Homework 11

Problems to turn in individually

1. Stock in the Haggis-n-Lutefisk Delicacies Co. has an expected annual return of 7%, an annual volatility of 20% and is currently trading at 30 dollars per share. Assume the risk-free continuously compounded interest rate is 5%. Determine V_0, the current worth of a financial instrument that, at $T =$ six months from now, has a payoff of

$$V_T(S_T, T) = \begin{cases} 50 - S_T & \text{if } S_T < 30 \\ S_T - 10 & \text{if } 30 \le S_T \le 50 \\ 2(S_T - 30) & \text{if } 50 < S_T. \end{cases}$$

   Hint: Look to replicate this instrument by combining other financial instruments (options, etc.) that you know how to value.

2. For a binomial tree with equity returns continuously compounded with $\sigma = 0.2$, and interest rates quarterly compounded at annual rate $r = 0.03$, what is the up shift in stock price, down shift and the risk-neutral probability of the up shift, if the interval on the tree is quarterly? Use the Cox-Ross-Rubinstein model for this problem.

Problems to turn in as a group

1. Recall from the last homework that the Black-Scholes PDE for a call option on a stock with dividends is

$$C_t + (r - D)SC_S + \frac{1}{2}\sigma^2 S^2 C_{SS} - rC = 0.$$

$$C(S, T) = \begin{cases} S - K & \text{if } S > K \\ 0 & \text{if } S \le K \end{cases}$$

   If we could just get rid of the "$-D$", we'd be back to our regular Black-Scholes PDE and we could pilfer our answer from class. Here's how we transform the PDE to remove the "$-D$": define $C(S,t) = V(X(S,t),t)$ where $X(S,t) = Se^{-D(T-t)}$. Now use the chain rule to find C_t, C_S, and C_{SS} in terms of V_t, V_X, and V_{XX}. When you substitute everything, you should get the regular Black-Scholes PDE for $V(X,t)$ and the same form for the final condition at time $= T$. This will allow you to use the results from class and then you only need to substitute $X(S,t) = Se^{-D(T-t)}$ to get your final result. As an aesthetic matter, it will be better to rewrite $\ln\left(\frac{Se^{-D(T-t)}}{K}\right)$ as $\ln\left(\frac{S}{K}\right) - D(T-t)$ in your final answer.

2. The domestic stock market loses money over a yearly period about 30% of the time. Unlike indexed mutual funds, a hedge fund can buy and sell options. Many hedge funds advertise that they will lose money significantly less often (only 10% of the time, 5% of the time, etc.). Explain a way (or ways) that they could make good on this type of promise. What is the downside of each of the strategies you determine?

3. This problem will require a spreadsheet or programming effort. The initial stock price is given to be \$100. We wish to price European calls and puts with strike price \$100. The option maturity is $T = 1$ year, the risk free rate of interest is 5% per annum. If the volatility is $\sigma = 0.40$, then price the call and the put using the Jarrow-Rudd model. Assume you use a Jarrow-Rudd binomial tree comprising $n = 30$ periods.

4. Using the same parameters as in the problem above, and the same Jarrow-Rudd tree, what are the prices of American calls and puts?

5. In problem 3 check that your solution satisfies put-call parity exactly.