Discrete Mathematics, Spring & Summer 2024

Midterm Exam (100 pts)

Class ID. ______________ Name: ______________ Score: ______________

1. (33 points) Determine whether the following statements are true or false. If it is true write a √ otherwise a × in the blank before the statement.

( ) (1) The proposition "1+1=3 if and only if 2+2=3" is True.

( ) (2) The proposition $((p \to q) \wedge \neg p) \to \neg q$ is a tautology.

( ) (3) The following system specifications are consistent.

Whenever the system software is being upgraded, users cannot access the file system.
If users can access the file system, then they can save new files.
If users cannot save new files, then the system software is not being upgraded.

( ) (4) If $P(A) \in P(B)$, then $A \in B$. ($P(S)$ is the power set of S.)

( ) (5) Suppose $A=\{a,b,c\}$, then $\{\{a\}\} \subset P(A)$. ($P(S)$ is the power set of S.)

( ) (6) $A \oplus A = A$ ($\oplus$ is the symmetric difference of two sets.)

( ) (7) Let f be a function from the set A to the set B, let S and T be subsets of A. then

$$f(S \cup T) \subseteq f(S) \cup f(T)$$

( ) (8) Let R be the set of real numbers. Then $|R|=|R \times R|$.

( ) (9) The set of all finite subsets of the set of positive integers is an uncountable set.

( ) (10) $\sum_{j=1}^{n}(j^3 + j)$ is $O(n^3)$.

( ) (11) In a group of five people (where any two people are either friends or enemies), there are either three mutual friends or three mutual enemies.

2. (27 points) Fill in the blanks.

(1) (2pts) The full disjunctive normal form of $(p \wedge q) \vee (r \wedge q) \vee \neg p$ is:

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(2) (2pts) The power set of $\{\varnothing, \{\varnothing\}\}$ is ______________________________

(3) (2pts) Suppose $g: A \to B$ and $f: B \to C$ where $A = \{1,2,3,4\}$, $B = \{a,b,c\}$, $C = \{2,6,10\}$, and f and g are defined by $g = \{(1,b),(2,a),(3,a),(4,b)\}$ and $f = \{(a,10),(b,7),(c,2)\}$. $f \circ g$ =

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(4) (2pts) How many different truth tables of compound propositions are there that involve the propositional variables p, q and s? ______________________________

(5) (2pts) Arrange the functions $2.5^n, n^{100}, (\log n)^3, \sqrt{n \log n}, (n!)^2, and\ n^{99} + n^{98}$ in a list so that each function is big-O of the next function.

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(6) (2pts) Suppose $|A| = 20$ and $|B| = 24$. The number of 1-1 functions $f: A \to B$ is__________

If $|A| = 24$ and $|B| = 20$, then the number of 1-1 functions $f: A \to B$ is __________

(7) (3pts) You have 10 books and 4 labeled boxes (labeled 1 2, 3, 4). In how many ways can you put the 10 books into the boxes if

(a) the books are distinct.__________

(b) the books are identical. __________

(c) the books are identical and no box can be left empty.__________

(8) (2pts) The number of ways to put 7 of the 8 letters in CHEMISTS in a row is__________

(9) (2pts) The next larger permutation in lexicographic order after 3254761 is

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(10) (2pts) The next larger 4-combinations of the set {1, 2. 3, 4, 5, 6, 7, 8} after {1, 2, 3, 5} is

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(11) (2pts) The number of bit strings of length n, where $n \geqslant 6$, contaning exactly three occurrences of 01 is __________

(12) (2pts) Let $V(n)$ be the number of strings of n letters of the ordinary alphabet that do not have adjacent vowels. Find a recurrence for $V(n)$ along with a sufficient number of initial cases.

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(13) (2pts) A particular solution of the linear nonhomogeneous recurrence relation $a_n = 2a_{n-1} + n \cdot 2^n$

has the form of __________

3. **(5 points)** Determine whether there exist an onto function f from S to $P(S)$, where S is a set and $P(S)$ is the power set of S. You need to **justify your answer.**

4. (5 points)

Consider the following argument:

She is a Math Major or a Computer Science Major.
If she does not know discrete math, she is not a Math Major.
If she knows discrete math, she is smart.
She is not a Computer Science Major.
Therefore, she is smart.

Let $\boldsymbol{p}$: "She is a Math Major;" $\boldsymbol{q}$: "She is a Computer Science Major;" $\boldsymbol{r}$: "She knows discrete math;" $\boldsymbol{s}$: "She is smart."

1) Write the **formal form** of the above argument.
2) Determine whether this argument is valid and justify your answer.

5. (10 points) Show that in any set of $n + 1$ positive integers not exceeding $2n$ there must be two that are relatively prime

6. (10 points) Prove the following identity using a combinatorial argument.

$$\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+r+1}{r} \quad (n \text{ and } r \text{ are positive integers})$$

7. (10 points) Suppose a vending machine accepts \$1 coins, \$1 bills, \$2 bills, \$5 bills and \$5 coins. Write a recurrence relation for the number of ways to put n dollars into the machine, assuming that the order that the coins and bills are entered matters. Include the initial conditions.