1) a)

$$\int_{-\infty}^{\infty} z^2 f_Z(z)\,dz = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} z^2 e^{-\frac{z^2}{2}}\,dz$$

$$u = z \qquad dv = ze^{-\frac{z^2}{2}}dz$$
$$du = dz \qquad v = -e^{-\frac{z^2}{2}}$$

$$\int u\,dv = uv - \int v\,du$$

$$\frac{1}{\sqrt{2\pi}}\left(-ze^{-\frac{z^2}{2}}\Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}}dz\right)$$

$$\downarrow$$

$$\int_{-\infty}^{\infty} f_Z(z)\,dz = 1$$

$$E[Z^2] = \frac{1}{\sqrt{2\pi}}\left(-ze^{-\frac{z^2}{2}}\Big|_{-\infty}^{\infty}\right) + 1$$

$$= \frac{1}{\sqrt{2\pi}}(0) + 1 = 1$$

b) $E[Z^4] = \frac{1}{\sqrt{2\pi}}\int z^4 e^{\frac{z^2}{2}}\,dz$

$$u = z^3 \qquad dv = ze^{-\frac{z^2}{2}}dz$$
$$du = 3z^2dz \qquad v = -e^{-\frac{z^2}{2}}$$

$$uv - \int v\,du$$

$$-z^2e^{-\frac{z^2}{2}} + \int e^{-\frac{z^2}{2}}(3z^2)\,dz$$

$$= -z^2e^{-\frac{z^2}{2}}\Big|_{-\infty}^{\infty} + 3\int_{-\infty}^{\infty} z^2 e^{-\frac{z^2}{2}}\,dz$$

$E[Z^2] = 1$

$$E[Z^4] = \frac{1}{\sqrt{2\pi}}\left(-z^2e^{-\frac{z^2}{2}}\Big|_{-\infty}^{\infty}\right) + 3$$

$= 0 + 3 = \boxed{3}$

c) $Var[(\Delta B)] = \Delta t$

$Var[(\Delta B)^2] = E[(\Delta B^2 - u)^2]$

$u = E[\Delta B^2]$

$= E[(\Delta B^2 - E(\Delta B^2))^2]$

$= E[(\Delta B^2 - \Delta t E[z^2])^2]$

$= E[(\Delta B^2 - \Delta t)^2]$

$(\Delta B^2 - \Delta t)(\Delta B^2 - \Delta t)$

$\hookrightarrow \Delta B^4 - 2\Delta B^2 \Delta t + (\Delta t)^2$

$= E[\Delta B^4] - 2E[\Delta B^2 \Delta t] + E[\Delta t^2]$

$= 3\Delta t^2 - 2\Delta t^2 (\cancel{E[z^2]}) + \Delta t^2$

$= \boxed{2\Delta t^2}$

$(\sqrt{\Delta t}\, z)^4 = \Delta t^2 \cdot z^4$

$\Delta t^2 \cdot E(z^4) = \Delta t^2 \cdot 3$

2) a) T-Distribution

b) i) −. So that it brings the term closer to the mean.

ii) +. Same as the standard model

c) Negatively correlated.