<u>Group 4A</u>
Sissi Zhang
Nicholas Calzada
Lechen Wang
Zun Cao

Group Project 3

1. Use appropriate data and Newton's Divided Differences method to construct a polynomial of degree one to estimate the value of the temperature at h = −7.5 m.

(a) Which data points did you choose and why?

First, in order to construct a degree 1 polynomial, 1 = n - 1, or n = 2 points are needed. Given that the values (in the form (h, T(h)) (-8, 11.7) and (-7, 17.6) are closest to the intended point of approximation h = -7.5 m, they were chosen to create the interpolating polynomial of degree 1.

(b) Complete the Newton's Divided Differences triangle by hand. Calculations may be verified using newtonDD, but there is no need to provide the output.

x	f(x)	
-8	11.7	
		$\dfrac{17.6 - 11.7}{-7-(-8)} = 5.9$
-7	17.6	

$$\boxed{T_1(h) = 11.7 + 5.9(x+8)}$$

(c) What is the interpolating polynomial (name it T_1(h))?

$$T_1(h) = 11.7 + 5.9(h + 8)$$

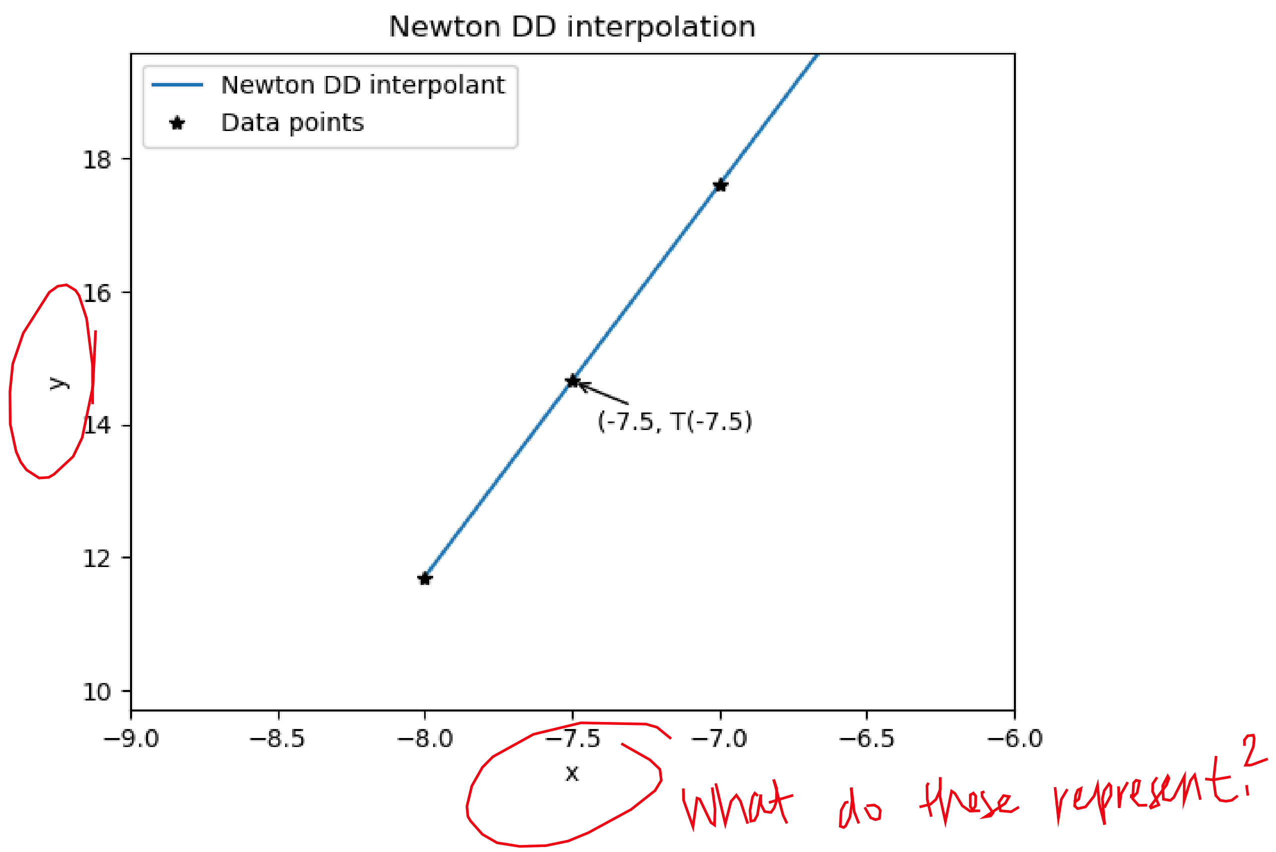
$$T_1(h) = 5.9h + 58.9$$

(d) What is the estimate of the temperature at h = −7.5 m?

The estimate at h = -7.5 m is:

$$T_1(-7.5) = 5.9(-7.5) + 58.9$$
$$= 14.65℃$$

(e) Plot the interpolating polynomial T1(h), with markers at the nodes, as well as at (−7.5, T_1(−7.5)). Annotate the figure correspondingly.



