Module 1

CHAPTER 2

Simulation Examples

2.1 System Simulation using a Simulation Table

- You will not really run the simulations manually in practice.
- System simulation using a simulation table gives you an insight on how the system evolves in time and helps you implement or verify a simulator (computer program).
- Simulation using a Table can be done via pen-and-paper or by using a spreadsheet

The simulation of a given system is carried by the following three steps :

- Determine the characteristics of each input to the simulation. Often, inputs are random variables, and are characterized by probability distributions (probability density functions – pdf)
- Construct a simulation table consisting of :
  - **p** inputs $x_{ij} = 1, 2, \ldots, p$
  - one response y_i, $i = 1, 2, \ldots, n$
- For each repetition **i**, generate a value for each of the **p** inputs x_{ij} and calculate the response y_i.
- That is Columns in the table are inputs and response(s) and Rows are repetitions of system operation (i.e., the inputs are changing).

- Example of a generic simulation table :

Repetition	Inputs							Response
i	x_{i1}	x_{i2}	x_{i3}	...	x_{ij}	...	x_{im}	y_i
1								
2								
3								
|								
|								
|								
|								
n								

2.2 Queuing System

Q. Describe the simulation of Queuing system. **MU - May 05, May 06**

Q. Explain multiuser queuing system with suitable example. **MU - May 12**

Q. Describe queuing system. What is the condition that leads to its stability? **MU - Dec. 10, Dec. 11, Dec. 12**

- A queuing system is described by :
  - Calling population.
  - Arrival rate.
  - Service mechanism.
  - System capacity.
  - Queuing discipline.

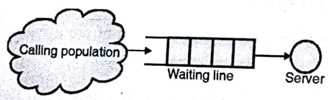

Fig. 2.2.1 : Queuing System

2.2.1 Single Server Queue System

Q. Draw the figures for service outcomes after service completion and potential unit actions upon arrival and the flow diagrams for unit-entering-system and service-just-completed flow for a queuing system. **MU - Dec. 10, Dec. 11**

- **Calling population** is infinite i.e. if a unit leaves the calling population and joins the waiting line or enters service; there is no change in the arrival rate of other units that could need service.
- **Arrival** defines the way customers enter the system. Mostly the arrivals are random with random intervals between two adjacent arrivals. Typically the arrival is described by a random distribution of intervals also called *Arrival Pattern*.

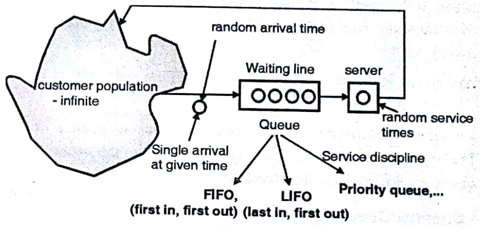

Fig. 2.2.2 : Single Server Queue

- **Service mechanism** used by the server for the units is FIFO. Services are according to distribution of service times. Service items are of random length as per a particular probability distribution.
- Service represents some activity that takes time and that the customers are waiting for. Again take it very generally. It may be a real service carried on persons or machines, but it may be a CPU time slice, connection created for a telephone call, being shot down for an enemy plane, etc. Typically a service takes random time. Theoretical models are based on random distribution of service duration also called Service Pattern.
- **System capacity** is the maximum number of customers that may wait in the queue (plus the one(s) being served). Queue is always limited, but some theoretical models assume an unlimited queue length. If the queue length is limited, some customers are forced to renounce without being served. System capacity has no limit means any number of units can wait in the line.

- **Arrival rate** must be less than service rate for a stable system. Otherwise waiting line grows unbounded and system will be unstable. Such systems are called explosive.
- **Queue discipline** refers to the rule that a server uses to choose the next customer from the queue when the server completes the service of the current customer. Queue disciplines used are FIFO, FCFS, LIFO, LCFS, Priority, etc.
  - FIFO (First In First Out) also called FCFS (First Come First Serve) - queue.
  - LIFO (Last In First Out) also called LCFS (Last Come First Serve) - stack.
  - SIRO (Serve In Random Order).
  - Priority Queue that may be viewed as a number of queues for various priorities.
  - Many other more complex queuing methods that typically change the customer's position in the queue according to the time spent already in the queue, expected service duration, and/or priority. These methods are typical for computer multi-access systems.
- **State** of the system is the number of units in the system and status of the server, busy or idle.
- **Event** is a set of circumstances that causes instantaneous change in the state of the system. The two possible events that can affect the state of the system in case of a single channel queue are arrival event and departure event.

