems to turn in individually

1. Let X be a normal random variable with a mean of 100 and a standard deviation of 50. Let Z be a standard normal random variable. Note that $\frac{X-100}{50}$ has a Z distribution. (Recall that any normal random variable minus its mean divided by its standard deviation has a standard normal distribution.) Use the Python command `norm.cdf(`z`)`$= P(Z < z)$ (or an equivalent command in R or Excel, et.) to help determine

   (a) $P(Z < 1.78)$
   (b) $P(Z > -.54)$
   (c) $P(X > 80)$
   (d) $P(65 < X < 83)$

   ANS:

   (a) $P(Z < 1.78) = \texttt{norm.cdf}(1.78) = .962$
   (b) $P(Z > -.54) = 1 - \texttt{norm.cdf}(-0.54) = .705$
   (c) $P(X > 80) = 1 - \texttt{norm.cdf}\left(\frac{80-100}{50}\right) = .655$
   (d) $P(65 < X < 83) = \texttt{norm.cdf}\left(\frac{83-100}{50}\right)$
   . $- \texttt{norm.cdf}\left(\frac{65-100}{50}\right) = .125$

2. Recall in class that we said if $P(X = 1) = p$ and $P(X = 0) = q = 1-p$ then X's mean and standard deviation are p and $\sqrt{pq}$ respectively. Use the discrete formulas defining the mean and variance from class to prove that p and $\sqrt{pq}$ are correct.

   ANS:

   $E[X] = 0 * q + 1 * p = p$
   $Var[X] = (0-p)^2 q + (1-p)^2 p = p^2 q + q^2 p = pq(p+q) = pq$
   $\Rightarrow SD[X] = \sqrt{Var[X]} = \sqrt{pq}$

3. Assume each day that the stock of "Cocktail Monkeys and Umbrellas, Inc." has a 70% chance of increasing 0.5% (note that's *half* a percent, not 50 percent!) and a 30% chance of decreasing 0.6%. Assuming the movement on any given day has no influence upon the movement on any other day, determine the probability that after the next 252 trading days the stock will have neither lost more than a third of its original value nor gained over a half of its original value. Use the Python command `norm.cdf(`z`)`$= P(Z < z)$ (or its equivalent in R or Excel, et.) to approximate this using the central limit theorem and then compare this approximation to the exact value.

ANS:

Increasing 50%:

$$\begin{aligned}(1.005)^r(.994)^{252-r} &= 1.5\\ \left(\frac{1.005}{.994}\right)^r &= \frac{1.5}{(.994)^{252}}\\ \Rightarrow r = \frac{\ln\left(\frac{1.5}{(.994)^{252}}\right)}{\ln\left(\frac{1.005}{.994}\right)} &= 174.6\end{aligned}$$

Decreasing by a third:

$$r = \frac{\ln\left(\frac{\frac{2}{3}}{(.994)^{252}}\right)}{\ln\left(\frac{1.005}{.994}\right)} = 100.9$$

Now define

$$X_i = \begin{cases} 1 & \text{if increases} \sim .5\% \text{ on day } i\\ 0 & \text{if decreases} \sim .6\% \text{ on day } i\end{cases}$$

So,

$$\begin{aligned}P(101 < \sum_{i=1}^{252} X_i < 174) &= P\left(\frac{\frac{101}{252}-.7}{\frac{\sqrt{.3*.7}}{\sqrt{252}}} < \overbrace{\frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}}^{Z} < \frac{\frac{174}{252}-.7}{\frac{\sqrt{.3*.7}}{\sqrt{252}}}\right)\\ &= P\left(-\overbrace{10.36}^{\text{huge!}} < Z < -.3299\right)\\ &\approx P(Z < -.3299) = \boxed{.3707}\end{aligned}$$

4. Download daily data for your favorite stock from the internet. Grab at least 5 years of data. Using this data calculate the following using R. Write a program to read in your data file and run all the analyses. Do not use the command line for conducting the analyses in this question.

   (a) What is the mean daily return? Compute the mean annual return. Report both values.

   (b) Compute the daily standard deviation of returns, and the annual standard deviation as well.

   (c) Assuming that the returns are normally distributed, simulate a histogram of the returns on the stock for a horizon of 1 year. Choose a suitable number of random draws from the distribution of returns.

   (d) Repeat the analysis for the same stock using a horizon of 5 years. Compare the graph in this question with the one from the previous question. What can you say about the two graphs?

