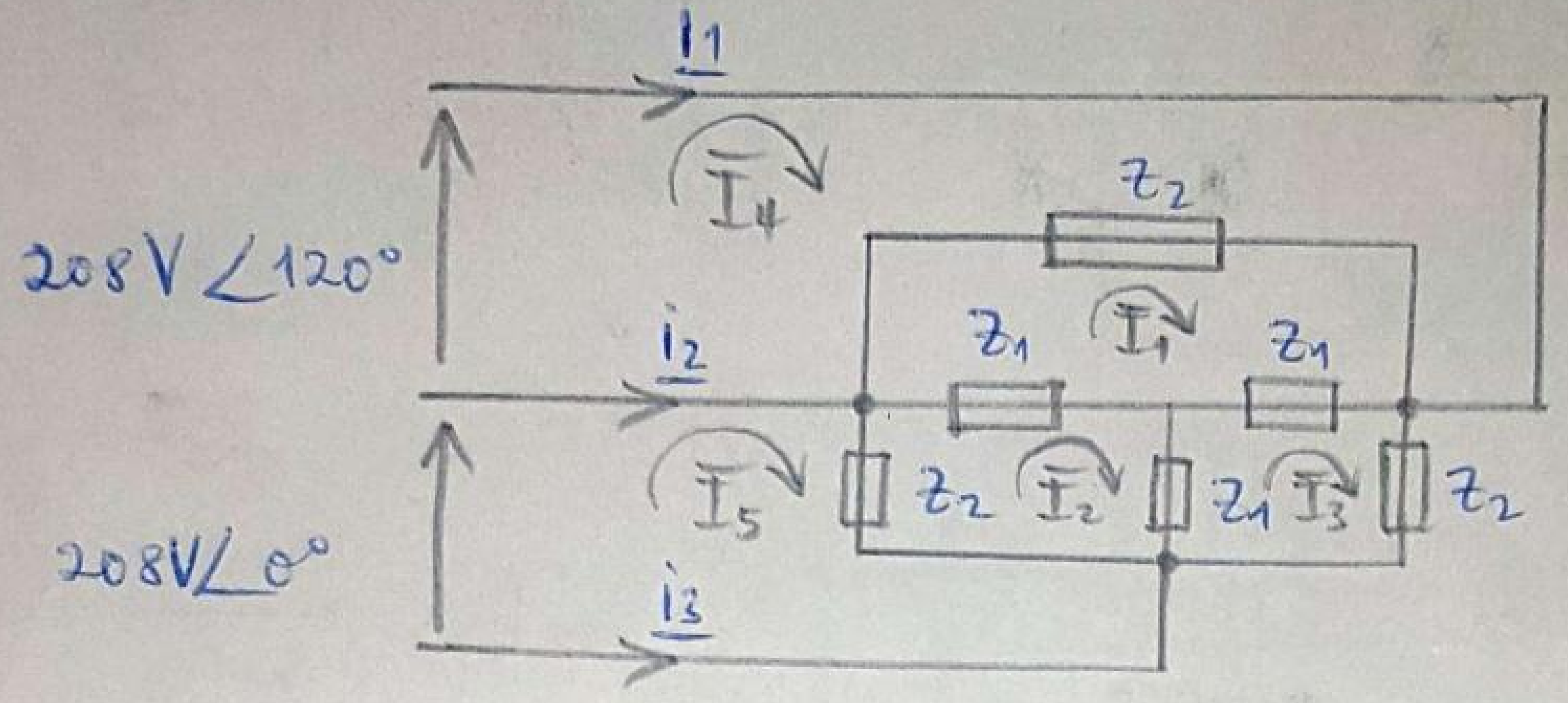
SN Electrocinétique 2019/2020

Exercice 1

1) Equations de maille du circuit en utilisant z_1 et z_2 :



Maille 1 : $(z_2\bar{I}_1 - z_2\bar{I}_4) + (z_1\bar{I}_1 - z_1\bar{I}_3) + (z_1\bar{I}_1 - z_1\bar{I}_2) = 0$

$$\Rightarrow (2z_1 + z_2)\bar{I}_1 - z_1\bar{I}_2 - z_1\bar{I}_3 - z_2\bar{I}_4 = 0$$

Maille 2 : $(z_2\bar{I}_2 - z_2\bar{I}_5) + (z_1\bar{I}_2 - z_1\bar{I}_1) + (z_1\bar{I}_2 - z_1\bar{I}_3) = 0$

$$\Rightarrow -z_1\bar{I}_1 + (z_2 + 2z_1)\bar{I}_2 - z_1\bar{I}_3 - z_2\bar{I}_5 = 0$$

Maille 3 : $(z_1\bar{I}_3 - z_1\bar{I}_2) + (z_1\bar{I}_3 - z_1\bar{I}_1) + z_2\bar{I}_3 = 0$

$$\Rightarrow -z_1\bar{I}_1 - z_2\bar{I}_2 + (2z_1 + z_2)\bar{I}_3 = 0$$

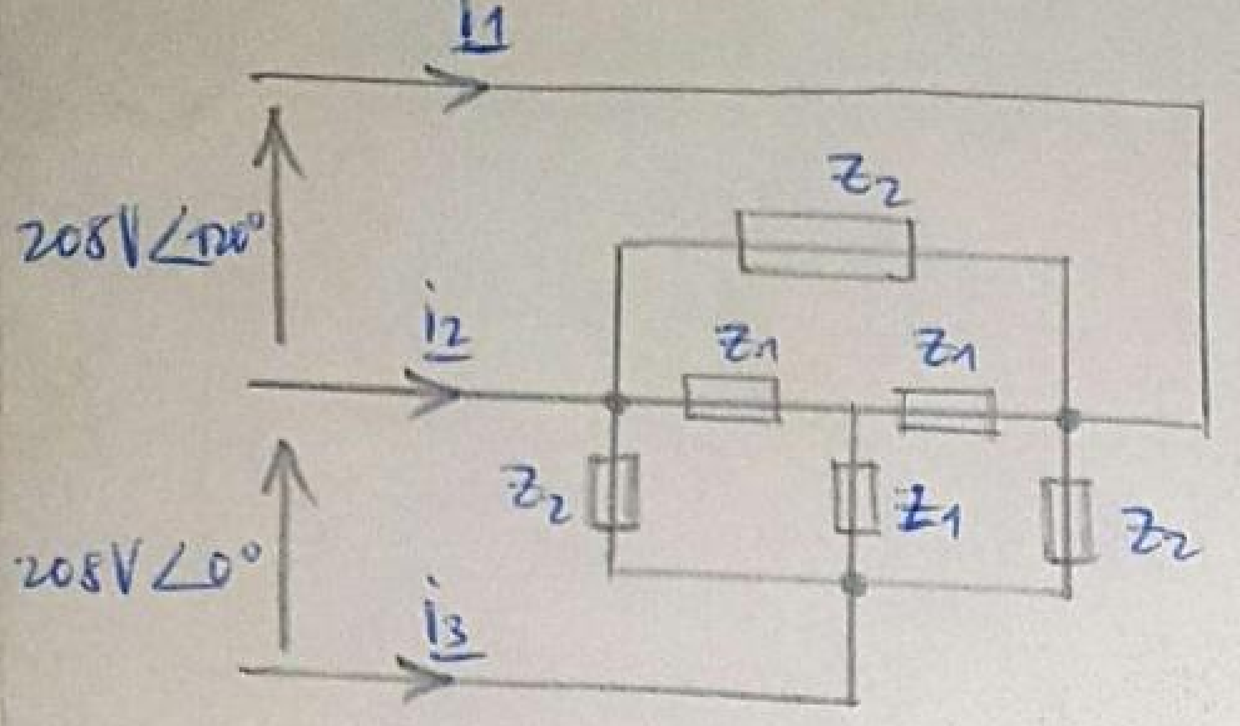
Maille 4 : $-z_2\bar{I}_1 + z_2\bar{I}_4 = 208V\angle 120°$