2.2.2 Single Channel Queue System

The Single Channel Queue System includes the server, the unit being serviced and the units in the queue (if any are waiting). The following are the components :

- **Entities :** Server, queue.
- **State :**
  a. Number of units in the system, Q (e.g. customers in bank)
  b. Server status: busy/idle, S = {B,I}
- **Events :** Arrivals, Departures.
- **Simulation Clock :** Tracks simulated time.
- **Actions :** Different actions, depending on the type of the event and the current system state.

2.2.2.1 Arrival Event

- The arrival event is set to occur when a unit enters the system.

- The flow diagram for the arrival event is as shown in Fig. 2.2.3 :

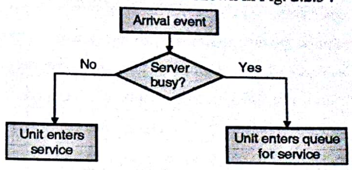

Fig. 2.2.3 : Arrival Event

- The unit on entering the system will find the server either idle or busy.
- If server is idle and the queue is empty, then the unit will begin service immediately, else it enters the queue for the server.
- It is not possible for the server to be idle while queue is not empty.

		Queue status	
		Not empty	Empty
Server	Busy	Enter queue	Enter queue
status	Idle	Impossible	Enter service

2.2.2.2 Departure Event

- Departure event is set to occur at the time of completion of a service for a particular unit.
- The flow diagram for the departure event is as shown in Fig. 2.2.4 :

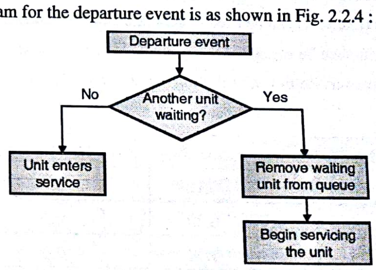

Fig. 2.2.4 : Departure Event

- When server completes the service for unit, the server either becomes idle or remains busy with the next unit.

- Server will check the queue; if queue is empty then server becomes idle, otherwise another unit will enter the server and server becomes busy.
- Following two condition are not possible in queuing system
  - o Queue is empty after service completion, and server become busy.
  - o Queue is not empty after service completion, and server become idle.

		Queue status	
		Not empty	Empty
Server outcome	Busy		Impossible
	Idle	Impossible	

2.2.2.3 Generation of Random Numbers from Random Digits

- The randomness is needed to imitate real life and this is done by using random numbers.
- The distribution of random numbers is uniform and independent on the interval {0, 1} as against random digits that are uniformly distributes on the set {0, 1, 2… 9}.
- Random numbers are generated by using random digits by selecting the appropriate number of digits for each random number and then placing decimal point to the left of the value selected.
- The exact number of digits depends on the accuracy of the data being used for input purposes.
- In a single channel queuing system, interarrival times and service times are generated using the distributions of these random variables.
- Rand() function (Excel, C) can be used to generate uniform random numbers in (0,1)
- Random digits can then be generated using rand() by using a simple algorithm.
- For example, if the service time distribution is specified as in the Table 2.2.1 :

Table 2.2.1

Service time (minutes)	Probability	Cumulative probability
1	0.10	0.10
2	0.20	0.30
3	0.30	0.60
4	0.25	0.85
5	0.10	0.95
6	0.05	1.00

Then random digit allocation is done by following algorithm/code :

```
int service_time(void) {
r = rand()/RAND_MAX;
if(r<0.1) return(1);
else if (r<0.3) return(2);
    else if (r<0.6) return(3);
        else if (r<0.85) return(4);
            else if (r<0.95) return(5);
                else return(6);
}
```

2.2.3 Manual Simulation of Single Channel Queue (Bank)

Consider bank that has only one counter. Bank customer arrives in bank at random from 1 to 8 minutes apart. The service time vary from 1 to 6 minutes with probabilities shown below. We will analyze this problem for 20 customers.

Table 2.2.2 shows service time and probabilities.

Table 2.2.2

Service time	Probability
1	0.10
2	0.20
3	0.30
4	0.25
5	0.10
6	0.05

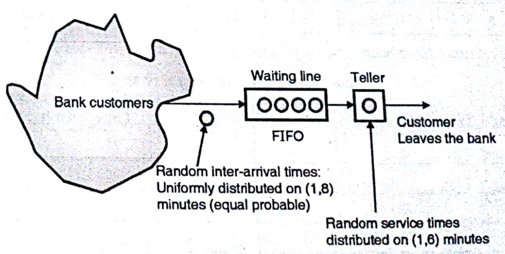

Fig. 2.2.5 : Bank with one teller

Inter-arrival times are uniformly distributed on (1, 8) minutes (equal probable). We construct distribution of time between Arrivals as shown in Table 2.2.3.

