Math221 – Mathematics for Computer Science

S3-2024 Assignment 1

(Each question carries 10 marks)

1. Let Z be the set of integers, $\mathbb{R}$ be the set of real numbers, N be the set of natural numbers, $A = \{x \in \text{R}: -5 < x \leq 5\}$ and $\text{B} = \{x \in \text{N}: -5 < x \leq 5\}$. Find the following and specify them either directly or using set builder notations:

   (i) (Z – R)

   (ii) (N – R)

   (iii) (N – Z).

   (iv) $(A - B)$

   (v) $\{9, 3, 8\} \cap (\{-9, 9\} \cup \{1, 10\})$

2. Use element argument method to prove that for any two sets A and B, if $\text{A} \subseteq \text{B}$, then $\text{P(A)} \subseteq \text{P(B)}$, where P(A) and P(B) are power sets of A and B respectively. You must state your reasons clearly for every statement in your proof.

3. Prove using mathematical induction that if a and r are real numbers and $r \neq 1$,

$$\sum_{i=0}^{n} ar^i = \frac{a(r^{n+1} - 1)}{r - 1}$$

   for all $n \geq 0$.

4. Let $x \in \text{R}$. Prove that if $b^2 + 7b + 777$ is an irrational number, then b irrational numbers.

5. Using substitution method to prove the following equivalence:

   $(\sim x \Rightarrow y \vee z) \equiv (\sim x \wedge \sim y \Rightarrow z)$

6. Let *P*, *Q* and *R* be simplest statements. Determine whether the following statement is a tautology, contradiction or contingent statement:

   $(Q \wedge P \Rightarrow R) \Rightarrow Q \wedge R$

7. Prove that for any integer $m \in Z$, $m^2 + 3m + 9$ is odd.

8. Specify the following statement in symbolic form:

   (i) The sum of an odd integer and an even integer is odd.

   (ii) Some integers are not natural numbers.

   (iii) There are computes that do not use any Microsoft product.

9. Determine whether the following statements are True or False.

   (i) $\{a, b\} \not\subset \{a, b\}$;

   (ii) $\varnothing \subseteq P(\{a, b\})$;

   (iii) $\varnothing \in P(\{a, b\})$;

   (iv) $\varnothing \notin P(\{a, b\})$;

   (v) $\varnothing \in \{\{\varnothing\}\}$;

   (vi) $\varnothing \notin \{\varnothing, a, b\}$;

   (vii) $\{\varnothing\} \in P(\{a, b\})$;

10. Start from vertex e, find a minimum spanning tree (MST) of the following graph using *Prim*'s algorithm: Show the minimum weight and the sequence of edges in the MST according to the order added.