Locally Extractable Energy in Quantum Measurement: A Derivation from the Wave-Particle-Correlation Triality

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We derive how the energy extractable from a quantum system through local operations depends on measurement-induced correlations with the apparatus. Starting from two-path wave–particle–entanglement (concurrence) triality relations of the form $\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1$ [6–8], with related experimental demonstrations of wave/particle/entanglement conservation principles [2], and the theory of quantum ergotropy, we prove that for a two-level system measured under the unambiguous quantum state discrimination (UQSD) protocol, the locally extractable energy satisfies $E_{\text{loc}} = E_0\sqrt{1-\mathcal{C}^2}$, where E_0 is the qubit excitation energy (energy gap, with ground set to zero) and $\mathcal{C}$ quantifies system–apparatus correlation. The total energy of the system–apparatus composite is conserved—what varies with measurement strength is *access*, not existence. At maximum correlation ($\lambda = 0.25$, where λ parameterizes measurement strength), only 70.7% of the system's energy is locally extractable; the remaining 29.3% requires joint operations on system and apparatus. In the correlation-free limits ($\lambda = 0$ or $\lambda = 1$), the locally extractable energy equals the isolated-system ergotropy for the chosen initial preparation (e.g., $E_{\text{loc}} = E_0$ for initial $|1\rangle$). We present experimental protocols to test these predictions using existing weak measurement technology. The mathematical structure parallels that appearing in black hole information dynamics under square-root dampening, suggesting a deeper geometric origin, though this connection remains conjectural.

I. INTRODUCTION

Einstein's mass-energy equivalence $E = mc^2$ is among the most precisely verified equations in physics [1]. Yet this relation is silent on a fundamental aspect of quantum

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reality: the role of measurement. When a quantum system is measured, it becomes correlated with the measuring apparatus. Does this correlation affect the energy available to an observer?

We demonstrate that it does. Using wave–particle–entanglement triality relations [6–8], with related experimental demonstrations of triad conservation [2], and the theory of quantum ergotropy [3], we derive that the *locally extractable energy*—the maximum work obtainable through operations on the system alone—depends on the measurement-induced correlation:

$$E_{\text{loc}} = E_0\sqrt{1-\mathcal{C}^2}, \tag{1}$$

where E_0 is the system energy scale (qubit excitation energy / energy gap, taking the ground energy as the zero reference) and $\mathcal{C}$ is the correlation resource from the triality.

Critical clarification: The total energy of the system–apparatus composite is conserved. What Eq. (1) describes is how much work is *accessible through local operations* on the system, given measurement-induced system–apparatus correlations. Throughout, E_0 denotes the qubit excitation energy (energy gap, taking the ground energy as zero), which sets the relevant work/energy scale. The "missing" work is stored in system–apparatus correlations and can be recovered through joint operations.

This result applies specifically to:

1. Two-level quantum systems (qubits)
2. Measurements satisfying the unambiguous quantum state discrimination (UQSD) protocol
3. Symmetric configurations with equal priors

Outside this regime, the specific numerical predictions may differ, though the qualitative dependence of accessible energy on correlation should persist.

A. Structure of This Paper

Section II reviews the triality and related experimental support. Section III introduces quantum ergotropy. Section IV derives the main result. Section V presents numerical predictions. Section VI proposes experimental tests. Section VII discusses the parallel with black hole physics. Section VIII addresses implications and limitations.

II. THE WAVE-PARTICLE-CORRELATION TRIALITY

A. Theoretical Foundation

The Englert-Greenberger-Yasin duality relation [4, 5] established that path distinguishability $\mathcal{D}$ and interference visibility $\mathcal{V}$ satisfy:

$$\mathcal{D}^2 + \mathcal{V}^2 \leq 1. \tag{2}$$

Recent theoretical work [6–8] extended this to include system-detector correlations, yielding a three-term conservation law:

$$\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1, \tag{3}$$

where $\mathcal{C}$ quantifies the system–apparatus entanglement resource (identified with concurrence/I-concurrence in the two-path setting), rather than coherence.

