

Conversion Electromécanique

19.02.13

Livre: Traité d'Électrotechnique IX Electromécanique (Jufar) → libre service

Examen: 3 exos / droit à H

1.1. Équations de Maxwell

$$\text{a)} \quad \text{Rot } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

\vec{J} = Densité de courant $[\text{A}/\text{m}^2]$

$$\text{Rot } \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

\vec{H} = Champ Magnétique

$$\text{Div } \vec{B} = 0$$

\vec{B} = Champ d'Induction Magnétique

$$\vec{B} = \mu \vec{H}$$

\vec{E} = Champ électrique

$$\vec{D} = \text{Déplacement électrique}$$

Hypothèse: Régime quasi-statique $\Rightarrow \frac{\partial \vec{D}}{\partial t} = 0$

Reste: $\text{Rot } \vec{H} = \vec{J}$

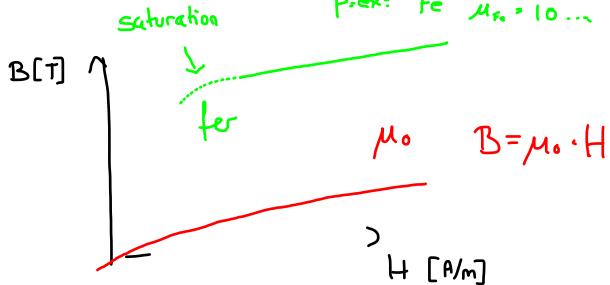
$$\text{Rot } \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{B} = \mu \vec{H}$$

μ = perméabilité magnétique $\left[\frac{\text{Vs}}{\text{Am}} \right]$ constante du vide $\mu_0 = 4\pi \cdot 10^{-7}$

tous les matériaux ont μ_0 comme grandeur. Exception: Fe, Co, Ni

Par ex: Fe $\mu_r > 10 \dots 10'000 \mu_0 \Rightarrow \mu$ n'est pas une constante!



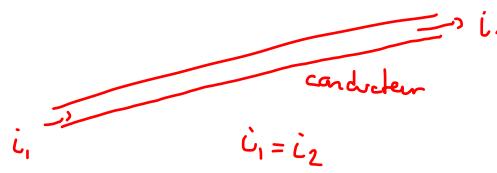
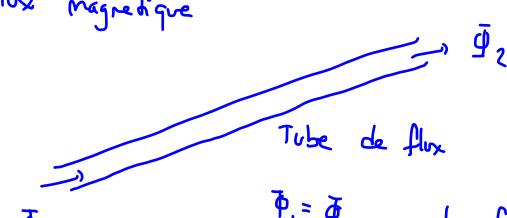
Circuits imbriqués:



2 choix : 2 démarches analytiques

- a) Modèle de Kirchoff, Analyser un circuit magnétique $\rightarrow F$
- b) Modèle de Maxwell, Analyse $\vec{H} \rightarrow F$

1.2 Analogie

| Électrique | Magnétique |
|--|--|
| Densité de courant : $J \text{ A/m}^2$ | Densité de flux (Induction) $B [\text{T}] \left(\frac{\text{Vs}}{\text{m}^2} \right)$ |
| $i = \int J \cdot ds$ | Flux : $\Phi = \int B \cdot ds$ |
| Si $S = \text{cte} \Rightarrow i = J \cdot S$ | Si $S = \text{cte} \Rightarrow \Phi = B \cdot S$ |
| Conservation du courant : | Flux magnétique |
|  |  |
| $i_1 = i_2$ | $\Phi_1 = \Phi_2 \quad \text{le flux est conservé}$ |

Câble électrique :

$$U_{12} = \int_1^2 E \cdot dl = \int_1^2 \underbrace{\rho}_{\text{résistivité}} \cdot J \cdot dl = \int_1^2 \frac{\rho \cdot J \cdot S}{S} dl = \int_1^2 \frac{\rho \cdot i}{S} dl = i \underbrace{\int_1^2 \frac{\rho \cdot dl}{S}}_{R_{12}}$$

$$\Rightarrow U_{12} = R_{12} i$$

$$R_{12} = \int_1^2 \frac{\rho \cdot dl}{S} [\Omega] \quad \rho \text{ peut être nulle ou infinie}$$

Côté magnétique

$$\text{rot } \vec{H} = \vec{J}$$

$$\oint \vec{H} d\vec{l} = \int \vec{J} ds = \Theta [A]$$

Θ : potentiel magnétique (solenation) (MMF, FMM = force magnétomotrice)

Différence de potentiel magnétique

$$\Theta_{12} = \int_1^2 \vec{H} d\vec{l} = \int_1^2 \frac{\vec{B}}{\mu} d\vec{l} = \int_1^2 \frac{\vec{B} \cdot s}{\mu s} d\vec{l} = \phi \int_1^2 \frac{d\vec{l}}{\mu s}$$

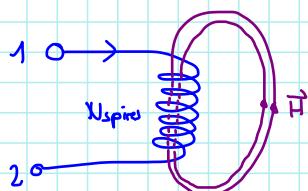
Définition: $\int_1^2 \frac{ds}{\mu s} = R = R_m$ résistance

$$\Theta_{12} = R_m \phi \quad 0 < R_m < R_{m,\max}$$

⚠ μ ne peut pas être nul \Rightarrow pas d'isolant magnétique !

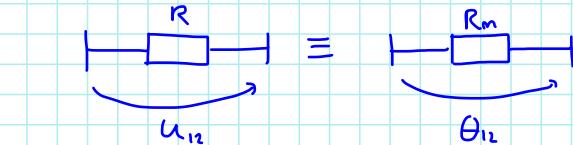
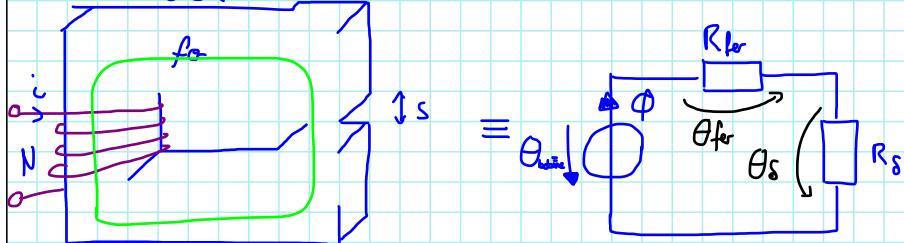
$$\lambda = \frac{1}{R_m} = \int \mu \frac{ds}{l} [H] \quad \text{perméance}$$

(lambda)



$$\oint \vec{H} d\vec{l} = \Theta = \int_S \vec{J} ds = N \cdot i = \Theta = R_m \phi$$