   (e) Convert each daily return into an annualized value, thereby generating a time series of annualized returns. Now plot the histogram of this time series. Overlay this graph on the one from question 4c. Explain what you observe from this comparison. What can you say about the normality assumption?

   (f) Using the mean and variance you calculated previously for daily returns, calculate the probability that the stock will return -10% or worse after 20 trading days. Use the normal distribution for your calculation.

   (g) *Bootstrap*: You will now undertake a simulation using the original data to calculate the probability that a stock will lose at least 10% of its value in 20 trading days. One sample draw is as follows – randomly select 20 of the returns from the downloaded data. Sum the returns to see if they are -10% or less. Repeat this 100,000 times. Keep track of how many times the condition is satisfied to determine the answer to the question. (This is known as bootstrapping from empirical data). Compare your answer to that of the previous question. Explain what you find.

   (Note: answers will vary depending on the stock that you downloaded).

ANS:

No common solution as answers will vary with data. The R program is as follows:

```
stkret = read.table("Data.txt",header=TRUE)
n = length(stkret)
ny = n/5
mean_daily_return = mean(stkdata)
mean_annual_return = mean_daily_return*ny

sd_daily = sd(stkret)
sd_annual = sd_daily*sqrt(ny)

m=100000
simret = rnorm(m)*sd_annual + mean_annual_return
hist(simret,100)

simret = rnorm(m)*sd_annual*sqrt(5) + mean_annual_return*5
hist(simret,100)

aret = stkret*ny
hist(aret,100)

mu = mean_daily_return*20
sig = sd_daily*sqrt(20)
z = (-0.1-mu)/sig
print(pnorm(z))

count_loss = 0
for (j in 1:m) {
    w1=rnorm(n)
    w2=sort(w1,index.return=TRUE)
    w3 = which(w2<=21)
    sumret = sum(stkret(w3))
    if (sumret < -0.20) { count_loss = count_loss + 1 }
}
print(count_loss)
```

Problems to turn in as a group

1. Recall that if the random variable X represents the number of times a class of i.i.d. events occur within a fixed time period, then X will have a Poisson distribution. For Poisson distributions, the mean happens to be equal to the variance. On average, each hour, 14 daytraders in the world will go broke. Assuming the time each trader goes broke is i.i.d., approximate the chance that between 2300 and 2400 will go broke next week.

   ANS:

   $X_i =$ number of day traders going broke in hour i, where $i = 1, 2, 3, ..., 168$.
   $\mu = 14, \sigma^2 = 14 \Rightarrow \sigma = \sqrt{14}$ and so

$$\begin{aligned} P\left(2300 < \sum_{i=1}^{168} X_i < 2400\right) &= P\left(\frac{\frac{2300}{168} - 14}{\frac{\sqrt{14}}{\sqrt{168}}} < Z < \frac{\frac{2400}{168} - 14}{\frac{\sqrt{14}}{\sqrt{168}}}\right) \\ &= P\left(-1.07 < Z < .990\right) \\ &= \boxed{.697} \end{aligned}$$

2. During each trading hour, stock in "Pink Flamingoes and Garden Gnomes R Us" has a 25% chance of losing a dollar, a 25% chance of staying the same, and a 50% chance of gaining 50 cents. Approximate the chance that the stock will neither gain nor lose a total of more than 10 dollars over the next 300 trading hours.

   ANS:

$$\begin{aligned} P\left(X_i = -1\right) &= .25 \\ P\left(X_i = 0\right) &= .25 \\ P\left(X_i = .5\right) &= .50 \end{aligned}$$

   implies

$$\begin{aligned} \mu &= -1 * .25 + 0 * .25 + .5 * .5 = 0 \\ \sigma^2 &= (-1 - 0)^2 * .25 + (0 - 0) * .25 + (.5 - 0)^2 * .5 = .375 \\ \sigma &= \sqrt{.375} = .6124 \end{aligned}$$

and so

$$
\begin{aligned}
P\left(-10 < \sum_{i=1}^{300} X_i < 10\right) &= P\left(\frac{\frac{-10}{300} - 0}{\frac{.6124}{\sqrt{300}}} < Z < \frac{\frac{10}{300} - 0}{\frac{.6124}{\sqrt{300}}}\right) \\
&= P\left(-.943 < Z < .943\right) \\
&= \boxed{.654}
\end{aligned}
$$