Group 11

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1. we assume $\Pi = v(S,t) + nS + c$

$$d\Pi = dV + d(nS) + dc_{buy/sell} + dc_{int} + dc_{div}$$

$$= V_t dt + V_s dS + \frac{1}{2} V_{ss}(dS)^2 + n dS + S dn + dn dS$$

$$+ dc_{B/S} + dc_{int} + dc_{div}$$

$$= V_t dt + V_s dS + \frac{1}{2} V_{ss}(dS)^2 + n dS + dc_{int} + dc_{div}$$

$$= V_t dt + V_s dS + \frac{1}{2} V_{ss}(dS)^2 + n dS + rc dt + nSD dt$$

let $dS = (\mu - D) S dt + \sigma S dB$

$$(dS)^2 = \sigma^2 S^2 dt$$

$$d\Pi = (V_t + \frac{1}{2}\sigma^2 S^2 + rc + nSD) dt + (V_s + n) dS$$

we choose $n = -V_s$ s.t. $(V_s + n) = 0$

$$d\Pi = (V_t + \frac{1}{2}\sigma^2 S^2 + rc - V_s SD) dt$$

$$d\Pi = r\Pi dt$$

$$r\Pi = V_t + \frac{1}{2}\sigma^2 S^2 + rc - V_s SD$$

$$r(V - V_s S + c) = rV - rV_s S + rc$$

$$rV - rV_s S + \cancel{rc} = rV - rV_s S + \cancel{rc} - V_s SD$$

$$\boxed{V_t + (r - D) S V_s + \frac{1}{2}\sigma^2 S^2 V_{ss} - rV = 0}$$

2.

$$\begin{tikzcd}[row sep=small] & & 80 \\ & 60 \arrow[ur] \arrow[dr] & \\ 45 \arrow[ur] \arrow[dr] & & 50 \\ & 30 \arrow[ur] \arrow[dr] & \\ & & 20 \end{tikzcd}$$

$R = 1.02$

$$C = \frac{1}{R}\left[p C_u + (1-p) C_d \right]$$

$$p = \frac{R-d}{u-d}$$

For node 45:

$$\begin{tikzcd}[row sep=small] & 60 \\ 45 \arrow[ur] \arrow[dr] & \\ & 30 \end{tikzcd} \qquad u = \frac{4}{3}, \quad d = \frac{3}{4}$$

$$\Rightarrow \quad P_{45} = \frac{1.02 - \frac{3}{4}}{\frac{4}{3} - \frac{3}{4}} = 0.46286$$

For node 60:

$$\begin{tikzcd}[row sep=small] & 80 \\ 60 \arrow[ur] \arrow[dr] & \\ & 50 \end{tikzcd} \qquad u = \frac{4}{3}, \quad d = \frac{5}{6}$$

$$\Rightarrow \quad P_{60} = \frac{1.02 - \frac{5}{6}}{\frac{4}{3} - \frac{5}{6}} = 0.37333$$

For node 30:

$$\begin{tikzcd}[row sep=small] & 50 \\ 30 \arrow[ur] \arrow[dr] & \\ & 20 \end{tikzcd} \qquad u = \frac{5}{3}, \quad d = \frac{2}{3}$$

$$\Rightarrow \quad P_{30} = \frac{1.02 - \frac{2}{3}}{\frac{5}{3} - \frac{2}{3}} = 0.35333$$

They are not the same. Thus, the tree is invalid.

3. $S = 100 \quad u = 1.10 \quad d = 0.90 \quad K = 100$

a. For European put option when maturity time increases, the insurance value of option increases but the time value decreases

$R = 1.02$. At given high interest rate, one period put value is 3.92, two period put value is 3.38 which is smaller than one period put. because the insurance value effect is outweighed by the time-value effect.

b. $R = 1.00$

$$q = \frac{R-d}{u-d} = \frac{1-0.9}{1.0-.9} = 0.5 \leftarrow \text{risk neutral probability}$$

$$P_u = \max\left\{0, K-uS\right\} = \left\{0, 100-(100\times 1.10)\right\} = \left\{0,-10\right\} = 0$$

$$P_d = \max\left\{0, 100(-100\times .9)\right\} = \left\{0, 10\right\} = 10$$

$$P_{uu} = \max\left\{0, K-u^2S\right\} = \max\left\{0, 100-(100\times 1.10^2)\right\} =$$

$$\left\{0, -21\right\} = 0$$

$$P_{ud} = \max\left\{0, K-udS\right\} = \max\left\{0, 100-1.10(.9)(100)\right\} = \max\left\{0, 1\right\}$$

$$= 1$$

$$P_{dd} = \max\left\{0, K-d^2S\right\} = \max\left\{0, 100-(.90)^2(100)\right\}$$

$$= \max\left\{0, 19\right\} = 19$$

One-period put

$$P(1) = \frac{1}{R}\left[q P_u - (1-q) P_d\right]$$

$$= \frac{1}{1.00}\left[.5(0) + (1-.5)10\right] = 5$$

2-period put

$$P(2) = \frac{1}{R^2}\left[q^2 P_{uu} + 2q(1-q) P_{ud} + (1-q)^2 P_{dd}\right]$$

$$= \frac{1}{1.00}\left[(.5)^2(0) + 2(.5)(.5)(1) + (.5)^2 19\right]$$

$$= 5.25 \qquad 5.25 > 5$$

∴ The value of the two-period put is more than the one period put