CHAPTER 7

Module 3

Random Variate Generation

7.1 Introduction

- A random variate is a particular outcome of a random variable: the random variates which are other outcomes of the same random variable would have different values. Random variates are used when simulating processes driven by random influences (stochastic processes). In modern applications, such simulations would derive random variates corresponding to any given probability distribution from computer procedures designed to create random variates corresponding to a uniform distribution, where these procedures would actually provide values chosen from a uniform distribution of pseudorandom numbers.
- Procedures to generate random variates corresponding to a given distribution are known as procedures for random variate generation. In probability theory, a random variable is a measurable function from a probability space to a measurable space of values that the variable can take on. In that context, and in statistics, those values are known as a random variates, or occasionally random deviates, and this represents a wider meaning than just that associated with pseudorandom numbers.
- Random variate is a variable generated ~~form~~ from uniformly distributed random numbers. It refers to a particular value of a random variable. There are various techniques used for the generation of the random numbers. They are : Inverse-transform method, Acceptance-rejection method, Convolution method, etc.

MU - May 16

7.2 Inverse Transform Technique

Q. By using Inverse Transform Technique which of the distributions random variates can be generated ? Write the procedure for sampling. **MU - Dec. 05, May 06, Dec. 07, May 08**

> **Q.** Write short notes on: Inverse Transform Technique.
> May 10, May 11, May 12, Dec. 12, May 13, May 15

- **Theorem :** Let Y have a distribution that is U(0,1). Let F(x) have the properties of a distribution function of the continuous type with F(a) = 0 and F(b) = 1, and suppose that F(x) is strictly increasing on the support $a < x < b$, where a and b could be $-\infty$ and ∞, respectively. Then the random variable X defined by $X = F^{-1}(Y)$ is a continuous random variable with distribution function F(x).
- It is used to sample from the exponential, the uniform, the Weibull, the triangular distributions and empirical distributions.
- The inverse-transform technique can be used in principle for any distribution, but most useful when the CDF **F(x)** has an inverse **F -1(x)** which is easy to compute.
- This is a very straightforward technique, though not always the most computationally efficient.
- The required steps in this technique are :
  - Compute the CDF of the desired random variable X.
  - Set F(X) = R on the range X.
  - Solve equation F(X) = R for X in terms of R.
  - Generate uniform random numbers $R_1, R_2, R_3 \ldots$ and compute the desired random variate by $X_i = F^{-1}(R_i)$

7.2.1 Exponential Distribution

> **Q.** Describe the procedure to generate samples from exponential distribution.
> MU - Dec. 06, May 07
>
> **Q.** Using inverse Transform method derive random variates for exponential distribution.
> MU - Dec. 12
>
> **Q.** Generate random variates of exponential distribution.
> MU - Dec. 14

- A random variable X is exponentially distributes with parameter $\lambda > 0$, if its PDF is given by :

$$F(x) = \lambda e^{-\lambda x} \quad ; x \geq 0$$
$$F(x) = 0 \quad ; \text{Elsewhere}$$

- The CDF is given by :

$$F(x) = 0 \quad ; x < 0$$

$$F(x) = \int_0^x \lambda e^{-\lambda t}\, dt = 1 - e^{-\lambda x} \quad ; x \geq 0$$

- The various steps to generate samples from exponential distribution are :

i. Here the CDF = F(x) = $1 - e^{-\lambda x}$, $x \geq 0$

ii. Set F(x) = R on the range of X.