What do these represent?

2. Use appropriate data and Newton's Divided Differences method to construct a polynomial of degree two to estimate the value of the temperature at h = −7.5 m.

(a) Which data points did you choose and why?

For a second-degree polynomial (d=2), we need three data points (n = d+1 = 3). We should choose the points around h=−7.5 meters to minimize the interpolation error.
I chose the data points (-8, 11.7), (-7, 17.6), and (-6, 18.2). [in the form (h, T(h))]
That's because these three points at depths of -8m, -7m, and -6m are the closest available data to -7.5m.

(b) Complete the Newton's Divided Differences triangle by hand. Calculations may be verified using newtonDD but there is no need to provide the output.

Newton's Divided Difference triangle:

-8	11.7		
		$\frac{17.6-11.7}{-7-(-8)} = \boxed{5.9}$	
-7	17.6		$\frac{0.6-5.9}{-6-(-8)} = \boxed{-2.65}$
		$\frac{18.2-17.6}{-6-(-7)} = \boxed{0.6}$	
-6	18.2		

$$P_2(X) = 11.7 + 5.9(X-(-8)) - 2.65(X-(-8))(X-(-7))$$

$$P_2(X) = 11.7 + 5.9(X+8) - 2.65(X+8)(X+7)$$

$$\boxed{T_2(h) = 11.7 + 5.9(h+8) - 2.65(h+8)(h+7)}$$

(c) What is the interpolating polynomial (name it $T_2(h)$)?

$$T_2(h) = 11.7 + 5.9(h + 8) - 2.65(h + 8)(h + 7)$$

(d) What is the estimate of the temperature at h = −7.5 m?