B. Experimental Verification

Ding *et al.* [2] experimentally demonstrated a universal wave–particle–entanglement conservation law on silicon-integrated nanophotonic quantum chips, providing strong experimental support for the broader family of complementarity/triality conservation frameworks in which wave behavior, which-path information, and system–detector entanglement are quantitatively linked. In this work, we use the squared triality form written in Eq. (3) as it arises in two-path analyses where the third resource is identified with an entanglement monotone (e.g., concurrence or I-concurrence) [6–8].

C. The UQSD Regime and Linear Complementarity

Under unambiguous quantum state discrimination (UQSD) [9, 10], where the observer accepts inconclusive outcomes to guarantee zero error on conclusive results, visibility and distinguishability satisfy *linear* complementarity:

$$\mathcal{V} = 1 - \mathcal{D}. \tag{4}$$

This is stronger than the general inequality (2). Substituting into the triality:

$$\begin{aligned} \mathcal{C}^2 &= 1 - \mathcal{D}^2 - (1-\mathcal{D})^2 \\ &= 2\mathcal{D}(1-\mathcal{D}). \end{aligned} \tag{5}$$

Defining the measurement strength parameter $\lambda := \mathcal{D}^2$, so that $\mathcal{D} = \sqrt{\lambda}$:

$$\mathcal{C}^2 = 2\sqrt{\lambda}(1-\sqrt{\lambda}). \tag{6}$$

Theorem II.1 (Correlation Maximizer [11])**.** *The correlation resource $\mathcal{C}^2$ in Eq.* (6) *is uniquely maximized at $\lambda = 0.25$, where $\mathcal{C}^2_{\max} = 0.5$.*

Proof. Let $u = \sqrt{\lambda}$, so $\mathcal{C}^2 = 2u(1-u)$. Then $d(\mathcal{C}^2)/du = 2 - 4u = 0$ gives $u = 1/2$, hence $\lambda = 1/4$. The second derivative $-4 < 0$ confirms this is a maximum. □

At $\lambda = 0.25$: $\mathcal{D} = \mathcal{V} = 0.5$ and $\mathcal{C}^2 = 0.5$. This is the point of maximum system-apparatus correlation.

III. QUANTUM ERGOTROPY

A. Definition and Physical Meaning

The *ergotropy* of a quantum state ρ with Hamiltonian H is the maximum work extractable through unitary operations [3]:

$$\mathcal{W}(\rho) = \mathrm{Tr}(\rho H) - \min_U \mathrm{Tr}(U\rho U^\dagger H). \quad (7)$$

For a pure state $|\psi\rangle$ in the excited energy eigenstate with eigenvalue E_0:

$$\mathcal{W}(|\psi\rangle\langle\psi|) = E_0 - E_{\text{ground}}. \quad (8)$$

The full energy is extractable because a pure eigenstate can be unitarily rotated to the ground state. Throughout this work, we set $E_{\text{ground}} = 0$ without loss of generality, so that the excited state energy E_0 equals the energy gap.

B. Ergotropy of Mixed States

When a system becomes entangled with an apparatus, the system's reduced density matrix is mixed:

$$\rho_S = \mathrm{Tr}_A(|\Psi\rangle_{SA}\langle\Psi|). \quad (9)$$

For a mixed state, the ergotropy is strictly less than the energy:

$$\mathcal{W}(\rho_S) < \mathrm{Tr}(\rho_S H_S). \quad (10)$$

The "missing" extractable work is not lost—it resides in the correlations between system and apparatus and can be recovered through joint operations.

C. Ergotropy and Purity

For a qubit system with energy gap E_0, the ergotropy of a state with purity $\gamma = \mathrm{Tr}(\rho^2)$ is [12]:

$$\mathcal{W} = E_0 \cdot f(\gamma), \quad (11)$$

where $f(\gamma)$ is a monotonically increasing function with $f(1) = 1$ (pure state: full extraction) and $f(1/2) = 0$ (maximally mixed: no extraction).

For a qubit in a mixed state diagonal in the energy eigenbasis (as arises from measurement-induced decoherence in the present setup), and restricted to the *active* regime (excited-state population $\geq 1/2$, which holds for the initially excited preparation in the considered UQSD-induced mixing regime), the ergotropy depends only on the purity:

$$f(\gamma) = \sqrt{2\gamma - 1} \quad \text{for } \gamma \geq 1/2. \quad (12)$$

Under these conditions, the optimal unitary work-extraction protocol yields $\mathcal{W}/E_0 = \sqrt{2\gamma - 1}$ [12].

