GROUP PROJECT 1: COMPARISON OF SOLVERS

Project Rules

- Your class will be split into small groups (3-4 students). Please remember your group number (e.g. group 3A). Assign a scribe and a coordinator.
- **The scribe** should create a Google Docs file and share it with all group members and your instructors: Dr. Pencheva (gergina@utexas.edu) and Eboni (eboniwilliams@utexas.edu). Please share with us by adding us as "commentors" and restrict access so that only those people with access can open the link. The name of the file should be "GroupProject1 Group<your group number>" (for the example above, **"GroupProject1 Group3A"**). Please **do NOT change the file name.**
- The scribe should also reply to the Ed Discussion **post** under **category 'Group Projects'**, **subcategory 'GP1'**. The title of the post is "Section <your section number> Group Project 1 Groups". In the post include the names of all group members, your group number, and a link to the shared Google Docs file. Please **double-check that the GoogleDoc access has been restricted**.
- **The coordinator** will be responsible for collecting contact information from the group members and setting place/time for communication (GroupMe, Discord, Slack, Zoom, etc.) The coordinator should contact Dr. Pencheva in the case of unresponsive team member.
- The group project will be only **a part of one class period**. You are **not expected to finish the entire project during class**. In fact, group collaboration outside is strongly encouraged and even necessary. **I want to see evidence of group collaboration by the end of Saturday, February 10. Listing names and contact information only is NOT sufficient.** All team members might not get the same score. A component of the score will be determined based on the individual contribution.
- A list with tasks and questions is positioned in one place, at the bottom of the document. You are encouraged to provide only your answers at the top of your Google Docs file and keep the rest of the discussion separated. This will help for a quicker grading.
- Upload on Canvas the final version of your project report. **One submission per group is sufficient.** The Canvas assignment will be setup such that it shows submission for everyone in the group so long as one person in the group submits.

Project Description

We will study the convergence rates for four of the methods in Chapter 1. For that purpose, we will solve the following problem. For ideas on finding a good "true solution", please refer to the "Finding an Unknown True Solution" section of the Notes About Correct Decimal Places in Code page on Canvas.

In environmental engineering (a specialty area in civil engineering), the following equation can be used to compute the oxygen level c (mg/L) in a river downstream from a sewage discharge:

$$c = 10 - 20\left(e^{-0.2x} - e^{-0.75x}\right)$$

where x is the distance downstream in kilometers. We want to answer the following two questions:

(A) What is the distance downstream where the oxygen level first falls to a reading of 5 mg/L? Note that levels of oxygen below 5 mg/L are generally harmful to game-fish such as trout and salmon.

(B) What is the distance downstream at which the oxygen is at a minimum? What is the concentration at that location? What will happen to the fish?

We will use four solvers to answer these questions: Fixed Point Iteration, Newton's Method, Secant Method, and Newton-Bisection Method. Allow a sufficient number of iterations for each of the solvers to get **6 correct decimal places.**

For a general iterative method we know that the error (or error estimate) has the form $e_{i+1} = Ce_i^{\alpha}$. To confirm numerically the order of convergence α, we can take natural log of both side and get $\ln(e_{i+1}) = \ln C + \alpha \ln(e_i)$, i.e. plotting $\ln(e_{i+1})$ vs $\ln(e_i)$ should produce points forming **roughly** a line with slope α. You will use the four methods found on Canvas on page 'Programming' in module Resources. They store the error at each iteration in vector errors. If you are unsure how to get the **slope per each pair of points**, please refer to the following code snippets in MATLAB and Python. You are also welcome to use something else to calculate the slopes and generate the graphs. Please just make sure that you mention what you used if you didn't use the code below.

MATLAB:

```
x = log(errors(1:end-1));
y = log(errors(2:end));
plot(x,y,'bo-');
xlabel('log{(e_i)}');
ylabel('log{(e_{i+1})}');
grid on
dx = x(2:end)- x(1:end-1);
dy = y(2:end)- y(1:end-1);
slope = dy'./dx'
```

Python:

```
from numpy import log
import matplotlib.pyplot as pyp
x = log(errors[:-1])
y = log(errors[1:])
dx = x[1:]-x[:-1]
dy = y[1:]-y[:-1]
slopes = [dy[i]/dx[i] for i in range(len(dx))]
print("slopes = ",slopes)
pyp.plot(x,y,"bo-")
pyp.xlabel("log(e_i)")
pyp.ylabel("log(e_{i+1})")
pyp.grid(True)
# This saves to a file
pyp.savefig("./LogErrorsPlot.png")
# This shows it on your screen
# pyp.show()
```

The slopes should be roughly equal but don't get alarmed if you see some variation, even an outlier.