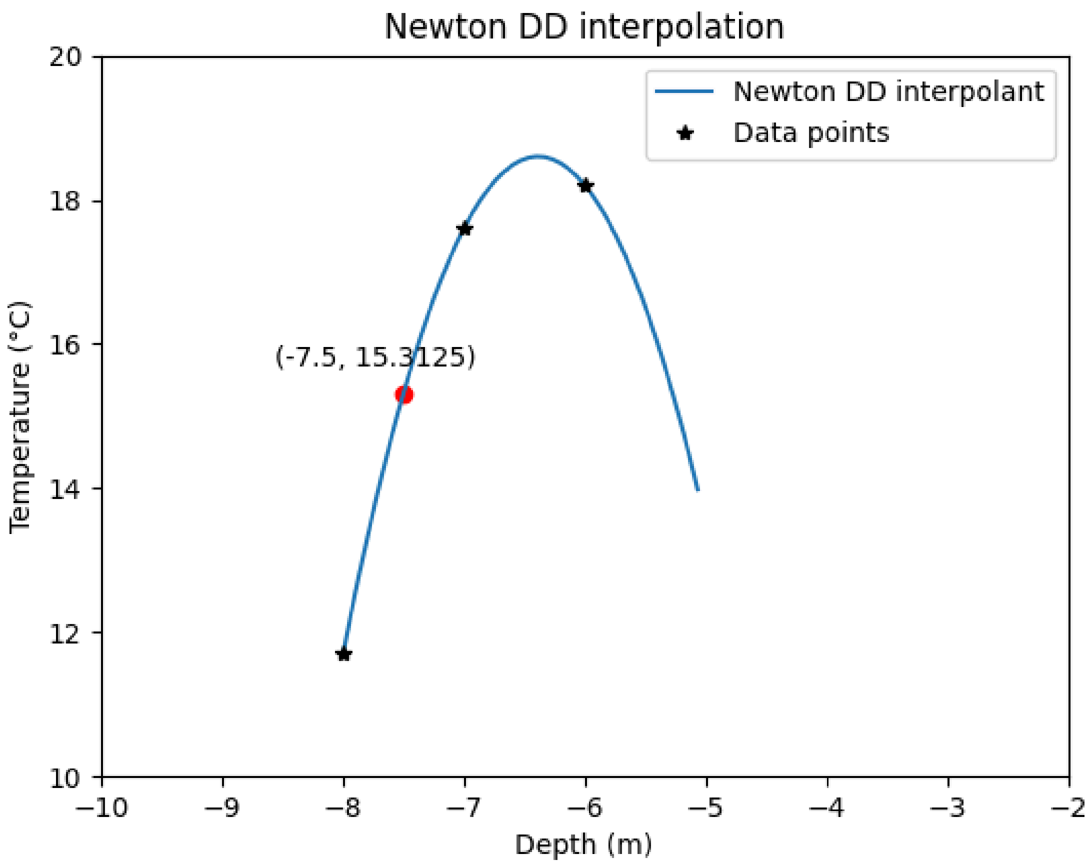
$$T_2(-7.5) = 11.7 + 5.9(8 - 7.5) - 2.65(8 - 7.5)(7 - 7.5)$$

$$T_2(-7.5) = 11.7 + 5.9(0.5) - 2.65(0.5)(-0.5)$$

$$T_2(-7.5) = 15.3125℃$$

The estimate of the temperature at h = -7.5m is 15.3125℃.

(e) Plot the interpolating polynomial T2(h), with markers at the nodes, as well as at (−7.5, T_2(−7.5)). Annotate the figure correspondingly.



The red point in the picture is (−7.5, T2(−7.5)).

3. Use appropriate data and Newton's Divided Differences method to construct a polynomial of degree three to estimate the value of the temperature at h = −7.5 m. You may use newtonDD for the calculations and the plot. If you choose another method, please include a description/code of your way.

(a) Which data points did you choose and why?
I use the following 4 points to construct the polynomial of degree 3:
(-9, 9.9), (-8, 11.7), (-7, 17.6), (-6, 18.2), where the coordinates are in the form (height (m) , temperature(°C)). These three points cover the interval [-10,0] relatively evenly.

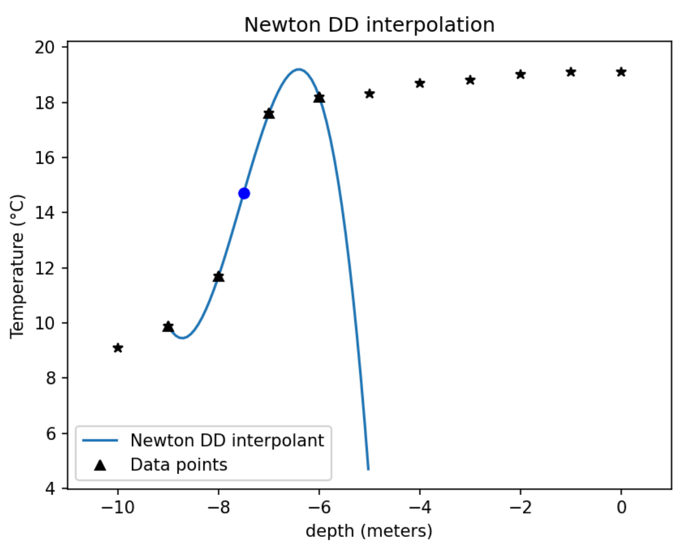
Further, among other possible choices that also spread evenly across this interval, the interpolating polynomial derived from these 4 points give the best fit for the data available.