Hence, $1 - e^{-\lambda x} = R$

iii. Solve the equation F(x) = R for X in terms of R, in this case, the solution is as :

$$1 - e^{-\lambda x} = R$$
$$e^{-\lambda x} = 1 - R$$
$$-\lambda X = ln(1 - R)$$
$$X = \frac{ln(1-R)}{-\lambda}$$
$$X = -\frac{ln(1-R)}{\lambda}$$

iv. Finally, Since R and (1 – R) are uniformly distributed on [0,1]. We can replace $(1 - R_i)$ by R_i for simplification. We get,

$$X = -\frac{ln(R)}{\lambda}$$

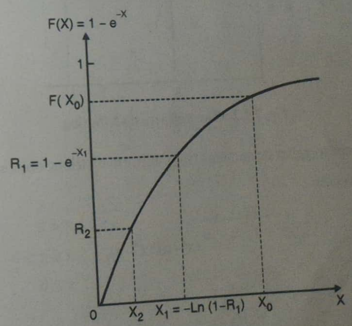

Fig. 7.2.1 : Inverse-transform technique for exp (λ = 1)

Ex. 7.2.1 : To generate a random numbers from exponential distribution with mean 10
$F(x) = 1 - e^{-x/10}$ and $x = F^{-1}(y) = -10\ ln(1-y)$

Soln. :

Use uniform $U(0, 1)$ random number generator to generate random numbers $y_1, y_2, \ldots, y_n$.

The exponentially distributed random numbers x_i's will be

$$x_i = -10\ ln(1-y_i),\ i = 1, \ldots, n.$$

Therefore, if the uniform random number generator generates a number **0.1514**, then **1.6417** $= -10 ln(1-0.1514)$ would be a random exponential observation.

7.2.2 Uniform Distribution

The probability density function of the uniform distribution is defined as follows:

Consider a random variable X that is uniformly distributed on the interval [a, b]

$$\text{pdf :} \quad f(X) = \begin{cases} 1/(b-a) & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

It is shown graphically in Fig. 7.2.2.

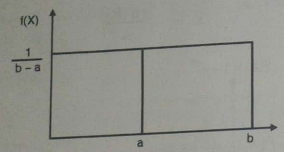

Fig. 7.2.2 : The uniform distribution

The various steps to generate samples from uniform distribution are as follows :

i. The CDF is given by :

$$\begin{aligned} F(x) &= 0, & x < a \\ &= (x-a)/(x-b), & a \le x \le b \\ &= 1, & x \ge b \end{aligned}$$

ii. Set $F(x) = R$

$$\frac{X-a}{b-a} = R$$

$$X - a = R(b-a)$$

iii. Solving for X in terms of R we get

$$X = a + (b-a)R$$

7.2.3 Weibull Distribution

The various steps to generate samples from Weibull distribution are as follows :

$1 - e^{-\left(\frac{x-v}{\alpha}\right)^{\beta}}$

i. The CDF is given by,

$$F(X) = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\beta}}$$

~~for x ≥ 0~~

ii. Set $F(x) = R$

$$F(X) = R$$

$$1 - e^{-\left(\frac{x}{\alpha}\right)^{\beta}} = R$$

iii. Solving for X in terms of R, we get,

$$F(X) = R$$

$$1 - e^{-\left(\frac{x}{\alpha}\right)^{\beta}} = R$$

$$e^{-\left(\frac{x}{\alpha}\right)^{\beta}} = 1 - R$$

$$-\left(\frac{x}{\alpha}\right)^{\beta} = \ln(1-R)$$

$$\frac{X^{\beta}}{\alpha^{\beta}} = -\ln(1-R)$$

$$X^{\beta} = -\alpha^{\beta} \cdot \ln(1-R)$$

$$X = \sqrt[\beta]{-\alpha^{\beta} \cdot \ln(1-R)}$$

$$X = \alpha \cdot \sqrt[\beta]{-\ln(1-R)}$$

$$X = \alpha[-\ln(1-R)]^{1/\beta}$$

even
$R \geq 0$

7.2.4 Triangular Distribution

i. The CDF is given by :

