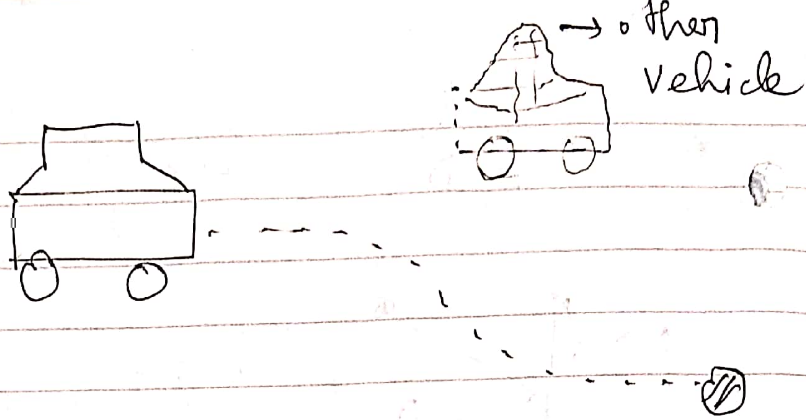
A Control Task:



→ **Given:**
Reference plan
State estimate

→ **Find:**
which control inputs achieve the plan?

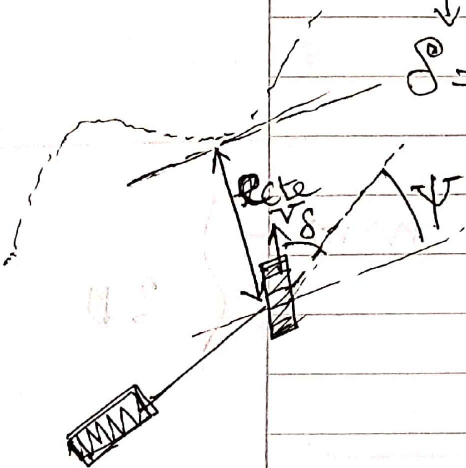
Reactive Control: decomposition of the problem in different dimension.

- **Longitudinal** – control via PID.

- **Lateral** — control via Stanley
  ↓ Steering command.

$$\delta = \psi + \tan^{-1}\left(\frac{k\, e_{cte}}{v}\right)$$

(cross track error)



Limitations of Reactive Control:

Non trivial for more complex systems, control gains are tuned manually, seperation into longitudinal & lateral controller ignores coupling.

No handling of constraints such as obstacles

Ignores future decision.

Reactive $\longrightarrow$ MPC {Since this uses optimal control}.

- optimal control? How to formulate?

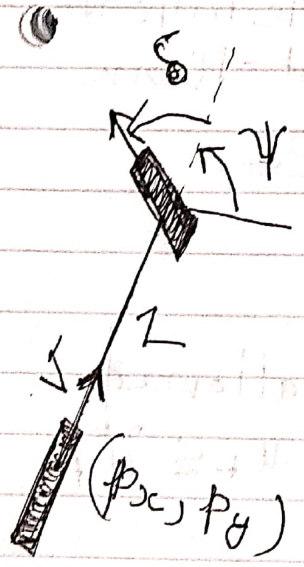
→ Model of system dynamics:

→ Predicts the evolution of states for a given sequence of inputs.

$$x_{t+1} = f(x_t, u_t).$$

- Control I/P $\xrightarrow{\text{dynamics}}$ State trajectory.

Model example: Discrete 2D Bicycle:



$$x_{t+1} = \begin{cases} p_{x,t+1} = p_{x,t} + \Delta t\, v_t \cos\psi_t & \text{Position} \\ p_{y,t+1} = p_{y,t} + \Delta t\, v_t \sin\psi_t & \\ \psi_{t+1} = \psi_t + \Delta t\, v/L \tan\delta & \text{heading angle} \\ v_{t+1} = v_t + \Delta t\, a_t & \text{velocity} \\ \delta_{t+1} = \delta_t + \Delta t\, w_t & \text{steering angle} \end{cases}$$

where $u_t = [a_t, w_t]^T$

Optimal Control: Objective:

