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Math 255 - Assignment 1

Q1 (i) $(\mathbb{Z}-\mathbb{R}) = \emptyset$ or $\{\}$ specify ~~α~~ directly
(ii) $(\mathbb{N}-\mathbb{R}) = \emptyset$ or $\{\}$ specify directly
(iii) $(\mathbb{N}-\mathbb{Z}) = \emptyset$ or $\{\}$ specify directly
(iv) $(A-B) = \{x \in \mathbb{R} : (-5 < x \leq -4) \vee (-4 \leq x \leq -3) \vee (-3 \leq x \leq -2) \vee (-2 \leq x \leq -1) \vee (-1 \leq x \leq 0) \vee (0 \leq x < 1) \vee (1 < x < 2) \vee (2 < x < 3) \vee (4 < x < 5)\}$
specify using set builder notation
(v) $\{9,3,8\} \cap (\{-9,9\} \cup \{1,10\}) = \{9,3,8\} \cap \{-9,9,1,10\}$
$= \{9\}$ specify directly.

Q2 If $A \subseteq B$ prove $P(A) \subseteq P(B)$
Let $x \in P(A)$,
if x is an element in $P(A)$, then $x \subseteq A$.

Since $x \subseteq A$ and $A \subseteq B$, then every element in A should be in B, $x \subseteq B$.
Which will leads to $x \in P(B)$
$\therefore$ Hence, $A \subseteq B$ if and only if $\forall x \in A \Rightarrow x \in B$.

Q3 $a, r = \mathbb{R}$ $r \neq 1$ $n \geq 0$

$$\sum_{i=0}^{n} ar^i = \frac{a(r^{n+1}-1)}{r-1}$$

Let CLAIM(n) represent the statement $\sum_{i=0}^{n} ar^i = \frac{a(r^{n+1}-1)}{r-1}$

Step ①: CLAIM(0)

$$\sum_{i=0} ar^0 = \frac{a(r^{0+1}-1)}{r-1}$$
$$a = \frac{a(r-1)}{r-1}$$
$$a = a \quad \#$$

$\therefore$ Therefore CLAIM(0) is true, LHS = RHS

Step ②: Assume CLAIM(k) is true for some $k \in \mathbb{N}$, that is

$$\sum_{i=0}^{k} ar^i = \frac{a(r^{k+1}-1)}{r-1} \quad (*)$$

Prove CLAIM (k+1) is true, that is, prove that

$$\sum_{i=0}^{(k+1)} ar^i = \frac{a(r^{(k+1)+1}-1)}{r-1} = \frac{a(r^{k+2}-1)}{r-1}$$

$$LHS = \sum_{i=0}^{(k+1)} ar^i$$
$$= ar^{k+1} + \sum_{i=0}^{k} ar^i$$

Sub $*$ into here

$$= ar^{k+1} + \frac{a(r^{k+1}-1)}{r-1}$$
$$= \frac{ar^{k+1}(r-1) + a(r^{k+1}-1)}{r-1}$$
$$= \frac{ar^{k+2} - ar^{k+1} + ar^{k+1} - a}{r-1}$$
$$= \frac{ar^{k+2} - a}{r-1}$$

= RHS of CLAIM (k+1) #

Hence claim(k) implies Claim(k+1).
Therefore, by mathematical induction, Claim(n) is true for all n. #

Q4 $b^2+7b+777$ irrational, then b is irrational $\quad b \in R$

We shall use the proof by contradiction
Let $b^2+7b+777$ be irrational. We assume that b is rational.
$b=\frac{p}{q}$, where $p,q \in Z$, from definition of rational number.
Hence, $(\frac{p}{q})^2 + 7(\frac{p}{q}) + 777$
$= \frac{p^2}{q^2} + \frac{7p}{7q} + 777$
$= \frac{p^2+7pq+777q}{q^2}$
rational → $= \frac{m}{n}$, where $m=p^2+7pq+777q$, $n=q^2$, $m,n \in Z$, $n \neq 0$
There is a contradiction in the given assumption that $b^2+7b+777$ is irrational number.
$\therefore$ Therefore, if b is irrational then $b^2+7b+777$ is also irrational. ∦