Maille 5 : $-z_2\bar{I}_2 + z_2\bar{I}_5 = 208V\angle 0°$

$$\Rightarrow \begin{bmatrix} 2z_1+z_2 & -z_1 & -z_1 & -z_2 & 0 \\ -z_1 & 2z_1+z_2 & -z_1 & 0 & -z_2 \\ -z_1 & -z_2 & 2z_1+z_2 & 0 & 0 \\ -z_2 & 0 & 0 & z_2 & 0 \\ 0 & -z_2 & 0 & 0 & z_2 \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \\ \bar{I}_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 208\angle 120° \\ 208\angle 0° \end{bmatrix}$$

2) $Z_1 = 15\,\Omega \angle 30°$, $Z_2 = 30\,\Omega \angle 45°$

a) Transformation en circuit triangle :



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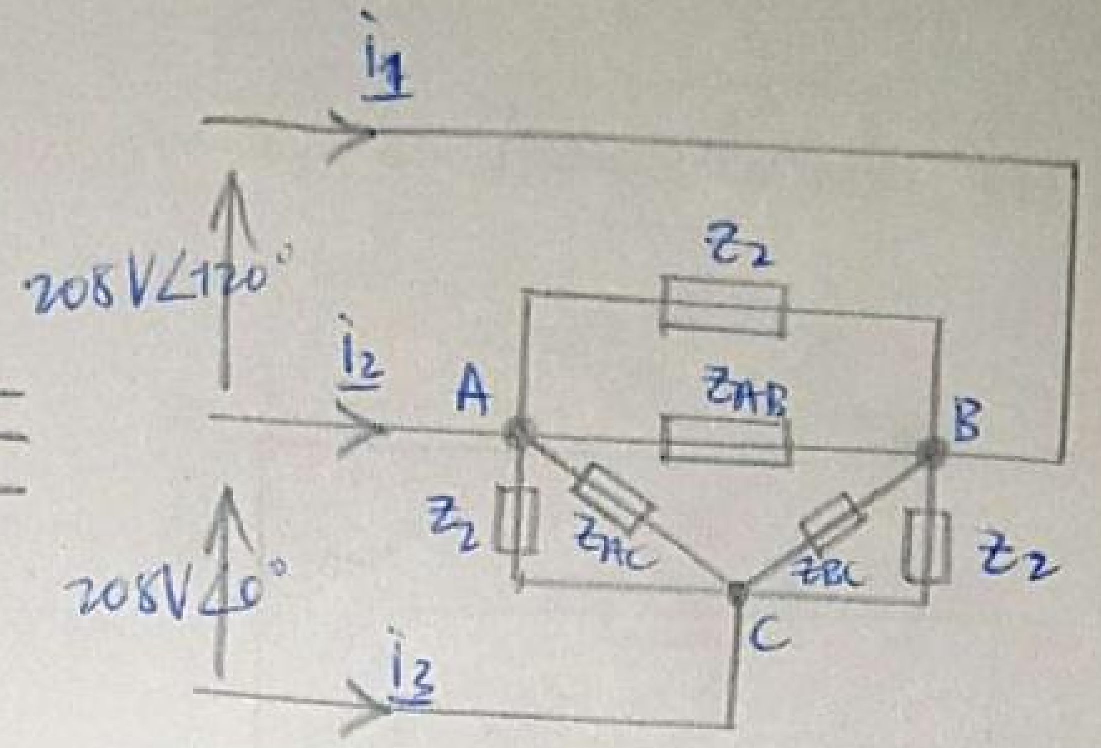

avec $\begin{cases} Z_{AB} = \dfrac{3Z_1^2}{Z_1} = 3Z_1 \\ Z_{BC} = 3Z_1 \\ Z_{AC} = 3Z_1 \end{cases}$

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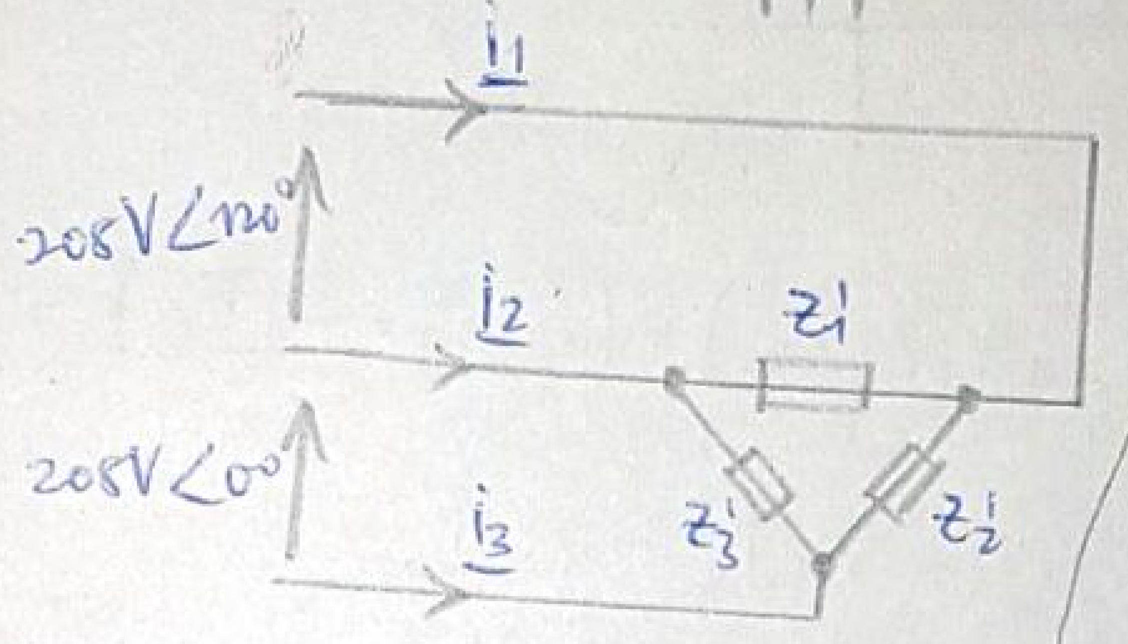
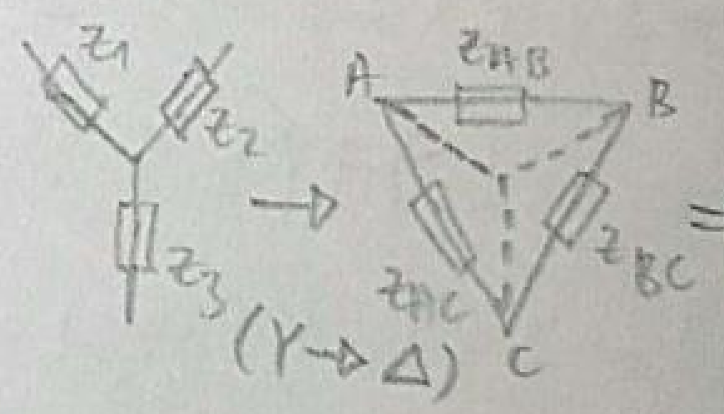


