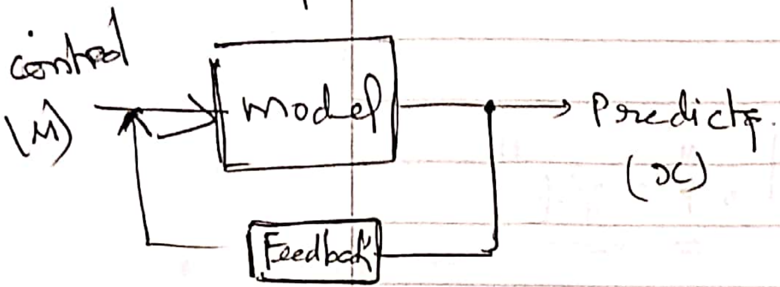
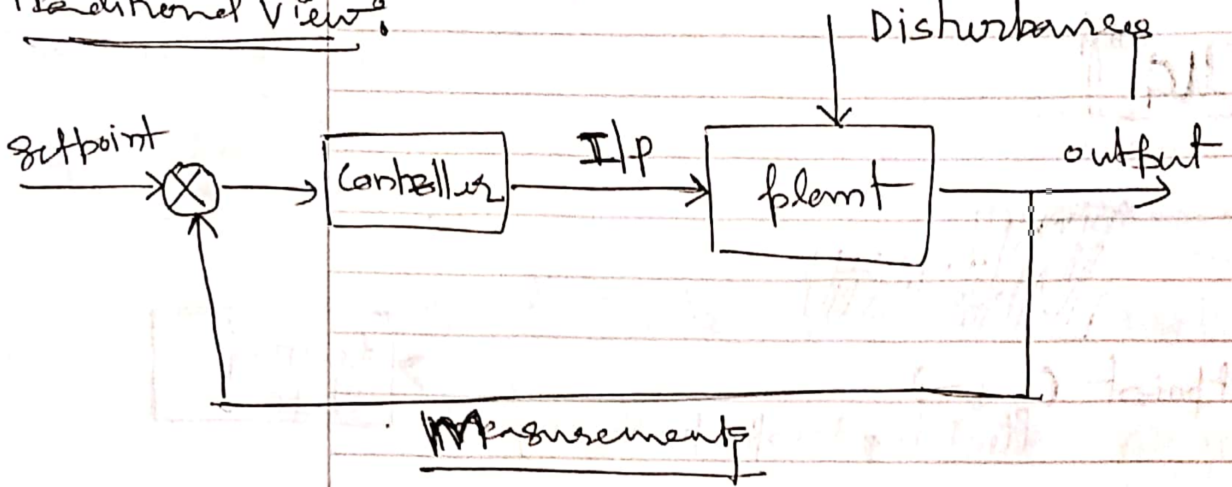
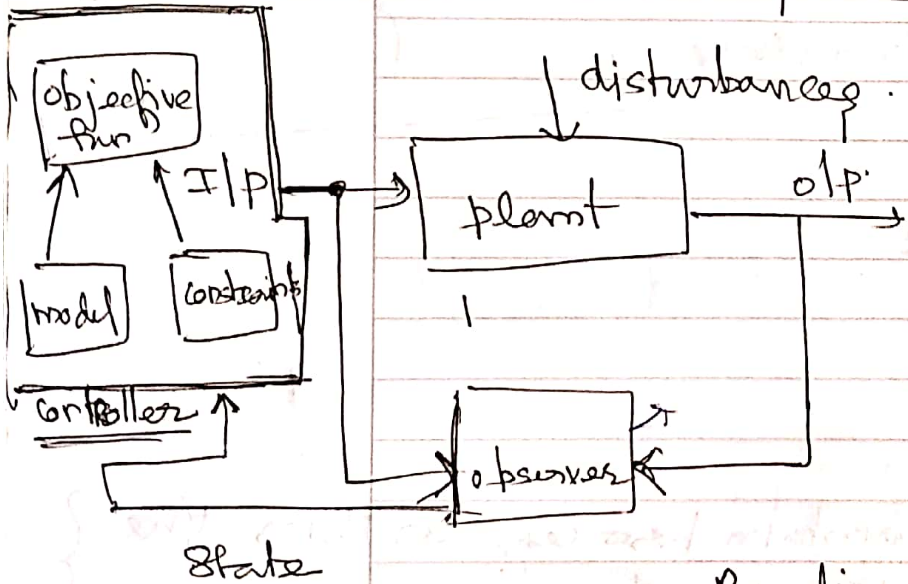
MPC