IV. DERIVATION OF THE MAIN RESULT

A. Setup: Energy Measurement as State Discrimination

Consider a qubit system with Hamiltonian H_S (excitation energy gap E_0). An apparatus couples to the system to implement tunable-strength state discrimination.

We model the measurement as a tunable-strength system–apparatus interaction that produces a pure joint state $|\Psi\rangle_{SA}$ whose reduced system state ρ_S is mixed due to measurement-induced correlations. The measurement strength is parameterized operationally by the UQSD distinguishability λ (equivalently by the observed $(\mathcal{D}, \mathcal{V}, \mathcal{C})$ triple), without committing to a specific microscopic interaction Hamiltonian beyond the assumption that a joint unitary realizing the required UQSD statistics exists.

Initially: $|\Psi_0\rangle = |1\rangle_S \otimes |\phi\rangle_A$, where $|\phi\rangle_A$ is a fiducial apparatus state.

After the interaction: $|\Psi\rangle_{SA}$ may be entangled; in particular one may write (up to local basis choice) a Schmidt-form state (choosing the system basis as the energy eigenbasis) $|\Psi\rangle_{SA} = \sqrt{p}\,|0\rangle_S|a_0\rangle_A + \sqrt{1-p}\,|1\rangle_S|a_1\rangle_A$, with $p \in [0,1]$ and correlation quantified (in the triality sense) by $\mathcal{C}$. In the present setting the measurement is an energy (eigenstate) discrimination, so the apparatus correlates with the system energy eigenbasis; consequently the reduced state ρ_S is diagonal in the energy basis.

We specifically consider energy measurement implemented via the UQSD protocol: the observer couples to the system with tunable strength and accepts inconclusive outcomes in exchange for zero error on conclusive results. This is not the only way to measure energy, but it is the protocol under which linear complementarity (Eq. 4) holds. Other measurement strategies (e.g., minimum-error discrimination) yield different complementarity relations and correspondingly different energy-correlation relationships. The UQSD protocol is experimentally realizable through weak measurement with post-selection [14].

B. Connecting Correlation to Purity

The correlation resource $\mathcal{C}$ in the triality is related to the entanglement between system and apparatus. For a pure bipartite state, the correlation $\mathcal{C}^2$ is connected to the purity of the reduced state.

Proposition IV.1. *For a two-level system measured under UQSD conditions, the purity of the reduced density matrix satisfies:*

$$\gamma_S = Tr(\rho_S^2) = 1 - \frac{\mathcal{C}^2}{2}. \tag{13}$$

Proof. For a pure bipartite state $|\Psi\rangle_{SA}$, the Schmidt decomposition gives:

$$|\Psi\rangle_{SA} = \sqrt{p}|s_0\rangle_S|a_0\rangle_A + \sqrt{1-p}|s_1\rangle_S|a_1\rangle_A. \tag{14}$$

The reduced density matrix:

$$\rho_S = p|s_0\rangle\langle s_0| + (1-p)|s_1\rangle\langle s_1|. \tag{15}$$

Its purity:

$$\gamma_S = p^2 + (1-p)^2 = 1 - 2p(1-p). \tag{16}$$

Under UQSD with symmetric priors (equal prior probabilities for the two measurement outcomes), the relationship between the Schmidt coefficients and the triality resources yields [6]:

$$\mathcal{C}^2 = 4p(1-p) = 2[1-\gamma_S]. \tag{17}$$

The correlation resource $\mathcal{C}$ in the triality is identified with the concurrence of the bipartite system-apparatus state [6, 7]. For the Schmidt decomposition above, concurrence equals $2\sqrt{p(1-p)}$, hence $\mathcal{C}^2 = 4p(1-p)$.

Solving for γ_S: $\gamma_S = 1 - \mathcal{C}^2/2$. □

C. Main Theorem

Theorem IV.2 (Locally Extractable Energy). *For a two-level system with excitation energy (gap) E_0, measured under UQSD conditions with correlation resource $\mathcal{C}$, the locally extractable energy[17] is:*

$$E_{loc} = E_0\sqrt{1-\mathcal{C}^2}. \tag{18}$$

Proof. From Proposition IV.1: $\gamma_S = 1-\mathcal{C}^2/2$.

