Name: Jeslyn Ho Ka yan, UOW ID: 8535 383
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Math 255 - Assignment 2

Q1.

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

Answer

x_i	f_i	$f_i x_i$	$(x_i-\bar{x})$	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
6	2	12	-13	169	338
10	4	40	-9	81	324
14	7	98	-5	25	175
18	12	216	-1	1	12
24	8	192	5	25	200
28	4	112	9	81	324
30	3	90	11	121	363
	$\Sigma f_i = N = 40$	$\Sigma f_i x_i = 760$			$\Sigma f_i(x_i-\bar{x})^2 = 1736$

Mean $= \bar{x}$

$= \frac{1}{N}\sum_{i=1}^{n} f_i x_i$

$= \frac{760}{40}$

$= 19$

Variance $= \sigma^2$

$= \frac{1}{N}\sum_{i=1}^{n}(x_i-\bar{x})^2$

$= \frac{1736}{40}$

$= 43.4$

Hence, Mean $=19$ and Variance $=43.4$ #

Q2 Let A be the event that student studys math $P(A) = \frac{90}{100} = 0.9$

Let B be the event that student studys physic. $P(B) = \frac{80}{100} = 0.8$

P(Student study both math and Physic) $= P(A \cup B)$ $P((A\cup B)') = \frac{5}{100} = 0.05$

$= 1 - 0.05 = 0.95$ $P(A\cup B) = 0.95$

$P(A\cap B) = 0.75$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(A\cap B) = P(A) + P(B) - P(A\cup B)$

$= 0.9 + 0.8 - 0.95 = 0.75$

(a) Students studies Mathematics given that he or she studies physics.

$P(A|B) = \frac{P(B\cap A)}{P(A)} = \frac{0.75}{0.8} = 0.9375$ #

(b) Students does not study physic given that he or she studies Mathematics

$P(A\cap B') = P(A) - P(A\cap B)$

$= 0.9 - 0.75 = 0.15$

$P(B'|A) = \frac{P(A\cap B')}{P(A)} = \frac{0.15}{0.9} = \frac{1}{6} = 0.1666667$

≈ 0.167 (3.sf) #

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Q3. $P(M_1) = 30\% = 0.3$ $P(M_2) = 45\% = 0.45$ $P(M_3) = 25\% = 0.25$

$P(D|M_1) = 0.02$ ← probability that a product made by Machine 1 is defective

$P(D|M_2) = 0.03$ ← probability that a product made by Machine 2 is defective

$P(D|M_3) = 0.02$ ← probability that a product made by Machine 3 is defective

$$P(M_3|D) = \frac{P(D|M_3) \cdot P(M_3)}{P(D|M_1) \cdot P(M_1) + P(D|M_2) \cdot P(M_2) + P(D|M_3) \cdot P(M_3)}$$

$$= \frac{0.02 \times 0.25}{(0.02 \times 0.30) + (0.03 \times 0.45) + (0.02 \times 0.25)}$$

$= 0.204081$

≈ 0.204 (3.sf) //

Q4. ① Verify that $f(x)$ is a valid PDF

PDF Conditions, (a) $f(x) \geq 0$ all x and (b) total area under the curve must be equal 1, $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(a) For $x \in (0,1)$, $5x^4$ is positive so $f(x) \geq 0$ holds true.

(b) $\int_0^1 5x^4\,dx = \left[\frac{5x^{4+1}}{4+1}\right]_0^1 = \left[\frac{5x^5}{5}\right]_0^1 = \frac{5(1)^5}{5} - \frac{5(0)^5}{5} = 1$

② Find $P(0.1 < x < 0.9)$

$F(x) = \int_0^x 5t^4\,dt = 5\left[\frac{t^5}{5}\right] = \left[t^5\right]_0^x = x^5 - 0^5 = x^5$

So, the CDF is $F(x) = x^5$

$F(0.9) = 0.9^5 = 0.59049$ $F(0.1) = 0.1^5 = 0.00001$

$P(0.1 < x < 0.9) = F(0.9) - F(0.1) = 0.59049 - 0.00001 = 0.59048$

Hence $f(x)$ is a valid PDF and the probability of $P(0.1 < x < 0.9)$ is 0.59048 //

Q5. $Z = \frac{x - \mu}{\sigma}$

$Z = \frac{42-50}{10} = -0.8$ $Z = \frac{62-50}{10} = 1.2$

$P(42 < X < 62) = P(-0.8 < Z < 1.2)$

$P(Z < -0.8) = \cancel{P(Z > 0.8)}$

$= 0.2119$

$P(Z < 1.2) = 1 - P(Z > 1.2)$

$= 1 - 0.1151 = 0.8849$

$P(Z < 1.2) - P(Z < -0.8)$

$= 0.8849 - 0.2119$

$= 0.6730$ //

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Q6 $\sigma = 7$ Mean $= 74$ $n = 100$