Traditional view:

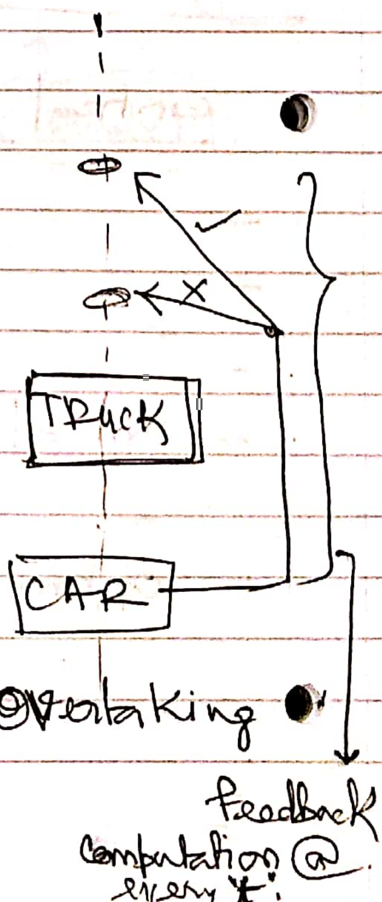


Decision under constraints with feedback:



"MIMO → structure"

Receding Horizon → [Approach after feedback update] → Overtaking



Linear MPCs:

- Linear controllers are most commonly used.
- MPC includes both feedback & feed forward actions.

→ MPC uses discrete time Linear models:

$$\boxed{x(K+1)} = A\,x(K) + B\,u(K)$$

looks like K.F

State space

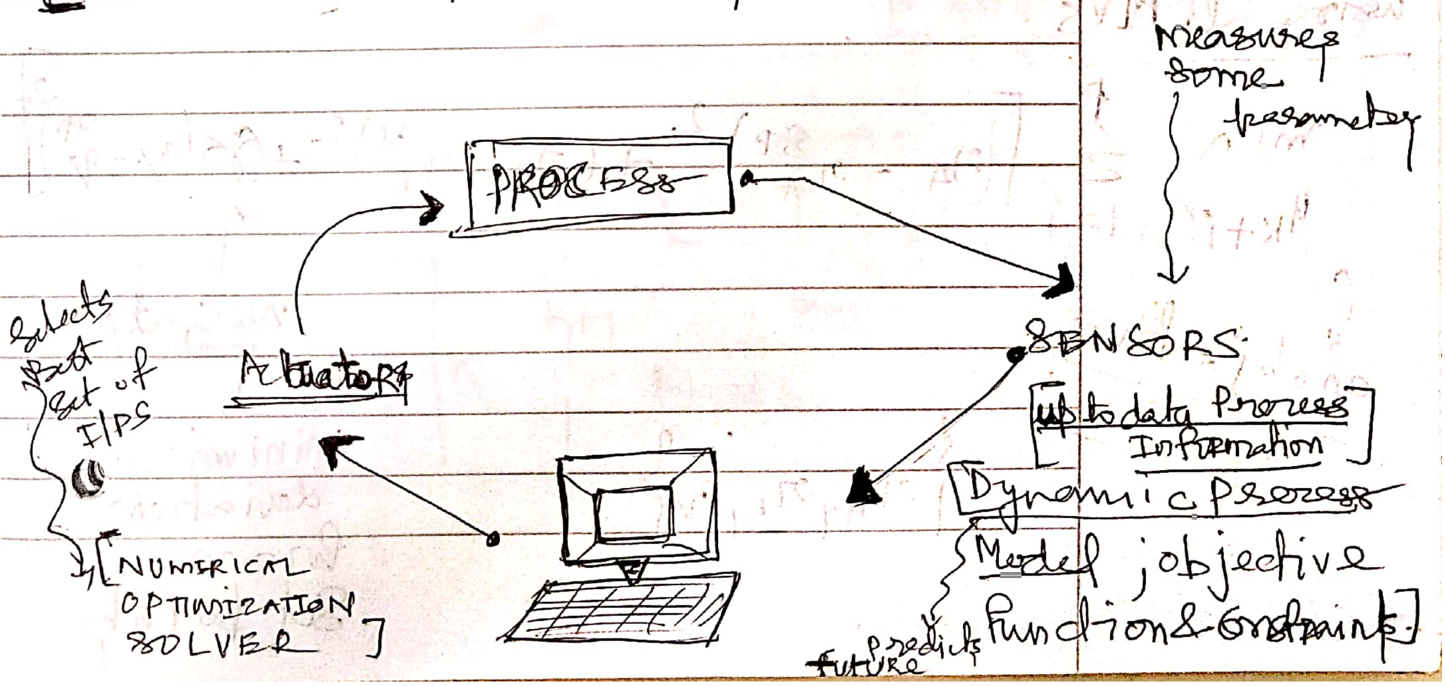
$$\frac{y(z)}{u(z)} = G(z)$$

T.F

$$y(K) = \sum_{i=1}^{n} h_i\, u(k-i)$$

step or Impulse response.

MPC → allows current time-slot to be optimized.
- ↳ has ability to anticipate future events.
- ↳ handles constraints.



