Kalkulus tugas pertemuan 13

No
Date

Nama : Rina Nabila
NIM : 20210810078
Kelas : TINFC - 2021 - 01 (A)

1. Hitunglah integral tentu berikut :

a. $\int_{-1}^{2} (3x^2 - 2x + 3)\, dx$

$= 3\int_{-1}^{2} x^2 - 2\int_{-1}^{2} x + 3\, dx$

$= \left[\frac{3}{3}x^3 - \frac{2}{2}x^2 + 3x\right]_{-1}^{2}$

$= \left[x^3 - x^2 + 3x\right]_{-1}^{2}$

$= (2^3 - 2^2 + 3(2)) - ((-1)^3 - (-1)^2 + 3(-1))$

$= (8 - 4 + 6) - (-1 - 1 - 3)$

$= 10 + 5$

$= 15$ //

b. $\int_{1}^{4} \frac{x^4 - 8}{x^2}\, dx$

$= \int_{1}^{4} \frac{x^4}{x^2} - \frac{8}{x^2}\, dx$

$= \int_{1}^{4} x^2 - 8x^{-2}\, dx$

$= \left[\frac{1}{3}x^3 - \frac{8}{-1}x^{-1}\right]_{1}^{4}$

$= \left[\frac{1}{3}x^3 + 8x^{-1}\right]_{1}^{4}$

$= \left(\frac{1}{3}(4)^3 + 8(4)^{-1}\right) - \left(\frac{1}{3}(1)^3 + 8(1)^{-1}\right)$

$= \left(\frac{64}{3} + 8 \cdot \frac{1}{4}\right) - \left(\frac{1}{3} + 8\right)$

$= \left(\frac{64}{3} + 2\right) - \left(\frac{1}{3} + 8\right)$

$= \left(\frac{64 + 6}{3}\right) - \left(\frac{1 + 24}{3}\right)$

$= \frac{70}{3} - \frac{25}{3} = \frac{45}{3} = 15$ //

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c. $\int_{\pi/6}^{\pi/2} 2 \sin t \; dt$

$= \left[-2 \cos t \right]_{30^\circ}^{90^\circ}$

$= (-2 \cos 90^\circ) - (-2 \cos 30^\circ)$

$= (-2 (0)) - (-2 (\tfrac{1}{2}\sqrt{3}))$

$= 0 + \sqrt{3}$

$= \sqrt{3}$ //

2. Misalkan bahwa : $\int_0^1 f(x)\,dx = 2$, $\int_1^2 f(x)\,dx = 3$, $\int_0^1 g(x)\,dx = -1$ dan $\int_0^2 g(x)\,dx = 4$. Hitunglah :

a. $\int_0^2 [2 f(x) + g(x)]\,dx$

$= 2\int_0^2 f(x)\,dx + \int_0^2 g(x)\,dx$

$= 2 \left[\int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx \right] + \int_0^2 g(x)\,dx$

$= 2 [2 + 3] + 4$

$= 10 + 4$

$= 14$ //

b. $\int_2^1 [2 f(x) + 5 g(x)]\,dx$

$= 2\int_2^1 f(x)\,dx + 5\int_2^1 g(x)\,dx$

$= 2 \left[-\int_1^2 f(x)\,dx \right] + 5 \left[-\int_1^2 g(x)\,dx \right]$

$= 2 \left[-\int_1^2 f(x)\,dx \right] + 5 \left[-\int_1^0 g(x)\,dx + \int_0^2 g(x)\,dx \right]$

$= 2 \left[-\int_1^2 f(x)\,dx \right] + 5 \left[-\left(-\int_0^1 g(x)\,dx + \int_0^2 g(x)\,dx \right] \right.$

$= 2(-3) + 5(-1 + 4)$

$= -6 + 15$

$= 9$ //

c. $\int_1^0 [3 f(x) + 2 g(x)]\,dx$

$= 3\int_1^0 f(x)\,dx + 2\int_1^0 g(x)\,dx$

$= 3 \left[-\int_0^1 f(x)\,dx \right] + 2 \left[-\int_0^1 g(x)\,dx \right]$

$= 3(-2) + 2(1)$

$= -6 + 2$

$= -4$ //

d. $\int_1^1 [2f(x) + 3g(x)]\, dx$

$= 2\int_1^1 f(x)\, dx + 3\int_1^1 g(x)\, dx$

$= 2\left[\int_0^1 f(x)\, dx + \int_1^2 f(x)\, dx\right] + 3\left[\int_0^1 g(x)\, dx + \left(-\int_1^0 g(x)\, dx\right)\right]$

$= 2(2+3) + 3(-1+1)$

$= 10 + 3$

$= 13$

3. Hitunglah : $\int_0^4 f(x)\, dx$, untuk fungsi :

$$f(x) = \begin{cases} 1 & \text{jika } 0 \le x < 1 \\ x & \text{jika } 1 \le x < 2 \\ 4-x & \text{jika } 2 \le x \le 4 \end{cases}$$

Jawab :

$\int_4^0 f(x)\, dx = \int_0^1 1\, dx + \int_1^2 x\, dx + \int_2^4 4 - x\, dx$

$= [x]_0^1 + \frac{1}{2}[x^2]_1^2 + \left[4x - \frac{1}{2}x^2\right]_2^4$

$= (1-0) + \left(\frac{1}{2} \cdot 2^2 - \frac{1}{2} \cdot 1^2\right) + \left(\left(4(4) - \frac{1}{2} \cdot 4^2\right) - \left(4(2) - \frac{1}{2} \cdot 2^2\right)\right)$

$= 1 + \left(2 - \frac{1}{2}\right) + ((16-8) - (8-2))$

$= 1 + \frac{3}{2} + 2$

$= \frac{2+3+4}{2}$

$= \frac{9}{2}$