From Eq. (12), the ergotropy is:

$$\begin{aligned}\mathcal{W} &= E_0\sqrt{2\gamma_S - 1} \\ &= E_0\sqrt{2(1-\mathcal{C}^2/2)-1} \\ &= E_0\sqrt{1-\mathcal{C}^2}.\end{aligned} \tag{19}$$

□

D. The Correlation Energy

Definition IV.3. *The correlation energy is the energy that is not locally extractable:*

$$E_{corr} := E_0 - E_{loc} = E_0(1-\sqrt{1-\mathcal{C}^2}). \tag{20}$$

Physical interpretation: E_{corr} is not "lost"—it exists in the system-apparatus correlations. It can be recovered through:

- Joint unitary operations on system and apparatus
- Measurement of the apparatus followed by conditional operations
- Thermodynamic protocols exploiting mutual information [13]

E. Verification of the Quadratic Structure

Corollary IV.4. *The locally extractable energy and the correlation resource satisfy:*

$$E_{loc}^2 + E_0^2\mathcal{C}^2 = E_0^2. \tag{21}$$

Proof. From Eq. (18):

$$E_{\text{loc}}^2 = E_0^2(1-\mathcal{C}^2) \tag{22}$$

$$E_{\text{loc}}^2 + E_0^2\mathcal{C}^2 = E_0^2(1-\mathcal{C}^2) + E_0^2\mathcal{C}^2 = E_0^2. \tag{23}$$

□

The quadratic structure emerges from the triality (which constrains squared quantities) combined with the square-root dependence of ergotropy on purity. This is derived, not assumed.

F. Correlation-Free Limit and an Interpretive Analogy

Corollary IV.5. *When $\mathcal{C}^2 = 0$, there is no correlation-induced loss of local accessibility, and the locally extractable energy equals the isolated-system ergotropy for the chosen initial preparation. In particular, if the system is initially in the excited energy eigenstate $|1\rangle$, then $E_{loc} = E_0$.*

This occurs at both measurement extremes:

- $\lambda = 0$ (no measurement): $\mathcal{D} = 0$, $\mathcal{V} = 1$, so $\mathcal{C}^2 = 0$ by Eq. (5)
- $\lambda = 1$ (projective measurement): $\mathcal{D} = 1$, $\mathcal{V} = 0$, so $\mathcal{C}^2 = 0$ by Eq. (5)

If one interprets E_0 as a rest-energy scale, the correlation-free limit is consistent with Einstein's mass–energy equivalence; in this work, however, E_0 is the qubit excitation energy.

V. NUMERICAL PREDICTIONS

A. Energy Accessibility vs. Measurement Strength

Using Eqs. (6) and (18):

λ	$\mathcal{D}$	$\mathcal{C}^2$	E_{loc}/E_0	E_{corr}/E_0
0.00	0.000	0.000	100.0%	0.0%
0.05	0.224	0.346	80.9%	19.1%
0.10	0.316	0.432	75.4%	24.6%
0.15	0.387	0.475	72.4%	27.6%
0.20	0.447	0.494	71.1%	28.9%
0.25	**0.500**	**0.500**	**70.7%**	**29.3%**
0.30	0.548	0.496	71.0%	29.0%
0.50	0.707	0.414	76.5%	23.5%
0.75	0.866	0.232	87.7%	12.3%
1.00	1.000	0.000	100.0%	0.0%

TABLE I. Energy accessibility as a function of measurement strength λ, for initial preparation in the excited eigenstate $|1\rangle$.

B. The Critical Point: $\lambda = 0.25$

At maximum correlation:

$$E_{\text{loc}} = E_0 \cdot \sqrt{0.5} = 0.707\, E_0, \tag{24}$$

$$E_{\text{corr}} = E_0 \cdot (1 - \sqrt{0.5}) = 0.293\, E_0. \tag{25}$$

At this point, 29.3% of the system's energy is inaccessible through local operations—it exists in the quantum correlations with the apparatus.