Current time

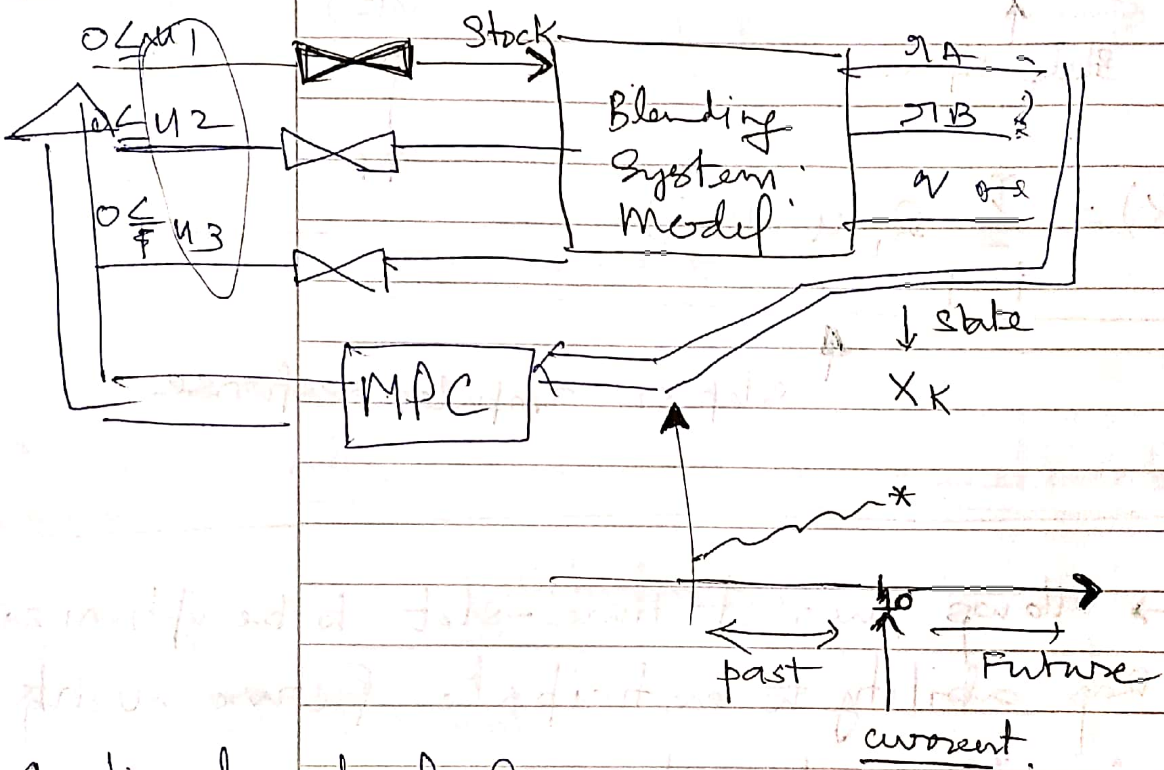
10t
happended

11t	12t	13t	14t	15t	→ ...

Prediction takes place

Known as a Horizon.

example1:



Combined control of r_A & r_B & blend flow (q)
using all MVs.

$$\min_{u_{k+i}} \sum_{i=1}^{p} \left[(r_A - r_A^{sp})^2 + (r_B - r_B^{sp})^2 + \gamma (q - q^{sp})^2 \right]$$

an objective fun^n

Weights

minimize deviation from set point

$\{ r_A, r_B, q \}$

example 2:

y_1, y_2 — setpoint

(y_7) — o/p →

temp {min} value

say (70°C)

u_1, u_2, u_3 } → i/p.

↳ heat recovery maximize.

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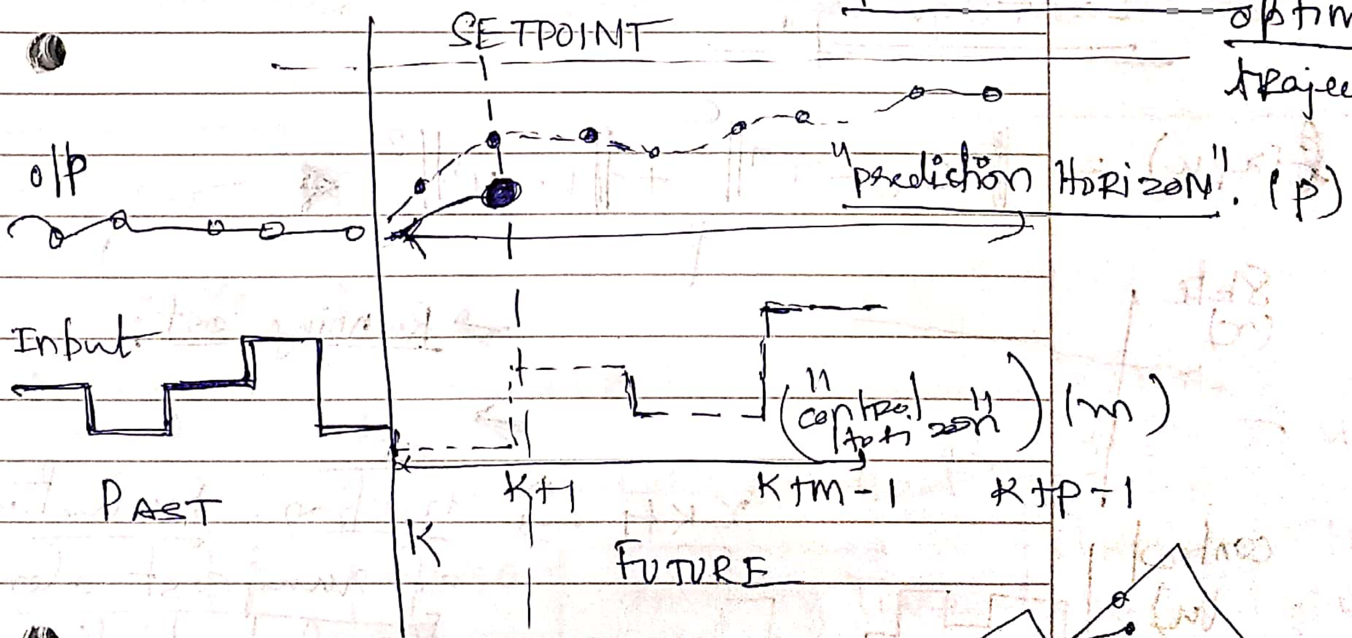
1. model → $\dot{x} = f(x, u, d)$
2. constraints → $x_{min} \le x \le x_{max}$
3. objective → max Σ something