Tasks and Questions

PART A: What is the distance downstream where the oxygen level first falls to a reading of 5 mg/L?

1. What equation are you solving? Why?
2. Plot the function f_1 (corresponding to your equation).
3. Which root r of f_1 are you finding? Why?
4. For the Fixed Point Iteration (fixedPoint_err):
   i. What is your choice of function g_1 with a fixed point at r? Motivate your choice.
   ii. What are your initial approximation and tolerance? Please choose them such that the method takes at least 4-5 iterations. Find the solution with **6 correct decimal places.**
   iii. What are the iteration errors?
   iv. Include the graph and the **vector with slopes** described in the project description above.
   v. What is the slope that the theory predicts for FPI? Why? Does it match your numerics?
5. For the Newton method (newton_err):
   i. What are your initial approximation and tolerance? Please choose them such that the method takes at least 4-5 iterations. Find the solution with **6 correct decimal places.**

ii. What are the iteration errors?

iii. Include the graph and the **vector with slopes** described in the project description above.

iv. What is the slope that the theory predicts for Newton's method? Why? Does it match your numerics?

6. For the secant method (secant_err):

i. What are your two initial approximations and tolerance? Please choose them such that the method takes at least 4-5 iterations. Find the solution with **6 correct decimal places.**

ii. What are the iteration errors?

iii. Include the graph and the **vector with slopes** described in the project description above.

iv. What is the slope that the theory predicts for secant method? Why? Does it match your numerics?

7. For the Newton-Bisection method (newtonBisection_err):

i. What are your initial bracketing interval and tolerance? Please choose them such that the method takes at least 4-5 iterations. Find the solution with **6 correct decimal places.**

ii. What are the iteration errors?

iii. Include the graph and the **vector with slopes** described in the project description above.

8. What is the distance downstream where the oxygen level first falls to a reading of 5 mg/L?

PART B: What is the distance downstream at which the oxygen is at a minimum? What is the concentration at that location? What will happen to the fish?

1. What equation are you solving? Why?
2. Plot the function f_2 (corresponding to your equation).
3. For the Fixed Point Iteration (fixedPoint_err):

i. What is your choice of function g_2 with a fixed point at the root of f_2? Motivate your choice.

ii. What are your initial approximation and tolerance? Please choose them such that the method takes at least 4-5 iterations. Find the solution with **6 correct decimal places.**

iii. What are the iteration errors?

iv. Include the graph and the **vector with slopes** described in the project description above.

v. What is the slope that the theory predicts for FPI? Why? Does it match your numerics?

4. For the Newton method (newton_err):

i. What are your initial approximation and tolerance? Please choose them such that the method takes at least 4-5 iterations. Find the solution with **6 correct decimal places.**

ii. What are the iteration errors?

iii. Include the graph and the **vector with slopes** described in the project description above.

iv. What is the slope that the theory predicts for Newton's method? Why? Does it match your numerics?

5. For the secant method (secant_err):

   i. What are your two initial approximations and tolerance? Please choose them such that the method takes at least 4-5 iterations. Find the solution with **6 correct decimal places.**

   ii. What are the iteration errors?

   iii. Include the graph and the **vector with slopes** described in the project description above.

   iv. What is the slope that the theory predicts for secant method? Why? Does it match your numerics?

6. For the Newton-Bisection method (newtonBisection_err):

   i. What are your initial bracketing interval and tolerance? Please choose them such that the method takes at least 4-5 iterations. Find the solution with **6 correct decimal places.**

   ii. What are the iteration errors?

   iii. Include the graph and the **vector with slopes** described in the project description above.

7. Answer the following questions.

   i. What is the distance downstream at which the oxygen is at a minimum?

   ii. What is the concentration at that location?

   iii. What will happen to the fish at the found concentration?

PART C: Give feedback.

1. Did you struggle with any part of the group project?
2. What do you think we should change so our next sessions run better?
3. Do you have any conceptual questions that didn't get answered?
4. Feel free to add any comments that you want to make but I haven't listed.