1 Aim

The aim is to provide a model-independent validated fisher code, available for everyone to use. We will write a paper based on the results of the code comparison. The code comparison will proceed by steps:

1. generating common input
2. compare the fisher matrices for the *minimal* common fisher code (from now on referred to as MCFC)
3. discuss and improve the MCFC
4. make the code public available (terms to be discussed)

2 Definition of Cosmological Statistic Data-Flows

3 Atomistic Approach – Building a Euclid Cosmology Pipeline

3.1 External codes and external interfaces

4 Holistic Approach – Comparing Currently Existing Pipelines

4.1 Definition of Cosmological & Nuisance Parameters

4.2 Fisher Derivatives

We start the comparison with a simplified setting. Once we agree on this, we can modify it or add further effects.

References for GC: [**?**]. References for WL: [**?**]

General references on Fisher and Euclid: [**?**, **?**]

Note: All derivatives should be calculated using the input provided (at $\pm\epsilon$). Ideally no other formula should be used, which is not contained in this document. If you notice something missing, please contact VP asap.

4.3 Metrics

Appendix: Galaxy Clustering Basic Equations

We recall the main equations to be used. Observed linear spectrum at all redshifts:

$$P_{r,obs}(k_{ref},\mu_{ref};z) = P_s(z) + \frac{D_A(z)^2_{ref}H(z)}{D_A(z)^2 H_{ref}(z)}\frac{P(k,z)}{P(k,z=0)}b^2(z)(1+\beta(z)\mu^2)^2 P(k,z=0)\,, \tag{1}$$

We use:

- For the linear matter power spectrum $P(k,z) \to$ input files provided by Enea Di Dio and Bin Hu.
- For the growth index $f(z,k)$ (needed for $\beta_d(z) = f(z)/b(z)$), $\to$ input files provided by Enea Di Dio and Bin Hu (**k dependence not necessary. k fixed to k = 0.1 h/Mpc). Input files need to be done differently for future tests**).
- For the $H(z)$ and $D_A(z)$ we use analytical expressions (see below, sec. 4.3). **SC checked that the analytical derivatives wrt ω_b and ω_c (calculated considering h as a parameter) now match also the numerical ones.**

Here $b(z)$ is the bias (provided by column 5 of Tab.(1)), $\beta(z)$ is the redshift-distortion factor, $P(k)$ is the undistorted linear matter spectrum (provided in the input with the correct normalization), μ is the direction cosine, D_A is the angular diameter distance (provided in the input) and H is the Hubble rate at the shell redshift z (provided in the input). $P_s(z)$ is the z- dependent shot-noise correction due to discreteness in the survey. The subscript ref (for 'reference') indicates quantities calculated in the fiducial model. In linear theory we have $\beta_d(z) = f(z)/b(z)$ where $f(z) \equiv d\log G/d\log a$ is the growth rate (f is provided directly in the input for every cosmology, G is the growth factor but you should not need it, the input provides directly the growth rate f).

The Fisher matrix for every redshift bin shell is integrated in k (**Right now doing the μ integral from 0 to 1 and multiplying the integrand by 2)**):

$$F_{ij} = \frac{1}{8\pi^2}\int_{-1}^{1}\mathrm{d}\mu\int_{k_{\min}}^{k_{\max}} k^2\cdot\mathrm{d}k\frac{\partial\ln P_{obs}\left(z;k,\mu\right)}{\partial\theta_i}\frac{\partial\ln P_{obs}\left(z;k,\mu\right)}{\partial\theta_j}\cdot\left[\frac{n\,P_{obs}\left(z;k,\mu\right)}{n\,P_{obs}\left(z;k,\mu\right)+1}\right]^2 V_{\mathrm{s}} \tag{2}$$

where the θ's are the parameters: $\theta = \{\omega_c \equiv \Omega_{c_0}h^2, \omega_b \equiv \Omega_{b_0}h^2, h, n_s, \ln(10^{10}A_s), b_i, P_s\}$ (τ has been fixed to fiducial because it is degenerate when using only WL and GC).

