3) $\Sigma = \begin{bmatrix} \sigma_S^2 & \sigma_{SF_1} & \sigma_{SF_2} \\ \sigma_{SF_1} & \sigma_{F_1}^2 & \sigma_{F_1F_2} \\ \sigma_{SF_2} & \sigma_{F_1F_2} & \sigma_{F_2}^2 \end{bmatrix}$

$\sigma_S^2 = (0.3)^2 = 0.09$

$\sigma_{F_1}^2 = (0.25)^2 = 0.0625$

$\sigma_{F_2}^2 = (0.15)^2 = 0.0225$

$\sigma_{SF_1} = (0.98757)(0.3)(0.25) = 0.0741$

$\sigma_{SF_2} = (0.82923)(0.3)(0.15) = 0.0373$

$\sigma_{F_1F_2} = (0.84939)(0.25)(0.15) = 0.0319$

$$\Sigma = \begin{bmatrix} 0.09 & 0.0741 & 0.0373 \\ 0.0741 & 0.0625 & 0.0319 \\ 0.0373 & 0.0319 & 0.0225 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 \\ h_1 \\ h_2 \end{bmatrix}$$

$$W^T \Sigma W =$$

$$\begin{bmatrix} 1 & h_1 & h_2 \end{bmatrix} \begin{bmatrix} 0.09 & 0.0741 & 0.0373 \\ 0.0741 & 0.0625 & 0.0319 \\ 0.0373 & 0.0319 & 0.0225 \end{bmatrix} \begin{bmatrix} 1 \\ h_1 \\ h_2 \end{bmatrix}$$

$$=$$

$$\begin{bmatrix} 1 & h_1 & h_2 \end{bmatrix} \begin{bmatrix} 0.09 & 0.0741h_1 & 0.0373h_2 \\ 0.0741 & 0.0625h_1 & 0.0319h_2 \\ 0.0373 & 0.0319h_1 & 0.0225h_2 \end{bmatrix}$$

$$= \Big[(0.09 + 0.0741h_1 + 0.0373h_2) \quad (0.0741h_1 + 0.0625h_1^2 + 0.0317h_1h_2) \quad (0.0373h_2 + 0.0319h_1h_2 + 0.0225h_2^2) \Big]$$

$$= 0.09 + 0.148h_1 + 0.0746h_2 + 0.0625h_1^2 + 0.0225h_2^2 + 0.0638h_1h_2$$

$$\frac{\partial}{\partial h_1}(0.09 + 0.148h_1 + 0.0746h_2 + 0.0625h_1^2 + 0.0225h_2^2 + 0.0638h_1h_2)$$

$$= 0.148 + 0.125h_1 + 0.0638h_2 = 0$$

$$\frac{\partial}{\partial h_2}(0.09 + 0.148h_1 + 0.0746h_2 + 0.0625h_1^2 + 0.0225h_2^2 + 0.0638h_1h_2)$$

$$= 0.0746 + 0.045h_2 + 0.0638h_1 = 0$$

$$(0.148 + 0.125h_1 + 0.0638h_2) - 1.959(0.0746 + 0.045h_2 + 0.0638h_1)$$

$$= 0.148 + \cancel{0.125h_1} + 0.0638h_2 - 0.146 - 0.088h_2 \quad \cancel{= 0.125h_1}$$

$$0.002 - 0.0242h_2 = 0$$

$$h_2 = 0.0826$$

$$0.148 + 0.125h_1 + 0.0638(0.0826) =$$

$$0.153 + 0.125h_1 = 0$$

$$h_1 = -1.226$$

Minimum variance hedge ratio for first future contract

$$= -1.226$$

Minimum variance hedge ratio for second future contract

$$= 0.0826$$

The least variance hedge includes a short position of 1.226 units for F_1 and a long position of 0.0826 units for F_2.