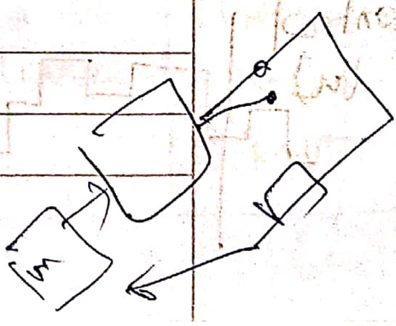
↳ Any funct^n: linear, N.L, Quadratic.

Receding Horizon:

Model predicts future

optimizer results optimal trajectory





Components of MPC:

(1) Dynamic Process model State: (stored in memory)
(2) Multi-step prediction equation
(3) Objectives & constraints.
(4) optimization solver.
(5) State update scheme. {observer, state estimator}

Objective function → with an optimization variable.

Assume 'o'

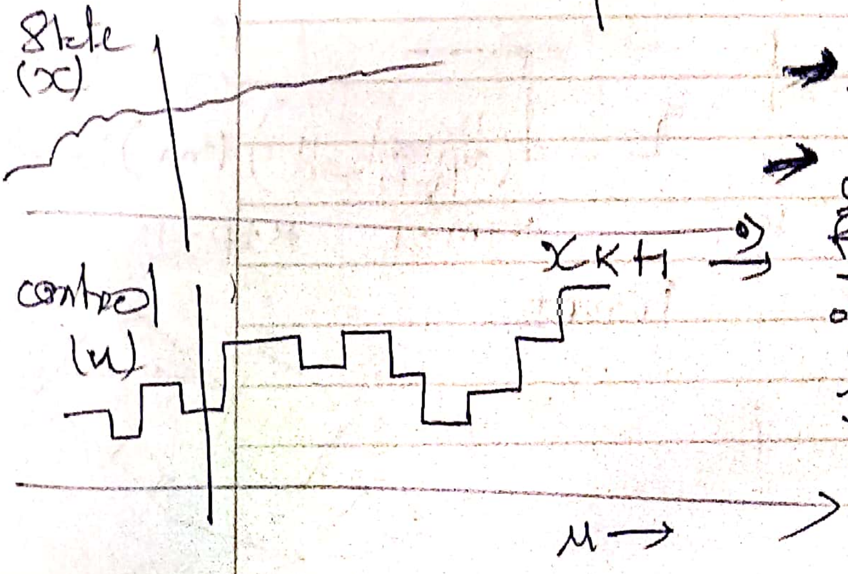
$$\boxed{obj = o x^2 + o y + 3}$$

↳ we need to minimize 'o' such that obj is minimized.

ex: Least squares:

$$\ell(x,u) = \|x_u - x^r\|^2_Q + \|u - u^r\|^2_R$$

→ Running cost;



→ cost function: Evaluation of running cost along the whole prediction horizon.

OCP → to find a minimizing control sequence

$$\min_{u} J_N(x_0, u) = \sum_{K=0}^{N-1} l(x_u(K), u(K))$$

$u \quad \forall K \in [0, N-1]$
$x \quad \forall K \in [0, N]$ → How? (For my problem)

Value function: min of cost function.

$$V_N(x) = \min_{u} J_N(x_0, u)$$

Questions:

→ How to set x, u constraints
→ How should N look like.
→ Numerical representation

?