$$\begin{cases} Z_1' = Z_{AB} \parallel Z_2 = \dfrac{Z_{AB} Z_2}{Z_{AB}+Z_2} = \dfrac{3Z_1 Z_2}{Z_2 + 3Z_1} \\ Z_2' = Z_{BC} \parallel Z_2 = \dfrac{Z_{BC} Z_2}{Z_{BC}+Z_2} = \dfrac{3Z_1 Z_2}{Z_2+3Z_1} \\ Z_3' = Z_{AC} \parallel Z_2 = \dfrac{3Z_1 Z_2}{Z_2+3Z_1} \end{cases}$$

Rappel :



$$\Rightarrow \begin{cases} Z_{AB} = \dfrac{Z_1Z_2+Z_1Z_3+Z_2Z_3}{Z_3} \\ Z_{BC} = \dfrac{Z_1Z_2+Z_1Z_3+Z_2Z_3}{Z_1} \\ Z_{AC} = \dfrac{Z_1Z_2+Z_1Z_3+Z_2Z_3}{Z_2} \end{cases}$$

$Y \leftarrow \Delta$ (L'inverse) $\Rightarrow \begin{cases} Z_1 = \dfrac{Z_{AB} Z_{AC}}{Z_{AB}+Z_{AC}+Z_{BC}} \\ Z_2 = \dfrac{Z_{AB} Z_{BC}}{Z_{AB}+Z_{AC}+Z_{BC}} \\ Z_3 = \dfrac{Z_{AC} Z_{BC}}{Z_{AB}+Z_{AC}+Z_{BC}} \end{cases}$

On a donc : $Z_1' = Z_2' = Z_3' = Z_\Delta$

$$\Rightarrow \boxed{Z_\Delta = \frac{3Z_1Z_2}{Z_2+3Z_1}}$$

A.N :

$$Z_\Delta = \frac{3(15\Omega\angle 30°)(30\Omega\angle 45°)}{3(15\Omega\angle 30°) + 30\Omega\angle 45°}$$

$$= \frac{(45\Omega\angle 30°)(30\Omega\angle 45°)}{(45\Omega\angle 30°) + (30\Omega\angle 45°)}$$

$$= \frac{1350\angle 75°}{(45\cos 30° + 30\cos 45°) + i(45\sin 30° + 30\sin 45°)}$$

$$= \frac{1350\angle 75°}{\frac{45\sqrt{3}+30\sqrt{2}}{2} + i\,\frac{(45+30\sqrt{2})}{2}}$$

or : $a + ib = \sqrt{a^2+b^2} \angle tg^{-1}\left(\frac{b}{a}\right)$

$$\Rightarrow \frac{45\sqrt{3}+30\sqrt{2}}{2} + i\,\frac{(45+30\sqrt{2})}{2} = 74{,}38\angle 36°$$

$$\Rightarrow Z_\Delta = \frac{1350\angle 75°}{74{,}38\angle 36°}$$

$$\Rightarrow \boxed{Z_\Delta = 18{,}15\,\Omega \angle 39°}$$

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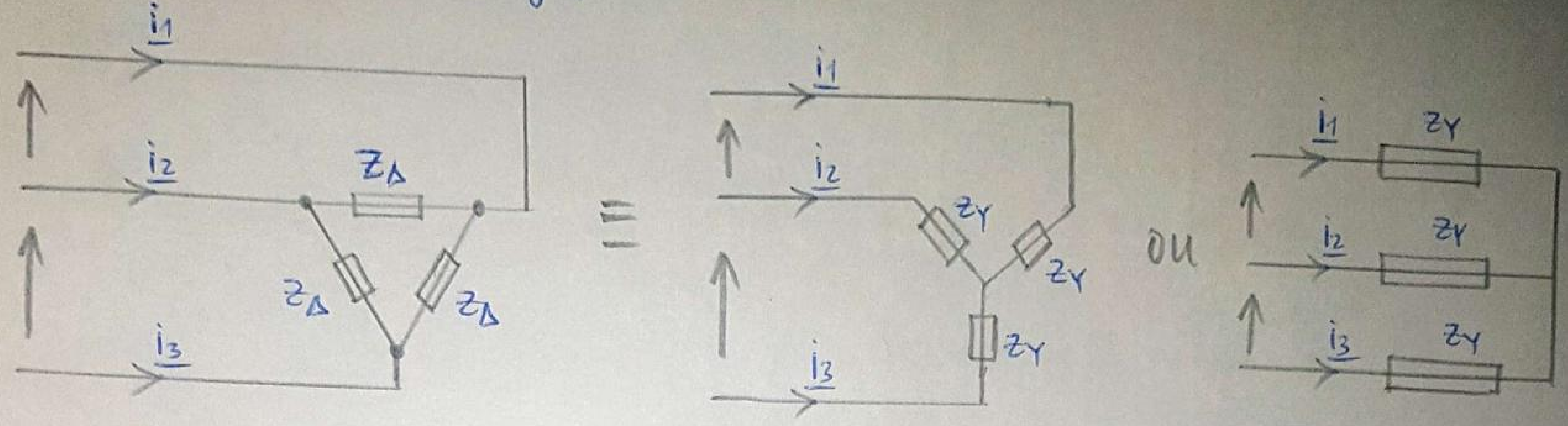
b) Transformation en circuit étoile :