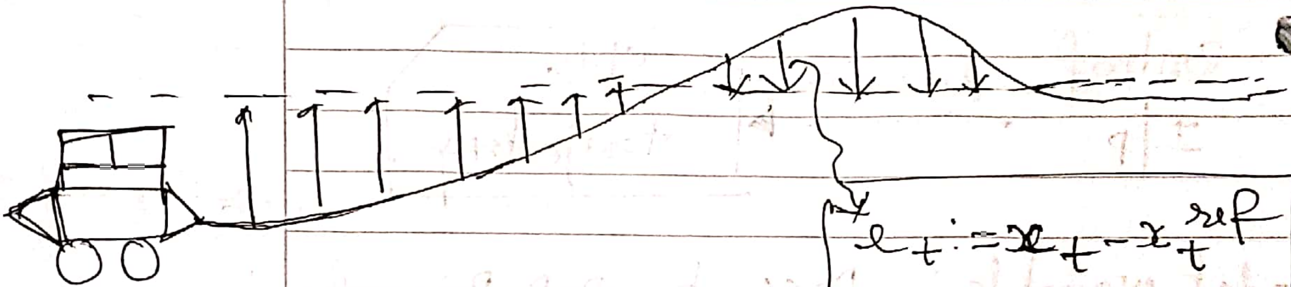
→ **Objective**: Assigns a cost to the trajectory

$$J(x_{1:T}, u_{1:T}) = \sum_{t \in [T]} g_t(x_t, u_t)$$

where $x_{1:T} := (x_1, \ldots, x_T)$, $u_{1:T} := (u_1, \ldots, u_T)$

→ **example**: deviation from a reference.

$$g_t(x_t, u_t) = e_t^T Q_t e_t + u_t^T R_t u_t$$



$$\boxed{e_t := x_t - x_t^{ref}}$$

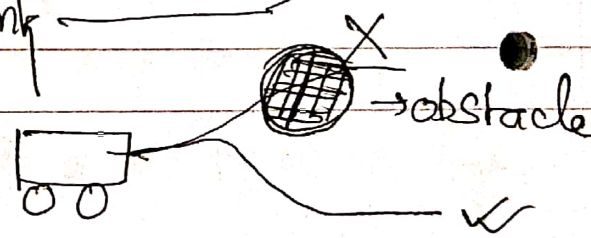
Q_t = weight matrix which tells which part of error is more important than other.

Optimal Control: Constraints

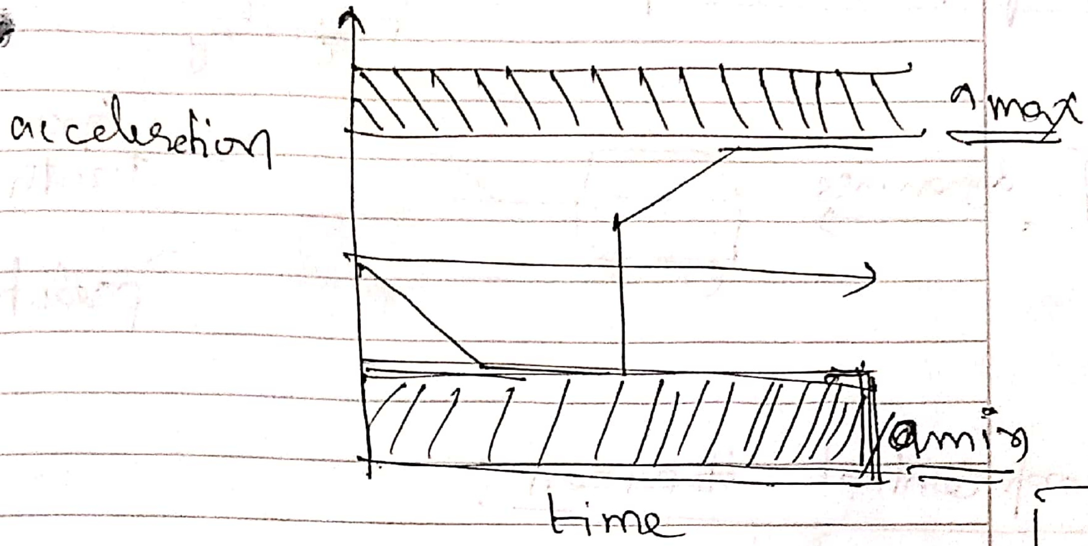
→ **Constraints**: Encode the domains of allowed states $x_t \in \mathcal{X}_t$ and inputs $u_t \in \mathcal{U}_t$

constraint

$$\mathcal{X}_t = \{x \mid p_x^2 + p_y^2 \geq r^2\}$$



$\mathcal{U}_t = \{u \mid a_{min} \leq a \leq a_{max}\}$.