The CDF of a Triangular Distribution with endpoints (0, 2) is given by,

$$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{x^2}{2} & 0 < x \le 1 \\ 1 - \frac{(2-x)^2}{2} & 1 < x \le 2 \\ 1 & x > 2 \end{cases}$$

ii. Set $F(x) = R$ for $0 \le X \le 1$

$$R = \frac{X^2}{2} \text{ and for } 1 \le X \le 2$$

$$R = 1 - \frac{(2-X)^2}{2}$$

iii. Solving for X in terms of R, we get,

$$X = \sqrt{2}R, \qquad 0 \le R \le \frac{1}{2}$$

$$X = 2 - \sqrt{2}\,(1-R) \qquad \frac{1}{2} \le R \le 1$$

i.e.

$$X = \begin{cases} \sqrt{2R} & 0 \le R \le \frac{1}{2} \\ 2 - (\sqrt{2(1-R)}) & \frac{1}{2} < R \le 1 \end{cases}$$

7.2.5 Geometric Distribution

Q. Design the generator for geometric distribution. **MU - Dec.11**

- The *PDF* is given by,

$$p(x) = p * (1-p)^x, \qquad x = 0, 1, 2, \ldots$$

$$0 < p < 1$$

- The CDF is given by,

$$F(x) = \sum_{j=0}^{x} p(1-p)^j$$

$$= \frac{p\{1-(1-p)^{x+1}\}}{1-(1-p)}$$

$$= 1-(1-p)^{x+1}$$

- Let $R = F(x)$, solve for *x* in term of *R*. Because this is a discrete random variate, use the inequality

$$F(x_{i-1}) = r_{i-1} < R \le r_i = F(x_i)$$

that is

$$F(x_{i-1}) = r_{i-1} = 1-(1-p)^x < R \le 1-(1-p)^{x+1} = r_i = F(x_i)$$

$$(1-p)^{x+1} \le 1-R < (1-p)^x$$

$$(x+1)\, ln\,(1-p) \le ln\,(1-p) < x\,ln\,(1-p)$$

$$ln\,(1-p) < 0$$

Notice that

$$\frac{ln\,(1-R)}{ln\,(1-p)} - 1 \le x < \frac{ln\,(1-R)}{ln\,(1-p)}$$

- Consider that *x* must be an integer, so

$$X = \left\lceil \frac{ln\,(1-R)}{ln\,(1-p)} - 1 \right\rceil$$

7.2.6 Erlang Distribution

Describe the procedure to generate samples from Erlang distribution

- An Erlang random variable X with parameters (k, θ) can be shown as sum of k independent exponential random variables, X_i, i = 1, 2 … k each having mean 1/kθ.
- The convolution approach is used to generate $X_1, X_2 \ldots X_k$, then we add them to get X. The convolution method involves adding together two or more random variables to obtain a random variable with desired probability.
- Each X_i can be generated by the Inverse Transform Technique equation :

$$X_i = -\frac{ln\,R_i}{\lambda} \quad \text{with } 1/\lambda = 1/k\theta$$

- Hence, an Erlang random variate can be generated by :

$$X = \sum_{i=1}^{K} \frac{-1}{K\theta} \ln R_i$$

$$= \frac{-1}{K\theta} \ln \left(\prod_{i=1}^{K} R_i \right)$$

7.2.7 Empirical Distribution

- This distribution is used when theoretical distribution is not applicable.
- To collect empirical data :
  - Resample the observed data.
  - Interpolate between observed data points to fill in the gaps.
- For a small sample set (size n) the following method is followed :

i. Arrange the data from smallest to largest as $x_1 \le x_2 \le \ldots \le x_n$

ii. Set $X_0 = 0$

iii. Assign the probability 1/n to each interval $x_{i-1} \le x \le x_i$

iv. The slope of each line segment is defined as

$$a_i = \frac{x_{(i)} - x_{(i-1)}}{1/n - (i-1)/n} = \frac{x_{(i)} - x_{(i-1)}}{1/n}$$

v. The inverse CDF is given by,

$$X = \hat{F}^{-1}(R) = x_{(i-1)} + a_i \left(R - \frac{(i-1)}{n} \right)$$

7.3 Direct Transformation for the Normal Distribution

Consider two standard normal random variables, Z1 and Z2, plotted as a point in the plane. Then (B, α) are the polar coordinates where B is radius of circle in polar form and α is angle of this radius with center.