Table 2.2.3 : Distribution of time between Arrivals

Time between Arrivals	Probability	Cumulative Probability	Random Digit Assignment
1	0.125	0.125	001-125
2	0.125	0.250	126-250
3	0.125	0.375	251-375
4	0.125	0.500	376-500
5	0.125	0.625	501-625
6	0.125	0.750	626-750
7	0.125	0.875	751-875
8	0.125	1.000	876-000

Table 2.2.4 : Distribution of Service time

Service time	Probability	Cumulative Probability	Random Digit Assignment
1	0.10	0.10	01-10
2	0.20	0.30	11-30
3	0.30	0.60	31-60
4	0.25	0.85	61-85
5	0.10	0.95	86-95
6	0.05	1.00	96-00

Arrival times have been generated randomly as discussed in Table 2.2.5.

Table 2.2.5 : Time between arrivals

Customer	Time between arrivals (min)
1	–
2	8
3	6
4	1
5	8
11	1
12	1
13	5
14	6
15	3

Customer	Time between arrivals (min)
6	3
7	8
8	7
9	2
10	3
16	8
17	1
18	2
19	4
20	5

Service times have been generated randomly as described in Table 2.2.6.

Table 2.2.6 : Service Times

Customer	Service time (min)	Customer	Service time (min)
1	4	11	3
2	1	12	5
3	4	13	4
4	3	14	1
5	2	15	5
6	4	16	4
7	5	17	3
8	4	18	3
9	5	19	2
10	3	20	3

Table 2.2.7 : Simulation table for the bank teller example

Customer	Time since last arrival	Arrival time	Service time	Time service begins	Time in queue	Time service ends	Customer time in system	Idle time for server
1	–	0	4	0	0	4	4	0
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2

Customer	Time since last arrival	Arrival time	Service time	Time service begins	Time in queue	Time service ends	Customer time in system	Idle time for server
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
11	1	47	3	53	6	56	9	0
12	1	48	5	56	8	61	13	0
13	5	53	4	61	8	65	12	0
14	6	59	1	65	6	66	7	0
15	3	62	5	66	4	71	9	0
16	8	70	4	71	1	75	5	0
17	1	71	3	75	4	78	7	0
18	2	73	3	78	5	81	8	0
19	4	77	2	81	4	83	6	0
20	5	82	3	83	1	86	4	0
Total	82		68		56		124	18

System Statistics

- **Average time between arrivals :**

  = (Sum of all inter-arrival times) / (Number arrivals – 1)

  = 82/19

  = 4.3 min.

- **Expected time between arrivals :**

  E(T) = t*p(t)

  = 1/8 (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8)

  = 4.5 min.

- ***Average service time :***

  $\bar{s}$ = (Total service time) / (Total number of customers)

  = 68/20

  = 3.4 min

- **Expected service time :**

  E(s) = 1*0.1 + 2*0.2 + 3*0.3 + 4*0.25 + 5*0.1 + 6*0.05

  = 3.2 min

- **Average waiting time :**

  $\bar{w}_{waited}$ = (Total waiting time in queue) / (Number of customers who wait)

  = 56/13

  = 4.3 min.

- **Probability that a customer has to wait in a queue :**

  P(wait) = (Number of customers that wait) / (Total number of customers)

  = 13/20

  = 0.65

- **Fraction of idle time for server :**

  P(idle) = (Total idle time) / (Total simulation time)

  = 18/86

  = 0.21

- **Average time spent in the system :**

  $\bar{t}$ = (Total time that customers spend in the system)/(Total number of customers)

  = 124/20

  = 6.2 min.

Ex. 2.2.1 : When simulation is done for 100 customer statistics obtained are given below.

Customer	Interarrival time [Minutes]	Arrival time [Clock]	Service time [Minutes]	Time service begins [Clock]	Time service ends [Clock]	Waiting time in queue [Minutes]	Time customer in system [Minutes]	Idle time of server [Minutes]
1	–	0	4	0	4	0	4	0
2	1	1	2	4	6	3	5	0
3	1	2	5	6	11	4	9	0
4	6	8	4	11	15	3	7	0

Customer	Interarrival time [Minutes]	Arrival time [Clock]	Service time [Minutes]	Time service begins [Clock]	Time service ends [Clock]	Waiting time in queue [Minutes]	Time customer in system [Minutes]	Idle time of server [Minutes]
5	3	11	1	15	16	4	5	0
6	7	18	5	18	23	0	5	2
				...				
100	5	415	2	416	418	1	3	0
Total	415		317			174	491	101

Soln. :

System Statistics when simulation is done for 100 customer.

