Homework 5

Problems to turn in individually

1. Stock A's (annual) expected return is 10% and its (annual) volatility is 15%.

   (a) What is the stock's expected return over a half-year? (This means the expected value of $\int_0^{.5} \frac{dS}{S}$.)

   (b) What is the stock return's volatility over a half-year? (This means the standard deviation of $\int_0^{.5} \frac{dS}{S}$.)

2. (Note: We'll use the results from this problem in class on the day this homework set is due to solve Markowitz mean-variance portfolio optimization.) An $n \times n$ matrix P is called "positive semi-definite" if, for any vector x, we have that

   $$x^T P x \geq 0.$$

   For example, the identity matrix I is clearly positive semi-definite. In class we defined the matrix Σ to equal $\sigma\sigma^T$, where σ is the volatility matrix.

   (a) Show that Σ is positive semi-definite.

   (b) Show that Σ is symmetric. That is, show that $\Sigma^T = \Sigma$.

3. In class you learned that diversification comes quickly when we invest equally in n stocks in a portfolio. What if we do not invest equally? Say $n = 100$, and we invest a certain amount in the first stock. We invest twice as much in the second stock, and thrice as much in the third stock and so on. Do we still get adequate diversification from this lopsided allocation scheme?

Problems to turn in as a group

1. Navigate to the Fama-French data web site (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`) and download the weekly return series for the risk free rate (Rf), the excess stock return (Rm-Rf), SMB and HML for the past 50 years from this file: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_Research_Data_Factors_weekly_TXT.zip`. Use this data to compute the following results:

   (a) Compute the covariance matrix of returns for each of the five 10-year blocks of data. Do you see a trend in the variances? How about the covariances?

   (b) What is the empirical relation between the variance values in each 10-year block and the covariances?

2. Using the WRDS data web site, download the returns on a bond index and a stock index of your choosing. Do so for the past 50 years (daily). I have done this for you and placed the files in the course site as follows:

   - `SPReturn_1950_2015.csv` : this file contains the value-weighted return on the S&P500 from January 1950 to December 2015. The returns are expressed in decimals (be careful with your units!).
   - `BondReturn_1961_2014.csv` : this file contains bond returns from January 1961 to December 2014. Returns are in the column "retadj" and are expressed in percentage. Make sure to use the same units across the two data sets.

   Organize the data so that both series are merged into one data file with the same dates and the data runs from 1961-01 through 2014-31. Then, answer the following questions (use any means you like, Python, R, Excel).

   (a) Compute the mean and standard deviation of returns of the two indexes.

   (b) Suppose you invested \$1 in each index 50 years ago and every day re-invested the returns. Plot the value of the \$1 in each index over the 50-year period. How much does each \$1 become after 50 years?

(c) Now suppose you were clairvoyant, and knew in advance which index would perform better each day, and invested all your cumulated money each day in the better index. How much would your \$1 be worth after 50 years? Compare your answer to the ones in the previous part and comment.