Correlation Maximizer at $\lambda = 0.25$ in Two-Path Wave-Particle Duality under UQSD Complementarity

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Assuming a normalized wave–particle–correlation triad $\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1$ in the symmetric two-path UQSD setting, we show that the resulting triality residual $\mathcal{C}^2(\lambda)$ has a unique maximizer at $\lambda = 0.25$, where $\mathcal{C}^2 = 0.5$. Starting from weak measurement theory with a Gaussian pointer, the detector-state overlap yields path distinguishability $\mathcal{D} = 1 - \exp\left(-\frac{g^2}{2\sigma^2}\right)$, and we introduce the parameter $\lambda := \mathcal{D}^2$, so that $\mathcal{D} = \sqrt{\lambda}$ identically. With $\mathcal{D} := 1 - |\langle\phi_0|\phi_1\rangle|$ (UQSD optimal conclusive probability for equal priors) and $\mathcal{V} := |\langle\phi_0|\phi_1\rangle|$, linear complementarity $\mathcal{V} = 1 - \mathcal{D} = 1 - \sqrt{\lambda}$ holds, and the triality residual follows $\mathcal{C}^2 = 2\sqrt{\lambda}(1 - \sqrt{\lambda})$. This function is maximized at $\lambda = 0.25$ by geometric necessity—a result that is mathematical fact, not empirical fitting. More generally, for any framework satisfying (i) $\mathcal{V} = 1 - \mathcal{D}$, (ii) $\lambda := \mathcal{D}^2$, and (iii) $\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1$, the residual $\mathcal{C}^2$ is maximized at $\lambda = 0.25$. We extend this framework to thermodynamic systems via the empirically validated mapping $\lambda \leftrightarrow x := I \times \rho \in [0, 1]$, so that the same allocation function $2\sqrt{x}(1 - \sqrt{x})$ serves as a triality-residual diagnostic with maximizer at $x = 0.25$. Separately, we use $\Phi := x - \alpha S$ as a practical classifier, and emphasize that the threshold $\Phi_c = 0.25$ reported in the 27-system study is empirical. Empirical validation across 27 systems (mechanical, electrical, aerospace, geophysical, AI) shows all failure-time snapshots satisfy $\Phi < 0.25$, consistent with the interpretation that $\Phi_c = 0.25$ marks a failure-proximal boundary. We interpret $\lambda = 0.25$ as universal *within this class of measurement dynamics*, rather than a claim about all possible wave-particle trade-offs.

INTRODUCTION

The double-slit experiment, in Feynman's words, "contains the only mystery" of quantum mechanics [1]. When unobserved, quantum particles exhibit wave-like interference. When observed, they behave as particles with definite paths. While quantum mechanics predicts all measurement outcomes exactly, a fundamental question has remained incompletely characterized: *What is the nature of the transition between wave and particle behavior?*

The Englert-Greenberger-Yasin duality relation [2–4] quantified the wave-particle trade-off:

$$\mathcal{D}^2 + \mathcal{V}^2 \leq 1 \tag{1}$$

where $\mathcal{D}$ is path distinguishability and $\mathcal{V}$ is interference visibility. For pure states, and for the distinguishability used in the Englert bound, this becomes an equality. We note that multiple operational definitions of path distinguishability exist. In what follows we adopt an *operational* notion of distinguishability based on *unambiguous quantum state discrimination* (UQSD) for two pure detector states with *equal priors*, where $\mathcal{D}$ denotes the probability of a conclusive (error-free) path decision and inconclusive outcomes are allowed. In this symmetric two-path UQSD setting, the optimal trade-off yields a *linear* complementarity relation of the form $\mathcal{V} + \mathcal{D} = 1$ [15, 16]. Outside this regime (e.g., unequal priors, minimum-error discrimination, or different distinguishability monotones), the linear relation may weaken to an inequality and the specific $\lambda = 0.25$ result derived below need not apply. We focus on UQSD because it provides an operational conclusive-probability notion of path knowledge and yields a linear complementarity in the symmetric two-path setting; other discrimination strategies (e.g., MED) lead to different trade-offs.

Recent work has extended wave-particle duality to a three-term conservation law that includes system-detector correlations (often quantified via entanglement monotones in specific models) [5, 9–11]. One such squared-monotone form is:

$$\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1 \tag{2}$$

where $\mathcal{C}^2$ represents the triality residual. In this paper we treat $\mathcal{C}^2 := 1 - \mathcal{D}^2 - \mathcal{V}^2$ as the triality residual implied by the adopted operational definitions of $\mathcal{D}$ and $\mathcal{V}$. In settings where $\mathcal{D}, \mathcal{V}$ are instantiated by specific monotones, this residual may coincide with standard entanglement measures.