- Average waiting time :

$$\bar{w} = \frac{\sum \text{Waiting time in queue}}{\text{Number of customers}}$$

$$\bar{w} = \frac{174}{100} = 1.74 \text{ min.}$$

- Probability that a customer has to wait :

$$p(\text{wait}) = \frac{\text{Number of customer who wait}}{\text{Number of customers}}$$

$$p(\text{wait}) = \frac{46}{100} = 0.46$$

- Proportion of server idle time :

$$p(\text{idle server}) = \frac{\sum \text{Idle time of server}}{\text{Simulation run time}}$$

$$p(\text{idle server}) = \frac{101}{418} = 0.24$$

- Average service time :

$$\bar{s} = \frac{\sum \text{Service time}}{\text{Number of customers}}$$

$$\bar{s} = \frac{317}{100} = 3.17 \text{ min.}$$

- Expected service time :

$$E(s) = \sum_{s=0}^{\infty} s \cdot p(s) = 0.1 \cdot 10 + 0.2 \cdot 20 + \ldots + 0.05 \cdot 6 = 3.2 \text{ min}$$

- Average time between arrivals :

$$\bar{\lambda} = \frac{\sum \text{Time between arrivals}}{\text{Number of arrivals} - 1}$$

$$\bar{\lambda} = \frac{415}{99} = 4.19 \text{ min.}$$

$$E(\lambda) = \frac{a+b}{2} = \frac{1+8}{2} = 4.5 \text{ min}$$

- Average waiting time of those who wait :

$$\bar{w}_{\text{waited}} = \frac{\sum \text{Waiting time in queue}}{\text{Number of customers that wait}}$$

$$\bar{w}_{\text{waited}} = \frac{174}{54} = 3.22 \text{ min}$$

- Average time a customer spends in system :

$$\bar{t} = \frac{\sum \text{Time customers spends in system}}{\text{Number of customers}}$$

$$\bar{t} = \frac{491}{100} = 4.91 \text{ min.}$$

OR

Average time a customer spends in system = Average waiting time+ Average service time

$$\bar{t} = \bar{w} + \bar{s} = 1.74 + 3.17 = 4.91 \text{ min.}$$

Ex. 2.2.2 A baker is trying to determine how many dozens of bagels to bake each day. The probability distribution of the number of bagel customers is as follows :

No. of Customers/Day	8	10	12	14
Probability	0.35	0.30	0.25	0.10

Customers order 1, 2, 3, or 4 dozen bagels according to the following probability distribution :

No. of Dozens Ordered/Customer	1	2	3	4
Probability	0.4	0.3	0.2	0.1

Bagels sell for Rs. 5.4/- per dozen. They cost Rs. 3.8/- per dozen to make. All bagels not sold at the end of the day are sold at half price to a local grocery store. Based on 5 days of simulation, how many dozen (to the nearest 10 bagels) bagels should be baked each day ? In the above problem identify random inputs, decision variables, the mathematical model and the output. **MU - Dec. 05**

Soln. :

Using the probability of dozens ordered we can assign the random digit to it and is given in Table P. 2.2.2. Similarly, we can assign the random digit to number of customers/day by using its probability and is given in Table P. 2.2.2(a).

Table P. 2.2.2

Dozens Ordered	Probability	Cumulative Probability	Random Digit Assignment
1	0.4	0.4	1 – 4
2	0.3	0.7	5 – 7
3	0.2	0.9	8 – 9
4	0.1	1.00	0

Table P. 2.2.2(a)

No of Customer/Day	Probability	Cumulative Probability	Random Digit Assignment
8	0.35	0.35	01 – 35
10	0.30	0.65	36 – 65
12	0.25	0.90	66 – 90
14	0.10	1.00	91 – 00

Here, the problem is to find out the optimal dozens of bagels that should be baked on each day. We can get the optimal solution by simulating the demand for 5 days and recording the profits from sale on each day. The profit is given by :

Profit = (Revenue from sale) – (Cost of bagels baked)

+ (Revenue from grocery store sale) – (Lost profit)

Let Q = number of dozens baked/day

$S = \sum O_i$ where O_i = Order quantity in dozens for the i^{th} customer

If $Q > S$ then $Q - S$ and represents the grocery store sales in dozens else $S - Q$ and it represents dozens of excess demand.

$$\text{Profit} = (5.40 \times \min(S, Q)) - (3.80 \times Q) + (2.70 \times (Q - S)) - (1.60 \times (S - Q))$$

Pre-analysis :