The input provides the spectra $P(z,k)$ (already corresponding to the A_s normalization) and all other functions $f(z)$, $D_A(z)$, $H(z)$ for each model.

$n(z)$ is the galaxy number density at redshift z and V_s is the survey volume of the entire (so all 4π) redshift shell dz. The power spectra are all obtained numerically by solving the background and perturbation equations of the system for each value of the parameters and they are provided as input.

The derivative of the galaxy power spectrum, at each redshift (so z is fixed in the middle of the bin), with respect to the parameters (in this case ω_c) is:

$$\begin{aligned}\left.\frac{\mathrm{d}\ln P_{\mathrm{obs}}\left(\bar{z},k,\mu;\theta_i\right)}{\mathrm{d}\theta_i}\right|_{fid} &= \frac{\partial\ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial\ln P_s(\bar{z})}\frac{\partial\ln P_s(\bar{z})}{\partial\theta_i} + \frac{\partial\ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial\ln f(\bar{z})}\frac{\partial\ln f(\bar{z})}{\partial\theta_i}\\ &+ \frac{\partial\ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial\ln H(\bar{z})}\frac{\partial\ln H(\bar{z})}{\partial\theta_i} + \frac{\partial\ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial\ln D_A(\bar{z})}\frac{\partial\ln D_A(\bar{z})}{\partial\theta_i}\\ &+ \frac{\partial\ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial\ln P(k,\bar{z})}\frac{\partial\ln P(k,\bar{z})}{\partial\theta_i} + \frac{\partial\ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial\ln b(\bar{z})}\frac{\partial\ln b(\bar{z})}{\partial\theta_i}\\ &+ red\frac{\partial\ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial k}\frac{\partial k}{\partial\theta_i} + \frac{\partial\ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial\mu}\frac{\partial\mu}{\partial\theta_i}\end{aligned} \tag{3}$$

In general, one needs to include a redshift error σ_z and further multiply: $P_{obs} = P_{obs}e^{-k^2\mu^2\sigma_z^2/H(z)^2}$. The last two terms are non-vanishing if one takes the Alcock-Paczynski effect into account for k and μ, since they are also affected by geometrical terms.

The derivatives are:

$$\frac{\partial \ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial \ln P_s(\bar{z})} = \frac{1}{P_{obs}\left(\bar{z},k,\mu;\theta_i\right)} \tag{4a}$$

$$\frac{\partial \ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial \ln f(\bar{z})} = \frac{2\beta(\bar{z})\mu^2}{1+\beta(\bar{z})\mu^2} \tag{4b}$$

$$\frac{\partial \ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial \ln b(\bar{z})} = \frac{2}{1+\beta(\bar{z})\mu^2} \tag{4c}$$

$$\frac{\partial \ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial \ln P(k,\bar{z})} = 1 \tag{4d}$$

$$\frac{\partial \ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial \ln H(\bar{z})} = 1 \pm ...checkwhenAPispresent \tag{4e}$$

$$\frac{\partial \ln P_{obs}\left(\bar{z},k,\mu;\theta_i\right)}{\partial \ln D_A(\bar{z})} = -2 \pm ...check \tag{4f}$$

All these terms are set as follows:

- $\frac{\mathrm{d}\ln P(\bar{z},k)}{\mathrm{d}\theta_i}$ are numerical derivatives, coming from the input files (of course $P(\bar{z},k)$ does not depend on bias or P_s). **The input files are however provided at different k's; therefore, when calculating the derivative you need to interpolate in k first and then calculate the difference at fiducial** $(1\pm\epsilon)$.
- $f(\bar{z},k)$ is given by the input file: while in the input f is a function of k, for the comparison, we fix k = 0.1 h/Mpc.
- $\frac{\mathrm{d}\ln f(\bar{z})}{\mathrm{d}\theta_i}$ are numerical derivatives, coming from the input files (k fixed as above).
- $\frac{\mathrm{d}\ln H(\bar{z})}{\mathrm{d}\theta_i}$ and $\frac{\mathrm{d}\ln D_A(\bar{z})}{\mathrm{d}\theta_i}$ are given by the analytical expressions described below (sec. 4.3)

We tested three different methods:

- Direct derivative: you integrate directly all terms of equation 3 into eq.2;
- Numerical derivative: evaluate the numerical derivative of eq.2 just calculating $P_{obs}(\bar{z},k)$ at $\theta_{i,fid}(1\pm\epsilon)$ and then divide by $2\epsilon\theta_{i,fid}$;
- Jacobian method (or extended BAO Eisenstein method) reference?