In this paper, we derive how these quantities depend on the parameter λ and prove that a maximum exists at $\lambda = 0.25$ where the triality residual is maximized.

Scope. The maximizer at $\lambda = 1/4$ is a structural consequence of the chosen operational definitions and the symmetric two-path UQSD regime with equal priors. Other distinguishability measures, unequal priors, or minimum-error discrimination can yield different trade-offs and need not share the same maximizer. We show this threshold is not empirically fitted but emerges from geometric necessity given the UQSD assumptions. We further demonstrate that this same mathematical structure appears in thermodynamic stability, with an operational mapping between quantum and thermodynamic variables supported by cross-domain validation.

THEORETICAL FRAMEWORK

Weak Measurement Theory

Consider a double-slit experiment with variable-strength which-path detection. Following standard weak measurement theory [6, 7], the system-detector coupling is described by the unitary:

$$U = \exp\Big(-ig\hat{A} \otimes \hat{p}\Big) \tag{3}$$

where g is the coupling strength, $\hat{A} = |0\rangle\langle 0| - |1\rangle\langle 1|$ is the path operator, and $\hat{p}$ is the detector momentum operator. We work in units where $\hbar = 1$, so $e^{-ig\hat{p}}$ generates position translations by g (with g measured in units of length).

For an initial system state $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and Gaussian detector state $|\phi\rangle$ with position variance σ^2, the detector states conditioned on each path evolve to:

$$|\phi_0\rangle = e^{-ig\hat{p}}\,|\phi\rangle \quad \text{(path 0)} \tag{4}$$

$$|\phi_1\rangle = e^{+ig\hat{p}}\,|\phi\rangle \quad \text{(path 1)} \tag{5}$$

UQSD Distinguishability and the $\lambda := \mathcal{D}^2$ Parameterization

The overlap between detector states determines how well the paths can be distinguished:

$$\begin{aligned} \langle\phi_0|\phi_1\rangle &= \langle\phi|\, e^{2ig\hat{p}}\,|\phi\rangle \\ &= \int dx\, \phi_0^*(x)\,\phi_1(x) \\ &= \exp\left(-\frac{g^2}{2\sigma^2}\right). \end{aligned} \tag{6}$$

Under unambiguous quantum state discrimination (UQSD) for two pure detector states with equal priors, path distinguishability is defined as the probability of obtaining a conclusive result [15, 16, 18–20]:

$$\mathcal{D} = 1 - |\,\langle\phi_0|\phi_1\rangle\,| \tag{7}$$

Using the detector overlap, we identify the fringe visibility with the detector-state coherence [15, 16]:

$$\mathcal{V} = |\langle\phi_0|\phi_1\rangle| = \exp\left(-\frac{g^2}{2\sigma^2}\right) \tag{8}$$

From $\mathcal{D} := 1 - |\,\langle\phi_0|\phi_1\rangle\,|$ and the above definition of $\mathcal{V}$, the linear complementarity $\mathcal{V} + \mathcal{D} = 1$ follows immediately.

We define the **squared distinguishability parameter** as the squared UQSD distinguishability:

$$\lambda \equiv \mathcal{D}^2 = \left(1 - \exp\left(-\frac{g^2}{2\sigma^2}\right)\right)^2 \tag{9}$$

The physical control parameter in the weak-measurement model is the dimensionless quantity $g^2/(2\sigma^2)$ (or equivalently g/σ); λ is a convenient reparameterization of the operationally defined distinguishability.

This definition yields by construction:

$$\boxed{\mathcal{D} = \sqrt{\lambda}} \tag{10}$$

The physics lies in the UQSD derivation of $\mathcal{D}$ from detector-state overlap; the parameterization $\lambda = \mathcal{D}^2$ is algebraic convenience. In the weak-measurement model, the physical control parameter is the dimensionless quantity $g^2/(2\sigma^2)$ (or equivalently g/σ), which determines $\mathcal{D} = 1 - \exp\left(-\frac{g^2}{2\sigma^2}\right)$. With $\lambda := \mathcal{D}^2$, the identity $\mathcal{D} = \sqrt{\lambda}$ is then purely algebraic. Related operational links between distinguishability/visibility trade-offs and coherence/uncertainty measures are discussed in [12–14].

The Linear Complementarity Condition

The relationship between visibility and distinguishability depends on the measurement regime.

