GROUP PROJECT 3: DATA INTERPOLATION

Project Rules

- Your class will be split into small groups (3-4 students). Please remember your group number (e.g. group 3A). Assign a scribe and a coordinator.
- **The scribe** should create a Google Docs file and share it with all group members and your instructors: Dr. Pencheva (gergina@utexas.edu) and Eboni (eboniwilliams@utexas.edu). Please share with us by adding us as either "commentors" or "editors" and restrict access so that only those people with access can open the link. The name of the file should be "GroupProject3 Group<your group number>" (for the example above, **"GroupProject3 Group3A"**). Please **do NOT change the file name.**
- The scribe should also reply to the Ed Discussion **post** under **category 'Group Projects'**, **subcategory 'GP3'**. The title of the post is "Section <your section number> Group Project 3 Groups". In the post include the names of all group members, your group number, and a link to the shared Google Docs file. Please **double-check that the GoogleDoc access has been restricted**.
- **The coordinator** will be responsible for collecting contact information from the group members and setting place/time for communication (GroupMe, Discord, Slack, Zoom, etc.) The coordinator should contact Dr. Pencheva in the case of unresponsive team member.
- The group project will be only **a part of one class period**. You are **not expected to finish the entire project during class**. In fact, group collaboration outside is strongly encouraged and even necessary. **I want to see evidence of group collaboration by the end of Sunday, March 31. Listing names and contact information only is NOT sufficient.** Collaboration must demonstrate that your group has started thinking about how to approach the project and work together. There are multiple ways to demonstrate collaboration. Examples include providing meeting notes, showing a screenshot of a group message where the project is being discussed, a short recording of a virtual or in-person collaboration session, or even sharing a link to a FigJam or Google Jamboard session (include version history). **All team members might not get the same score. A component of the score will be determined based on the individual contribution and attendance on the class project day.**
- A list with tasks and questions is positioned in one place, at the bottom of the document. You are encouraged to provide only your answers at the top of your Google Docs file and keep the rest of the discussion separated. This will help for a quicker grading.
- Upload on Canvas the final version of your project report. **One submission per group is sufficient.** The Canvas assignment will be setup such that it shows submission for everyone in the group so long as one person in the group submits.

Project Description

We will practice data interpolation using Newton's Divided Differences and Cubic Splines. For that purpose, we will consider the following problem.

Application: You are planning to fish bass in a lake and you were told that to maximize the catch, you should throw the line to the depth of the thermocline. The characteristic feature of this area is the sharp decrease in temperature. You are given the temperature vs. depth data for a lake in Table 1. (See Figure 1 for a scatter plot of the data.)

Temperature, T (°C)	Depth, h (m)
19.1	0
19.1	-1
19.0	-2
18.8	-3
18.7	-4
18.3	-5
18.2	-6
17.6	-7
11.7	-8
9.9	-9
9.1	-10

Table 1: Temperature vs. depth for a lake

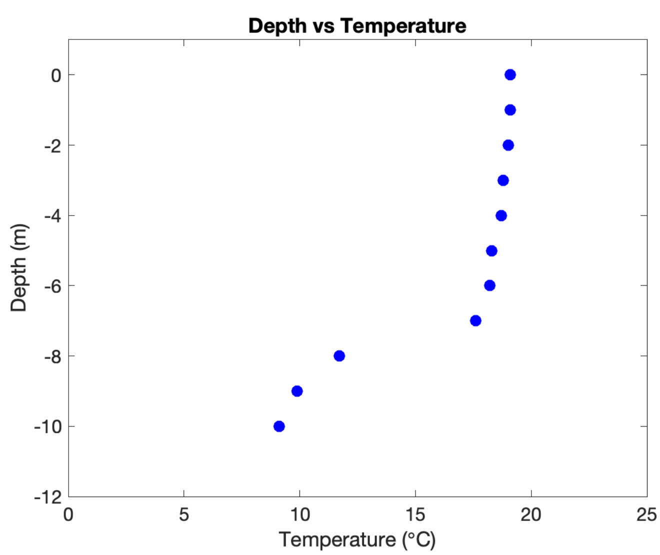

Figure 1: Temperature vs. depth for a lake

Using the given data, you see that the largest change in temperature is between $h = -8$ m and $h = -7$ m. You will determine the value of the temperature at $h = -7.5$ m using Newton's divided difference method of interpolation for polynomials of various degrees as well as cubic splines.