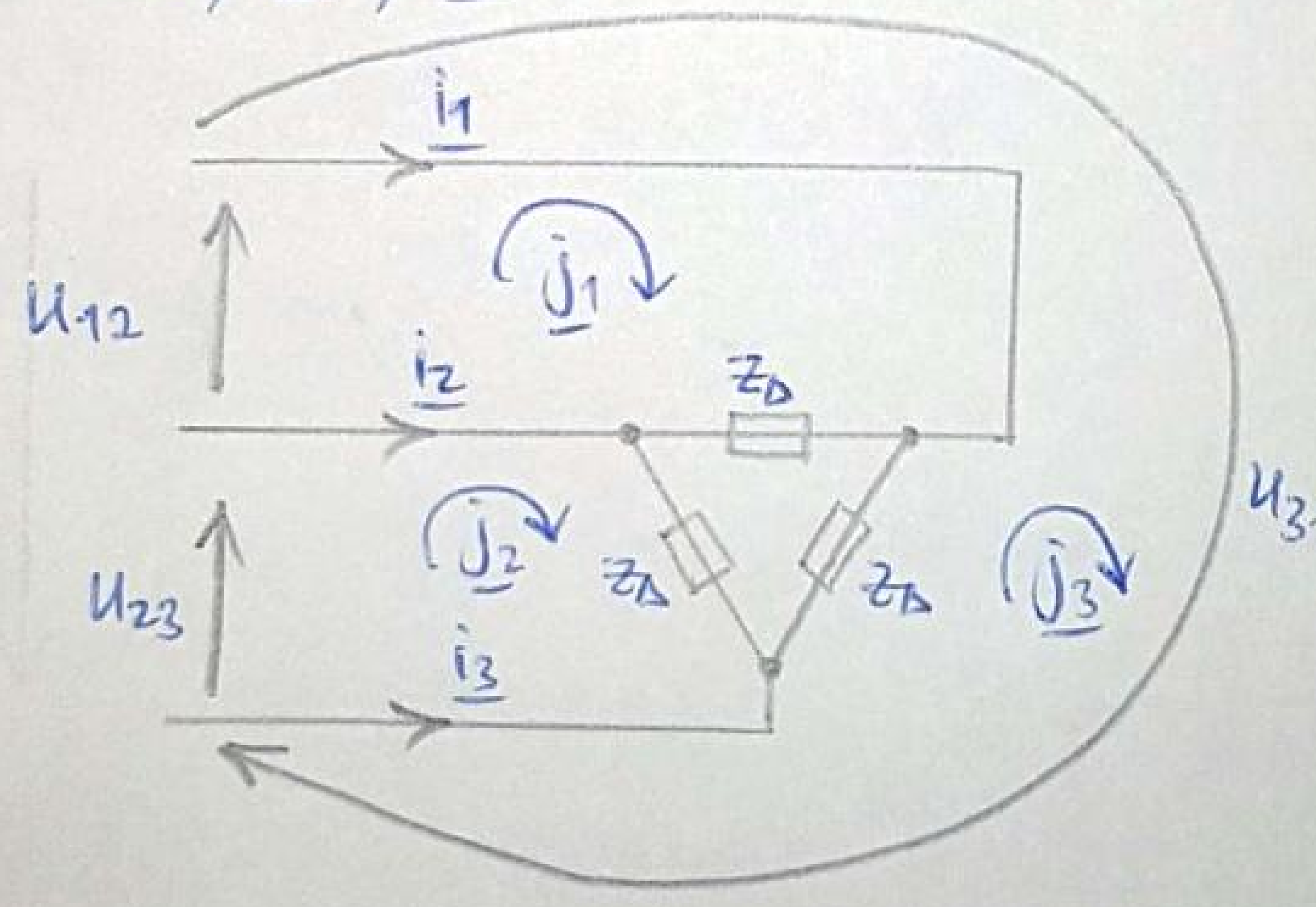
En utilisant notre circuit triangle, on a :



$$\text{avec } Z_Y = \frac{Z_\Delta^2}{3 Z_\Delta} = \frac{Z_\Delta}{3} \qquad \Rightarrow \boxed{Z_Y = \frac{Z_1 Z_2}{Z_2 + 3 Z_1}}$$

<u>A.N :</u> $Z_Y = \frac{18{,}15\,\Omega}{3} \angle 39° \Rightarrow \boxed{Z_Y = 6{,}05\,\Omega \angle 39°}$

c) Valeurs de $\underline{i_1}$, $\underline{i_2}$, $\underline{i_3}$:



$$\Rightarrow \begin{cases} \underline{j_1}\, Z_\Delta = u_{12} \\ \underline{j_2}\, Z_\Delta = u_{23} \\ \underline{j_3}\, Z_\Delta = u_{31} \end{cases}$$

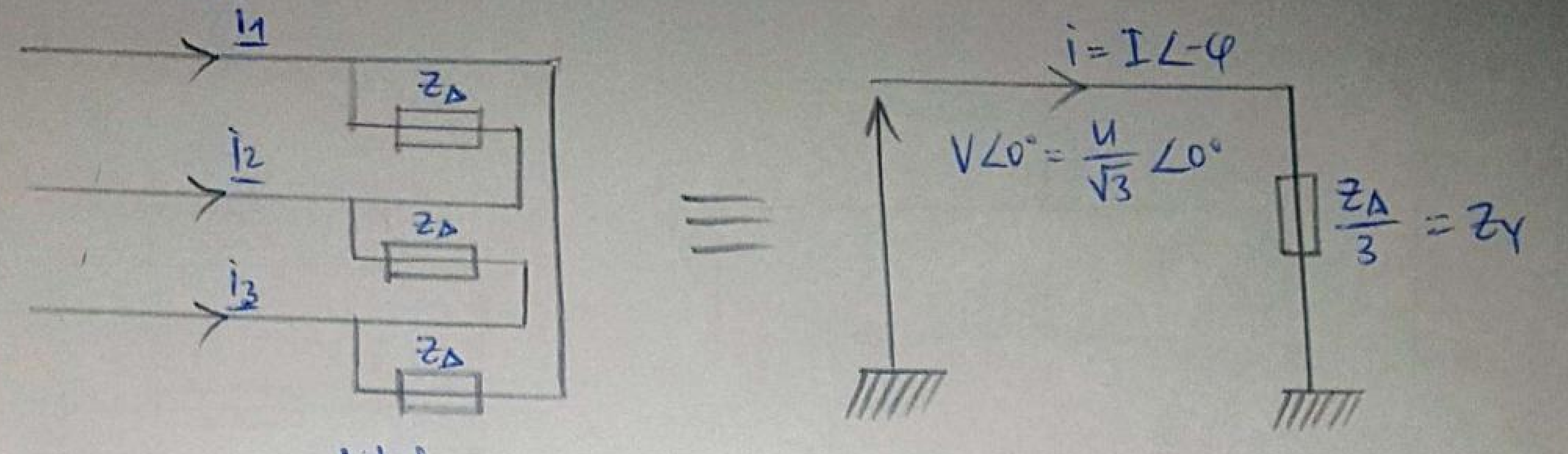
$$\Rightarrow \underline{j_1} = \frac{u_{12}}{Z_\Delta} = \frac{208\,V \angle 120°}{18{,}15\,\Omega \angle 39°} = 11{,}46\,A \angle 81°$$

$$\underline{j_2} = \frac{u_{23}}{Z_\Delta} = \frac{208\,V \angle 0°}{18{,}15\,\Omega \angle 39°} = 11{,}46\,A \angle -39°$$

$$\underline{j_3} = \frac{u_{31}}{Z_\Delta} = \frac{208\,V \angle -120°}{18{,}15\,\Omega \angle 39°} = 11{,}46\,A \angle -159°$$

$$\text{D'où :} \begin{cases} \underline{i_1} = \underline{j_1}\sqrt{3} \angle -30° \Rightarrow \\ \underline{i_2} = \underline{j_2}\sqrt{3} \angle -30° \Rightarrow \\ \underline{i_3} = \underline{j_3}\sqrt{3} \angle -30° \Rightarrow \end{cases} \boxed{\begin{array}{l} \underline{i_1} = 19{,}85\,A \angle 51° \\ \underline{i_2} = 19{,}85\,A \angle -69° \\ \underline{i_3} = 19{,}85\,A \angle -189° \end{array}}$$