Control as a optimization Problem:

→ minimizes cost (1)
→ dynamically feasible trajectory (2)
→ does not violate the constraints (3)

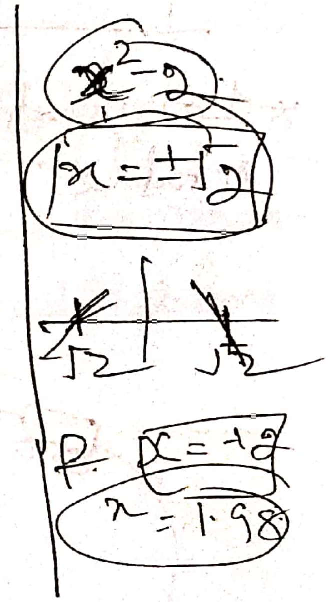
$$\min_{x_{1:T}, u_{1:T}} J(x_{1:T}, u_{1:T})$$

subject to $x_{t+1} = f(x_t, u_t)$; $\forall t \in [T-1]$

$u_t \in \mathcal{U}_t$, $\forall t \in [T]$

$x_t \in \mathcal{X}_t$, $\forall t \in [T]$

$x_1 = x_{init}$

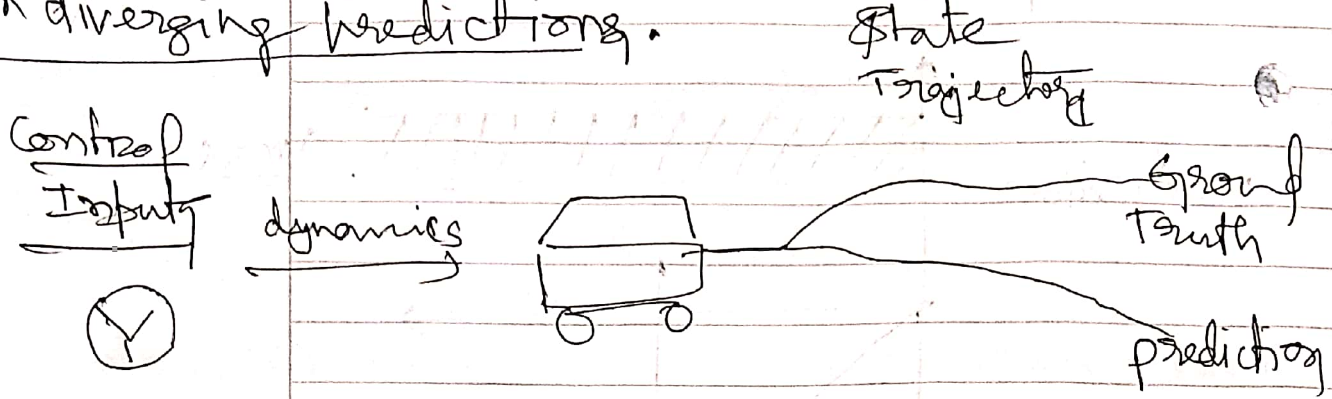


Solving Optimization Problem:

Open Loop Control : Model errors:

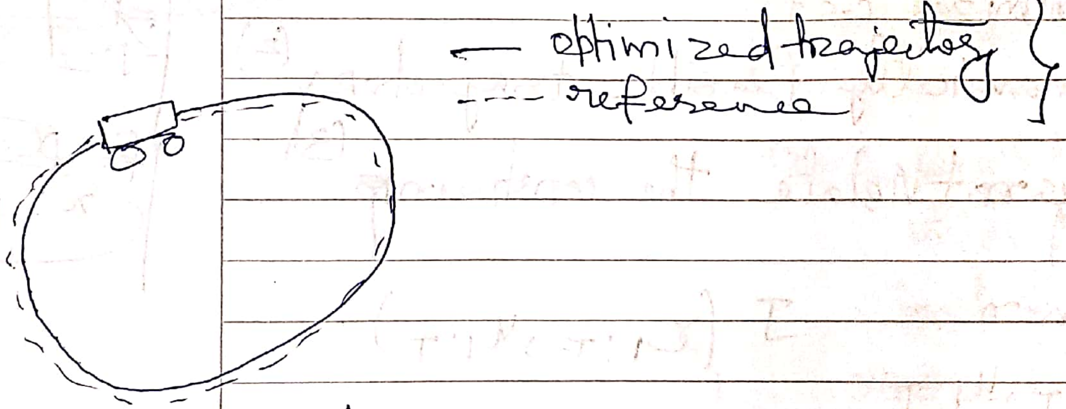
→ The prediction model with always be wrong to some extend.

