1) $dX = \mu X dt + \sigma X dB$

$f = Y = X^a$

$$d(X^a) = \frac{df}{dX} dX + \frac{1}{2}\frac{d^2 f}{dX^2}(dX)^2$$

$$= (aX^{a-1})dX + \frac{1}{2}\left((a)(a-1)X^{a-2}\right)(dX)^2$$

$$= (aX^{a-1})(\mu X dt + \sigma X dB) + \frac{1}{2}\left((a)(a-1)X^{a-2}\right)(\mu X dt + \sigma X dB)^2$$

$$\mu^2 X^2 \cancelto{0}{dt^2} + 2\mu\sigma X^2 \cancelto{0}{dt}\, dB + \sigma^2 X^2 \cancelto{dt}{dB^2}$$

$$= (aX^{a-1})(\mu X dt + \sigma X dB) + \frac{1}{2}\left((a)(a-1)X^{a-2}\right)(\sigma X^2 dt)$$

$$d(Y) = d(X^a)$$

$$= \left(aX^{a-1}\mu X + \frac{1}{2}(a)(a-1)X^{a-2}\sigma X^2\right)dt + (aX^{a-1}\sigma X)dB$$

2) a) $df = (f_t + \frac{1}{2}f_{BB})dt + f_B dB$

$dg = (g_t + \frac{1}{2}g_{BB})dt + g_B dB$

$$d(fg) = f\left((g_t + \tfrac{1}{2}g_{BB})dt + g_B dB\right) + g\left((f_t + \tfrac{1}{2}f_{BB})dt + f_B dB\right)$$
$$+ \left((f_t + \tfrac{1}{2}f_{BB})dt + f_B dB\right)\left((g_t + \tfrac{1}{2}g_{BB})dt + g_B dB\right)$$

$$= \left(f(g_t + \tfrac{1}{2}g_{BB}) + g((f_t + \tfrac{1}{2}f_{BB}) + f_B g_B\right)dt + (f g_B + g f_B)dB$$

b) $df = \left(f_t + \frac{1}{2} f_{BB_1}\right) dt + f_{B_1} dB_1$

$dg = \left(g_t + \frac{1}{2} g_{BB_2}\right) dt + g_{B_2} dB_2$

$$d(fg) = f\left(\left(g_t + \frac{1}{2} g_{BB_2}\right) dt + g_{B_2} dB_2\right) + g\left(\left(f_t + \frac{1}{2} f_{BB_1}\right) dt + f_{B_1} dB_1\right) + \left(\left(f_t + \frac{1}{2} f_{BB_1}\right) dt + f_{B_1} dB_1\right)\left(\left(g_t + \frac{1}{2} g_{BB_2}\right) dt + g_{B_2} dB_2\right)$$

$$= \left(f\left(g_t + \frac{1}{2} g_{BB_2}\right) + g\left(f_t + \frac{1}{2} f_{BB_1}\right)\right) dt + \left(f g_{B_2}\right) dB_2 + \left(g f_{B_1}\right) dB_1$$