3) Schéma équivalent à un conducteur :



* Puissance active totale et facteur de puissance ($\cos\varphi$) :

$$Z_Y = 6{,}05\,\Omega \angle 39°$$

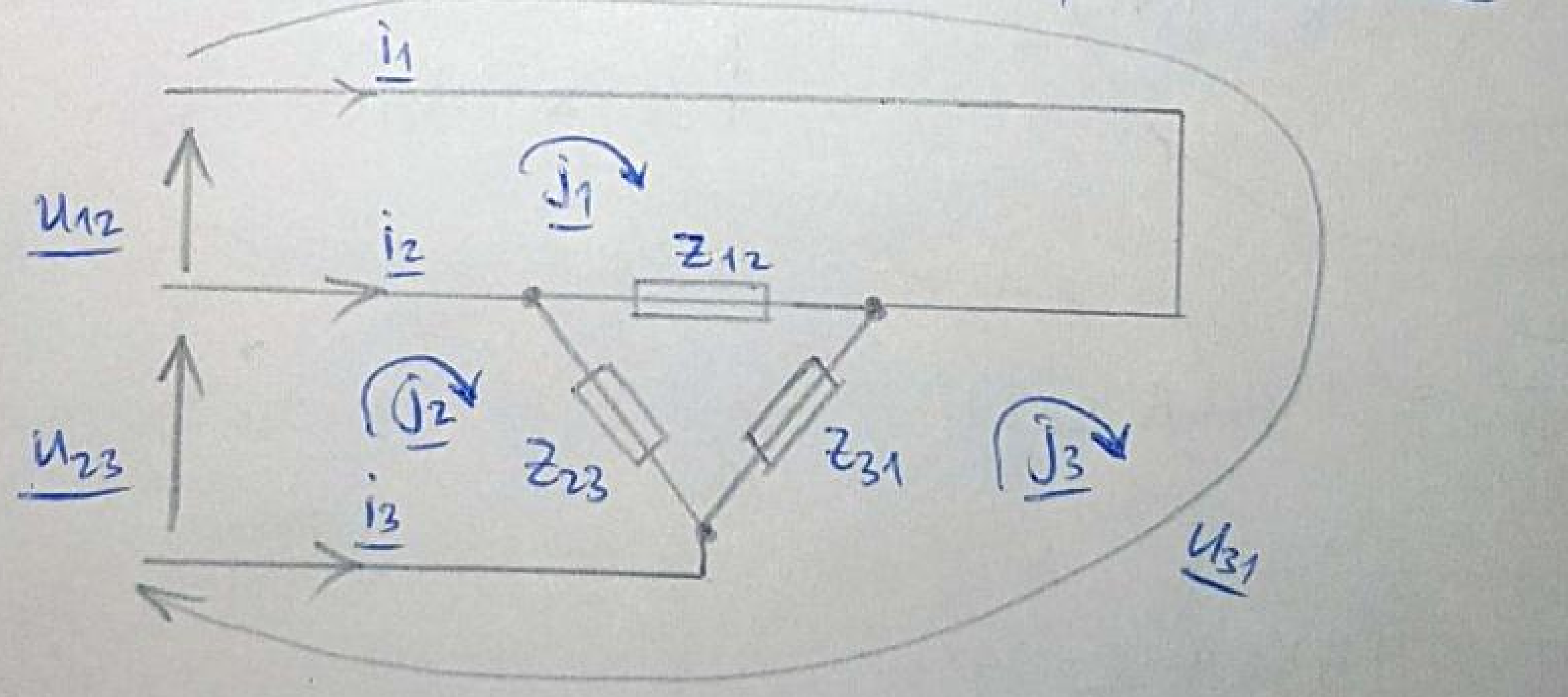
$$P = 3VI\cos\varphi = \frac{3V^2}{Z_Y}\cos\varphi$$

or $V = \frac{U}{\sqrt{3}} = \frac{208\,V}{\sqrt{3}} = 120\,V$ et $I = \frac{V}{Z_Y} = 19{,}85 \angle -39° \Rightarrow \varphi = 39°$

D'où : $\boxed{P = 5552\,W \quad \text{et} \quad \cos\varphi = 0{,}78}$

Exercice 2

$U = 381\,V$ $\quad Z_{12} = 25\,\Omega\angle 90°$, $Z_{23} = 15\,\Omega\angle 30°$, $Z_{31} = 20\,\Omega\angle 0°$



1) Calcul de j_1, j_2, j_3 :

$$j_1 Z_{12} = U_{12} \Rightarrow j_1 = \frac{U_{12}}{Z_{12}} = \frac{381\,V\angle 0°}{25\,\Omega\angle 90°} = \underline{15{,}24\,A\angle -90°}$$

$$j_2 Z_{23} = U_{23} \Rightarrow j_2 = \frac{U_{23}}{Z_{23}} = \frac{381\,V\angle -120°}{15\,\Omega\angle 30°} = \underline{25{,}4\,A\angle -150°}$$

$$j_3 Z_{31} = U_{31} \Rightarrow j_3 = \frac{U_{31}}{Z_{31}} = \frac{381\,V\angle -240°}{20\,\Omega\angle 0°} = \underline{19{,}05\,A\angle -240°}$$

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2) Les courants $\underline{i_1}$, $\underline{i_2}$, $\underline{i_3}$:

$$\underline{i_1} = \underline{j_1} - \underline{j_3} = 15{,}24A \angle -90° - 19{,}05A \angle -240°$$

$$= [15{,}24 \cos(-90) - 19{,}05 \cos(-240)] + i[15{,}24 \sin(-90) - 19{,}05 \sin(-240)]$$

$$= 9{,}525 + i(-31{,}74)$$

$$\Rightarrow \boxed{\underline{i_1} = 33{,}14 \angle -73{,}3°}$$

$$\underline{i_2} = \underline{j_2} - \underline{j_1} = 25{,}4A \angle -150° - 15{,}24A \angle -90°$$

$$= [25{,}4 \cos(-150) - 15{,}24 \cos(-90)] + i[25{,}4 \sin(-150) - 15{,}24 \sin(-90)]$$

$$= -22 + i(2{,}54)$$

$$\Rightarrow \boxed{\underline{i_2} = 22{,}1A \angle -6{,}6°}$$

$$\underline{i_3} = \underline{j_3} - \underline{j_2} = 19{,}05A \angle -240° - 25{,}4A \angle -150°$$

$$\Rightarrow \boxed{\underline{i_3} = 31{,}75A \angle 66{,}87°}$$

3) Puissance active :

