

Binary Quadratic Functions, an introduction

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We begin with a few theorems

Theorem 1. *Arithmetic Progression* $f(x, y) = ax^2 + bxy + cy^2$ has the property that $f(x_1 + x_2, y_1 + y_2) + f(x_1 - x_2, y_1 - y_2) = 2(f(x_1, y_1) + f(x_2, y_2))$

The proof is trivial by expansion.

The automorph of a form is a transformation matrix that satisfies $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} w & x \\ y & z \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix},$

$$\det \begin{pmatrix} w & x \\ y & z \end{pmatrix} = 1$$