Surface totale de Cuirre

Modélisation

$$\Theta_{bob} = Ni = \Phi \cdot R_{meq}$$

Avantage: On connaît la manière de résoudre les circuits

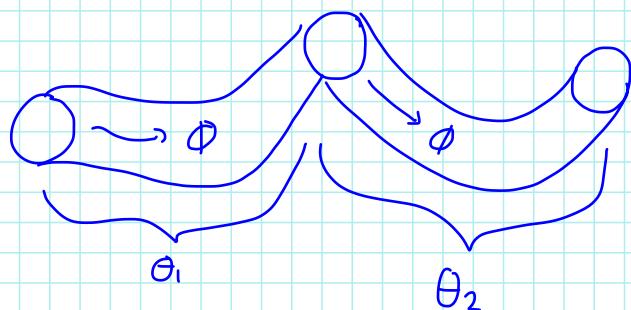
$$\text{Kirchoff} \quad \sum H dl = \Theta \quad \text{Maille}$$

$$\sum \phi = 0 \quad \text{Nœuds}$$

Principe de superposition (si linéaire)

Thévenin / Norton

1.4. Mise en série et en // des réductances et perméances



$$\Theta_1 = R_{m1} \phi$$

$$\Theta_2 = R_{m2} \phi$$

$$\Theta_1 + \Theta_2 = (R_{m1} + R_{m2}) \phi = R_{\text{req}} \phi$$

En série: $R_{\text{req}} = \sum_i R_{mi}$

En parallèle: $\frac{1}{R_{\text{req}}} = \sum_i \frac{1}{R_{mi}}$

Pour les perméances:

$$\text{En série: } \Lambda_{\text{eq}} = \frac{1}{\sum_i 1/\Lambda_i}$$

$$\text{En } //: \quad \Lambda_{\text{eq}} = \sum_i \Lambda_i$$

1.5. Résumé et exemple

Électrique

$R [\Omega]$

$Y [1/\Omega]$

$I [A]$

$U [V]$

\vec{J}

\vec{E}

$$\operatorname{div} \vec{J} = 0$$

$$i_1 = i_2$$

$$U_{12} = \int_a^b \vec{E} \cdot d\vec{r}$$

Magnétique

$R_m [1/H]$

$\Lambda [H]$

$\phi [Wb] [Vs]$

$\theta [A] !!$

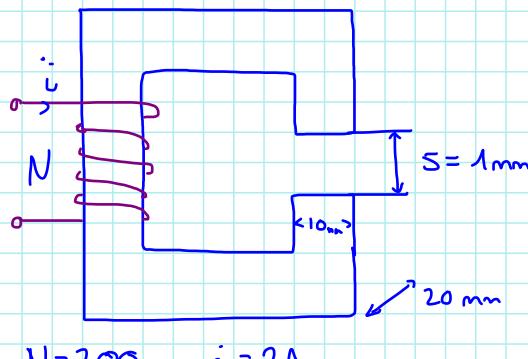
\vec{B}

\vec{H}

$$\operatorname{div} \vec{B} = 0$$

$$\phi_1 = \phi_2$$

$$\Theta_{12} = \int_a^b \vec{H} \cdot d\vec{r}$$



Calculer ϕ_s et B_s , on suppose que les réductances de fer sont nulles. $R_m = \frac{1}{\mu S}$

$$\equiv \begin{array}{c} \phi \\ \downarrow \\ NI \end{array}$$

$$B = \frac{\phi}{S} = 0,5 [T]$$

$$R_{ms} = \frac{1 \cdot 10^{-3}}{4\pi \cdot 10^{-7} \cdot 10 \cdot 10^{-3} \cdot 20 \cdot 10^{-3}} = 3,98 \cdot 10^6 [1/H]$$

$$\theta = 400 [A]$$

$$\Theta = R_{ms} \phi \quad \Lambda = \frac{\mu S}{l} = 2,5 \cdot 10^{-3} [H] \quad \phi = \frac{\theta}{R_m} = \theta \Lambda = 10^{-4} [V]$$

Autre méthode:

$$\oint \vec{H} \cdot d\vec{s} = \Theta \quad B_S = \frac{NI \cdot \mu_0}{S} = \frac{400 \cdot 4\pi \cdot 10^{-7}}{10^{-3}} = 0,5 T$$

$$H_S \cdot S = NI$$

$$\frac{B_S}{\mu_0} \cdot S = NI$$

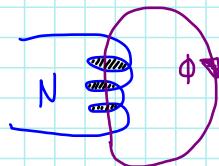
1.6. Définition du Flux totalisé

$$\text{rot } \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

$$\text{Stokes: } \oint \vec{E} \cdot d\vec{s} = \int_S -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

Système indéformable

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\vec{B}}{dt} \cdot d\vec{s} - \frac{d}{dt} \int_S \vec{B} \cdot d\vec{s} = - \frac{d\psi}{dt} \quad \underline{\psi = N\phi} \quad \text{flux totalisé} \quad \frac{d\psi}{dt} \quad \text{tension induite}$$



1.7. Loi de la tension induite

Diagram of a coil with two terminals labeled 1 and 2. A current i flows through the coil.

$$\oint \vec{E} \cdot d\vec{s} = \int_1^2 \vec{E} \cdot d\vec{l} + \int_2^1 \vec{E} \cdot d\vec{l} = \int_1^2 \rho J dl - U_{12} = R_{12} i - U_{12} = - \frac{d\psi}{dt}$$

$$U_{12} = R_{12} i + \frac{d\psi}{dt}$$

1.8. Définition de l'induction propre et mutuelle

$$\psi = N\phi \quad \phi = \lambda \theta = \lambda Ni$$

$$\psi = N \lambda Ni = \underbrace{N^2 \lambda}_L i = Li$$

Inductance $L = \frac{\psi}{i}$ (mesure de la capacité à créer du flux totalisé par un courant donné).

$$U = Ri + \frac{d\psi}{dt} \quad U = Ri + \frac{d}{dt} Li = Ri + \frac{di}{dt} i + L \frac{di}{dt}$$

sys. lin.
+ immobile. } $L = \text{cste}$

$$U = Ri + L \frac{di}{dt}$$

$$L = \frac{U^2}{R_m} = \frac{\psi^2}{i^2} = N^2 \lambda \quad [\text{H}]$$

Rappel:

$$\text{Flux magnétique } \Phi = B \cdot S \quad (B = \text{densité de flux})$$

$$\text{Flux totalisé } \Psi = N \cdot \Phi \quad (N = \text{nb de spires})$$

$$\text{Lai d'Ohm Magnétique: } \Phi R = \Theta \quad (\text{potential})$$

\hookrightarrow Résistivité

$$\frac{1}{R} = \lambda \quad \text{perméance}$$

$$\Rightarrow \Phi = \Theta \lambda$$

$$\Psi = N \cdot \Phi = N \lambda \Theta = N \lambda N i = N^2 \lambda i$$

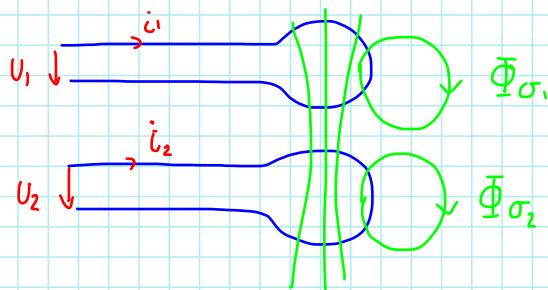
$$\rightarrow U = R_i + \frac{d\Psi}{dt} \quad \text{pour une bobine.}$$