$P(Z > z) = 0.123$

$P(Z < 1.16) = 0.123$ ∴ $z = 1.16$

$x = \mu + z\sigma$

$= 74 + (1.16 \times 7)$

$= 82.12$

≈ 83

Ans: Lowest Possible A is 83

Highest Possible B is 82 #

Q7 Step 1: Hypothesis → $H_0: \mu = 0$ $H_1: \mu \neq 0$

step 2: Significant level → $\alpha = 0.001$

Step 3: Critical value and Rejection region → The critical Z-value for $\alpha = 0.001$ is a one-tailed test is approximately 3.09

step 4: Test Statistic → $Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{1.4 - 0}{3.6/\sqrt{100}} = \frac{1.4}{0.36} = 3.89$

step 5: Conclusion

Since $3.89 > 3.09$, we shall reject the null hypothesis.

step 6: State and Conclude in words

There is sufficient evidence to conclude that the drug increase heart rate, the company should considered marketing the drug as it appears to be efficient.

Q8. $\bar{x} = \frac{(59 + 62 + 59 + 74 + 70 + 61 + 62 + 66 + 62 + 75)}{10} = 65$

Sample standard deviation $S = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}}$

$S = \sqrt{\frac{[(59-65)^2 + (62-65)^2 + (59-65)^2 + (74-65)^2 + (70-65)^2 + (61-65)^2 + (62-65)^2 + (66-65)^2 + (62-65)^2 + (75-65)^2]}{10-1}}$

≈ 5.98

$df = n - 1 = 10 - 1 = 9$, $\alpha = 0.05$

$t_{\alpha/2} = t_{0.025} = 2.262$

$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 65 \pm 2.262 \frac{5.98}{\sqrt{10}}$ ∴ $60.72 \leq \sigma \leq 69.28$

Hence the 95% t-confidence interval for the mean score σ is $(60.72, 69.28)$ #

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Q9 $N(N, 25)$, $\alpha = 0.05$

varience $\sigma^2 = 25$

Standard deviation $\sigma = \sqrt{25} = 5$

$Z_{0.05/2} = 1.96$

$$\text{Width} = 2 Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$= 2 \times 1.96 \times \frac{5}{\sqrt{n}} = 1$$

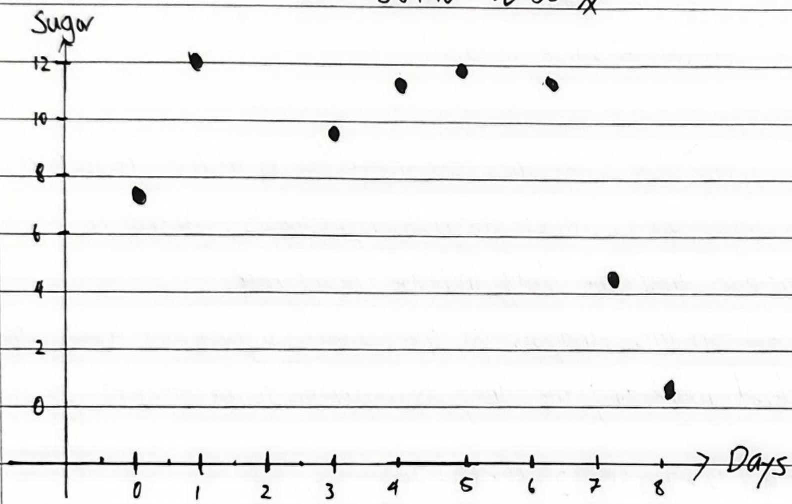
$$\frac{1}{2 \times 1.96} = \frac{5}{\sqrt{n}}$$

$$\frac{1}{3.92} = \frac{5}{\sqrt{n}}$$

$$\sqrt{n} = 5 \times 3.92$$

$$n = 384.16 \approx 385 \#$$

Q10



$$\sum_{i=1}^{n} x_i y_i = (0 \times 7.9) + (1 \times 12.0) + (3 \times 9.5) + (4 \times 11.3) + (5 \times 11.8) + (6 \times 11.3) + (7 \times 4.2) + (8 \times 0.4)$$

$$= 245.1$$

$$\sum_{i=1}^{n} x_i = 0 + 1 + 3 + 4 + 5 + 6 + 7 + 8 = 34 \qquad \bar{x} = 34 \div 8 = 4.25$$

$$\sum_{i=1}^{n} y_i = 7.9 + 12 + 9.5 + 11.3 + 11.8 + 11.3 + 4.2 + 0.4 = 68.4 \qquad \bar{y} = 68.4 \div 8 = 8.55$$

$$\sum_{i=1}^{n} x_i^2 = 7.9^2 + 12^2 + 9.5^2 + 11.3^2 + 11.8^2 + 11.3^2 + 42^2 + 0.4^2 = 709.08$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i - \frac{\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n}}{\sum_{i=1}^{n} x_i^2 \quad \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}} = \frac{245.1 - \frac{(34 \times 68.4)}{8}}{709.08 \times \frac{(34)^2}{8}} = \frac{245.1 - 290.7}{709.08 \times 144.5} = -4.45 \times 10^{-4}$$

$$= -0.000445$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 8.55 - (-0.000445)(4.25) = 8.55$$

$$\therefore \hat{y} = 8.55 + -0.000445x \#$$