E (Number of customers) = 0.35(8) + 0.30(10) + 0.25(12) + 0.10(14) = 10.20

E (Dozens ordered) = 0.4(1) + 0.3(2) + 0.2(3) + 0.1(4) = 2.0

E (Dozens sold) = S = (10.20) (2.0) = 20.4

E (Profit) = 5.40 min (S, Q) – 3.80Q + 2.70(Q – S) – 1.60(S – Q)

= 5.40 min (20.4, Q) – 3.80Q + 2.70(Q – 20.4) – 1.60(20.4 – Q)

E (Profit| Q=0) = 0 – 0 + 1.60(20.4) = – 32.64

E (Profit| Q=10) = 5.40 (10) – 3.80 (10) + 0 – 1.60(20.4 – 10) = – 0.64

E (Profit| Q=20) = 5.40 (20) – 3.80 (20) + 0 – 1.60(20.4 – 20) = Rs. 31.36

E (Profit| Q=30) = 5.40 (20.4) – 3.80 (30) + 2.70 (30 – 20.4) – 0 = Rs. 22.08

E (Profit| Q=40) = 5.40 (20.4) – 3.80 (40) + 2.70 (40 – 20.4) – 0 = Rs. 11.08

E (Profit| Q=50) = 5.40 (20.4) – 3.80 (50) + 2.70 (50 – 20.4) – 0 = Rs. 0.08

The pre – analysis, based on expectation only, indicates that simulation of the policies Q = 20, 30, 40 and 50 should be sufficient to determine the policies. The simulation should begin with Q = 20, then proceed to Q = 30, then most likely to Q = 40 and Q = 50.

Initially, conduct a simulation for Q = 20, 30, 40 and 50. If the profit is maximized when Q = 20, it will become the policy recommendation. The problem requests that the simulation for each policy should run for 5 days. This is a very short run length to make a policy decision.

Here, we are simulating this experiment for Q = 40 dozens.

Given that :

Cost price of 1 dozen bagel = Rs. 3.80

Selling price of 1 dozen bagel = Rs. 5.40

So, cost price of 40 dozens bagel = Rs. 3.80 x 40 = Rs. 152.00

Hence, Profit = (Revenue from sale) – 152.00 +

(Revenue from grocery store sale) – (Lost profit)

Ex. 2.2.3 Calculate the output statistics for the queuing system whose interarrival and service times for ten arrivals are given below :

Interarrival time	–	8	6	1	8	3	8	7	2	3
Service time	4	1	4	3	2	4	5	4	5	3

MU - Dec. 07. Dec. 08

Soln :

The simulation table for the system will be :

Customer	Inter arrival time	Arrival time	Service time	Time service begins	Time customer waits in queue	Time service ends	Time customer spends in system	Idle time of server
1	–	0	4	0	0	4	4	0
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	4
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
	Σ 46		Σ 33		Σ 8		Σ 44	Σ 20

Output statistics for queuing system

i. Average waiting time for a customer = 8/10 = 0.8

ii. Probability that customer has to wait in queue = 3/10 = 0.3

iii. Probability of idle server = 20/53 = 0.37

iv. Average service time = 33/20 = 1.65

v. Average time between arrival = 46/9 = 5.11

vi. Average waiting time for customers who wait = 8/3 = 2.67

vii. Average time customer spends in the system = 44/10 = 4.4

2.2.4 Manual Simulation of Multi Channel Queue

Consider bank having two counters. Bank customer arrives in bank at random from 1 to *8 minutes apart. The service* time for teller1 vary from 1 to 6 minutes with probabilities shown *in Table 2.2.8.*

Table 2.2.8 : Teller1 service time

Service time	Probability
1	0.10
2	0.20
3	0.30
4	0.25
5	0.10
6	0.05

The service time for teller 2 vary from 3 to 6 minutes with probabilities shown in Table 2.2.9.

Table 2.2.9 : Teller 2 service time

Service time (min)	Probability
3	0.35
4	0.25
5	0.20
6	0.20

Service policy : If both tellers are idle, Teller 1 serves the next customer Otherwise, the customer is served by the teller 2.

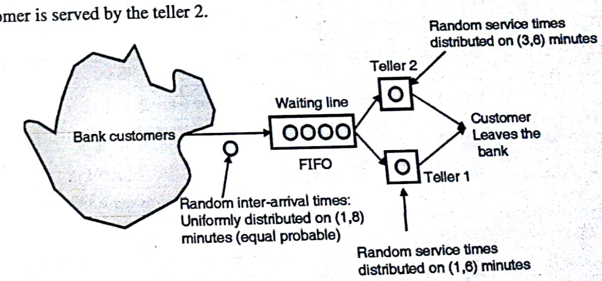

Fig. 2.2.6 : Multi-channel queue

Inter-arrival times are uniformly distributed on (1, 8) minutes (equal probable). We construct distribution of time between Arrivals.