$$U = R_i + N^2 \lambda \frac{di}{dt} \quad (\text{Si } \lambda = \text{cte})$$

$$\text{Définition} \quad L = N^2 \lambda$$

$$L = \frac{\Psi}{i}$$

1.9 Inductance Mutuelle:

Flux Φ_n : Flux commun Φ_σ : Flux de file

$$U_1 = R_1 i_2 + \frac{d\Psi_1}{dt}$$

$$U_2 = R_2 i_1 + \frac{d\Psi_2}{dt}$$

$$\Psi_1 = N_1 \Phi_1 = N_1 (\Phi_n + \Phi_{\sigma_1})$$

$$\Psi_2 = N_2 \Phi_2 = N_2 (\Phi_n + \Phi_{\sigma_2})$$

$$\Phi_{\sigma_1} = \lambda_{\sigma_1} \cdot N_1 \cdot i_1$$

$$\Phi_{\sigma_2} = \lambda_{\sigma_2} \cdot N_2 \cdot i_2 \quad (\Phi = \lambda \cdot \Theta)$$

$$\Phi_n = \lambda_n (i_1 + i_2) = \lambda_n (N_1 i_1 + N_2 i_2)$$

$$\text{Cas particulier} \quad \begin{array}{l} i_1 \neq 0 \\ i_2 = 0 \end{array} \quad \text{hyp: } \Phi_\sigma = 0$$

$$U_2 = R_2 i_2 + \frac{d\Psi_2}{dt}$$

$$\Psi_2 = N_2 \lambda_n N_1 \cdot i_1 = N_1 N_2 \lambda_n \cdot i_1$$

$$U_2 = \underbrace{N_1 N_2 \lambda_n}_{\text{L}_M} \frac{di_1}{dt} \quad (\text{Si } \lambda_n = \text{cte})$$

Inductance Mutuelle, $L_{12} \leftarrow$ passe par bobine 2

$$\text{Par définition} \quad L_{12} = \frac{\Psi_{21}}{i_1} \leftarrow \text{créé par bobine 1}$$

Cas Général:

$$\Psi_1 = N_1 [\Phi_0 + \Phi_{\sigma_1}]$$

$$= N_1 [\lambda_{\sigma_1} \theta_1 + \lambda_h (\theta_1 + \theta_2)]$$

$$\theta_1 = N_1 i_1 \quad \theta_2 = N_2 i_2$$

$$\Psi_1 = N_1^2 \lambda_{\sigma_1} i_1 + N_1^2 \lambda_h i_1 + N_1 N_2 \lambda_h i_2$$

$$\Psi_1 = \underbrace{N_1^2 [\lambda_{\sigma_1} + \lambda_h] i_1}_{L_{11}} + \underbrace{N_1 N_2 \lambda_h i_2}_{L_{12}}$$

$$\Psi_1 = \underbrace{N_1^2 \lambda_{\sigma_1} i_1}_{L_{11}} + \underbrace{N_1 N_2 \lambda_h i_2}_{L_{12}}$$

$$\text{Inductance propre: } L_{11} = \frac{\Psi_1}{i_1}$$

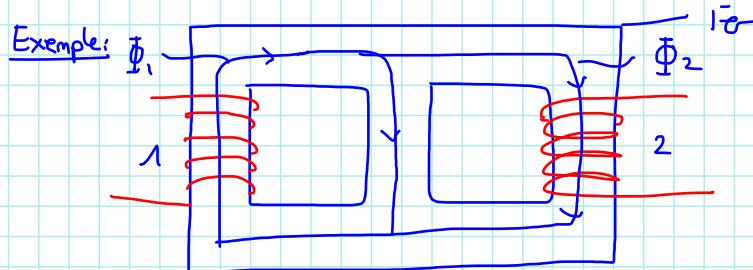
$$\text{Inductance mutuelle: } L_{12} = \frac{\Psi_2}{i_1}$$

Si on a deux bobines:

$$U_1 = R_1 i_1 + \frac{d\Psi_1}{dt} = R_1 i_1 + L_1 \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

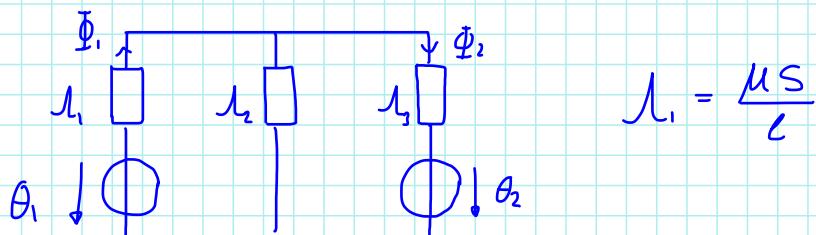
$$U_2 = R_2 i_2 + \frac{d\Psi_2}{dt} = R_2 i_2 + L_{21} \frac{di_2}{dt} + L_{12} \frac{di_1}{dt}$$

$$L_{12} = L_{21}$$



$$L_{11} = \frac{\Psi_1}{i_1}$$

$$L_{12} = \frac{\Psi_2}{i_1}$$



$$L_1 = \frac{\mu_r S}{L} \quad S = 100 \text{ mm}^2 = 10^{-4} \text{ m}^2$$

$$L = 105 \text{ mm}$$

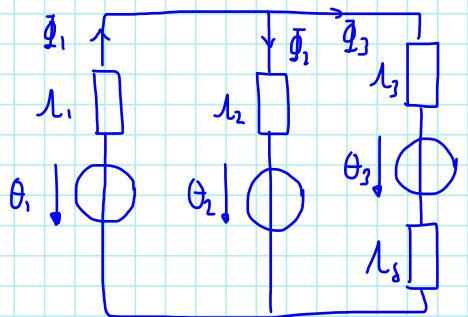
$$L_1 = \frac{250 \cdot 10^{-4} \cdot 4\pi \cdot 10^{-7}}{0,105} = 2,95 \cdot 10^{-7}$$

$$L_1 = \frac{\mu_s S}{c}$$

$$L_2 = \frac{250 \cdot 5 \cdot 10^{-5} \cdot 4\pi \cdot 10^{-7}}{0,05} = 3,14 \cdot 10^{-7}$$

$$\pi \cdot 10^{-7}$$

Exercice: Inductance propres & mutuelles



$$M_1 = \frac{\mu_0 \mu_0 S}{l} = \frac{250 \cdot 4\pi \cdot 10^{-7} \cdot 10 \cdot 10 \cdot 10^{-6}}{(50+27,5+27,5) \cdot 10^{-3}} = 2,99 \cdot 10^{-7} \text{ H}$$

$$M_2 = \frac{250 \cdot 4\pi \cdot 10^{-7} \cdot 10 \cdot 5 \cdot 10^{-6}}{50 \cdot 10^{-3}} = 3,14 \cdot 10^{-7} \text{ H}$$

$$M_3 = \frac{250 \cdot 4\pi \cdot 10^{-7} \cdot 10 \cdot 10 \cdot 10^{-6}}{104,5 \cdot 10^{-3}} = 3 \cdot 10^{-7} \text{ H}$$

$$M'_3 = \frac{L_3 L_8}{L_3 + L_8} = 1,37 \cdot 10^{-7} \text{ H}$$

$$M_8 = \frac{4\pi \cdot 10^{-7} \cdot 10 \cdot 10 \cdot 10^{-6}}{0,5 \cdot 10^{-3}} = 2,51 \cdot 10^{-7}$$