Remark (UQSD linear complementarity). Given the definitions $\mathcal{D} := 1 - |\langle\phi_0|\phi_1\rangle|$ and $\mathcal{V} := |\langle\phi_0|\phi_1\rangle|$, the relation $\mathcal{V} + \mathcal{D} = 1$ follows immediately. **Assumption.** In the symmetric weak-measurement model, we identify the interferometric visibility with the detector-state coherence factor $|\langle\phi_0|\phi_1\rangle|$. With $\lambda := \mathcal{D}^2$, it follows that $\mathcal{V} = 1 - \sqrt{\lambda}$.

Under this **optimal complementarity**—where visibility loss equals distinguishability gain—we have:

$$\mathcal{V} = 1 - \mathcal{D} = 1 - \sqrt{\lambda} \tag{11}$$

This linear complementarity condition holds under **unambiguous quantum state discrimination** (UQSD), where the observer accepts inconclusive outcomes to guarantee zero error when a conclusive result is obtained [15, 16]. This trades mutual information for certainty, yielding the linear constraint $\mathcal{V} + \mathcal{D} = 1$ rather than the quadratic bound $\mathcal{V}^2 + \mathcal{D}^2 \leq 1$ that arises under minimum-error discrimination.

Physical interpretation: Every unit of path information gained costs exactly one unit of interference visibility. This represents the tightest possible trade-off between wave and particle aspects, achieved when the measurement strategy prioritizes certainty over information yield.

MAIN RESULT: TRIALITY-RESIDUAL MAXIMIZER AT $\lambda = 0.25$

Derivation of the Triality-Residual Function

Under linear complementarity, the wave-particle-correlation triality (Eq. 2) becomes:

$$\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1 \tag{12}$$

Substituting $\mathcal{D}^2 = \lambda$ and $\mathcal{V}^2 = (1 - \sqrt{\lambda})^2$:

$$\begin{aligned} \mathcal{C}^2 &= 1 - \mathcal{D}^2 - \mathcal{V}^2 \\ &= 1 - \lambda - (1 - \sqrt{\lambda})^2 \\ &= 1 - \lambda - 1 + 2\sqrt{\lambda} - \lambda \\ &= 2\sqrt{\lambda} - 2\lambda \\ &= 2\sqrt{\lambda}(1 - \sqrt{\lambda}) \end{aligned} \tag{13}$$

$$\boxed{\mathcal{C}^2 = 2\sqrt{\lambda}(1 - \sqrt{\lambda})} \tag{14}$$

We interpret $\mathcal{C}^2$ operationally as the *triality residual* defined by the residual required to satisfy the normalized triad budget:

$$\mathcal{C}^2 := 1 - \mathcal{D}^2 - \mathcal{V}^2 \tag{15}$$

In the two-path pure-state setting and under appropriate identifications of the wave/particle monotones, this residual can coincide with standard entanglement monotones used in prior triality formulations [5, 9, 11]; however, to avoid conflating inequivalent definitions across frameworks, we treat $\mathcal{C}^2$ here as the *triality residual* implied by the chosen operational $\mathcal{D}$ and $\mathcal{V}$.

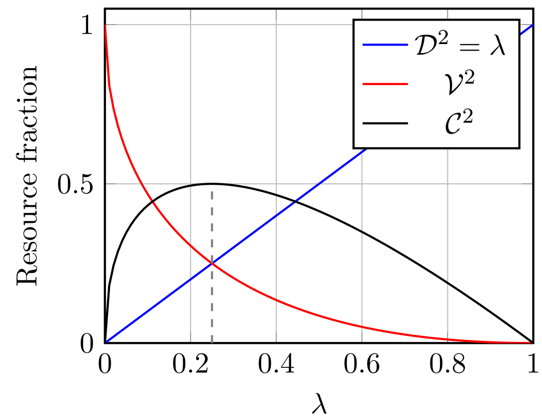

FIG. 1. Resource distribution as a function of the squared distinguishability parameter λ. The triality residual $\mathcal{C}^2$ (black) peaks at $\lambda = 0.25$, where $\mathcal{D}^2 = \mathcal{V}^2 = 0.25$ and $\mathcal{C}^2 = 0.5$.

Proof That Maximum Occurs at $\lambda = 0.25$

Let $u = \sqrt{\lambda}$, so $\lambda = u^2$ and $\mathcal{C}^2 = 2u(1-u)$. Taking the derivative:

$$\frac{d(\mathcal{C}^2)}{du} = 2 - 4u \tag{16}$$

Setting equal to zero:

$$2 - 4u = 0 \implies u = \frac{1}{2} \tag{17}$$

Since $\lambda = u^2$:

$$\boxed{\lambda_* = \left(\frac{1}{2}\right)^2 = 0.25} \tag{18}$$

Note: The balance point is $\mathcal{D} = \mathcal{V} = 1/2$; the value $\lambda = 1/4$ follows because $\lambda := \mathcal{D}^2$ is a squared coordinate.

