**Math255 – Mathematics for Computing**

**S3-2024 Assignment 2**

**(Each Question carries 10 marks)**

1. Find the mean and variance for ethe following frequence distribution:

| $x_i$ | 6 | 10 | 14 | 18 | 24 | 28 | 30 |
|---|---|---|---|---|---|---|---|
| $f_i$ | 2 | 4 | 7 | 12 | 8 | 4 | 3 |

2. In a group of 100 students, 90 study Mathematics, 80 study Physics, and 5 study none of these subjects. Find the probability that a randomly selected student: (a) studies Mathematics given that he or she studies Physics, and (b) does not study Physics given that he or she studies Mathematics.

3. A certain assembly plant, three machines, Machine 1, Machine 2 and Machine 3 make 30%, 45% and 25% respectively of the products. It is known from the quality control data collected from the past that 2%, 3% and 2% of the products made by each machine respectively, are defective. If a product is chosen randomly chosen and found to be defective, what is the probability that it is made by Machine 3?

4, A function f is defined as follows:

$$f(x) = \begin{cases} 5x^4, & \text{x} \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$$

Show that f(x) is a valid pdf and find P(0.1 < X < 0.9).

5. Given a random variable X having a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that X assumes a value between 42 and 62.

6. The average mark for an exam is 74 with standard deviation = 7. All the marks are integer values between 0 and 100 inclusively. If the highest 12.3% of the marks are to be awarded with the highest grade A and marks are curved to follow a normal distribution, what is the lowest possible A and the highest possible for the next lower grade B?

7. A drug company is testing a drug intended to increase heart rate. A sample of 100 yielded a mean increase of 1.4 beats per minute, with a population standard deviation known to be 3.6. Since the company wants to avoid marketing an ineffective drug, it proposes a 0.001 significance level. Should it market the drug? (*Hint:* If the drug doesn't work, increase will be zero.)

8. *Suppose that against a certain opponent the number of points the MIT basketball team scores is normally distributed with unknown mean $\theta$ and unknown variance $\sigma^2$.*
*Suppose that over the course of the last 10 games between the two teams MIT scored the following points:*

59, 62, 59, 74, 70, 61, 62, 66, 62, 75

*Compute a* 95% *t–confidence interval for* $\theta$.

9. The volume in a set of wine bottles is known to follow a $N(\mu, 25)$ distribution. You take a sample of the bottles and measure their volumes. How many bottles do you have to sample to have a 95% confidence interval for with width 1?

10. The data below show the sugar content of a fruit (SUGAR) for different numbers of days after picking (DAYS).

| Days | Sugar |
|---|---|
| 0 | 7.9 |
| 1 | 12.0 |
| 3 | 9.5 |
| 4 | 11.3 |
| 5 | 11.8 |
| 6 | 11.3 |
| 7 | 4.2 |
| 8 | 0.4 |

Draw the scatterplot and obtain the estimated regression line to predict sugar content based on the number of days after picking.