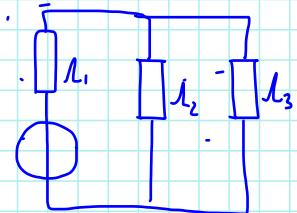
Inductances propres:

$$L_{11} = N_1^2 M_{11}$$

$$L_{22} = N_2^2 M_{22}$$

$$L_{33} = N_3^2 M_{33}$$

$$M_{11} :$$



$$M_{11} = \frac{L_1(L_2 + L_3)}{L_1 + L_2 + L_3} = 1,78 \cdot 10^{-7} \text{ H}$$

$$L_{11} = N_1^2 M_{11} = 178 \text{ mH} \checkmark$$

$$L_{11} = \frac{N_1 \Phi_{11}}{i_1} \quad \Phi_{11} = \Phi_1 M_{11} = \Phi_1 \frac{M_1 (M_2 + M_3)}{M_1 + M_2 + M_3}$$

$$L_{22} = N_2^2 \frac{L_2 (M_1 + M_3)}{M_1 + M_2 + M_3} = 46 \text{ mH} \checkmark$$

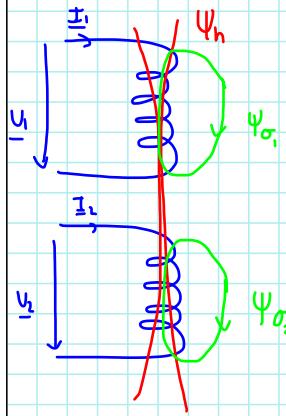
$$L_{33} = N_3^2 \frac{L_3 (M_1 + M_2)}{M_1 + M_2 + M_3} = 28 \text{ mH} \checkmark$$

Inductances mutuelles ?

$$L_{21} = \frac{U_2 \Phi_{21}}{i_1} = \frac{N_2}{i_1} \Phi_{11} \frac{M_2}{M_1 + M_3} = N_2 \frac{\Phi_1}{i_1} \frac{M_1 M_2}{M_1 + M_2 + M_3} = N_2 U_1 \underbrace{\frac{M_1 M_2}{M_1 + M_2 + M_3}}_{L_{12}} = 62,6 \text{ mH} \checkmark$$

$$L_{31} = \frac{N_3 \Phi_{31}}{i_1} = \frac{N_3}{i_1} \Phi_{11} \frac{M_3}{M_1 + M_3} = N_3 \frac{\Phi_1}{i_1} \frac{M_1 M_3}{M_1 + M_2 + M_3} = N_3 U_1 \underbrace{\frac{M_1 M_3}{M_1 + M_2 + M_3}}_{L_{13}} = 27,3 \text{ mH} \checkmark$$

$$L_{23} = \frac{N_2 \Phi_{23}}{i_3} = \frac{N_2}{i_3} \Phi_{22} \frac{M_3}{M_1 + M_3} = N_2 N_3 \frac{M_2 M_3}{M_1 + M_2 + M_3} = 14,3 \text{ mH} \checkmark$$



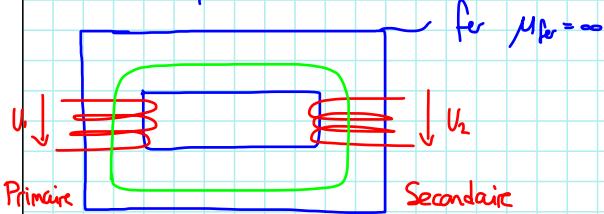
$$\Psi_h = L_h (i_1 + i_2)$$

$$\Psi_{\sigma_1} = L_{\sigma_1} i_1$$

$$\Psi_{\sigma_2} = L_{\sigma_2} i_2$$

1.10 Transformateur

a) Transformateur idéal:



$$Hyp: -M_{fe} = \infty$$

- pas de fuite
- $R_1, R_2 = 0$

$$U_1 = R_i i_1 + \frac{d\Psi_1}{dt}$$

$$U_1 = \frac{d\Psi_1}{dt} \quad U_2 = \frac{d\Psi_2}{dt}$$

$$\Psi_1 = L_h (N_1 i_1 + N_2 i_2) \quad N_1 = \Phi_1 \cdot M$$

$$\Psi_2 = \Phi_2 \cdot N_2 = N_2 \cdot \Phi_1 = N_2 \Phi$$

$$\frac{U_1}{U_2} = \frac{d\Phi/dt}{d\Phi/dt} \frac{N_1}{N_2} = \frac{N_1}{N_2} \quad (\text{seulement puisque } R_1 = R_2 = 0)$$

$$\oint H dl = \sum V \cdot i = 0 \quad H = \frac{B}{\mu} \underset{R_\infty}{\approx} 0$$

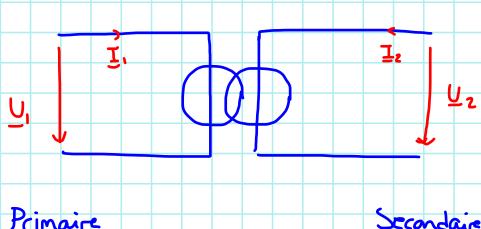
$$N_1 i_1 - N_2 i_2 = 0$$

$$\Rightarrow \frac{i_1}{i_2} = \frac{N_2}{N_1}$$

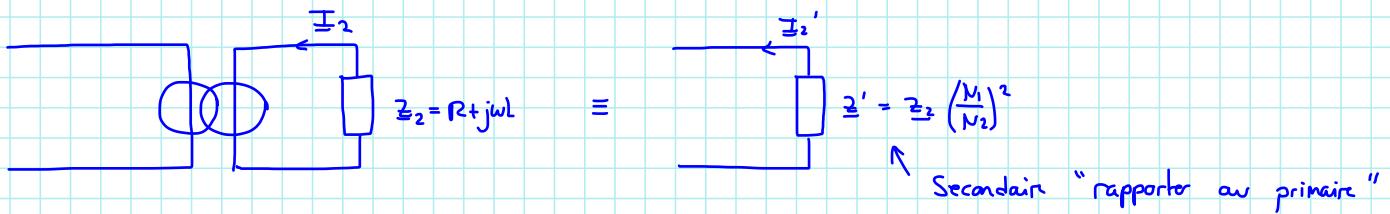
Cas particulier: Sinus

$$\frac{U_1}{U_2} = \frac{N_1}{N_2} \quad \frac{I_1}{I_2} = \frac{N_2}{N_1} \quad \frac{U_2}{I_2} = Z_2 \rightarrow \frac{U_1}{I_1} = \frac{U_2 N_1 / N_2}{I_2 N_2 / N_1} = Z_1 = Z_2 \left(\frac{N_1}{N_2} \right)^2$$