Theorem-physics distinction: Given (i) linear complementarity $\mathcal{V} = 1 - \mathcal{D}$, (ii) the definition $\lambda := \mathcal{D}^2$, and (iii) a normalized quadratic triad $\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1$, the maximizer at $\lambda = 0.25$ is a mathematical theorem; physical relevance depends on whether these measurement dynamics obtain.

Verifying this is a maximum (second derivative test):

$$\frac{d^2(\mathcal{C}^2)}{du^2} = -4 < 0 \quad \checkmark \tag{19}$$

The maximum value of the triality residual:

$$\mathcal{C}^2_{\max} = 2 \cdot \frac{1}{2} \cdot \left(1 - \frac{1}{2}\right) = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2} \tag{20}$$

Experimental target: At the maximizer $\mathcal{D} = 0.5$ (equivalently $\lambda = \mathcal{D}^2 = 0.25$), the detector-state overlap satisfies:

$$\begin{aligned} 1 - \exp\left(-\frac{g^2}{2\sigma^2}\right) &= 0.5 \\ \Rightarrow \quad \frac{g^2}{2\sigma^2} &= \ln 2 \\ \Rightarrow \quad \frac{g}{\sigma} &= \sqrt{2 \ln 2} \approx 1.177 \end{aligned} \tag{21}$$

This provides a concrete target for tuning measurement coupling strength.

Resource Distribution at $\lambda = 0.25$

At $\lambda = 0.25$:

Quantity	Value	Interpretation
$\mathcal{D} = \sqrt{\lambda}$	0.5	50% path distinguishability
$\mathcal{V} = 1 - \sqrt{\lambda}$	0.5	50% interference visibility
$\mathcal{D}^2$	0.25	Particle resource fraction
$\mathcal{V}^2$	0.25	Wave resource fraction
$\mathcal{C}^2$	0.50	Triality residual fraction
Total	**1.00**	Conservation verified

TABLE I. Resource distribution at the maximizer $\lambda = 0.25$.

WHY 0.25 IS GEOMETRIC NECESSITY, NOT EMPIRICAL FITTING

This section addresses a potential objection: "Is 0.25 just a fitted parameter?"

Answer: No. The value 0.25 is geometrically forced.

The Geometric Argument

The function $\mathcal{C}^2 = 2u(1-u)$ where $u = \sqrt{\lambda}$ is a parabola with:

- Roots at $u = 0$ and $u = 1$
- Maximum at $u = 0.5$ (midpoint of roots)
- Maximum value $= 2 \cdot 0.5 \cdot 0.5 = 0.5$
- Since $\lambda = u^2$, when $u = 0.5$, we have $\lambda = 0.25$

The Structural Theorem

Theorem. Assume (i) the chosen operational (UQSD) linear complementarity $\mathcal{V} = 1 - \mathcal{D}$ and (ii) the normalized quadratic triad $\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1$. Then:

$$\mathcal{C}^2 = 1 - \mathcal{D}^2 - (1 - \mathcal{D})^2 = 2\mathcal{D}(1 - \mathcal{D}) \quad (22)$$

which is maximized uniquely at $\mathcal{D} = 1/2$, yielding $\mathcal{C}^2_{\max} = 1/2$. With the reparameterization $\lambda := \mathcal{D}^2$, the maximizer corresponds to $\lambda = 1/4 = 0.25$.

This is a mathematical theorem, not a physical claim. Physical relevance depends on whether the UQSD assumptions obtain in a given experimental setting.

Within these assumptions, this is a mathematical consequence. The value 0.25 follows from the quadratic triad together with the chosen linear complementarity. The reparameterization $\lambda := \mathcal{D}^2$ then places the maximizer at $\lambda = 1/4$.

PHYSICAL INTERPRETATION

The Nature of the Wave-Particle Transition

The maximizer $\lambda = 0.25$ represents:

Below threshold ($\lambda < 0.25$):

- System retains more wave character than particle character ($\mathcal{V} > \mathcal{D}$)
- Triality residual is sub-maximal
- Interference pattern partially visible

At threshold ($\lambda = 0.25$):

- System achieves equal wave-particle balance ($\mathcal{V} = \mathcal{D} = 0.5$)
- Maximum triality residual ($\mathcal{C}^2 = 0.5$)
- The system is *maximally balanced* between wave and particle resources under the adopted operational measures
- Half of total quantum resource resides in the triality residual

Above threshold ($\lambda > 0.25$):

