1) It should be the same since you are still minimizing

$L = V_\pi$

$V_\pi = w^T \Sigma w$

$= \sum_{k,l=1}^{n} w_k \Sigma_{kl} w_l$

1) $\frac{\partial}{\partial x_i}\left(L - \sum_{j=1}^{m} \lambda_j g_j\right) = 0$

2) $\frac{\partial}{\partial \lambda_i}\left(L - \sum_{j=1}^{m} \lambda_j g_j\right) = 0$

$g_1 = \sum_{k=1}^{n} w_k \mu_k - E_\pi = 0$

$g_2 = \sum_{k=1}^{n} w_k - 1 = 0$

$\frac{\partial}{\partial w_i}\left(L - \lambda_1 g_1 - \lambda_2 g_2\right) = 0$

$\left(\frac{\partial w_k}{\partial w_i} \Sigma_{kl} w_l + w_k \Sigma_{kl} \frac{\partial w_l}{\partial w_i}\right) - \lambda_1 \sum_{k=1}^{n} \frac{\partial w_k}{\partial w_i} \mu_k - \lambda_2 \sum_{k=1}^{n} \frac{\partial w_k}{\partial w_i} = 0$

$= \left(\Sigma w + \Sigma^T w\right) - \lambda_1 \mu - \lambda_2 \mathbb{1} = 0$

$= 2(\Sigma w) - \lambda_1 \mu - \lambda_2 \mathbb{1} = 0$

$w = \frac{1}{2}\left(\lambda_1 \mu \Sigma^{-1}\right) + \frac{1}{2}\left(\lambda_2 \mathbb{1} \Sigma^{-1}\right)$

a) $g_1 = 0 \Rightarrow w^T \mu = E_\pi \Rightarrow \mu^T w = E_\pi$

$\frac{1}{2} \lambda_1 \mu^T \Sigma^{-1} \mu + \frac{1}{2} \lambda_2 \mu^T \Sigma^{-1} \mathbb{1} = E_\pi$

$\lambda_1 \underbrace{\mu^T \Sigma^{-1} \mu}_{B > 0} + \lambda_2 \underbrace{\mu^T \Sigma^{-1} \mathbb{1}}_{A} = 2(E_\pi)$

b) $g_2 = 0 \Rightarrow W^T = 1 \Rightarrow \mathbb{1}^T W = 1$

$$\frac{1}{2}\lambda_1 \mathbb{1}^T \Sigma^T \mu + \frac{1}{2}\lambda_2 \mathbb{1}^T \Sigma^{-1} \mathbb{1} = 1$$

$$\lambda_1 \underbrace{\mathbb{1}^T \Sigma^T \mu}_{A} + \lambda_2 \underbrace{\mathbb{1}^T \Sigma^{-1} \mathbb{1}}_{C>0} = 2$$

$$\lambda_1 B + \lambda_2 A = 2E_\pi$$

$$\lambda_1 A + \lambda_2 C = 2$$

$$\lambda_1 = 2 \cdot \frac{CE_\pi - A}{BC - A^2} \qquad \lambda_2 = 2\,\frac{B - AE_\pi}{BC - A^2}$$

$$W = \frac{1}{2}\left(\lambda_1 \mu \Sigma^{-1}\right) + \frac{1}{2}\left(\lambda_2 \mathbb{1} \Sigma^{-1}\right)$$

$$W = \frac{1}{BC - A^2}\left(-A\Sigma^{-1}\mu + B\Sigma^{-1}\mathbb{1}\right) + \frac{1}{BC - A^2}\left(C\Sigma^{-1}\mu - A\Sigma^{-1}\mathbb{1}\right)E_\pi$$

The result is the same as the one in class