Schéma du transfo idéal:



| |
|--|
| $\frac{U_1}{U_2} = \frac{N_1}{N_2}$ $\frac{I_2}{I_1} = \frac{N_1}{N_2}$ $\frac{Z_1}{Z_2} = \left(\frac{N_1}{N_2} \right)^2$ |
|--|



b) Transformateur réel: Plus d'hypothèse!

$$U_1 = R_1 I_1 + \frac{d}{dt} [N_1^2 L_h i_1 + N_1 N_2 L_h i_2 + N_1^2 L_{\sigma} i_1]$$

$$U_1 = R_1 i_1 + N_1^2 L_h \left[\frac{di_1}{dt} + \left(\frac{N_2}{N_1} \right) \frac{di_2}{dt} \right] + N_1^2 L_{\sigma} \frac{di_1}{dt}$$

↓ complexe

$$U_1 = R_1 I_1 + j\omega N_1^2 L_h (I_1 + I_2') + j\omega N_1^2 L_{\sigma} I_1$$

X_h X_{σ}

$$U_2 = R_2 I_2 + j\omega N_2^2 L_h \left(\frac{N_2}{N_1} I_1 + \left(\frac{N_2}{N_1} \right)^2 I_2 \right) + j\omega N_2^2 L_{\sigma_2} I_2$$

$$\downarrow \cdot \frac{N_1}{N_2}$$

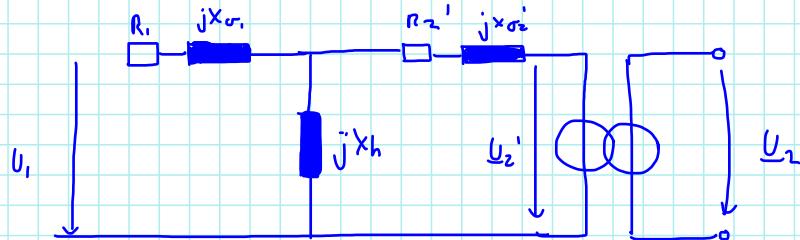
$$U_2' = R_2 \frac{N_2^2}{N_1^2} I_2 + j X_h \left(I_1 + I_2' \right) + j X_{\sigma_2} I_2$$

R_2' I_2' X_h X_{σ_2}' I_2'

$$\text{Donc: } U_1 = R_1 I_1 + j X_h (I_1 + I_2') + j X_{\sigma_1} I_1$$

$$U_2' = R_2' I_2' + j X_h (I_1 + I_2') + j X_{\sigma_2}' I_2'$$

Schéma:



1.1 Pertes fer

(Pertes se transforment en énergie active \rightarrow chaleur!)

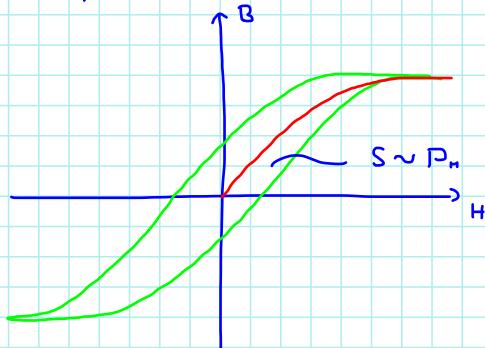
- Pertes par courant de Foucault

$$P_F \sim mf \quad (\text{masse du fer})$$

$$P_F \sim f^2 \quad (\text{fréquence})$$

$$P_F \sim B^2 \quad (\text{champs magnétique})$$

- Pertes par hystérésis



$$P_H \sim mf$$

$$P_H \sim f$$

$$P_H \sim B^2$$

Eq. Empirique tenant compte des deux phénomènes:

$$P_{fer} = C_{fer} \cdot \left(\frac{f}{f_0} \right)^n \cdot \left(\frac{B}{B_0} \right)^2 \cdot mf$$

↑
coeffcient
chiffre de perte
0,1 - 5 w/kg (Par f, B₀)

2. Conversion Électro-mécanique:

2.1. Propriétés

a) rendement élevé

b) Réversibilité

c) Fiabilité - durée de vie

d) Gamme de Puissance: $\mu\text{W} \rightarrow \text{GW}$

caractéristiques

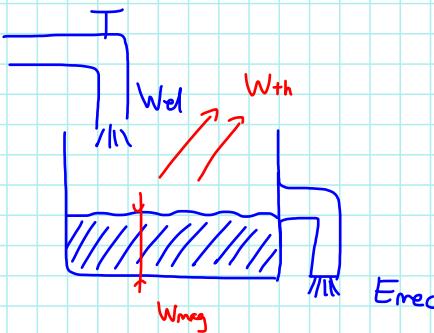
a) Interaction entre 2 courants

b) Interaction entre 1 courant & 1 circuit magnétique

c) Interaction entre 1 aimant & 1 circuit magnétique ou courant

L'expérience montre qu'il existe une forme intermédiaire de l'énergie \rightarrow E. magnétique

2.2. Bilan Énergétique:



$$dW_{el} = dW_{mec} + dW_{th} + dW_{mag}$$

Hyp: 1 degré de liberté dans l'axe x , 1 bobine

$$dW_{mec} = F_x \cdot dx$$

$$dW_{th} = R \cdot i^2 dt$$

$$dW_{el} = U \cdot i \cdot dt$$

$$(a) U \cdot i \cdot dt = F_x \cdot dx + R \cdot i^2 \cdot dt + dW_{mag}$$

$$(b) U = R \cdot i + \frac{d\psi}{dt}$$

$$\cancel{Ri^2 dt} + i \frac{d\psi}{dt} dt = F_x dx + \cancel{Ri^2 dt} + dW_{mag}$$

$$i d\psi = F_x dx + dW_{mag}$$

$$W_{mag} = W_{mag_1}(\psi, x) + W_{mag_2}(i, x)$$

$$dW_{mag} = \frac{\partial W_{mag}}{\partial \psi} \cdot d\psi + \frac{\partial W_{mag}}{\partial x} \cdot dx$$

$$\underline{i} \underline{d\psi} = \underline{F_x dx} + \underline{\frac{\partial W_{mag}}{\partial \psi} d\psi} + \underline{\frac{\partial W_{mag}}{\partial x} dx}$$

$$\left(\underline{i} - \underline{\frac{\partial W_{mag}}{\partial \psi}} \right) d\psi - \left(\underline{F_x} + \underline{\frac{\partial W_{mag}}{\partial x}} \right) dx = 0$$