$$P = V_1 I_1 \cos\varphi_1 + V_2 I_2 \cos\varphi_2 + V_3 I_3 \cos\varphi_3$$

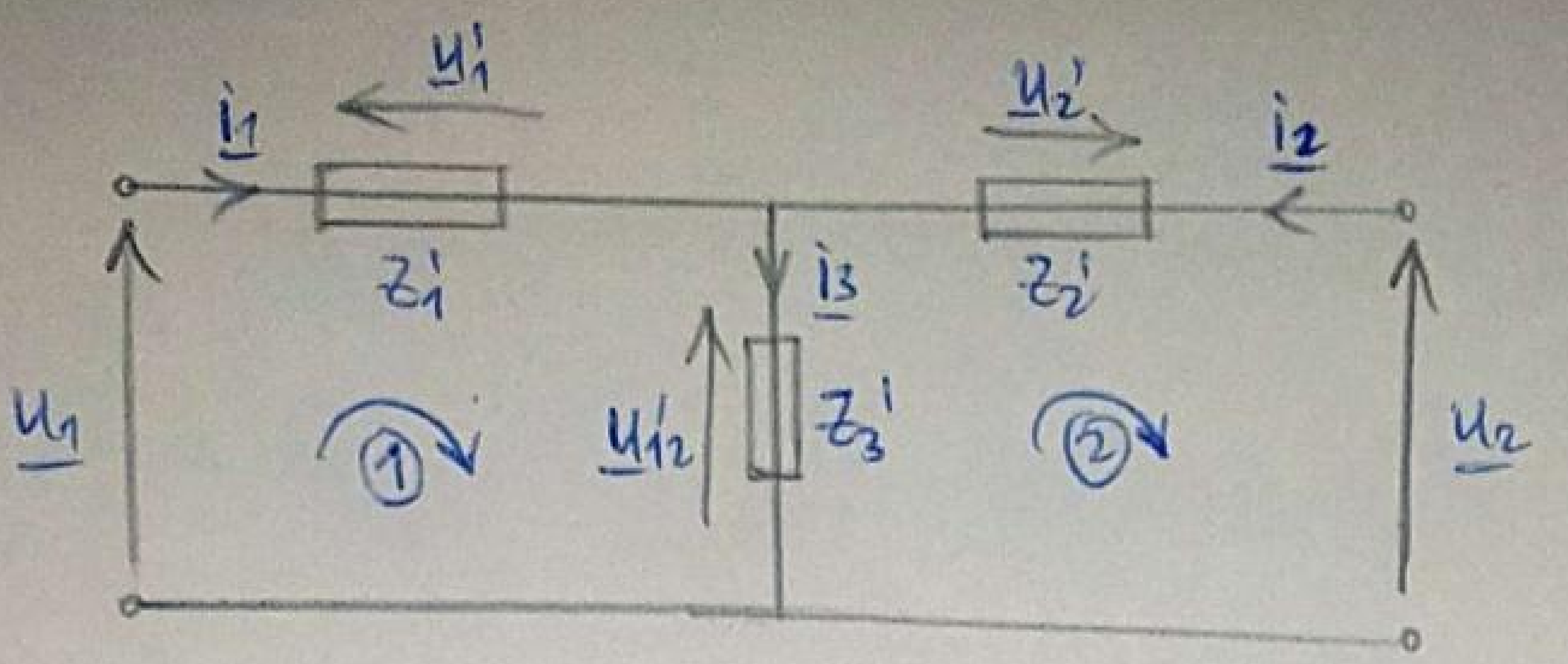
avec
$$\begin{cases} V_1 = \dfrac{U_{12}}{\sqrt{3}} \angle -30° = 219{,}97V \angle -30° \\ V_2 = \dfrac{U_{23}}{\sqrt{3}} \angle -30° = 219{,}97V \angle -150° \\ V_3 = \dfrac{U_{31}}{\sqrt{3}} \angle -30° = 219{,}97V \angle -270° \end{cases}$$
et
$$\begin{cases} \varphi_1 = \varphi_{V_1} - \varphi_{\underline{i_1}} = 43{,}3° \\ \varphi_2 = \varphi_{V_2} - \varphi_{\underline{i_2}} = \cancel{6{,}6} -144° \\ \varphi_3 = \varphi_{V_3} - \varphi_{\underline{i_3}} = -336{,}78° \end{cases}$$

$$\Rightarrow P = [219{,}97 \times 33{,}14 \cos(43{,}3)] + [219{,}97 \times 22{,}1 \cos(-144)] + [219{,}97 \times 31{,}75 \cos(-336{,}78)]$$

$$\Rightarrow \boxed{P = 16432{,}26\ W}$$

Exercice 3

A.



1) Z_1', Z_2', Z_3' en fonction des paramètres ~~d~~impédances :

$① \Rightarrow \underline{u}_1 - \underline{u}_1' - \underline{u}_{12}' = 0$

$\Rightarrow \underline{u}_1 = \underline{u}_1' + \underline{u}_{12}' = Z_1' \underline{i}_1 + Z_3'(i_1 + i_2)$ car $i_3 = i_1 + i_2$

$\Rightarrow u_1 = (Z_1' + Z_3')\underline{i}_1 + Z_3' i_2$

$② \Rightarrow -\underline{u}_2 + \underline{u}_2' + \underline{u}_{12}' = 0$

$\Rightarrow \underline{u}_2 = \underline{u}_2' + \underline{u}_{12}' = Z_2' \underline{i}_2 + Z_3'(\underline{i}_1 + \underline{i}_2)$

$\Rightarrow u_2 = Z_3' \underline{i}_1 + (Z_2' + Z_3')\underline{i}_2$

or $\begin{cases} \underline{u}_1 = Z_{11}\underline{i}_1 + Z_{12}\underline{i}_2 \\ \underline{u}_2 = Z_{21}\underline{i}_1 + Z_{22}\underline{i}_2 \end{cases}$

Par identification, on a $\begin{cases} Z_{11} = Z_1' + Z_3' \\ Z_{12} = Z_3' \\ Z_{21} = Z_3' \\ Z_{22} = Z_2' + Z_3' \end{cases} \Rightarrow \boxed{\begin{cases} Z_1' = Z_{11} - Z_{21} \\ Z_2' = Z_{22} - Z_{21} \\ Z_3' = Z_{21} \end{cases}}$

2) Paramètres hybrides en fonction de Z_1', Z_2' et Z_3' :

Pour une matrice hybride directe, on a :

$\begin{cases} \underline{u}_1 = h_{11}\underline{i}_1 + h_{12}\underline{u}_2 \\ \underline{i}_2 = h_{21}\underline{i}_1 + h_{22}\underline{u}_2 \end{cases}$

or $\begin{cases} \underline{u}_1 = (Z_1' + Z_3')\underline{i}_1 + Z_3'\underline{i}_2 & (1) \\ \underline{u}_2 = Z_3'\underline{i}_1 + (Z_2' + Z_3')\underline{i}_2 & (2) \end{cases}$

$(2) \Rightarrow \underline{i}_2 = \dfrac{\underline{u}_2 - Z_3'\underline{i}_1}{Z_2' + Z_3'}$ (*) ~~dans (1)~~