MPC For Diff. Drive MR

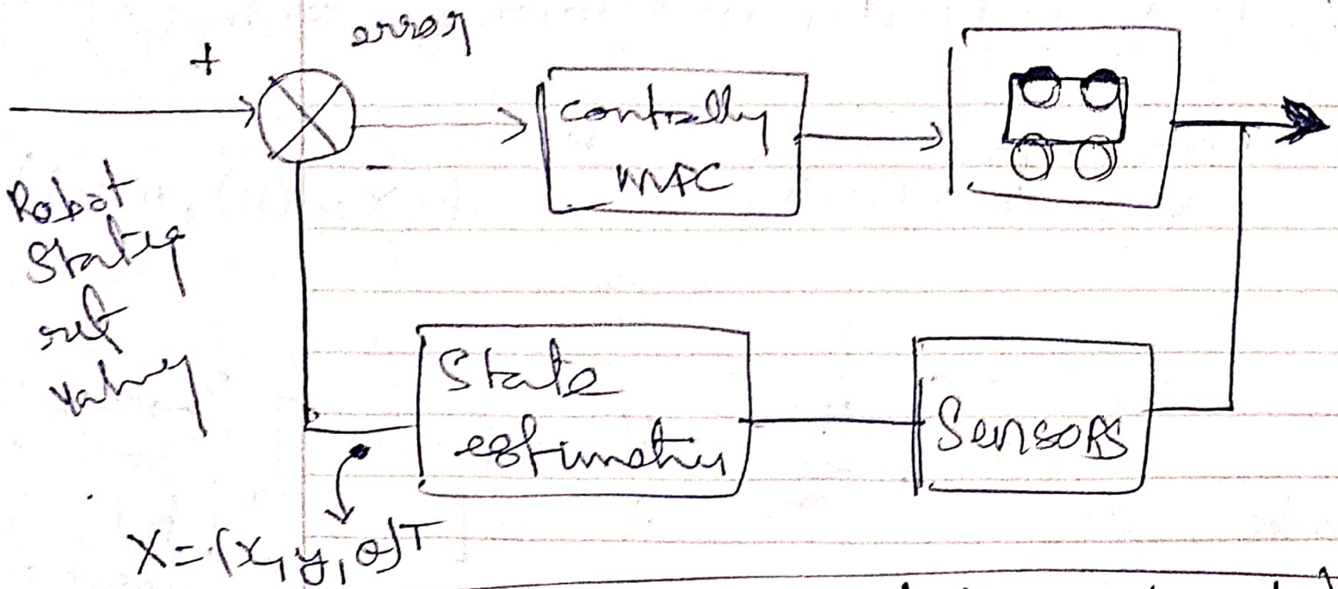
System Model:

$$\dot{x}(t) = f_t(x(t), u(t))$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \end{bmatrix} \xrightarrow[\text{Sampling Time } (\Delta T)]{\text{Euler discretization}}$$

$$x(K+1) = f(x(K), u(K))$$

$$\begin{bmatrix} x(K+1) \\ y(K+1) \\ \theta(K+1) \end{bmatrix} = \begin{bmatrix} x(K) \\ y(K) \\ \theta(K) \end{bmatrix} + \Delta T \begin{bmatrix} v(K)\cos\theta(K) \\ v(K)\sin\theta(K) \\ \omega(K) \end{bmatrix}$$



MPC controller:

(with constraints)

$$J_N(x_0, u) = \sum \ell(x(k), u(k))$$

[Running costs + OCP]

$$\ell(x,u) = \|x - x^{ref}\|_Q^2 + \|u - u^{ref}\|_R^2 .$$

OCP $\rightarrow$ Shooting methods:
1) Single shooting
2) Multiple shooting
⋮
$\rightarrow$ NLP.