Puisque ψ et x sont indépendants \Rightarrow chaque terme $= 0$

$$i = \frac{\partial W_{mag}}{\partial \psi}$$

$$i = \frac{dW_{mag}}{d\psi} \Big|_{x=cte}$$

$$F_x = - \frac{\partial W_{mag}}{\partial x}$$

$$F_x = - \frac{dW_{mag}}{dx} \Big|_{\psi=cte}$$

Un bilan d'énergie

$$i = \frac{dW_{mag}}{d\psi} \Big|_{x=\text{cste}}$$

$$F_x = - \frac{dW_{mag}}{dx} \Big|_{\psi=\text{cste}}$$

cas particulier: cas linéaire $\rightarrow \mu = \text{cste}$

$$\Psi = L i \quad dW_{mag} = i d\Psi$$

$$W_{mag} = \int_0^L L i di = L \frac{i^2}{2} = \frac{1}{2} \Psi i = \frac{1}{2} \frac{\Psi^2}{L}$$

2.3. Calcul d'une force, couple:

a) Non Linéaire :

$$dW_{mag} = i d\Psi$$

$$W_{mag} = \int_0^\Psi i d\Psi$$

$$F_x = - \frac{dW_{mag}}{dx} \Big|_{\psi=\text{cste}} = - \frac{d}{dx} \left[\int_0^\Psi i d\Psi \right] \Big|_{\psi=\text{cste}}$$

b) Cas linéaire

$$W_{mag} = \frac{1}{2} \frac{\Psi^2}{L}$$

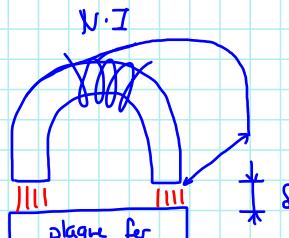
$$F_x = - \frac{1}{2} \frac{d(\frac{\Psi^2}{L})}{dx} \Big|_{\psi=\text{cste}} = - \frac{1}{2} \Psi^2 \frac{d \frac{1}{L}}{dx} = - \frac{1}{2} \Psi^2 \left(-\frac{1}{L^2} \right) \frac{dL}{dx} \quad (\Psi^2 = L^2 \cdot i^2)$$

$$= \underline{\underline{\frac{1}{2} \frac{dL}{dx} i^2}}$$

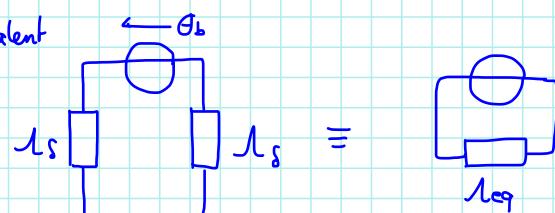
2.4. Exemple et Démarche

a) Poser les hypothèses

- pas de fuite
- $\mu_{air} = \infty$



b) Schéma équivalent



c) calcul des pertinences

$$\lambda_s = \frac{\mu \cdot S}{l} = \frac{\mu_0 \cdot a \cdot b}{s} \quad \lambda_{eq} = \frac{\mu_0 \cdot a \cdot b}{2s} = \frac{1}{2} \lambda_s \quad L_{prop} = L_s + L \cdot N^2 \lambda_{eq} = \frac{N^2 \mu_0 a \cdot b}{2s}$$

d) Calcul de la force:

$$F_x = \frac{1}{2} \frac{dL}{dx} i^2 \longrightarrow F_s = \frac{1}{2} \frac{dL}{ds} \cdot i^2$$

$$= -\frac{1}{4} N^2 \frac{\mu_0 ab}{s^2} i^2$$

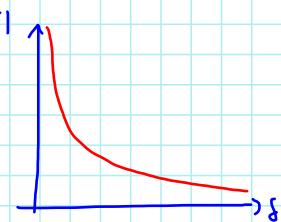
Si $s=0 \Rightarrow F_s \rightarrow \infty !!$

Impossible \rightarrow do aux hypothèses

Force: $F_x = \frac{1}{2} \frac{dL}{dx} i^2$

Couple: $M = \frac{1}{2} \frac{dL}{da} i^2 = L N^2 \cdot L$

$$= \frac{1}{2} \frac{dL}{da} \theta^2$$



Rappel: Dérivée de l'énergie.

$$F_x = - \frac{dW_{mag}}{dx} \Big|_{\psi=k}$$

↪ 1 bobine, 1 direction (x):

$$F_x = \frac{1}{2} \frac{dL}{dx} i^2 = \frac{1}{2} \frac{dL}{dx} \Theta_b^2$$

$$M = \frac{1}{2} \frac{dL}{dx} i^2 = \frac{1}{2} \frac{dL}{dx} \Theta_b^2$$

2.5. Généralisation: Plusieurs bobines

$$F_x = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{dL_{jp}}{dx} i_j \cdot i_p$$

Ex avec 2 bobines :

$$F_x = \frac{1}{2} \frac{dL}{dx} i_1^2 + 2 \cdot \frac{1}{2} \underbrace{\frac{dL_{12}}{dx} i_1 \cdot i_2 + \frac{1}{2} \frac{dL_{22}}{dx} i_2^2}$$

$$M_x = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{dL_{jp}}{dx} i_j i_p = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{dL_{jp}}{dx} \Theta_i \Theta_p$$

2.6. Forme locale de l'Energie Magnétique

$$dW_{mag} = i d\psi = N i d\Phi = \Theta d\bar{\Phi}$$

$$\Theta = \int H d\ell$$

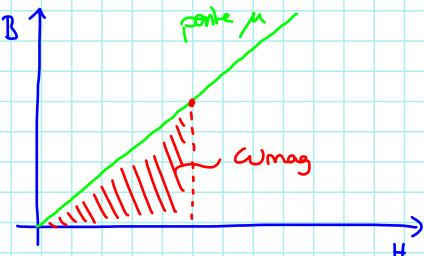
$$d\bar{\Phi} = \int dB \cdot ds$$

$$dW_{mag} = \int_V \underbrace{H d\bar{B}}_{dW_{mag}} dv$$

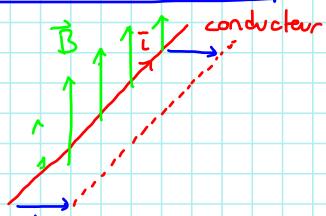
$$dW_{mag} = HdB [J/m^3] \quad \text{densité d'énergie magnétique}$$

$$w_{mag} = \int_B H d\bar{B} = \mu \int_H H d\bar{H} = \frac{1}{2} \mu H^2 \quad (B = \mu H)$$

linéaire



2.7 Force de Laplace:



Interaction entre un conducteur
et un champs externe B

$$d\bar{\Phi} = B \cdot \ell \cdot dx$$

$$dW_{mag} = i d\psi = i d\bar{\Phi} = i B \ell dx$$

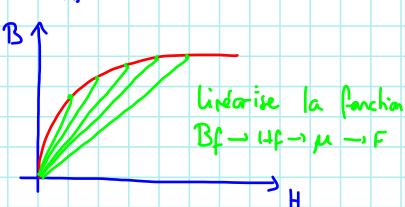
$$F_x = - \frac{dW_{mag}}{dx} = -i L B \quad \text{② due aux référentiels}$$

$$\text{Laplace: } \vec{F} = i \vec{\ell} \times \vec{B} \quad \text{i.e.: } F = 0 \text{ si } \vec{B} = 0$$