In polar coordinates two points represented as

$$Z_1 = B\cos(\alpha)$$

$$Z_2 = B\sin(\alpha)$$

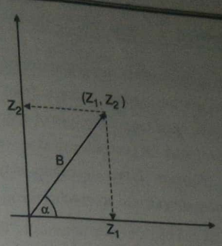

Fig. 7.3.1

Given k independent N(0, 1) random variables $X_1, X_2, \ldots, X_k$, then the sum is according to the Chi-square distribution.

$$X_k^2 = \sum_{i=1}^{k} X_i^2$$

$$f(x, k) = \frac{1}{\Gamma\left(\frac{k}{2}\right) 2^{k/2}} X^{k/2-1} e^{-x/2}$$

It is known that $B^2 = Z_1^2 + Z_2^2$ has the chi-square distribution with 2 degrees of freedom, which is equivalent to an exponential distribution with mean 2

$Y = \lambda e^{-\lambda t}$, $t \ge 0$

$E[Y] = 2 = \lambda$

Thus the radius *B* can be generated using $X_i = -\frac{1}{\lambda} \ln R_i$

$$B = \sqrt{-2 \ln R}$$

The radius B and angle α are mutually independent.

So a normal distribution can be generated by any one of the following :

$$Z_1 = \sqrt{-2 \ln R_1} \cos(2\pi R_2)$$

$$Z_2 = \sqrt{-2 \ln R_1} \cos(2\pi R_2)$$

where R_1 and R_2 are uniformly distributed over (0,1).

To obtain normal variants X_i with mean μ and variance σ^2, transform

$$X_i = \mu + \sigma Z_i$$

7.4 Convolution Method

- **Convolution random number generator** is a pseudo-random number sampling method that can be used to generate random variates from certain classes of probability distribution. The particular advantage of this type of approach is that it allows advantage to be taken of existing software for generating random variates from other, usually non-uniform, distributions. However, faster algorithms may be obtainable for the same distributions by other more complicated approaches.
- A number of distributions can be expressed in terms of the (possibly weighted) sum of two or more random variables from other distributions. (The distribution of the sum is the convolution of the distributions of the individual random variables).
- To generate a random variable X that can be represented as $X = + Y_1 + Y_2 + \ldots + Y_n$,
- Where the Y_i's are independent random variables, an obvious strategy is to generate the Y_i 's independently, and sum them to obtain X. This is known as the convolution method because the distribution function of X can be computed analytically as the convolution of the distributions of the Y_i's.
- Examples of random variables that can be expressed as sums like this include the Erlang, and binomial distributions.
- The probability distribution of a sum of two or more independent random variables is called a *convolution* of the distributions of the original variables. Thus, convolution refers to adding things up Erlang random variable X with parameters (k, θ) can be depicted as the sum of k independent exponential random variables Xi , i = 1, …, k each having mean 1/(k θ)

$$X = \sum_{i=1}^{k} X_i$$

- Using equation $X_i = -\frac{1}{\lambda} \ln R_i$ we can generate exponential variable, an Erlang variate can be generated by

$$X = \sum_{i=1}^{k} \frac{-1}{K\theta} \ln R_i = \frac{-1}{K\theta} \ln \left(\prod_{i=1}^{k} R_i \right)$$

7.5 Acceptance – Rejection Technique

combined related dis & uniform dis.

Q. Write short note on Acceptance-Rejection technique. MU - May 08, Dec. 08

- The acceptance-rejection method is an algorithm for generating random samples from an arbitrary probability distribution, given as ingredients random samples from a related distribution and the uniform distribution.
- The acceptance-rejection method's chief advantage over the inverse CDF method of generating random numbers is that it requires neither the cumulative distribution function nor its inverse to be computed. So in many cases it can run faster.