Table 2.2.10 : *Distribution of time between Arrivals*

Time between Arrivals	Probability	Cumulative Probability	Random Digit Assignment
1	0.125	0.125	001-125
2	0.125	0.250	126-250
3	0.125	0.375	251-375
4	0.125	0.500	376-500
5	0.125	0.625	501-625
6	0.125	0.750	626-750
7	0.125	0.875	751-875
8	0.125	1.000	876-000

Table 2.2.11 : Distribution of Service time for teller 1

Service time	Probability	Cumulative Probability	Random Digit Assignment
1	0.10	0.10	01-10
2	0.20	0.30	11-30
3	0.30	0.60	31-60
4	0.25	0.85	61-85
5	0.10	0.95	86-95
6	0.05	1.00	96-00

Table 2.2.12 : Distribution of Service time for teller 2

Service time	Probability	Cumulative Probability	Random Digit Assignment
3	0.35	0.35	0-35
4	0.25	0.60	36-60
5	0.20	0.80	61-80
6	0.20	1.00	81-00

Simulation table for the bank teller example – two tellers

Customer	Arrival time	Service time	Teller 1 Time service begins	Teller 1 Time service ends	Teller 2 Time service begins	Teller 2 Time service ends	Time in queue	Idle time T1	Active time T2
1	0	4	0	4			0	0	0
2	8	1	8	9			0	4	0
3	14	4	14	18			0	5	0
4	15	3			15	18	0	0	3
5	23	2	23	25			0	5	0
6	26	4	26	30			0	1	0
7	34	5	34	39			0	4	0
8	41	4	41	45			0	2	0
9	43	6			43	49	0	0	6
10	46	5	46	51			0	1	0
11	47	4			49	53	2	0	4
12	48	3	51	54			3	0	0
13	53	4			53	57	0	0	4
14	59	3	59	62			0	5	0
15	62	5	62	67			0	0	0
16	70	4	70	74			0	3	0
17	71	4			71	75	0	0	4
18	73	1	74	75			1	0	0
19	77	5	77	82			0	2	0
20	82	4	82	86			0	0	0
Total							6	32	21

Queuing statistics comparisons

Scenario	Queuing time (min)	Fraction of idle time teller 1 (min)	Fraction of active time teller 2 (min)	Fraction of idle time teller 2 (min)
1 Teller	4.3	0.21		
2 Tellers	2	0.372	0.244	0.756

2.3 Simulation Inventory System

MU - May 16

MU - May 06

Q. Describe the simulation of Inventory system.

Q. Discuss various costs that are involved in inventory system Explain the policy and goal of inventory system. MU - Dec. 11, Dec. 12

Q. Describe the input model for an inventory system if the lead time and demand are related. MU - May 13, Dec. 13, May 14

Fig. 2.3.1 : A simple Inventory System

- In an inventory system, one is mainly concerned with making decisions in order to minimize the total cost of the operation. These decisions are mainly related to the quantity of inventory to be acquired (or produced) and the frequency of acquisitions.
- Inventory systems are one of the most important classes of model simulation problems.
- Inventory control objectives are to maximize customer service, and to minimize costs.
- A simple inventory system is represented in Fig. 2.3.2.

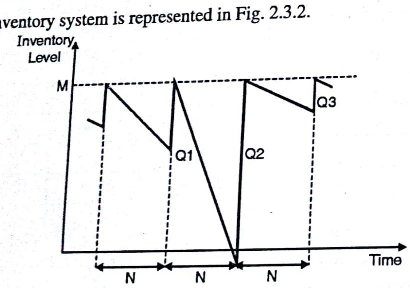

Fig. 2.3.2 : Probabilistic Order Level

- The inventory in system is checked (reviewed) after certain time. In above figure review period is shown as N. At the end of each review period order of quantity Q is placed to bring the inventory upto the level M.
- At the end of the first review period, an order quantity, Q1, is placed. At the end of the second review period, an order quantity, Q2, is placed to bring the inventory upto the level M. Demands are uncertain so order quantities are probabilistic in nature.