Q5
$(\sim x \Rightarrow y \vee z) \equiv \sim(\sim x) \vee (y \vee z)$ Implication Law
$\equiv x \vee (y \vee z)$ Double Negation
$\equiv (x \vee y) \vee z$ Associative Law
$\equiv (\sim\sim(x \vee y)) \vee z$ Double Negation
$\equiv \sim(x \vee y) \Rightarrow z$ Implication Law
$\equiv \sim x \wedge \sim y \Rightarrow z$ DeMorgan's Law

Q6 $(Q \wedge P \Rightarrow R) \Rightarrow Q \wedge R$

Q	P	R	$Q \wedge P$	$Q \wedge P \Rightarrow R$	$Q \wedge R$	$(Q \wedge P \Rightarrow R) \Rightarrow Q \wedge R$
T	T	T	T	T	T	T
F	F	F	F	T	F	F
T	F	F	F	T	F	F
T	T	F	T	F	F	T
F	F	T	F	T	F	F
F	T	T	F	T	F	F
F	T	F	F	T	F	F
T	F	T	F	T	T	T

$\therefore$ Since there is only 3 true in $(Q \wedge P \Rightarrow R) \Rightarrow Q \wedge R$, this statement is a contingent statement

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Q7 $m \in \mathbb{Z}$, $m^2 + 3m + 9$ is odd

~~m^2+3m+9~~ Using proof by cases method

Case 1: Let m be even. Then $m = 2p$ for some $p \in \mathbb{Z}$

$$(2p)^2 + 3(2p) + 9 = 4p^2 + 6p + 9$$
$$= 4p^2 + 6p + 8 + 1$$
$$= 2(2p^2 + 3p + 4) + 1$$
$$= 2x + 1 \quad , \text{ where } x = 2p^2 + 3p + 4, \ x \in \mathbb{Z}$$

Hence if m is even, then $m^2 + 3m + 9$ is odd.

Case 2: Let m be odd. Then $m = 2q + 1$ for some $q \in \mathbb{Z}$.

$$(2q+1)^2 + 3(2q+1) + 9 = 4q^2 + 4q + 1 + 6q + 3 + 9$$
$$= 4q^2 + 10q + 13$$
$$= 4q^2 + 10q + 12 + 1$$
$$= 2(2q^2 + 5q + 6) + 1, \text{ where } y = 2q^2 + 5q + 6, \ y \in \mathbb{Z}$$

Here if m is odd, then $m^2 + 3m + 9$ is odd.

$\therefore$ Therefore from this 2 cases we can see that any integer $m \in \mathbb{Z}$, $m^2 + 3m + 9$ is odd #

Q8 (i) $\forall x \in \mathbb{Z}, \forall y \in \mathbb{R}$, $(x \text{ is odd}) \wedge (y \text{ is even}) \Rightarrow (x+y) \text{ is odd}$

(ii) $\forall x \in \mathbb{Z}, x \notin \mathbb{N}$

(iii) $\exists$ computer x, $\forall$ Microsoft product y, x do not use y

Q9 (i) $\{a, b\} \notin \{a, b\}$ True

(ii) $\emptyset \subseteq P(\{a, b\})$ True

(iii) $\emptyset \in P(\{a, b\})$ True

(iv) $\emptyset \notin P(\{a, b\})$ False

(v) $\emptyset \in \{\{\emptyset\}\}$ False

(vi) $\emptyset \notin \{\emptyset, a, b\}$ False

(vii) $\{\emptyset\} \in P(\{a, b\})$ False

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Q10

	vertex added	Edge added	Weight
0	e		
1	f	e-f	5
2	h	e-h	9
3	b	f-b	6
4	g	f-g	7
5	a	g-a	8
6	c	b-c	9
7	d	c-d	7