$\Rightarrow \underline{i}_2 = \dfrac{1}{Z_2' + Z_3'}\underline{u}_2 - \dfrac{Z_3'}{Z_2' + Z_3'}\underline{i}_1 \Rightarrow h_{21} = -\dfrac{Z_3'}{Z_2' + Z_3'}$ et $h_{22} = \dfrac{1}{Z_2' + Z_3'}$

(*) dans (1) $\Rightarrow \underline{u}_1 = (Z_1' + Z_3')\underline{i}_1 + \dfrac{Z_3'(\underline{u}_2 - Z_3'\underline{i}_1)}{Z_2' + Z_3'}$

$\Rightarrow \underline{u}_1 = \left[Z_1' + Z_3' - \dfrac{Z_3'^2}{Z_2' + Z_3'}\right]\underline{i}_1 + \dfrac{Z_3'}{Z_2' + Z_3'}\underline{u}_2$

$\Rightarrow \underline{u}_1 = \left[Z_1' + \dfrac{Z_2' Z_3'}{Z_2' + Z_3'}\right]\underline{i}_1 + \dfrac{Z_3'}{Z_2' + Z_3'}\underline{u}_2$

$\Rightarrow h_{11} = Z_1' + \dfrac{Z_2' Z_3'}{Z_2' + Z_3'}$ et $h_{12} = \dfrac{Z_3'}{Z_2' + Z_3'}$

D'où $\boxed{\begin{cases} h_{11} = Z_1' + \dfrac{Z_2' Z_3'}{Z_2' + Z_3'} \\ h_{12} = Z_3' / Z_2' + Z_3' \\ h_{21} = -Z_3' / Z_2' + Z_3' \\ h_{22} = 1 / Z_2' + Z_3' \end{cases}}$

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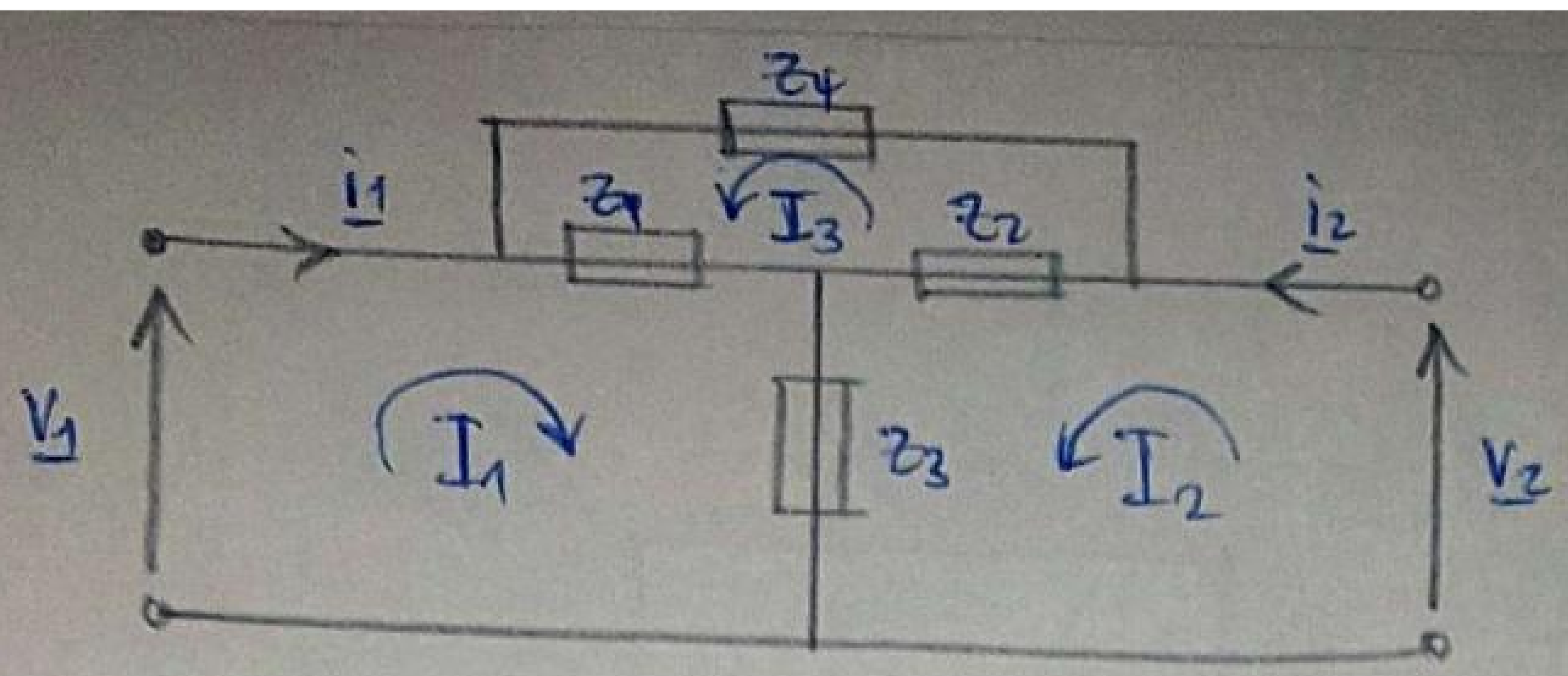
B.



1) Matrice impédance du quadripôle en T :

Maille 1 : $Z_1 I_1 + Z_1 I_3 + Z_3 I_1 + Z_3 I_2 = \underline{V}_1$

$\Rightarrow (Z_1+Z_3) I_1 + Z_3 I_2 + Z_1 I_3 = \underline{V}_1$

Maille 2 : $Z_2 I_2 - Z_2 I_3 + Z_3 I_2 + Z_3 I_1 = \underline{V}_2$

$\Rightarrow Z_3 I_1 + (Z_2+Z_3) I_2 - Z_2 I_3 = \underline{V}_2$

Maille 3 : $Z_1 I_3 + Z_1 I_1 + Z_2 I_3 - Z_2 I_2 + Z_4 I_3 = 0$

$\Rightarrow Z_1 I_1 - Z_2 I_2 + (Z_1+Z_2+Z_4) I_3 = 0$

Or $I_1 = \underline{i}_1$ et $I_2 = \underline{i}_2$

$$\Rightarrow \begin{cases} (Z_1+Z_3)\underline{i}_1 + Z_3 \underline{i}_2 + Z_1 I_3 = \underline{V}_1 & (1) \\ Z_3 \underline{i}_1 + (Z_2+Z_3)\underline{i}_2 - Z_2 I_3 = \underline{V}_2 & (2) \\ Z_1 \underline{i}_1 - Z_2 \underline{i}_2 + (Z_1+Z_2+Z_4) I_3 = 0 & (3) \end{cases}$$

$$(3) \Rightarrow I_3 = \frac{Z_2 \underline{i}_2 - Z_1 \underline{i}_1}{Z_1+Z_2+Z_4} \qquad (4)$$

