1) $\Pi = nS + C$

$S dn = dS d_1 + dC = 0$

$\Pi = V(S,t) + nS + C$

$dS = \mu S dt + \sigma(S,t) dB$

$dS^2 = (\sigma(S,t))^2 S^2 dt$

$d\Pi = \frac{dV}{dt} dt + \frac{dV}{dS} dS + \frac{1}{2}\frac{d^2V}{dS^2}(dS)^2 + n dS + rC dt$

$d\Pi = (V_t + \frac{1}{2}(\sigma(S,t))^2 S^2 V_{SS} + rC) dt + (n + V_S) dS$

$= (V_t + \frac{1}{2}(\sigma(S,t))^2 S^2 V_{SS} + rC) dt$

$d\Pi = r\Pi dt$

$r\Pi dt = (V_t + \frac{1}{2}(\sigma(S,t))^2 S^2 V_{SS} + rC) dt$

$r\Pi = V_t + \frac{1}{2}(\sigma(S,t))^2 S^2 V_{SS} + rC$

$r(V(S,t) + nS + C) = V_t + \frac{1}{2}(\sigma(S,t))^2 S^2 V_{SS} + rC$

$0 = V_t + rSV_S + \frac{1}{2}(\sigma(S,t))^2 S^2 V_{SS} - rV(S,t)$

2) $V(S,T) = S(T)$ at all times

$V = S$

$\frac{dV}{dt} dt + \frac{dV}{dS} dS + \frac{1}{2}\frac{\partial^2 V}{dS^2}(dS)^2 + n dS + rC dt$

$\frac{dV}{dS} = 1 \qquad \frac{d^2V}{dS^2} = 0$

$\frac{dV}{dT}dt + dS + n\,dS + r\,c\,dt$

$= (V_t + rc)dt + (1+n)\,dS \qquad n = -1$

$r\pi\,dt = (V_t + rc)dt \qquad \pi = V + nS + C$

$r(V + nS + C) = V_T + rc$

$rV - rS + rC = V_T + rc \qquad V_T = S_T$

$rV = V_T + rS$

$V = \frac{S_T + rS}{r}$

a) Guess: $V = S^2 \qquad g(S) = S^2$