VI. PROPOSED EXPERIMENTAL TESTS

A. Test 1: Ergotropy vs. Measurement Strength

Objective: Measure the work extractable from a qubit as a function of measurement coupling strength.

Protocol:

1. Prepare a qubit with excitation energy (gap) E_0 in the excited energy eigenstate $|1\rangle$, so that the baseline local ergotropy scale equals E_0.
2. Couple to a detector with tunable measurement strength, characterized operationally by λ.
3. Extract work from the qubit via optimal unitary.
4. Measure extracted work $W_{\rm ext}$.
5. Compare to prediction: $W_{\rm ext} = E_0\sqrt{1-\mathcal{C}^2(\lambda)}$.

Prediction:

- At $\lambda = 0$ and $\lambda = 1$: $W_{\rm ext} \approx E_0$
- At $\lambda = 0.25$: $W_{\rm ext} \approx 0.707\,E_0$

Required precision: $\pm 5\%$ should clearly distinguish the predicted non-constant dependence from a constant-extraction null model.

B. Test 2: Correlation Energy Recovery

Objective: Demonstrate that "missing" energy can be recovered through joint operations.

Protocol:

1. Prepare system-apparatus in correlated state at $\lambda = 0.25$.
2. Measure local ergotropy: should yield $\sim 70.7\%$ of E_0.
3. Perform joint unitary on system + apparatus.
4. Extract additional work: should yield remaining $\sim 29.3\%$.
5. Verify total: $W_{\rm local} + W_{\rm joint} = E_0$.

Significance: Confirms energy conservation while demonstrating the correlation-energy concept.

C. Test 3: Resource–Work Link Across Platforms

Objective: Verify the predicted link between the correlation resource $\mathcal{C}$ and locally extractable work.

Protocol:

1. Use a photonic or interferometric platform (e.g., Ding *et al.* [2]) to measure $\mathcal{D}$, $\mathcal{V}$, and a concurrence (or I-

concurrence) proxy consistent with the triality framework at multiple λ values.

2. Use a work-extraction platform (e.g., superconducting qubits or NMR) to engineer system–apparatus correlations whose measured concurrence proxy yields the same $\mathcal{C}$ values as in step (1), and measure ergotropy/work extraction at those settings.

3. Verify the functional dependence: $E_{\rm loc}/E_0 = \sqrt{1-\mathcal{C}^2}$.

D. Feasibility Assessment

Tests 1 and 2 are achievable with current technology:

- Weak measurement with tunable coupling: demonstrated [14, 15]
- Ergotropy measurement: demonstrated in NMR and superconducting qubits [12]
- Triality measurement: demonstrated [2]

Estimated timeline: 1–2 years for Tests 1 and 2.

VII. PARALLEL WITH BLACK HOLE INFORMATION DYNAMICS

A. The Mathematical Parallel

The fraction 29.3% $= 1 - \sqrt{0.5}$ appears independently in black hole physics. Under square-root dampening of information release [16], the information radiated by a black hole is:

$$I_{\rm rad} = I_{\rm total}(1 - \sqrt{M/M_0}), \tag{26}$$

where M is current mass and M_0 is initial mass.

At the Page time ($M = 0.5\, M_0$):

$$I_{\rm rad} = I_{\rm total}(1 - \sqrt{0.5}) = 0.293\, I_{\rm total}. \tag{27}$$

B. Status of the Connection

We emphasize: **this parallel is currently structural, not derived.**

Both systems exhibit:

- A $\sqrt{}$ dependence on a normalized parameter
- A critical point at parameter value 0.25 or 0.5
- The fraction 29.3% appearing at the critical point

A rigorous connection would require showing that:

1. Black hole horizons satisfy a triality-like conservation law
2. Hawking radiation dynamics imply linear complementarity
3. Both emerge from holographic dimensional reduction (3D $\rightarrow$ 2D)

This remains an open problem. The mathematical similarity is striking but not yet explained.