2.8. Systèmes non linéaire

$$F_x = - \frac{dW_{mag}}{dx} \Big|_{\mu_{rach}}$$

cas où $\mu_f \neq \mu_r$:



Décomposons l'E. Magnétique:

$$W_{mag} = W_{mag_{for}} + W_{mag_s}$$

$$\hookrightarrow f(B_f, H_f)$$

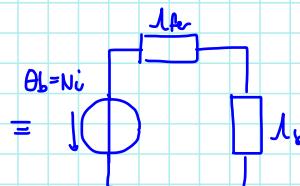
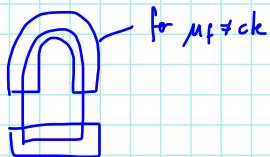
$$F_x = - \frac{dW_{mag}}{dx} = - \underbrace{\frac{dW_{mag_{for}}}{dx}}_0 - \underbrace{\frac{dW_{mag_s}}{dx}}_{(\neq 0)}$$

Sur volume indéformable

$$F_x = - \frac{dW_{mag}}{dx} \Big|_{\mu=\mu_r} = \frac{1}{2} \frac{dL_s}{dx} \cdot \theta_b^2$$

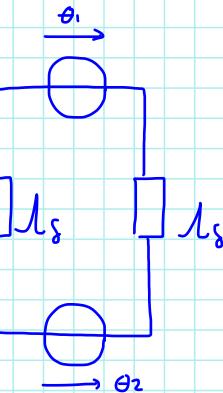
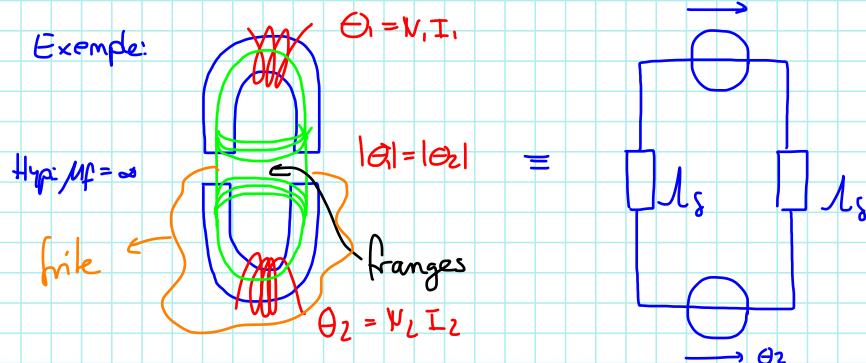
~~$$F_x = \frac{1}{2} \frac{dL}{dx} \Rightarrow \frac{1}{2} \frac{dL_{eq}}{dx} \theta_b^2$$~~

$$F_x = \frac{1}{2} \frac{dL_s}{dx} \theta_b^2$$



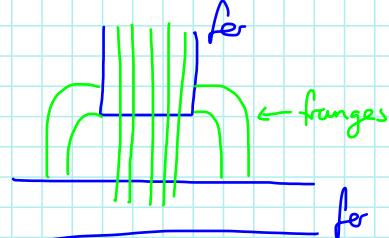
2.9. Franges & Fuites

Exemple:

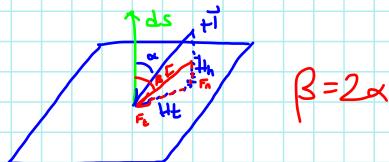


Franges: lignes de champs qui participent à la Force

Fuites: lignes de champs qui ne participent pas à la Force

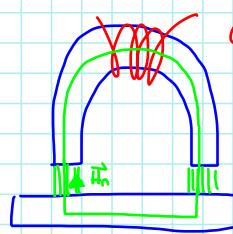
En Général:2.10. Tenseur de Maxwell

Tenseur → Matrice → Matrice des pression magnétiques.



$$\text{Définition: } \frac{dF_n}{ds} = \frac{1}{2} \mu_0 (H_n^2 - H_t^2)$$

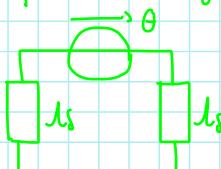
$$\cdot \frac{dF_t}{ds} = \mu_0 H_n H_t$$

Exemple

$$\Theta = NI$$

$$\text{Hyp: } M_f = \infty$$

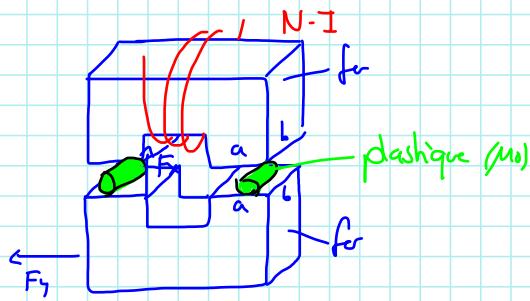
pas de franges/ fuites

Puisque franges & franges sont négligées $\Rightarrow H_t = 0$

$$\oint H_d l = \Theta = NI = 2S H_n \Rightarrow H_n = \frac{NI}{2S}$$

$$F_n = \frac{1}{2} \mu_0 H_n^2 \cdot 2 \cdot S_s = \frac{\mu_0 S_s}{4\delta^2} \cdot N^2 I^2$$

2.11. Autre exemple et comparaison des méthodes



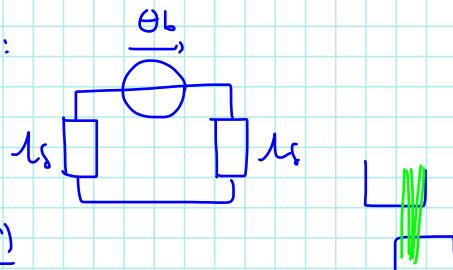
Hypothèses: $\mu_f = \infty$

pas de fuites/ Franges

lignes de champs + \vec{B}

a) Dérivée de l'énergie:

- Dessin, schéma

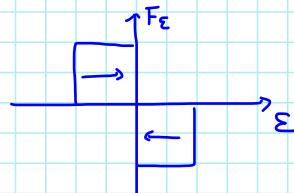


$$1_S = \frac{\mu_0 S}{\delta} = \frac{\mu_0 b(a-\varepsilon)}{\delta}$$

$$1_{eq} = \frac{\mu_0 b(a-\varepsilon)}{2\delta}$$

$$\left| \begin{array}{l} F_E = \frac{1}{2} \frac{d1_{eq}}{d\varepsilon} \theta_b^2 = -\frac{1}{2} \theta_b^2 \frac{\mu_0 b}{2\delta} \\ F_S = \frac{1}{2} \frac{d1_{eq}}{dS} \theta_b^2 = -\frac{1}{4} \frac{\mu_0 b(a-\varepsilon)}{\delta^2} \theta_b^2 \end{array} \right. \quad (0 \leq \varepsilon \leq a)$$

La Force selon ε est une force de rappel:



2^e Méthode: Tenseur de Maxwell:

$$\rightarrow H_z = 0$$

$$T_S = \frac{1}{2} \mu_0 H_n^2 \cdot 2 \cdot S_S$$

$$H_n = \frac{NI}{2\delta} \quad S_S = b(a-\varepsilon)$$

$$F_S = \frac{\mu_0 b(a-\varepsilon)}{4\delta^2} N \cdot I^2$$

$$F_E = \mu_0 H_n \cdot H_E = 0$$

Problème: Si on fait l'hypothèse des franges nulles, on obtient toujours une force $F_E = 0$!!