- In Fig. 2.3.2 the lead time is zero, but in reality lead time is random of some positive length.
- In Fig. 2.3.2 demands are shown as uniform over this time period, but in reality demand fluctuate over time.
- In the second cycle, there is shortage as amount in the inventory drops below zero. Buffer stocks need to be carried out to avoid shortage.
- There are various costs involved with the inventory system. They are as :
  a. Holding cost
  b. Shortage cost
  c. Setup cost
- The holding cost is related to the cost of keeping one item of inventory over a period of time. One of the most important components of this cost is that of the invested capital. Holding cost includes cost involved in the renting of space, hiring of guards and so on.
- The setup cost is related to the cost in placing a new order or changes to production. To carry out a high inventory we need to make frequent reviews and consequently more purchases or replenishments. The cost associated with order is called as ordering/ setup cost.
- Finally, the shortage cost is associated with the cost of not having available a unit of inventory when demanded. This cost may be in the form of transportation charges (i.e., expediting deliveries), increased overtime, and loss of future business.
- Let It be the inventory at time t. Let S be the quantity added in the system between time t and t´. Also, let D be the demand between these time instances. Then, the inventory at time t´ is

$$It' = It + S - D$$

- If It´ is below a certain value, then an order is placed. The time it takes for the ordered stock to arrive is known as the lead time. We assume that the daily demand and the lead time follow known arbitrary distributions.
- The inventory level is checked at the end of each day. If it is less than or equal to the re-ordering level, an order is placed. The lead time for the order begins to count from the following day. Orders arrive in the morning and they can be disposed of during the same day. During stock out days, orders are backlogged. They are satisfied on the day the order arrives.
- The simulation model is described in the flow chart given in Figs. 2.3.3 and 2.3.4.

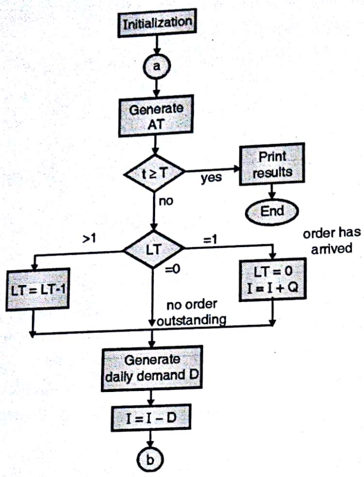

Fig. 2.3.3

- The model estimates the total cost of the inventory system for specific values of the reordering point and the quantity ordered. The model keeps track of It on a daily basis. In view of this, the model was developed using the unit-time advance design. We note that this design arises naturally in this case. A unit of time is simply equal to one day. The lead time is expressed in the same unit time.
- The basic input parameters to the simulation model are the following :
  - ROP, reordering point,
  - Q, quantity ordered,
  - BI, the beginning inventory.
  - Probability distributions for variables D and LT representing the daily demand and lead time, respectively,
  - T, the total simulation time, and
  - C1, C2, C3, representing the holding cost per unit per unit time, the setup cost per order, and the shortage cost per unit per unit time, respectively.

- The output parameters are TC1, TC2, TC3 representing the total holding, setup and shortage costs respectively.

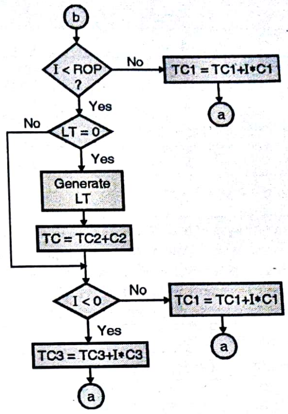

Fig. 2.3.4 : A unit-time simulation design of an inventory system

Ex. 2.3.1 : Perform the simulation of the inventory system. Daily demand is represented by the random numbers 4, 1, 8, 5, 2 and the demand probability is given by :

0	0.2
1	0.5
2	0.3

If the initial inventory is 4 units, determine on which day the shortage condition occurs. **MU - May 07. May 09**

Soln. :

Demand probability is given by :

Demand	Probability	Cumulative Probability	Random Digit Assignment
0	0.2	0.2	1 – 2
1	0.5	0.7	3 – 7
2	0.3	1.0	8 – 0

To determine shortage condition :

Day	Beginning Inventory	Random Digits for Demand	Demand	Ending Inventory	Shortage Quantity
1	4	4	1	3	0
2	3	1	0	3	0
3	3	8	2	1	0
4	1	5	1	0	0
5	0	2	0	0	0

So, shortage occurs in the inventory.

Ex. 2.3.2 : Consider an inventory system which has a periodic review of length N = 5 days, and inventory level M = 10 items. Estimate by simulation average ending item in inventory and number of days when shortage occur. Probability distribution of daily demand and lead time is given in table. Simulate 3 weeks of this system. Start with 4 items and order of 6 item scheduled to arrive in 2 days time.