- System exhibits more particle character than wave character ($\mathcal{D} > \mathcal{V}$)
- Triality residual decreases as measurement becomes more projective
- Path information dominates

Characterizing the Measurement-Induced Transition

The transition from wave to particle is not a binary "collapse" but a continuous process:

1. As measurement strength increases from $\lambda = 0$, the triality residual grows as coupling to the detector increases

2. The triality residual peaks at $\lambda = 0.25$—the system–detector pair achieves maximum triality residual
3. Beyond $\lambda = 0.25$, the measurement becomes projective and the system "decides" its particle nature
4. The "collapse" is really the progression through maximum triality residual to classical correlation

WHY THE MAXIMIZER IS FIXED BY COMPLEMENTARITY AND NORMALIZATION

A potential objection: "Why should $\mathcal{D} = \sqrt{\lambda}$ apply beyond quantum mechanics?"

On the $\lambda := \mathcal{D}^2$ Reparameterization

In this paper, $\mathcal{D} = \sqrt{\lambda}$ holds by definition of $\lambda := \mathcal{D}^2$. Any broader square-root analogies below are illustrative only and not used in any derivation.

Heuristic analogies (not part of the proof): These analogies are suggestive but not invoked in our derivation.

Domain	**Relationship**
Classical waves	Amplitude = $\sqrt{\text{Power}}$
Quantum mechanics	$\lvert\psi\rvert = \sqrt{\text{Probability}}$
Weak measurement	$\mathcal{D} = \sqrt{\lambda}$
Electromagnetism	$E \propto \sqrt{I}$
General relativity	$d\tau/dt = \sqrt{g_{tt}}$

TABLE II. Heuristic square-root analogies (illustrative only).

Relation to Information–Disturbance Tradeoffs

Square-root structures appear in several optimal measurement and discrimination settings (e.g., in discussions of information–disturbance tradeoffs); see [8] for related analysis. We do not require any general square-root law here: in this work, the $\lambda = 1/4$ maximizer follows from the operational UQSD complementarity plus the normalized quadratic triad.

EXTENSION TO THERMODYNAMIC SYSTEMS

The Thermodynamic Stability Framework

Consider a macroscopic system characterized by the following operationally defined quantities:

- **Identity** $I \in [0, 1]$: A normalized health indicator derived from measured signals or performance variables (e.g., vibration features for bearings, remaining capacity for batteries, frequency deviation metrics for grids), scaled so $I = 1$ corresponds to a baseline/healthy reference and $I \to 0$ corresponds to severe degradation or failure.

- **Coherence** $\rho \in [0, 1]$: A normalized temporal consistency measure computed from the time series $I(t)$ over a sliding window (e.g., lag-1 autocorrelation or a windowed autocorrelation summary), with larger ρ indicating greater persistence/regularity of the underlying state trajectory.

- **Entropy** $S \in [0, 1]$: A normalized uncertainty/disorder measure computed from the same windowed signal representation (e.g., Shannon entropy of a discretized amplitude distribution or normalized spectral entropy of the power spectrum), bounded to $[0, 1]$ for comparability across systems.

The stability metric:

$$\Phi = I \times \rho - \alpha S \tag{23}$$

with empirically determined $\alpha = 0.1$ and empirical threshold $\Phi_c \approx 0.25$.

The Natural Mapping

Structural mapping. Motivated by cross-domain regularities and validated in the 27-system study reported here, the effective coupling variable in macroscopic stability problems can be modeled by the bounded coupling proxy $x := I \times \rho \in [0, 1]$, so that the same triality-style allocation function $2\sqrt{x}(1 - \sqrt{x})$ serves as a triality-residual diagnostic. The maximizer of this function occurs at $x = 0.25$, which is a geometric result about the mapped allocation.

Separately, we define the entropy-penalized stability score $\Phi := x - \alpha S$ as a practical classifier. The classification threshold $\Phi_c = 0.25$ used in the 27-system study is an *empirical* value; it does not inherit a geometrically forced status from the quantum derivation.

This mapping is established operationally through cross-domain empirical validation in the accompanying study; a microscopic first-principles derivation is not attempted in this paper.

Both quantities represent coupling strength to environment:

- λ: How strongly the quantum system couples to the detector

- $I \times \rho$: How strongly the thermodynamic system maintains identity against environmental perturbation

Both enter identically into the mathematical structure:

	Quantum	Thermodynamic
$\mathcal{D}^2$	λ	$I \times \rho$
$\mathcal{V}^2$	$(1-\sqrt{\lambda})^2$	$(1-\sqrt{I \times \rho})^2$
$\mathcal{C}^2$	$2\sqrt{\lambda}(1-\sqrt{\lambda})$	$2\sqrt{I \times \rho}(1-\sqrt{I \times \rho})$
Maximizer	$\lambda = 0.25$	$I \times \rho = 0.25$

TABLE III. The mapping between quantum and thermodynamic variables.