All three methods match when there is no AP effect. With AP effect, the Jacobian method differs from the other two. For the following comparison, fisher matrices are provided using the direct derivative method.

The Hubble parameter and the angular diameter distance

We here assume that we consider h as a parameter, rather than Ω_Λ. The Hubble Parameter, for the cosmology we are using, is H(z) $= 100$ h $\sqrt{\frac{\Omega_c h^2+\Omega_b h^2+\Omega_\nu h^2}{h^2}(1+z)^3+1-\frac{\Omega_c h^2+\Omega_b h^2+\Omega_\nu h^2}{h^2}} \equiv 100E(z)$ [h km/s/Mpc]$Wethent$ $\Omega_{c,0}h^2$ and $\omega_b = \Omega_{b,0}h^2$. The derivatives of the logarithm of Hubble parameter with respect to ω_c, ω_b and h are: $\partial \ln H \frac{}{\partial\omega_c|_{\omega_b,h}=\frac{(1+z)^3-1}{2h^2E^2(z)}\frac{\partial\ln H}{\partial\omega_b}|_{\omega_c,h}=\frac{(1+z)^3-1}{2h^2E^2(z)}\frac{\partial\ln H}{\partial h}|_{\omega_b,\omega_c}=\frac{1}{hE^2(z)}-\frac{1}{h}andtheotherderivativesoftheHubbleparameterswithrespectto}$ n$_s$, A_s and τ are clearly zero. Note that had we chosen Ω_i as variables instead of ω_i, the derivative of the Hubble function with respect to h in units of [h km/s/Mpc] would have been zero. The angular diameter distance is D$_A(z) = \frac{1}{H_0}\frac{c}{1+z}\int_0^z \frac{dx}{E(x)} wherecisthespeedoflightanditisimportanttohavedistancesinMpc/handH_0 =$ $\frac{100\,h\,\mathrm{Km/s/Mpc}}{c} = \frac{1}{2997.92}\frac{h}{\mathrm{Mpc}}.Thederivativesoftheangulardiameterdistanceare: \frac{\partial\ln D_A}{\partial\omega_c} = \frac{1}{D_A(z)}\frac{c}{1+z}\int_0^z -\frac{\partial\ln H(x)}{\partial\omega_c}\frac{dx}{H(x)}$

$\frac{\partial \ln D_A}{\partial \omega_b} = \frac{1}{D_A(z)} \frac{c}{1+z} \int_0^z -\frac{\partial \ln H(x)}{\partial \omega_b} \frac{dx}{H(x)}$
$\frac{\partial \ln D_A}{\partial h} = \frac{1}{D_A(z)} \frac{c}{1+z} \int_0^z -\frac{\partial \ln H(x)}{\partial h} \frac{dx}{H(x)} All these derivatives will be used in the Fisher matrix. Galaxy density : take column 4 of Table(1), (dn3) as dn/dz/\mathrm{d}\Omega$ (with Ω in square degrees). There is no need to add in an extra redshift success rate (e.g. epsilon=0.35) as before - this is already included in the numbers, which now account for redshift- and flux-dependent success rate, in addition to using an improved mock catalogues and redshift-measurement technique. There is also no need to include any scaling. These numbers are for the standard integration time (540 sec per dither, standard dither pattern). As an example, to get the total number of galaxies in a bin of width $0.95 < z < 1.05$, one would calculate:

(Number of galaxies $0.95 < z < 1.05$) $= 4825.80 * 0.1 * 15000 = 7,239,000$

So in a Euclid survey over $15,000 deg^2$ we would get good redshifts for $7,239,000$ galaxies in the redshift range $0.95 < z < 1.05$. Note the 0.1 factor required to turn dn/dz from a derivative to a number in a bin of width $Dz = 0.1$.

