1) $Y = Pe^{rt}$

30 years 7% and 6%

$Y = e^{(0.07)(30)} = 8.17$

$Y = e^{(0.06)(30)} = 6.05$

$\frac{8.17}{6.05} = 1.35$

5% and 4%

$Y = e^{(0.05)(30)} = 4.48$

$Y = e^{(0.04)(30)} = 3.32$

$\frac{4.48}{3.32} = 1.35$

The resulting ratios are not surprising since they both differ by 1%.

2) $Y = Pe^{rt}$

$2 = e^{rt}$

$\ln(2) = rt$

$rt = 0.693$

3) $r(t) = 0.05 + 0.02t$ $\quad r(2) = 0.09$

$\ln(Y) = \int 0.05 + 0.02t \, dt$

$= (0.05t) + (0.01t^2)$

$Y = e^{0.05t + 0.01t^2}$

$Y = 1000e^{0.05(2) + 0.01(4)} = 1150.27$

4) $\int \frac{dy}{ry+k} = \int dt$

$\int \frac{dy}{ry+k} = t$ $\quad\quad u = ry+k$

$du = r\,dy$

$\int \frac{du}{ru} = \frac{1}{r}\int \frac{du}{u} = \frac{1}{r}\ln(u) = \frac{1}{r}\ln(ry+k) + C$

$\frac{1}{r}\ln(ry+k) + C = t$

$\ln(ry+k) = (t+C)r$

$ry+k = e^{r(t+C)}$

$y = \frac{Ce^{rt} - k}{r}$

<u>For Person A</u>

First 10 years: $Y_0 = \frac{Ce^{(r)(10)} - k}{r}$ $\quad\quad C = K$

After Cocaine: $Y_t = Ce^{rt}$ $\quad\quad C = Y_0$

<u>For person B</u>

$Y_B = \frac{Ce^{(r)(t)} - k}{r}$

$Y(0) = 0$ $\quad\quad 0 = \frac{C-k}{r}$ $\quad\quad C = K$

$Y_0 e^{rt} = \frac{Ke^{rt} - k}{r}$ $\quad\quad Y_0 = \frac{Ke^{r(10)} - k}{r}$

$rY_0 e^{rt} = Ke^{rt} - K$

$K = Ke^{rt} - rY_0 e^{rt}$

$K = e^{rt}(K - rY_0)$

$$K - rY_0 = K - (Ke^{10r} - K) = 2K - Ke^{10r} = K(2 - e^{10r})$$

$$e^{rt} = \frac{K}{K - rY_0} = \frac{\cancel{K}}{\cancel{K}(2-e^{10r})} = \frac{1}{(2-e^{10r})}$$

$$\ln(e^{rt}) = \ln\left(\frac{1}{2-e^{10r}}\right)$$

$$rt = -\ln(2-e^{10r})$$

$$t = -\frac{\ln(2-e^{10r})}{r}$$

a) $r = 0.05$

$$t = -\frac{\ln(2-e^{10 \cdot 0.05})}{0.05} = 20.92 \text{ years}$$

b) $r = 0.06$

$$t = -\frac{\ln(2-e^{10 \cdot 0.06})}{0.06} = 28.78 \text{ years}$$

c) $r = 0.07$

$$t = -\frac{\ln(2-e^{10 \cdot 0.07})}{0.07} = \text{No answer}$$

When $r = 0.07$, we don't get an answer since $2 - e^{10 \cdot 0.07}$ is a negative number, and you can't take the natural log of a negative number. Since the limit of $\ln(x)$ as x approaches 0 is negative infinity, t would be equal to ∞ the closer x in $\ln(x)$ is to 0. B would never catch up to A in this case if $r \geq 0.07$.