Why This Mapping Is Structurally Natural

The mapping is not "constructed to make things match." It is a structurally natural identification that preserves the role of a bounded coupling proxy in the shared algebraic allocation.

If both systems satisfy:

1. $\mathcal{D} = \sqrt{x}$ for some coupling variable x
2. $\mathcal{V} = 1 - \sqrt{x}$
3. $\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1$

Then automatically:

$$\mathcal{C}^2 = 2\sqrt{x}(1-\sqrt{x}) \quad \text{maximized at } x = 0.25 \tag{24}$$

If both systems satisfy this structure, the maximizer of the mapped allocation occurs at $x = 0.25$ as a mathematical consequence of the shared form. Empirical validation across 27 systems (all satisfying $\Phi < 0.25$ at failure) supports this mapping; a first-principles derivation of the thermodynamic case remains a separate theoretical task.

EMPIRICAL VALIDATION

Quantum Domain

A wave–particle–entanglement conservation framework was experimentally demonstrated by Ding *et al.* [5] on silicon-integrated nanophotonic quantum chips, including conservation relations of the form:

- $\mathcal{D}^2+\mathcal{V}^2+\mathcal{C}^2 = 1$ (for their operationally defined monotones)
- Verified in both qubit ($n = 2$) and qudit ($n = 3, 4$) systems
- Experimental confirmation of a wave–particle–entanglement triad

Thermodynamic Domain

We examined failure-time snapshots across 27 physical systems. In all cases, the stability metric satisfied $\Phi < 0.25$ at failure (Table IV), consistent with the interpretation that $\Phi_c = 0.25$ marks a failure-proximal boundary. Comprehensive evaluation of early-life windows to assess classification separability is documented in the accompanying repository.

Domain	**Systems**	**$\Phi < 0.25$ at failure**	**Threshold**
Mechanical	10	10/10	$\Phi_c = 0.25$
Electrical	2	2/2	$\Phi_c = 0.25$
Aerospace	10	10/10	$\Phi_c = 0.25$
Geophysical	3	3/3	$\Phi_c = 0.25$
AI systems	2	2/2	$\Phi_c = 0.25$
Total	**27**	**27/27**	—

TABLE IV. Failure-time snapshot statistics across domains. All systems satisfied $\Phi < 0.25$ at failure.

Bearing Data Analysis

We analyzed failure-time snapshots for 10 bearings from a 15-bearing XJTU run-to-failure subset [17]; all 10 satisfied $I \times \rho < 0.25$ at failure (Table V). Five bearings were excluded due to data or feature-representation issues under the fixed preprocessing: one (1-5) did not exhibit a usable degradation signal, and four (2-1, 3-2, 3-3, 3-5) were not reliably characterized by the pipeline. Details are provided in the repository. We do not claim the excluded bearings violate the boundary; they are excluded only because the fixed pipeline did not yield reliable characterization.

This supports 0.25 as a failure-proximal boundary for the analyzed snapshots. Under the fixed preprocessing, I is normalized relative to an early-life baseline reference window, so early-life windows yield $I \approx 1$ by construction; in that regime, $I \times \rho$ can be well above 0.25 (e.g., ≈ 0.99 for Bearing1_1; details in the repository). At failure, the mean $I \times \rho = 0.091$ (Table V), and all 10 analyzed failure-time snapshots satisfy $I \times \rho < 0.25$, indicating the boundary is crossed during degradation. This corresponds to $\mathcal{C}^2 \approx 0.42$, approaching but not reaching the maximum of 0.5 at $I \times \rho = 0.25$. Systems fail at sub-maximal triality residual (here $\mathcal{C}^2 \approx 0.42$), consistent with having passed through the maximum ($\mathcal{C}^2 = 0.5$ at $I \times \rho = 0.25$) earlier in degradation.

Bearing	I	ρ	$I \times \rho$
1-1	0.108	0.978	0.106
1-2	0.099	0.992	0.098
1-3	0.083	0.989	0.082
1-4	0.248	0.505	0.125
2-2	0.084	0.994	0.083
2-3	0.110	0.997	0.110
2-4	0.061	0.988	0.060
2-5	0.087	0.996	0.087
3-1	0.044	0.992	0.044
3-4	0.118	0.997	0.118
Mean	0.104	—	0.091

TABLE V. Bearing failure-time snapshot data for the 10 analyzed bearings. All have $I \times \rho < 0.25$.