(b) What is the interpolating polynomial (name it $T_3(h)$)?
The interpolating polynomial is:

$$T_3(h) = 9.9 + 1.80000(h + 9) + 2.05000(h + 9)(h + 8) - 1.56667(h + 9)(h + 8)(h + 7)$$

(c) What is the estimate of the temperature at h = −7.5 m?
The temperature estimate is, according to the program, is 14.73°C

(d) Plot the interpolating polynomial T3(h), with markers at the nodes, as well as at (−7.5, T_3(−7.5)). Annotate the figure correspondingly.



Blue point is (-7.5,14.73)

4. Construct the (unique) natural cubic spline using all data in Table 1 to estimate the value of the temperature at h = −7.5 m. You may use cubicSpline (Note that the code expects the first coordinates of the interpolated points to be given in increasing order.) If you choose another method, please include a description/code of your way.

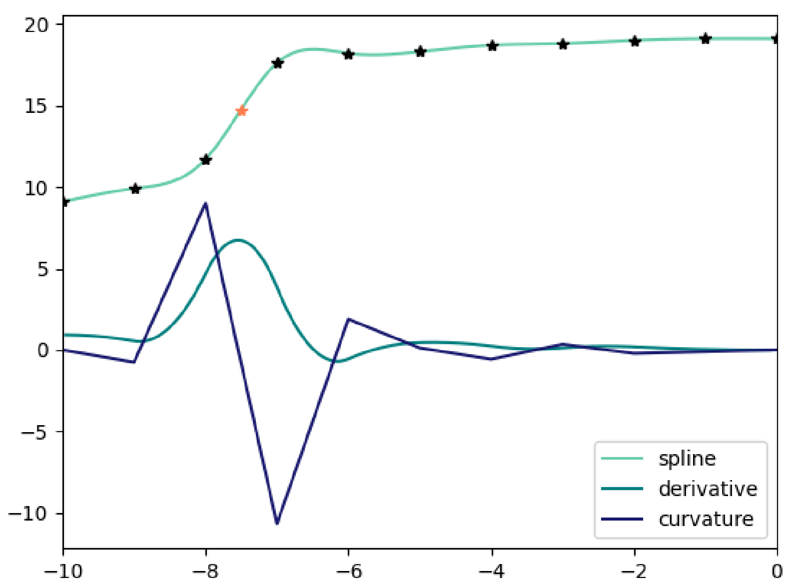
(a) Write the spline in the correct format (name it T_{sp}(h)). Don't forget to include the interval for each piece!

$T_{sp[1]}(x)$ = 9.1 + 0.9252637*(x+10) + 0.0000000*(x+10)^2 - 0.1252637*(x+10)^3 on [-10, -9]
$T_{sp[2]}(x)$ = 9.9 + 0.5494726*(x+9) - 0.3757911*(x+9)^2 + 1.6263184*(x+9)^3 on [-9, -8]
$T_{sp[3]}(x)$ = 11.7 + 4.6768458*(x+8) + 4.5031642*(x+8)^2 - 3.2800100*(x+8)^3 on [-8, -7]
$T_{sp[4]}(x)$ = 17.6 + 3.8431441*(x+7) - 5.3368659*(x+7)^2 + 2.0937217*(x+7)^3 on [-7, -6]
$T_{sp[5]}(x)$ = 18.2 - 0.5494224*(x+6) + 0.9442993*(x+6)^2 - 0.2948769*(x+6)^3 on [-6, -5]
$T_{sp[6]}(x)$ = 18.3 + 0.4545455*(x+5) + 0.0596685*(x+5)^2 - 0.1142140*(x+5)^3 on [-5, -4]
$T_{sp[7]}(x)$ = 18.7 + 0.2312406*(x+4) - 0.2829734*(x+4)^2 + 0.1517328*(x+4)^3 on [-4, -3]
$T_{sp[8]}(x)$ = 18.8 + 0.1204922*(x+3) + 0.1722250*(x+3)^2 - 0.0927172*(x+3)^3 on [-3, -2]
$T_{sp[9]}(x)$ = 19.0 + 0.1867906*(x+2) - 0.1059267*(x+2)^2 + 0.0191361*(x+2)^3 on [-2, -1]
$T_{sp[10]}(x)$ = 19.1 + 0.0323456*(x+1) - 0.0485183*(x+1)^2 + 0.0161728*(x+1)^3 on [-1, 0]