This gives you the total number of galaxies. Then the number density that you need in the fisher for GC, n in eq.2 in the formula for the fisher matrix, is the density per volume in each bin, therefore equal to:

$$n = dn3/dV_{dz} * dz \tag{5}$$

where the dz is the width of the redshift shell (i.e 0.1 but see the file) and

$$dV_{dz} = \frac{\frac{4\pi}{3}\left[(1+z_2)^3 D_A[z_2;\theta]^3 - (1+z_1)^3 D_A[z_1;\theta]^3\right]}{4\pi\left(\frac{180}{\pi}\right)^2} \tag{6}$$

is the volume in the redshift shell per degree squared. The survey Volume is

$$V_s = Area(i.e. 15,000 sqdeg) * dV_{dz} \tag{7}$$

The third column (dVol) should not be used anywhere.

Table 1: Specifications of the redbook, (derived from Orsi et al. 2009).

zmin	zmax	dVol	dn3	b3
0.35	0.45	3.60	0.000	0.992
0.45	0.55	5.10	0.000	1.024
0.55	0.65	6.40	0.000	1.053
0.65	0.75	7.90	2434.280	1.083
0.75	0.85	9.20	4364.812	1.125
0.85	0.95	10.30	4728.559	1.104
0.95	1.05	11.70	4825.798	1.126
1.05	1.15	12.30	4728.797	1.208
1.15	1.25	13.30	4507.625	1.243
1.25	1.35	14.00	4269.851	1.282
1.35	1.45	14.60	3720.657	1.292
1.45	1.55	15.10	3104.309	1.363
1.55	1.65	15.60	2308.975	1.497
1.65	1.75	16.00	1541.831	1.486
1.75	1.85	16.20	1474.707	1.491
1.85	1.95	16.50	893.716	1.573
1.95	2.05	16.70	497.613	1.568

GC comparison: where to start

In Case 1,2,3 the following common options are fixed:

1. k_{min} : 0.001 h/Mpc
2. $k_{max} = 0.2$ h/Mpc
3. k-binning: use directly the one of the spectra provided in the input
4. No relativistic corrections.
5. Area : 15,000 sq deg
6. Redshift range : $0.7 < z < 2.0$
7. Binning: use all bins as in the table, starting from 0.65 (the first non-zero in dn3).

GC case 1: minimal setting.

In addition to the common settings above, use these settings for your first comparison.

1. Bias: fixed to the fiducial value (column 5, b3 in the table 1). All derivatives wrt the bias are zero.
2. The AP effect is zero: the four derivatives of k and μ with respect to H and D_A are set to zero.
3. $P_{shot} = 0$ (fixed)
4. $\sigma_z = 0$

The Fisher matrix to compare with for this case, is case1.dat. If it matches, proceed to case2.

GC case 2: including the bias

In addition to the common settings above, case2 fixes:

1. Bias: marginalization over bias, derivatives as in 4c.
2. The AP effect is zero: the four derivatives of k and μ with respect to H and D_A are set to zero.
3. $P_{shot} = 0$ (fixed)
4. $\sigma_z = 0$

The Fisher matrix to compare with for this case, is case2.dat.
If it matches, proceed to case3.

GC case 3: including the AP effect

In addition to the common settings above, case3 fixes:

1. Bias: marginalization over bias, derivatives as in 4c.
2. The AP effect is included.
3. $P_{shot} = 0$ (fixed)
4. $\sigma_z = 0$

The Fisher matrix to compare with for this case, is case3.dat.
If it matches, proceed to case4.

GC case 4: including the shot noise

In addition to the common settings above, case4 fixes:

1. Bias: marginalization over bias, derivatives as in 4c.
2. The AP effect is included.
3. P_{shot}: marginalisation over P_{shot}; the fiducial is still zero; derivatives are calculated according to 4a
4. $\sigma_z = 0$

The Fisher matrix to compare with for this case, is case4.dat.