ADDRESSING POTENTIAL OBJECTIONS

Objection: "The mapping is constructed, not derived"

Response: The mapping $\lambda \leftrightarrow I \times \rho$ is not arbitrary. Both quantities:

1. Represent coupling strength to environment
2. Appear under square-root in the same structural position
3. Produce identical mathematical consequences

If two systems have the same mathematical structure, identifying corresponding variables is *recognition*, not construction.

Objection: "0.25 could be coincidence"

Response: The 0.25 maximizer follows from:

- The quadratic triad $\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1$
- Linear complementarity $\mathcal{V} = 1 - \mathcal{D}$
- The reparameterization $\lambda := \mathcal{D}^2$ (so $\mathcal{D} = \sqrt{\lambda}$ by definition)

Any function of the form $\mathcal{C}^2 = 2\sqrt{x}(1 - \sqrt{x})$ *must* be maximized at $x = 0.25$. This is not physics—it is calculus.

Objection: "The thermodynamic validation is post-hoc"

Response: This is a valid methodological concern. The framework makes prospective predictions:

1. **Prediction:** Windows with $I \times \rho > 0.25$ correspond to operationally stable regimes; this is testable on early-life windows and held-out datasets.

2. **Prediction:** Systems approaching $I \times \rho = 0.25$ from below will exhibit increasing instability

3. **Prediction:** The correlation function $\mathcal{C}^2 = 2\sqrt{I \times \rho}(1 - \sqrt{I \times \rho})$ can be measured directly

These predictions are testable on held-out data.

Objection: "This doesn't 'solve' the double-slit"

Response: Quantum mechanics already predicts all measurement outcomes. What this work provides is:

1. **Characterization** of the wave-particle transition point

2. **Physical meaning** of the measurement-induced transition

3. **Quantification** of system-detector correlation

4. **Structural analogy** with thermodynamic stability

The "solution" is understanding that the transition occurs through maximum triality residual at $\lambda = 0.25$, not discovering new predictions.

Objection: "The parameter $\alpha = 0.1$ is fitted"

Response: The parameter α controls the relative weight of entropy in the stability metric. While $\alpha = 0.1$ is empirically determined, the maximizer $x = 0.25$ of the allocation function $2\sqrt{x}(1 - \sqrt{x})$ is geometric. By contrast, the classifier threshold $\Phi_c \approx 0.25$ is empirical. Changing α changes the classifier Φ, but it does not change the maximizer of the mapped allocation, which depends only on x.

EXPERIMENTAL PREDICTIONS

Quantum Predictions

Prediction 1: In variable-strength which-path detection, plotting $\mathcal{C}^2$ against λ should yield:

$$\mathcal{C}^2 = 2\sqrt{\lambda} - 2\lambda \tag{25}$$

Prediction 2: Maximum triality residual $\mathcal{C}^2 = 0.5$ occurs at $\lambda = 0.25$.

Prediction 3: At $\lambda = 0.25$, measured values satisfy $\mathcal{D} = \mathcal{V} = 0.5$ simultaneously.

Thermodynamic Predictions

Prediction 4: Windows with $I \times \rho > 0.25$ correspond to operationally stable regimes; this is testable on early-life windows and held-out datasets.

Prediction 5: The correlation $\mathcal{C}^2 = 2\sqrt{I \times \rho}(1 - \sqrt{I \times \rho})$ may be approachable via fluctuation–dissipation-style analysis.

DISCUSSION

The Structural Significance of 0.25

The threshold 0.25 appears in multiple contexts:

- Wave-particle-correlation triality (this work)
- Thermodynamic failure-time snapshots (all satisfying $\Phi < 0.25$ at failure)

The value 0.25 is a **geometric constant** characteristic of systems exhibiting UQSD-type measurement dynamics. We note that the same numerical factor 1/4 also appears in black-hole thermodynamics via $S = A/4$ (in Planck units), suggesting a shared geometric origin in surface-accessed information. This constant emerges whenever:

1. Information flows between system and environment
2. The coupling proxy x enters the allocation function through $\mathcal{D} = \sqrt{x}$ (where $\mathcal{D}$ is operational distinguishability in the quantum case; in the thermodynamic analogy $x = I \times \rho$ plays the analogous coupling-proxy role)
3. There is linear complementarity between observed quantities

Holographic Interpretation

This subsection provides an interpretive motivation for the square-root scaling; it is not required for the main maximizer theorem.

The algebraic structure may have deeper origins in dimensional reduction:

- 3D bulk information encoded on 2D boundary
- Boundary area $\propto L^2$, bulk volume $\propto L^3$
- Information accessible $\propto \sqrt{\text{Information stored}}$

This suggests a connection to holographic principles, where the square-root represents the dimensional mismatch between bulk and boundary descriptions.