C. Holographic Conjecture

We conjecture (without proof) that the common origin is holographic: when information encoded in a 3D bulk is accessed through a 2D boundary, the accessible fraction follows $\sqrt{}$ scaling due to the dimensional mismatch:

$$\text{Accessible} \propto \sqrt{\text{Stored}}. \tag{28}$$

This would explain:

- The triality: boundary measurement of bulk quantum state
- Black holes: horizon encoding of interior information
- The common 0.25/29.3% values

Rigorous derivation is left for future work.

VIII. DISCUSSION

A. What This Result Does and Does Not Claim

Does claim:

- Locally extractable energy depends on measurement-induced correlation
- The dependence is $E_{\text{loc}} = E_0\sqrt{1-\mathcal{C}^2}$ for UQSD measurements
- Maximum correlation occurs at $\lambda = 0.25$, where $E_{\text{loc}} = 0.707\,E_0$
- This is derivable from established physics (triality + ergotropy)

Does not claim:

- Total energy is "partitioned" or reduced
- This applies to all measurement types (only UQSD)
- This applies to continuous-spectrum systems (only two-level)
- The black hole connection is proven (it is conjectural)

B. Scope and Limitations

The result applies specifically to:

1. **Two-level systems:** Extension to continuous spectra requires generalization of the triality

2. **UQSD protocol:** Other discrimination strategies yield different complementarity relations

3. **Symmetric priors:** Asymmetric cases modify the $\lambda = 0.25$ value

For macroscopic measurements (classical calorimetry), measurement back-action is effectively negligible and full energy is accessible.

C. Relationship to Mass–Energy Equivalence

In the correlation-free limits, where $\mathcal{C} = 0$, the locally extractable energy equals the full system energy scale E_0. This is consistent with the expectation that macroscopic calorimetry recovers the full energy of an effectively isolated system.

In this work, E_0 is the qubit excitation energy, not rest mass; any parallel with $E = mc^2$ is interpretive rather than a modification of relativistic mechanics.

What we have shown is that *intermediate-strength* quantum measurements create a regime where not all of this energy is locally accessible. The equation is not wrong; it describes the correlation-free limit.

D. Broader Significance of the 0.25 Threshold

The critical point $\lambda = 0.25$ derived here is not presented as an isolated numerical curiosity. In separate work by the author [11], the same threshold value appears in a thermodynamic stability framework and is reported to match 28/28 analyzed systems spanning mechanical, electrical, aerospace, geophysical, and computational domains. While the thermodynamic metric $\Phi = I \times \rho - \alpha S$ differs in form from the quantum triality, both frameworks yield the same critical boundary. This cross-domain consistency motivates the hypothesis that $\lambda = 0.25$ may reflect a broader geometric constraint on information–correlation trade-offs, though a rigorous derivation connecting the quantum and thermodynamic cases remains an open problem.

E. Physical Interpretation

When you measure a quantum system, you become correlated with it. Some of the system's extractable energy goes into establishing this correlation. The energy isn't gone—it's in the relationship between you and the system.

This is analogous to entanglement entropy: the information isn't lost when a system becomes entangled with its environment; it becomes inaccessible to local observers but remains in the correlations.

IX. CONCLUSION

We have derived that for two-level quantum systems measured under UQSD conditions, the locally extractable energy is:

$$E_{\text{loc}} = E_0\sqrt{1-\mathcal{C}^2}, \tag{29}$$

where $\mathcal{C}$ is the correlation resource from the triality.

The derivation rests on:

1. The triality $\mathcal{D}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1$ (theoretically established [6–8], with experimental support from related conservation frameworks [2])

2. Linear complementarity under UQSD (proven for this regime)

3. Quantum ergotropy theory (established)

4. The relationship between correlation and purity (derived)

At maximum correlation ($\lambda = 0.25$), only 70.7% of the system's energy is locally extractable. In the correlation-free limits, full energy is locally accessible.

Three experimental tests have been proposed, achievable with existing technology. The parallel with black hole information dynamics suggests a deeper geometric origin, though this connection remains to be established.

The energy is all there. It's just that some of it is in the relationship.

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[17] Maximum work obtainable by acting unitarily on the system alone with no access to the apparatus (i.e., the ergotropy of the reduced state $\rho_S = \mathrm{Tr}_A|\Psi\rangle\langle\Psi|$). Any additional work recovery requires access to the apparatus and/or joint system–apparatus operations.