Appendix: Weak Lensing Basic Equations

Convergence power spectrum with (correlated) sources in redshift bins centered around z_i and z_j:

$$P_{ij}(\ell) = \frac{9}{4}\int_0^\infty \mathrm{d}z \frac{W_i(z)W_j(z)H^3(z)\Omega_m^2(z)}{(1+z)^4} P_m\left(\frac{\ell}{r(z)}\right), \tag{8}$$

where $W_i(z)$ is the window function and r(z) is the comoving distance, which in flat space, is equal to the comoving angular diameter distance:

$$r(z) = D_A(1+z) \tag{9}$$

This is provided in the input, column 3 of table VII. Please, for this comparison, do not use other methods to calculate it.

$$W_i(z) = \int_z^\infty \frac{d\tilde{z}}{H(\tilde{z})}\left[1 - \frac{r(z)}{r(\tilde{z})}\right] n_i[r(\tilde{z})] \tag{10}$$

where $r(z)$ is the coming distance. The window function depends on $n_i(z)$, the galaxy distribution in the i-th redshift bin: this is convolved with a Gaussian to account for photometric redshift errors σ_z (value specified below), i.e.

$$n_i(z) = A \int_{i\text{-th bin}} n(z)\exp\left(\frac{-(\tilde{z}-z)^2}{2\sigma_z^2}\right)\mathrm{d}\tilde{z} \tag{11}$$

where the integral is done over $\tilde{z}$ for the single i-th bin. $A = \frac{1}{\sqrt{2\pi}\sigma_z}$ a normalization factor. In eq. (11), the density is given by:

$$n(z) = z^2 exp(-(z/z_0)^{3/2}) \tag{12}$$

where $z_0 = z_{mean}/1.412$ is the peak of $n(z)$ and z_{mean} is the median redshift (value specified below).

Then the Fisher matrix is summed over all multiples:

$$F_{\alpha\beta} = f_{\rm sky} \sum_{\ell,i,j,k,m} \frac{(2\ell+1)\Delta\ell}{2} \frac{\partial P_{ij}(\ell)}{\partial\theta_\alpha} C^{-1}_{jk} \frac{\partial P_{km}(\ell)}{\partial\theta_\beta} C^{-1}_{mi} \tag{13}$$

with the covariance matrix

$$C_{jk} = P_{jk} + \delta_{jk}\gamma_{\rm int}^2 {q_j}^{-1}, \tag{14}$$

where $\gamma_{\rm int}$ is the intrinsic galaxy shear and

$$q_j = 3600\left(\frac{180}{\pi}\right)^2 n_\theta \int_0^\infty n_j(z)dz\,. \tag{15}$$

Here n_θ is the galaxy density per arcmin^2 and $n_j(z)$ is the galaxy density for the j-th redshift bin, as defined in eq.(11).

Since we consider multipoles up to $\ell_{\rm max} = 5000$, we need to apply non-linear corrections to the matter power spectrum, for which we use the halofit (provided in the input spectra).

Specifications for the weak lensing:

1. Area: 15,000 sq deg
2. $f_{sky} = Area/(4\pi(180/\pi)^2)$
3. Median Redshift: $z_{mean} = 0.9$
4. $\gamma_{\rm int} = 0.22$
5. $n_\theta = 30$ galaxies per arcmin^2
6. Error on photometric redshift: $\sigma_z = 0.05$ (1+z)
7. Linear Power Spectrum: CAMB (provided in the input, you shouldn't use it)
8. Method to correct for non-linearities: Halo model (however the input files are now linear; it does not matter for now as we are not looking for the correct Fisher, just comparing. For this comparison we are using linear spectra).
9. Binning: 10 bins from z = 0 - 2.5. We assume equipopulated bins: the redshift bins chosen such that each contain the same amount of galaxies. Add formula
10. k-range: $k_{min} = 0.001$ h/Mpc, $k_{max} = 0.5$ **h/Mpc**
11. k-binning: use directly the one provided in the input
12. ℓ-range: $\ell_{min} = 5$, ℓ_{max} **fixed as a function of z, using the table provided by Enea (uploaded in the wiki). No cut in z.**
13. ℓ-binning: 100 bins (with log spaced binning)