(4) dans (1) et (2) $\Rightarrow \begin{cases} \underline{V}_1 = (Z_1+Z_3)\underline{i}_1 + Z_3 \underline{i}_2 + Z_1\left(\dfrac{Z_2\underline{i}_2 - Z_1\underline{i}_1}{Z_1+Z_2+Z_4}\right) \\ \underline{V}_2 = Z_3 \underline{i}_1 + (Z_2+Z_3)\underline{i}_2 - Z_2\left(\dfrac{Z_2\underline{i}_2 - Z_1\underline{i}_1}{Z_1+Z_2+Z_4}\right) \end{cases}$

$$\Rightarrow \begin{cases} \underline{V}_1 = \left(Z_1+Z_3 - \dfrac{Z_1^2}{Z_1+Z_2+Z_4}\right)\underline{i}_1 + \left(Z_3 + \dfrac{Z_1 Z_2}{Z_1+Z_2+Z_4}\right)\underline{i}_2 \\ \underline{V}_2 = \left(Z_3 + \dfrac{Z_1 Z_2}{Z_1+Z_2+Z_4}\right)\underline{i}_1 + \left(Z_2+Z_3 - \dfrac{Z_2^2}{Z_1+Z_2+Z_4}\right)\underline{i}_2 \end{cases}$$

D'où la matrice impédance :

$$\begin{bmatrix} Z_1+Z_3 - \dfrac{Z_1^2}{Z_1+Z_2+Z_4} & Z_3 + \dfrac{Z_1 Z_2}{Z_1+Z_2+Z_4} \\ Z_3 + \dfrac{Z_1 Z_2}{Z_1+Z_2+Z_4} & Z_2+Z_3 - \dfrac{Z_2^2}{Z_1+Z_2+Z_4} \end{bmatrix}$$

2) Déduire impédances ~~[illegible]~~ Z_1', Z_2', Z_3' en fonction de Z_1, Z_2, Z_3, Z_4 :

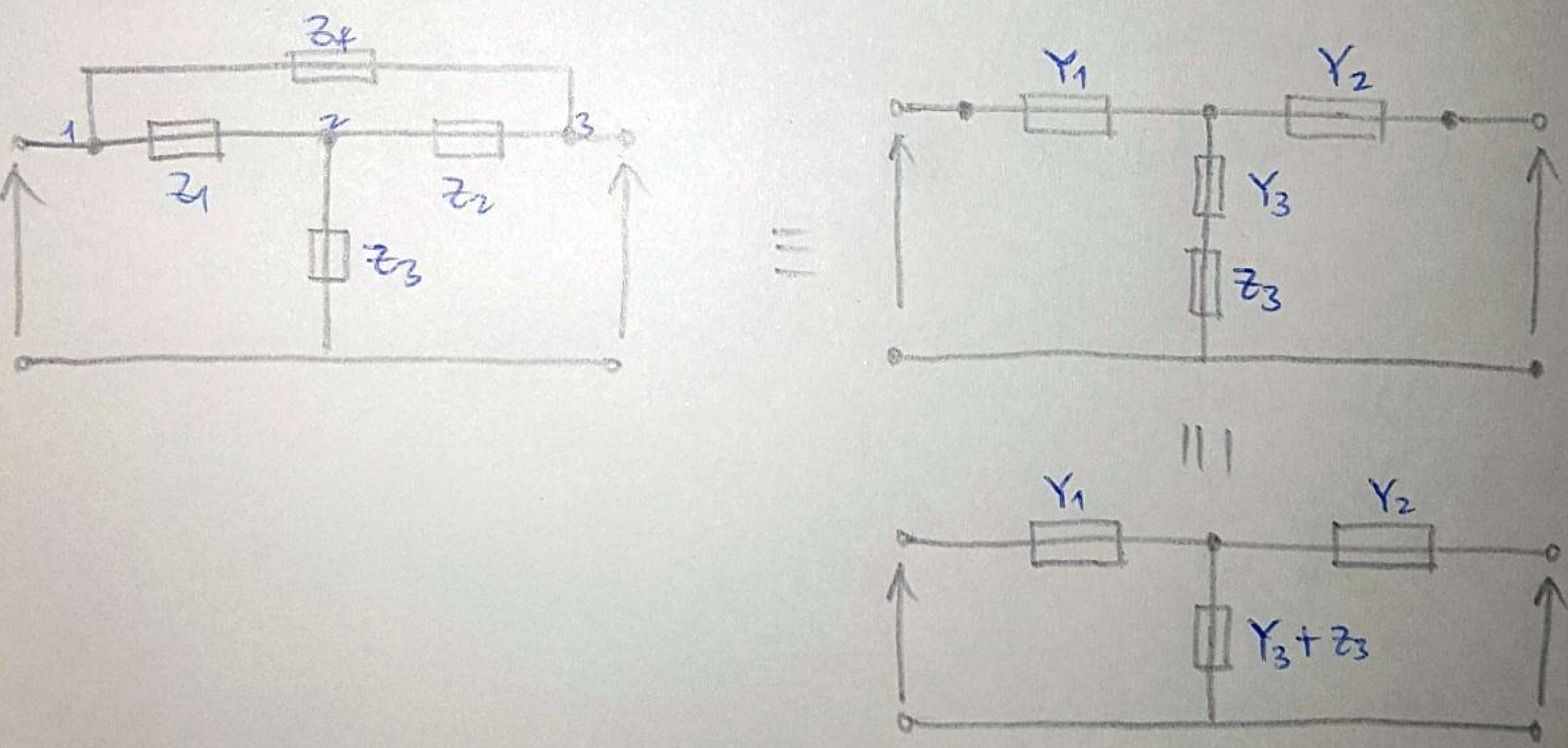
D'après la question A 1), on a :

$$Z_1' = Z_{11} - Z_{21} = Z_1 + Z_3 - \frac{Z_1^2}{Z_1+Z_2+Z_4} - Z_3 - \frac{Z_1 Z_2}{Z_1+Z_2+Z_4} \Rightarrow \boxed{Z_1' = \frac{Z_1 Z_4}{Z_1+Z_2+Z_4}}$$

$$Z_2' = Z_{22} - Z_{21} \Rightarrow \boxed{Z_2' = \frac{Z_2 Z_4}{Z_1+Z_2+Z_4}}$$

$$Z_3' = Z_{21} \Rightarrow \boxed{Z_3' = Z_3 + \frac{Z_1 Z_2}{Z_1+Z_2+Z_4}}$$

3) Retrouvons Z_1', Z_2', Z_3' en utilisant le théorème de Kenelly :



avec $Y_1 = \frac{Z_1 Z_4}{Z_1+Z_2+Z_4}$, $Y_2 = \frac{Z_2 Z_4}{Z_1+Z_2+Z_4}$, $Y_3 = \frac{Z_1 Z_2}{Z_1+Z_2+Z_4}$

D'où : $\boxed{Z_1' = Y_1 = \frac{Z_1 Z_2}{Z_1+Z_2+Z_4} \; ; \; Z_2' = Y_2 = \frac{Z_2 Z_4}{Z_1+Z_2+Z_4} \; ; \; Z_3' = Y_3 + Z_3 = \frac{Z_1 Z_2}{Z_1+Z_2+Z_4} + Z_3}$

On y retrouve le même résultats.