Daily demand	0	1	2	3	4
Probability	0.12	0.28	0.15	0.27	0.18

Lead Time	1	2	3
Probability	0.2	0.5	0.3

Random digits for demand : 81, 70, 44, 17, 8, 23, 63, 19, 88, 92, 21, 30, 58, 3, 83. Random digits for lead time : 2, 6, 4. **MU - May 07, May 09**

Soln :

Given : M = 10 items N = 5 days

Starting inventory = 4 items

Pending items = 6 items (2 days)

Demand

Demand	Probability	Cumulative Probability	Random Digit
0	0.12	0.12	1 – 12
1	0.28	0.40	13 – 40
2	0.15	0.55	41 – 55
3	0.27	0.82	56 – 82
4	0.18	1.00	83 – 00

Lead Time

Demand	Probability	Cumulative Probability	Random Digit
1	0.2	0.20	1 – 2
2	0.5	0.70	3 – 7
3	0.3	1.00	8 – 0

Cycle 1 :

Days	Random Digit for Demand	Demand	Starting Inventory	Ending Inventory	Shortage
1	81	3	4	1	0
2	70	3	1	0	2
3	44	2	0 + 6	2	0
4	17	1	2	1	0
5	8	0	1	1	0

Order = 9 items

Random digit for lead time = 2 = 1 day

Cycle 2 :

Days	Random Digit for Demand	Demand	Starting Inventory	Ending Inventory	Shortage
1	23	1	1	0	0
2	63	3	0 + 9	6	0
3	19	1	6	5	0
4	88	4	5	1	0
5	92	4	1	0	3

Order = 10 items

Random digit for lead time = 6 = 2 days

Cycle 3 :

Days	Random Digit for Demand	Demand	Starting Inventory	Ending Inventory	Shortage
1	21	1	0	0	4
2	30	1	0	0	5
3	58	3	0 + 10	2	0
4	3	0	2	2	0
5	83	4	2	0	2

Order = 10 items

Random digit for lead time = 4 = 2 days

Average ending unit = (Total ending inventory/ No of days) = 21/15 = 1.4 units

Shortage occurs in :

i. Cycle 1 at Day 2,

ii. Cycle 2 at Day 5,

iii. Cycle 3 at Day 1, 2 and 5.

2.4 Reliability Problem

Q. Write short notes on Reliability system. **MU - Dec. 11, Dec. 12**

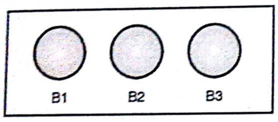

Fig. 2.4.1 : Reliability of manufacturing machine

- A large manufacturing machine has three different bearing, shown in Fig. 2.4.1 that fa in service.
- The distribution function of the life of each bearing is identical.
- When a bearing fails, the machine stops working. So, a repair person is called and a new bearing is installed.
- The delay time of repair person arriving at the manufacturing machine is also a random variable.
- The down time of machine is estimated as Rs. 'X' per minute.
- The site cost of the repair person is Rs. 'Y' per hours.
- It takes 'n1' minute to change one bearing, 'n2' minute to change two bearings and 'n3' minute to change three bearings where n1<n2<n3. The bearing cost is Rs. 'Z' each.
- The reliability problem can be solved using current method or proposed method.
- In current method, if the bearing fails only that bearing is replaced.
- In proposed method, if the bearing fails then replace all three bearing because their life distribution function is identical, other two can fail at any time near to the failed bearing.

Review Questions

Q. 1 Explain simulation of queuing system with relevant flow diagram.

Q. 2 Describe the simulation of Queuing system.

Q. 3 Give the input parameters, simulation variable, and output statistics for the queuing system. Calculate the output statistics for the queuing system whose interarrival and service times for ten arrivals are given below :

Interarrival time : 8 6 1 8 3 8 7 2 3

Service time : 4 1 4 3 2 4 5 4 5 3.

Q. 4 What are the costs associated with inventory system? Describe the inventory system when -

I. Lead time is zero.

II. Lead time is independent of demand.

III. Lead time is dependent on demand

Q. 5 Describe queuing and inventory system stating its inputs and output.

Q. 6 Perform the simulation of the inventory system. Daily demand is represented by the random numbers 4, 1, 8, 5, 2 and the demand probability is given by

Demand	Probability
0	0.2
1	0.5
2	0.3

If the initial inventory is 4 units, determine on which day the shortage condition occurs.

Q. 7 Explain Time-shared computer model.

Q. 8 Explain Job-shop model.

2.5 University Questions and Answers

Dec. 2010

Q. 1 Draw the event logic diagram for arrival event in single server queuing system. **(Section 2.2.1)** **(3 Marks)**

Dec. 2011

Q. 2 Describe briefly queuing, inventory and reliability systems. **(Sections 2.2, 2.3 and 2.4)** **(4 Marks)**