Tasks and Questions

1. Use **appropriate** data and Newton's Divided Differences method to construct a polynomial of **degree one** to estimate the value of the temperature at $h = -7.5$ m.
   (a) Which data points did you choose and why?
   (b) Complete the Newton's Divided Differences triangle **by hand**. Calculations may be **verified** using newtonDD but there is no need to provide the output.
   (c) What is the interpolating polynomial (name it $T_1(h)$)?
   (d) What is the estimate to the temperature at $h = -7.5$ m?
   (e) Plot the interpolating polynomial $T_1(h)$, with markers at the nodes, as well as at $(-7.5, T_1(-7.5))$. **Annotate** the figure correspondingly.
2. Use **appropriate** data and Newton's Divided Differences method to construct a polynomial of **degree two** to estimate the value of the temperature at $h = -7.5$ m.
   (a) Which data points did you choose and why?
   (b) Complete the Newton's Divided Differences triangle **by hand**. Calculations may be **verified** using newtonDD but there is no need to provide the output.
   (c) What is the interpolating polynomial (name it $T_2(h)$)?
   (d) What is the estimate to the temperature at $h = -7.5$ m?
   (e) Plot the interpolating polynomial $T_2(h)$, with markers at the nodes, as well as at $(-7.5, T_2(-7.5))$. **Annotate** the figure correspondingly.
3. Use **appropriate** data and Newton's Divided Differences method to construct a polynomial of **degree three** to estimate the value of the temperature at $h = -7.5$ m. You may use newtonDD for the calculations and the plot. If you choose another method, please include a description/code of your way.
   (a) Which data points did you choose and why?
   (b) What is the interpolating polynomial (name it $T_3(h)$)?
   (c) What is the estimate to the temperature at $h = -7.5$ m?
   (d) Plot the interpolating polynomial $T_3(h)$, with markers at the nodes, as well as at $(-7.5, T_3(-7.5))$. **Annotate** the figure correspondingly.
4. Construct **the (unique) natural cubic spline** using all data in Table 1 to estimate the value of the temperature at $h = -7.5$ m. You may use cubicSpline (Note that the code expects the first coordinates of the interpolated points to be given in **increasing order**.) If you choose another method, please include a description/code of your way.
   (a) Write the spline in the correct format (name it $T_{sp}(h)$). **Don't forget to include the interval for each piece!**
   (b) What is the estimate to the temperature at $h = -7.5$ m?
   (c) Plot the interpolating cubic spline $T_{sp}(h)$, with markers at the nodes, as well as at $(-7.5, T_{sp}(-7.5))$. **Annotate** the figure correspondingly.

5. The position of the thermocline is determined as the solution of the equation $\dfrac{d^2T}{dh^2} = 0$.

   (a) Use the cubic interpolating polynomial $T_3(h)$ found in question 3. What is the equation that you solved to get the thermocline? At what depth does the thermocline exist?

   (b) Use the cubic spline $T_{sp}(h)$ from question 4. What is the equation that you solved to get the thermocline? (It is sufficient to differentiate only the piece of $T_{sp}(h)$. Which one it is?) At what depth does the thermocline exist?

6. What are some potential issues that could arise from interpolating with polynomials of a higher degree? How would this affect the estimate?

7. Did you struggle with any part of the group project? What do you think we should change so our next sessions run better? Do you have any conceptual questions that didn't get answered? Feel free to add any comments that you want to make but I haven't listed them.

8. Submit on Canvas your **report** (.pdf, .doc, .docx, or .odt file).