1) a) $P(x,y) \approx y e^{-\frac{xy}{10000}}$ $(x,y) = (100,50)$

__First order__ $(x_0,y_0) = (103,48)$ $(x_1,y_1) = (100,50)$

$$P(x_0,y_0) = P(x_1,y_1) + P_x'(x_1,y_1)(x_0-x_1) + P_y'(x_1,y_1)(y_0-y_1)$$

$$P(103,48) = P(100,50) + P_x'(100,50)(3) + P_y'(100,50)(-2)$$

$$P(100,50) = 50e^{-\frac{(100\cdot 50)}{10000}} = 30.327$$

$$P_x'(100,50) = \frac{\partial}{\partial x}\left(ye^{-\frac{xy}{10000}}\right) = -\frac{y^2}{10000}e^{-\frac{xy}{10000}} = -0.152$$

$$P_y'(100,50) = \frac{\partial}{\partial y}\left(ye^{-\frac{xy}{10000}}\right) = \left(e^{-\frac{xy}{10000}}\right) + \left(-\frac{xy}{10000}e^{-\frac{xy}{10000}}\right)$$

$$= (0.607) + (-0.303) = 0.303$$

$$30.327 - 0.456 - 0.606 = 29.265$$

__Second Order__ $(x_0,y_0) = (103,48)$ $(x_1,y_1) = (100,50)$

$$P(x_0,y_0) = P(x_1,y_1) + P_x'(x_1,y_1)(x_0-x_1) + P_y'(x_1,y_1)(y_0-y_1)$$
$$+ \frac{1}{2}\left(P_{xx}(x_1,y_1)(x_0-x_1)^2 + 2P_{xy}(x_1,y_1)(x_0-x_1)(y_0-y_1) + P_{yy}(y_0-y_1)^2\right)$$

$$P_{xx} = \frac{\partial}{\partial x}(P_x) = \frac{\partial}{\partial x}\left(-\frac{y^2}{10000}e^{-\frac{xy}{10000}}\right) = \frac{y^3}{(10000)^2}e^{-\frac{xy}{10000}} = 7.58\cdot 10^{-4}$$

$$P_{xy} = \frac{\partial}{\partial y}(P_x) = \frac{\partial}{\partial y}\left(-\frac{y^2}{10000}e^{-\frac{xy}{10000}}\right) = -\frac{1}{10000}\left(-\frac{xy^2}{10000}e^{-\frac{xy}{10000}} + 2ye^{-\frac{xy}{10000}}\right) =$$

$$= 0.00152 - 0.00606 = -0.00455$$

$$P_{yy} = \frac{\partial}{\partial y}(P_y) = \frac{\partial}{\partial y}\left(\left(e^{-\frac{xy}{10000}}\right) + \left(-\frac{xy}{10000}e^{-\frac{xy}{10000}}\right)\right) = \frac{x^2y - 20000x}{(10000)^2 e^{\frac{xy}{10000}}} = -0.00910$$

$$29.265 + \frac{1}{2}\left((7.58\cdot 10^{-4})(3)^2 - 2(0.00455)(-6) - 0.00910(-2)^2\right.$$

$$= 29.278$$

b)

$$P(103,48) = 48e^{\frac{(103 \cdot 48)}{10000}} = \boxed{29.277}$$

The first order approximation was off by 0.012, while the second order approximation was off by 0.001.

2) $3375 - 2500 = 875$

$$P = \frac{875}{\#} + F$$

$$P = \frac{875}{1000} + 43 = 43.875$$

$$43.875 - 43 = \boxed{0.875}$$

3) $F = Se^{rt}$

$$F = 3.60e^{0.08 \cdot \frac{3}{12}} = \boxed{3.673}$$

3.673 is less than 3.9, there is an arbitrage opportunity

4) a) i) $PV = \$1e^{-(0.12)(\frac{1}{12})} = 0.99$

$$F = (S - PV)e^{rt}$$

$$= (90 - 0.99)e^{0.12(\frac{3}{12})}$$

$$= \boxed{91.72}$$

ii) $PV = 2e^{-(0.12)(\frac{4}{12})} = 1.92$

$$PV_{Both} = 1.92 + 0.99 = 2.91$$

$$F = (S - PV_{Both})e^{rt}$$

$$= (90 - 2.91)e^{0.12(\frac{6}{12})} = \boxed{92.48}$$

b) Yes. Quoted forward price is overvalued compared to spot price

1) Borrow 87.09 for repayment in 6 months
Borrow 1.92 for repayment in 4 months
Borrow 0.99 for repayment in 1 month
2) After 1 month, receive \$1 to repay loan for 1 month
3) After 4 months, receive \$2 to repay loan for 4 months
4) After 6 months, receive \$90 to repay loan for 6 months

5) a) $r = \frac{1}{T}[\ln F - \ln S]$

$r = \frac{1}{\left(\frac{3}{12}\right)}[\ln(90) - \ln(84)]$

$= 0.276$

b) Yes. The borrow rate is lower than the repo rate.