# Producing multistage survival curves

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- Run a Cox proportional hazards model for each individual cause of leaving the list, removing all other losses as censoring.
- For cause $i$ and patient with covariates $x$, we can then estimate a cumulative hazard model $H_i(t, x) = H_{i0}(t) \exp[a_i' x]$, where
  - $t$ is week post-listing (or this could be done in days if week is not smooth enough)
  - $a_i$ are the estimated log-hazard ratios for the $i$th cause of loss from the waiting list,
  - $H_{i0}(t)$ is a smooth model for the cumulative hazard, say through using fractional polynomials.
  - $h_i(t, x)$ is the instantaneous hazard at time $t$, ie the risk of leaving with cause $i$, given still on list at time $t$
- Assuming independent causes of leaving the list, the probability of still being on the list at time $t$ is
$$S(t, x) = \exp[-\sum_i H_i(t, x)].$$
- We want to estimate the probability $F_i(t, x)$ of an individual having left the list because of cause $i$ by time $t$, which is what we want to plot. A loss of type $i$ in time period $t, t + dt$ occurs with probability $h_i(t, x)S(t, x)dt$, so that
$$F_i(t, x) = \int h_i(t, x)S(t, x)dt.$$
- This is not available in closed-form, but can be obtained in discrete time. Set $S(1, x) = 1, F_i(1, x) = 0$. Now run through the following steps for $t = 1, 2, ....$
  - Assuming a patient is still on the list at the start of week $t$, estimate the probability of a loss of type $i$ as $h_i(t, x) = H_i(t + 1, x) - H_i(t, x)$.
  - Calculate $p_i(t, x) = h_i(t, x)S(t, x)$ as the probability of leaving the list with cause $i$
  - Add these to form $F_i(t + 1, x) = F_i(1, x) + p_i(t, x)$.
  - Set $S(t + 1, x) = S(t, x) - \sum_i p_i(t, x)$
- Then $S(t, x), F_i(t, x)$ will add to 1 and form the bands to be plotted.