Set-up

- Let X be a random variable with some other probability distribution that we know how to draw samples from -- that is, generate on a computer.
- Let U be a random variable uniformly distributed on the interval [0, 1].
- Let Y be the random variable that we want to be able to generate. Assume Y has a probability distribution that is absolutely continuous to the probability distribution for X, with density ρ.
- Further assume that the density ρ is bounded above by a constant c. So $\rho(x) \le c$ for all x in the range of X; and necessarily $c \ge 1$.
- In most applications, both X and Y will be continuous random variables with densities g and f respectively. In that case we have $p(x) = f(x) / g(x)$ and we require $f(x) \le cg(x)$.
- The random variables X and Y can be multi-variate.

Algorithm

- The procedure to generate a value for Y is :
  1. Generate a value for X.
  2. Generate a value for U.
  3. If $U \le \rho(X) / c$, then set Y = X ("accept").
  4. Otherwise, go back to step 1 ("reject"), repeating until we obtain a value of Y in step 3.
- Acceptance-Rejection technique is used when other methods of generating random variate fail.
- The efficiency of Acceptance-Rejection technique depends on how many generated numbers are rejected and the ability to minimize those rejections.

Efficiency ↑ No. of rejection (minimal)

- The flowchart explains how the technique actually works.
- Example : To generate uniform distribution between ¼ and 1:

Step 1 : Generate a random number R.

Step 2a : If R = ¼, accept X = R, go to step 3.

Step 2b : If R < ¼, reject R, go to step 1.

Step 3 : If another uniform variate is needed between ¼ and 1, repeat from step 1; if not STOP.

- Here step 2a is acceptance and step 2b is rejection.
- Random variate here will be generated until some condition (R > ¼) is satisfied.
- The accepted values here are conditioned values, that is, R itself does not have the desired distribution, but R conditioned on the event (R = ¼) does have the desired distribution.

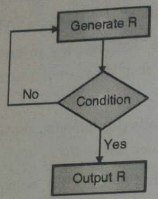

Fig. 7.5.1

- The random variate generated using above methods is indeed uniformly distributed over [1/4, 1].
- To prove this, use the definition. Take any $1/4 \le a < b \le 1$,

$$P(a < R \le b \mid 1/4 \le R \le 1) = \frac{P(a < R \le b)}{P(1/4 \le R \le 1)} = \frac{b-a}{3/4}$$

Which is the correct probability for a uniform distribution on [1/4,1].

Explanation

- When we generate X and U as prescribed in the algorithm, we are in fact picking the point (X, cU) in the rectangular box below. And the test $U \le \rho(X) / c$ determines that point lies below the graph of ρ. It seems plausible that if we keep only the points that fall under the graph of the density ρ, and ignore the points above, then the distribution of the abscissa should have density ρ.

- The acceptance-rejection method works more efficiently as the distribution of X and Y become similar enough that is, ρ(x) and its upper bound **c** are close to one. This makes the rejection region smaller, and so the algorithm is likely to go through fewer repetitions discarding the rejects.

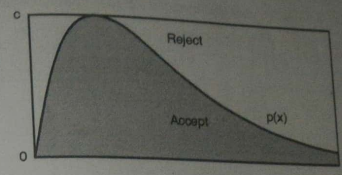

Fig. 7.5.2 : Acceptance and rejection regions for a density

Ex. 7.5.1 : Generating a random variable by using Acceptance – Rejection Technique from $f_x(x) = 3x^2,\ 0 \le x \le 1$

Soln. :

Assume : $g_x(x) = 1,\ 0 \le x \le 1$

Thus

$$\max\left(\frac{f_x(x)}{g_x(x)}\right) = 3 = c$$

$$\left(\frac{f_x(x)}{cg_x(x)}\right) = x^2$$

Algorithm

1) Generate two uniform random variables U_1 and U_2 from U(0, 1).
2) If $U_2 \le U_1^2$ accept U_1 as the random variable from $f_x(x)$, otherwise go to step 1.

