1) $\Pi = nS + C$

$S\,dn = dS\,dn + dc = 0$

$\Pi = V(S,t) + nS + C$

$dS = \mu S\,dt + \sigma(S,t)\,dB$

$dS^2 = (\sigma(S,t))^2 S^2\,dt$

$d\Pi = \frac{dV}{dt}dt + \frac{dV}{dS}dS + \frac{1}{2}\frac{d^2V}{dS^2}(dS)^2 + n\,dS + rc\,dt$

$d\Pi = (V_t + \frac{1}{2}(\sigma(S,t))^2 S^2 V_{SS} + rc)dt + \cancel{(n + V_S)dS}$

$= (V_t + \frac{1}{2}(\sigma(S,t))^2 S^2 V_{SS} + rc)dt$

$d\Pi = r\Pi\,dt$

$r\Pi\,\cancel{dt} = (V_t + \frac{1}{2}(\sigma(S,t))^2 S^2 V_{SS} + rc)\cancel{dt}$

$r\Pi = V_t + \frac{1}{2}((\sigma(S,t))^2 S^2 V_{SS} + rc$

$\cancel{r}(V(S,t) + nS + \cancel{c}) = V_t + \frac{1}{2}((\sigma(S,t))^2 S^2 V_{SS} + \cancel{rc}$

$0 = V_t + rSV_S + \frac{1}{2}(\sigma(S,t))^2 S^2 V_{SS} - rV(S,t)$

2) $V(S,T) = S(T)$ at all times

$V = S$

$\frac{dV}{dt}dt + \frac{dV}{dS}dS + \frac{1}{2}\frac{\partial^2V}{dS^2}(dS)^2 + n\,dS + rc\,dt$

$\frac{dV}{dS} = 1 \qquad \frac{d^2V}{dS^2} = 0$

$\frac{dV}{dt} dt + dS + n\,dS + rc\,dt$

$= (V_t + rc)\,dt + \cancel{(1+n)\,dS} \qquad n = -1$

$r\pi \cancel{dt} = (V_t + rc)\cancel{dt} \qquad \pi = V + nS + c$

$r(V + nS + c) = V_T + rc$

$rV - rS + \cancel{rc} = V_T + \cancel{rc} \qquad V_T = S_T$

$rV = V_T + rS$

$V = \frac{S_T + rS}{r}$

For no arbitrage to exist, the value of $V = S$. If $V > S$, then a riskless arbitrage is possible. Same for $V < S$.

a) $V = S^2 \qquad V(S,T) = S^2(T)$

$0 = \frac{1}{2}\sigma^2 S^2 (2) + rS(2S) + rS^2$

$= (\sigma^2 + r)S^2 \neq 0$

$V = S^2 h(t) \qquad V = S^2(T)\,h(T) \qquad h(T) = 1$