– Model errors accumulate over time & result in diverging predictions.



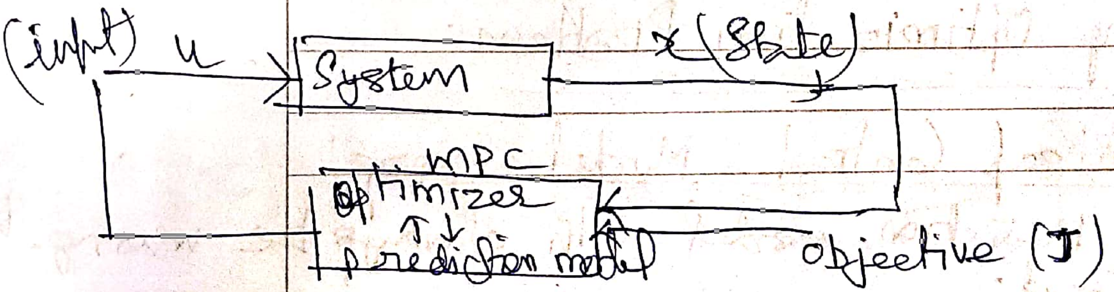
– Open Loop Control: Horizon:

* Long task-Horizons make the problem intractable. (within some time it is difficult to predict the route).



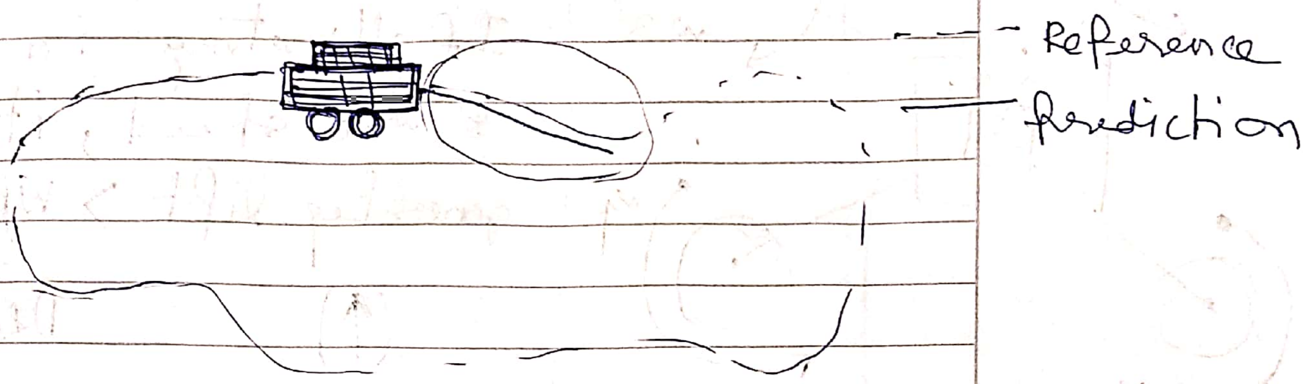
So we move to "Receding Horizon Control",

- Start from the current state
- Find controls for a limited preview into future.
- Apply only the first input, then replan.



MPC Schematic View:

* Plan @ $t = 1$ sec.



Advantages:

→ Accounts for error.
→ Reduced Problem Size.

MPC Design Parameters:

* Prediction Model
* Cost function
* Prediction horizon.
* Terminal constraints.

Prediction Model:

→ Trade-off in choice of model family: Model accuracy vs complexity.

→ Data-Driven approach:

- collect data of the real system behaviour.

$$D := \{(x'_d, x_d, u_d) \mid x'_d = f(x_d, u_d), d \in [N_d]\}.$$