(b) What is the estimate of the temperature at h = −7.5 m?

$$T_{sp}(-7.5) = T_{sp[3]}(-7.5) = 14.754212707182319$$

(c) Plot the interpolating cubic spline T_{sp}(h), with markers at the nodes, as well as at (−7.5, T_{sp}(−7.5)). Annotate the figure correspondingly.



5. The position of the thermocline is determined as the solution of the equation $d^2T/dh^2 = 0$.

(a) Use the cubic interpolating polynomial $T_3(h)$ found in question 3. What is the equation that you solved to get the thermocline? At what depth does the thermocline exist?

$T_3(h) = 9.9 + 1.80000(h + 9) + 2.05000(h + 9)(h + 8) - 1.56667(h + 9)(h + 8)(h + 7)$

$T_3(h) =- 1.56667h^3 - 35.5501h^2 - 262.584h - 615.902$

$T''_3(h) =- 9.4002h - 71.1002$

Equate the second derivative to zero we get: $h =- 7.564m$

(b) Use the cubic spline $T_{sp}(h)$ from question 4. What is the equation that you solved to get the thermocline? (It is sufficient to differentiate only the piece of $T_{sp}(h)$. Which one is it?) At what depth does the thermocline exist?

The equation that we solved to get the thermocline is :

$Tsp[3](x) = 11.7 + 4.6768458(x + 8) + 4.5031642(x + 8)^2 - 3.2800100(x + 8)^3$ on [-8, -7]

because it covers the interval [-8, -7], where the largest temperature change occurs.
Next, To find the thermocline, we need the second derivative of $T_{sp[3]}(x)$ and set it to zero.

$$T'sp[3](x) = 4.6768458 + 9.0063284x - 9.84003(x + 8)^2 \text{ on [-8, -7]}$$
$$T''sp[3](x) = 9.0063284 - 19.68006(x + 8) \text{ on [-8, -7]}$$
$$9.0063284 - 19.68006(x + 8) = 0$$
$$x \approx - 7.54236$$

The specific depth of $- 7.54236$ m was determined as the point where this second derivative equals zero, indicating the location of the thermocline within the lake. The thermocline exists at a depth of approximately -7.54 meters.

6. What are some potential issues that could arise from interpolating with polynomials of a higher degree? How would this affect the estimate?

With higher degree interpolating polynomials, there is the potential for Runge's phenomenon to occur. In other words, there could be highly oscillatory polynomials that exhibit very extreme levels of error towards the tail ends of the points that are to be interpolated.

7. Did you struggle with any part of the group project? What do you think we should change so our next sessions run better? Do you have any conceptual questions that didn't get answered? Feel free to add any comments that you want to make but I haven't listed them.

We didn't encounter any difficulties with any part of the group project. We believe that the instructions of this group project were clear and easy to understand, and there isn't much need to be changed. Conceptual questions were addressed and answered during our discussions.

Question division

1	2	3	4	5	6	7
Nick Calzada	Zun Cao	Leo Wang	Sissi Zhang	Leo Wang Zun Cao	Nick Calzada	Sissi Zhang