NT 9-13, NT-17

NT-9) i) $3x \equiv 2 \pmod{14}$

$\gcd(3,14) = 1$

$2 \equiv 3x \pmod{14}$ $\quad 3^{-1} \pmod{14}$

$14 = 3 \cdot 4 + 2$ $\quad 1 = 3 - 2 \cdot 1$

$3 = 2 \cdot 1 + 1$ $\quad 1 = 3 - (14 - 3 \cdot 4)$

$1 = 5 \cdot 3 - 1 \cdot 14$

$5 \cdot 3 \bmod 11 =$

$3^{-1} 2 = 5 \cdot 2 = 10$ $\quad x = 10$

$30 \equiv 2 \pmod{14}$

ii) $3x \equiv 2 \pmod{15}$

$\gcd(3,15) = 3$

$2 \equiv 3x \pmod{15}$

No solutions, $2 \notin \{0, 3, 6, 9, 12\}$

iii) $3x \equiv 6 \pmod{15}$

$6 \equiv 3x \pmod{15}$

$x \equiv 2 \pmod{5}$

$x = \{2, 7, 12\}$

iv) $37x \equiv 51 \pmod{100}$

$51 \equiv 37x \pmod{100}$

$x = 23$

$37 \cdot 23 = 851 \equiv 51 \pmod{100}$

NT-10) Sum of squares of 2 odd

n_1	n_2	$n_1^2 + n_2^2$	$(n_1^2 + n_2^2) \bmod 4$
1	9	82	2
3	11	130	2
5	13	194	2
7	15	274	2
9	17	370	2
11	19	482	2
13	21	610	2

n	n^2	$n^2 \bmod 4$
1	1	1
2	4	0
3	9	1
4	16	0

Squares mod 4 are always 1 or 0. The sum of two odds squared are always 2.

NT-11) If x is even, x^3 and x are even. Even + Even = Even. Even + 1 = odd.

If x is odd, x^3 is odd and x is odd. Odd + odd = Even. Even + 1 = odd.

NT-12) i) $32 = 2^5$ $\phi(2^5) = 2^4(2-1) = 16$

ii) $\phi(100) = \phi(2^2 \cdot 5) = 2(2-1)(5-1) = 8$

iii) $\phi(3600) = 2^4 \cdot 3^2 \cdot 5^2 = 2^3(2-1)\,3(3-1)\,5(5-1)$

$= 960$

iv) $\phi(35) = 5 \cdot 7 = (5-1)(7-1) = 24$

v) $= \phi(77) = 7 \cdot 11 = (7-1)(11-1) = 60$

NT-13) IF $n = pq$ then $\phi(n) = \phi(pq) = \phi(p)\phi(q)$

NT-17) The cases when $\phi(3n) = 3\phi(n)$ when $3 \mid n$. We are looking to see which numbers $\phi(q \cdot n) = q\phi(n)$ occurs when $q \mid n$.