Midterm 1

45/50

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1) $n^2 \lg n, 2^n, (2\lg n), (n), 3^n$

fastest: $2^n, 3^n$ is exponential

n and $n^2\lg n$ polynomial

slowest: $2\lg n$ is log

10

$$\lim_{n\to\infty} \frac{n^2 \cancel{\lg n}}{2\cancel{\lg n}} = \frac{n^2}{2} = \infty$$

$n^2\lg n$ faster than $2\lg n$

3^n faster than 2^n

$$\lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}^2 \lg n} = \frac{1}{n\lg n} = 0$$

n is slower than $n^2\lg n$ but faster than $2\lg n$

$$\boxed{(\lg n)^2, n, n^2\lg n, 2^n, 3^n}$$

2)

$$M(n) = \sum_{i=0}^{n-1} 4 \sum_{j=2i}^{n-1} (2) = 2\sum_{i=0}^{n-1} (n\cancel{-1} - 2i \cancel{+1}) =$$

9

$$2\sum_{i=1}^{n} n - 2i = 2\left(\sum_{i=1}^{n} n - \sum_{i=1}^{n} 2i\right) =$$

$$2\left(n^2 - \cancel{2}\left(\frac{n(n+1)}{\cancel{2}}\right)\right) = 2n^2 - 2n^2 + 2n$$

$$\boxed{= \cancel{1}2n}$$

3

3) $M(0) = 1$

$M(n) = \sum_{i=1}^{n-1} + 1 + M(n-1) = n + M(n-1) \quad n \geq 1$

7

$M(0) = 1$

$M(1) = 1+1 = 2$

$M(2) = 2+2 = 4$

$M(3) = 3+4 = 7$

$M(4) = 4+7 = 11$

$\boxed{M(n) = n + \cancel{(n-1)n}}$

$\frac{n^2+n+2}{2}$

$M(n) = n + a$

$-M(n) = n + b$

$0 \quad (a-b) = n$

$11 = n + b$

$2 = n + a$

$9 = b - a$

$n + (x) n = 2$

$n + (x) n = 4$

$an - bn = 2$

$(a-b) n = 2$

$\frac{n+(n-1)}{2} =$

$n + (n+1)$

$M(0) = 0$

4) $M(n) = 3 + 2M(\frac{n}{3}) \qquad n = 3^m$

9

$M(3^m) = 3 + 2M(\frac{3^m}{3}) = 3 + 2M(3^{m-1})$

$= 3 + 2(3 + 2M(3^{m-2})) = 3 + 2^1 \cdot 3 + 4M(3^{m-2})$

$= 2^0 \cdot 3 + 2^1 \cdot 3 + 4(3 + 2M(3^{m-3})) = 2^0 \cdot 3 + 2^1 \cdot 3 + 2^2 \cdot 3 \ldots$

$= 2^m(3 + 2M(3^{m-m})) \ldots = 2^m \cdot 3 + 2^m \cdot 0 \ldots$

$= 2^0 \cdot 3 + 2^1 3 \cdot 2^2 \cdot 3 \cdots + 2^m \cdot 3 =$

$$3\sum_{i=0}^{m} 2^i = 3\left(\frac{2^{m+1}-1}{2-1}\right) = 3(2^{\cancel{m+1}}-1)$$

$$= \boxed{3(2^{\log_3 n+1} - 1)} \qquad m = \log_3 n$$

5) 60

```
int arr[] divide(int q, int d) {
    int arr[2] = { };
    int sum = (25)q + (10)d
    for (int i = 0; i <= q; i++) {
        for (int j = 0; j <= d; j++) {
            if (10*j + 25*i == sum/2) {
                arr[0] = j;
                arr[1] = i;
            }
        }
    }
    if (arr[0] && arr[1] == NULL) {
        arr[0] = -1;
        arr[1] = -1;
    }
    return arr;
}
```