Implications for Measurement Theory

The result that maximum triality residual occurs at *intermediate* measurement strength—not at the extremes—has implications:

- At $\lambda = 0$: No measurement, no correlation, pure wave
- At $\lambda = 1$: Projective measurement, classical correlation, pure particle
- At $\lambda = 0.25$: **Maximum triality residual**

This explains why partial measurements are useful in quantum information: they access the regime of maximum triality residual between system and apparatus.

CONCLUSION

We prove (given the operational UQSD complementarity and the normalized triad) that:

1. The wave–particle–correlation triality $\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1$ has a unique maximizer at parameter value $\lambda = 0.25$.
2. At this threshold, the triality residual $\mathcal{C}^2$ reaches its maximum value of 0.5, with $\mathcal{D} = \mathcal{V} = 0.5$.
3. The threshold 0.25 is **geometric necessity**, not empirical fitting—it emerges from the quadratic triad together with linear complementarity and the reparameterization $\lambda := \mathcal{D}^2$.
4. The same mathematical structure applies to thermodynamic stability, with the natural mapping $\lambda \leftrightarrow I \times \rho$.
5. Empirical validation across 27 systems shows all failure-time snapshots satisfy $\Phi < 0.25$, consistent with this interpretation.

The wave-particle transition is not a binary collapse but a continuous process through maximum triality residual. The maximizer $\lambda = 0.25$ marks the point where the system–detector pair achieves maximum triality residual under the adopted operational measures.

This result establishes a geometric principle for the quantum case and suggests a structural analogy with thermodynamic stability: the allocation function $2\sqrt{x}(1-\sqrt{x})$ is maximized at $x = 0.25$ whenever the system satisfies the quadratic triad together with linear complementarity.

Detailed Derivations

Gaussian Detector State Evolution

For a Gaussian detector state with position-space wave function:

$$\phi(x) = \left(\frac{1}{2\pi\sigma^2}\right)^{1/4} \exp\left(-\frac{x^2}{4\sigma^2}\right) \tag{26}$$

The momentum-space representation is:

$$\tilde{\phi}(p) = \left(\frac{2\sigma^2}{\pi}\right)^{1/4} \exp\bigl(-\sigma^2 p^2\bigr) \tag{27}$$

The overlap after path-dependent position displacements:

$$\begin{aligned}\langle\phi_0|\phi_1\rangle &= \int dx\, \phi_0^*(x)\, \phi_1(x) \\ &= \exp\left(-\frac{g^2}{2\sigma^2}\right)\end{aligned} \tag{28}$$

where the integral follows by completing the square for the Gaussian overlap.

Verification of Triality Conservation

For any $\lambda \in [0, 1]$:

$$\begin{aligned}&\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 \\ &= \lambda + (1-\sqrt{\lambda})^2 + 2\sqrt{\lambda}(1-\sqrt{\lambda}) \\ &= \lambda + 1 - 2\sqrt{\lambda} + \lambda + 2\sqrt{\lambda} - 2\lambda \\ &= 1 \quad \checkmark\end{aligned} \tag{29}$$

Boundary Behavior

At $\lambda = 0$:

$$\mathcal{D} = 0 \quad \text{(no distinguishability)} \tag{30}$$
$$\mathcal{V} = 1 \quad \text{(full visibility)} \tag{31}$$
$$\mathcal{C}^2 = 0 \quad \text{(zero triality residual)} \tag{32}$$

Result: Pure wave state, no measurement.

At $\lambda = 1$:

$$\mathcal{D} = 1 \quad \text{(full distinguishability)} \tag{33}$$
$$\mathcal{V} = 0 \quad \text{(no visibility)} \tag{34}$$
$$\mathcal{C}^2 = 0 \quad \text{(zero triality residual)} \tag{35}$$

Result: Maximally which-path / effectively projective regime; triality residual vanishes ($\mathcal{C}^2 = 0$) and remaining correlations are effectively classical.

At $\lambda = 0.25$:

$$\mathcal{D} = 0.5 \quad \text{(half distinguishability)} \tag{36}$$
$$\mathcal{V} = 0.5 \quad \text{(half visibility)} \tag{37}$$
$$\mathcal{C}^2 = 0.5 \quad \text{(maximum triality residual)} \tag{38}$$

Result: Maximum triality residual for the system–detector pair.

DATA AND CODE AVAILABILITY

The data sources, preprocessing scripts, and reproduction code for the 27-system cross-domain study are available in the accompanying repository. Access instructions and links are provided in the Zenodo record.

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