Ex. 7.5.2 : Generating a random variable by using Acceptance – Rejection Technique having p.d.f. $f(x) = 20x(1-x)^3,\ 0 < x < 1$

Soln. :

Since this random variables (which is beta with parameters 2, 4) is concentrated in the interval (0, 1), let us consider the acceptance-rejection method with

$$g(x) = 1, \qquad 0 < x < 1$$

To determine the constant c such that f(x)/g(x) <= c, we use calculus to determine the maximum value of

$$f(x) / g(x) = 20x(1-x)^3$$

Differentiation of this quantity yields

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = 20\,((1-x)^3 - 3x\,(1-x)^2$$

Setting this equal to 0 shows that the maximal value is attained when x = 1/4, and thus

$$\frac{f(x)}{g(x)} \le 20\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^3 = \frac{135}{64} = c$$

Hence,

$$\frac{f(x)}{cg(x)} = \frac{256}{27}\,x\,(1-x)^3$$

and thus the simulation procedure is as follows :

1) Generate two random numbers U_1 and U_2.
2) If $U_2 \le \frac{256}{27}\,U_1\,(1-U_1)^3$ set $X = U_1$ and stop Otherwise return to step 1.

The average number of times that step 1 will be performed is c = 135/64.

Ex. 7.5.3 : Develope a random-variate generator for random variable X with the p.d.f.

$$f(x) = \begin{cases} e^{2x} & -\infty < X \le 0 \\ e^{-2x} & 0 < X < \infty \end{cases}$$

Soln. :

(i) Calculate the cdf by using the formula

$$\text{cdf} = F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$\text{Hence, cdf} = F(x) = e^{2x/2}, \quad -\infty < x \le 0$$
$$= 1 - e^{-2x}, \quad 0 < x < \infty$$

(ii) Set F(X) = R on $-\infty < X < \infty$.

(iii) Solve for X to obtain

$$X = \frac{1}{2}\ln(2R), \quad 0 < R \le \frac{1}{2}$$
$$= -\frac{1}{2}\ln(2-2R), \quad \frac{1}{2} < R < 1$$

Ex. 7.5.4 *Consider the following sequence of 5 numbers*
0.15, 0.94, 0.05, 0.51, 0.29
Use the kolmogorov – Smimov test to determine whether the Hypothesis of uniformity can be rejected. Given α = 0.05 and the critical value of D = 0.565.

SPPU - May 16, 10 Marks

Soln. :

I	1	2	3	4	5
R(i)	0.05	0.15	0.29	0.51	0.94
i/N	0.20	0.40	0.60	0.80	1.00
i/N – R(i)	0.15	0.26	0.32	0.29	0.06
R(i) – (I – 1)/N	0.05	–	-	-	0.14

$$D^+ = \max(i/N - R_{(i)})$$
$$= \{0.15, 0.26, 0.32, 0.29\ 0.06\} = 0.32$$
$$D^- = \max(R_{(i)} - (i-1)/N)$$
$$= \{0.05, 0.14\} = 0.14$$

Hence, $D\max(D^+, D^-) = 0.32$

Critical value of D for α = 0.05 and N = 5 is 0.565.

Since computed value 0.32 is less than tabulated critical value 0.565, the hypothesis that the distribution is uniform is not rejected.

7.6 Distinguish between Random Numbers and Random Variant

MU - May 16

- A random number is also called a random deviate or pseudo random number. It is a value that is generated by a computer, given a probability distribution. The uniform random number is distributed according to the uniform distribution with values from 0 to 1 and is often used to generate other numbers that follow other distributions.
- The relative frequency plot of random numbers generated using a particular distribution may not be exactly equal to the particular distribution, due to the limited number of points. But, as the number of random deviates increases, the frequency plot will more closely approximate the given distribution.
- Random variables have both an intuitive and mathematical definitions. Probability theory is based on sets of events. A throw of coin will have certain outcomes. The random variable, X, links or maps these events to values. A coin can come up heads, so the mapped random variable of this outcome can be 0, and for tails, the random variable of this outcome can be 1. The mathematical definition is a bit more complex- see related links.
- A random variable is associated with what is considered a random process. If we know the outcome without any uncertainty, we would call it an deterministic process and the outcomes deterministic variables.

Review Questions

Q. 1 How are random numbers generated from a probability distribution ?

Q. 2 By using Inverse Transform Technique which of the distributions random variates can be generated ? Write the procedure for sampling.

Q. 3 Describe the procedure to generate samples from exponential distribution.

Q. 4 Describe the procedure to generate samples from Ertang distribution.

Q. 5 How are random numbers generated from a probability distribution ? What are the other methods of getting input ?

Q. 6 Explain the distributions that are used to simulate reliability system.

Q. 7 Write short notes on acceptance/rejection technique.

Q. 8 By using Inverse Transform Technique which of the distributions random variates can be generated ? Develop a random – variate generator for a random variable X with the PDF.

$$f(x) = e^{2x} \qquad -\infty < x \leq 0$$
$$= e^{-2x} \qquad 0 < x < \infty$$

Q. 9 Describe the procedure to generate samples from :

a. Erlang distribution b. Binomial distribution.

7.7 University Questions and Answers

May 2010

Q. 1 Inverse transform Technique. **(Section 7.2)** **(10 Marks)**

Dec. 2011

Q. 2 Design the generator for geometric distribution. **(Section 7.2.5)** **(5 Marks)**

Q. 3 Explain the convolution method for random variate generation and design the generator for Erlang distribution. **(Sections 7.4 and 7.2.6)** **(6 Marks)**

May 2011

Q. 4 Explain Random- Variate generation using Inverse Transform technique. **(Section 7.2)** **(10 Marks)**

May 2012

Q. 5 Write short notes on : Inverse Transform Technique **(Section 7.2)** **(3 Marks)**

Q. 6 Distinguish between Random Numbers and Random Variant. **(Section 7.6)** **(3 Marks)**

Dec. 2012

Q. 7 Using inverse Transform method derive random variates for exponential distribution. **(Sections 7.2 and 7.2.1)** **(10 Marks)**

May 2013

Q. 8 Write short notes on Inverse transform technique. **(Section 7.2)** **(10 Marks)**

Dec. 2013

Q. 9 Compare random numbers and random variate. **(Section 7.6)** **(5 Marks)**

May 2014

Q. 10 Distinguish between Random numbers and random variates. **(Section 7.6)** **(3 Marks)**

Dec. 2014

Q. 11 Differentiate random variables and random variates. Generate random variates of exponential distribution. **(Sections 7.6 and 7.2.1)** **(10 Marks)**

May 2015

Q. 12 Write short note on Inverse Transform Technique. **(Section 7.2)** **(10 Marks)**

May 2016

Q. 1(b) Compare random numbers and random variate. **(5 Marks)**
(Ans. : Refer section 7.6)

Q. 2(b) Consider the following sequence of 5 numbers
0.15, 0.94, 0.05, 0.51, 0.29
Use the kolmogorov – Smirnov test to determine whether the Hypothesis of uniformity can be rejected. Given $\alpha = 0.05$ and the critical value of D = 0.565.
(Ans. : Refer Example 7.5.4) **(10 Marks)**

Q. 5(b) Explain inverse – Transform technique. **(10 Marks)**
(Ans. : Refer section 7.2)

❑❑❑

Scanned by CamScanner

Module 4

Module 5