Dérivée de l'énergie: prend pas en compte l'E totale du système \rightarrow plus robuste

Maxwell : Distribution locale de E \rightarrow ok par les T_n , ΔT_E !!

2.12. Ordres de grandeur :

Comparaison de Pressions

- Hydraulique 400 bar
- Magnétique 1 T
- Electrostatique $3 \cdot 10^6$ V/m

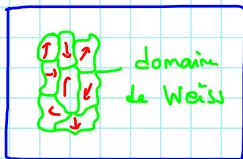
$$P_{\text{hydraulique}} = 400 \text{ bar} = 4 \cdot 10^7 \text{ N/m}^2$$

$$P_{\text{magnétique}} = \frac{1}{2} \mu_0 H^2 = \frac{1}{2} \frac{B^2}{\mu_0} = 4 \cdot 10^8 \text{ V/m}^2$$

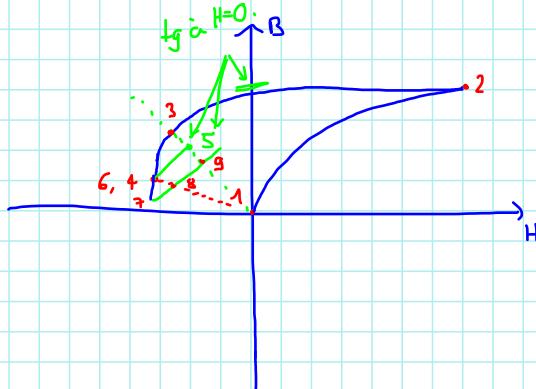
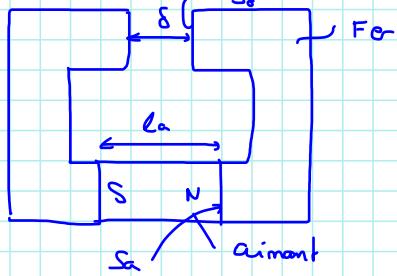
$$P_{\text{electrostatique}} = \frac{1}{2} \sum E^2 = 40 \text{ N/m}^2$$

(ck) $8,13 \cdot 10^{-12}$

3.1. Aimants permanents:



3.1. Aimant permanent:



1. aimant non magnétisé
2. Plonge l'aimant dans champs magn. Hextone

$$3. H_{extone} = 0, \delta = \delta_1$$

$$4. \delta = \delta_2 > \delta_1$$

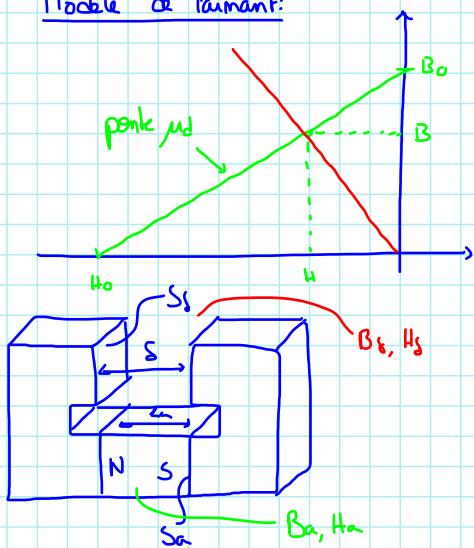
$$5. \delta = \delta_1$$

$$6. \delta = \delta_2$$

$$7. \delta = \delta_3 > \delta_2$$

$$8. \delta = \delta_2$$

$$9. \delta = \delta_1$$

Modèle de l'aimant:Droite de retour:

$$B = B_0 + \mu_d \cdot H \quad (\Delta H < 0)$$

$$\mu_d = \frac{B_0}{H_0}$$

$$\mu_{dr} = \frac{\mu_d}{\mu_0} \dots [1.05 \dots 3]$$

Hyp: pas de frotte
pas de franges
 $\mu_{fr} = \infty$

$$\bar{\Phi}_a = \bar{\Phi}_c \Rightarrow B_a \cdot S_a = B_c \cdot S_c$$

$$B_a \cdot S_a = \mu_0 H_a S_c \quad (1)$$

$$H_a = \frac{B_a S_c}{\mu_0 S_c}$$

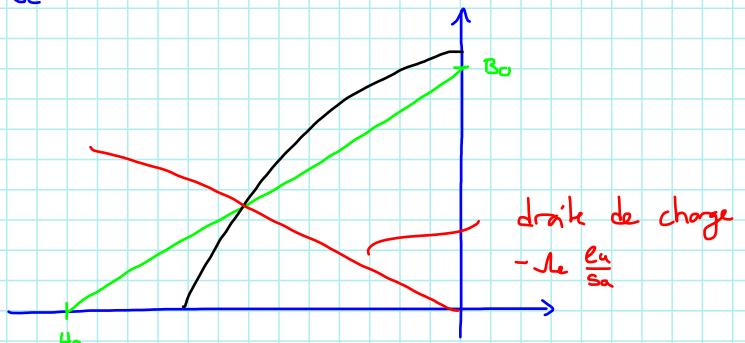
$$\oint H_a dL = \Theta_{lob} = 0$$

$$H_a \cdot L_a + H_c \cdot L_c = 0$$

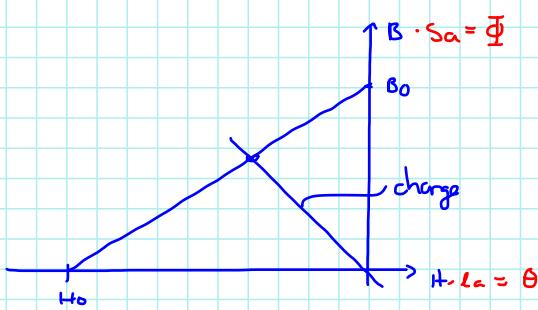
$$H_a \cdot L_a + \frac{B_a S_c}{\mu_0 S_c} L_c = 0$$

$$\frac{D_a}{H_a} = - \frac{\mu_0 S_c}{L_a} \cdot \frac{L_a}{S_a} = - \lambda_a \cdot \frac{L_a}{S_a}$$

$$\lambda_a$$



3.2. Modèle de l'aimant:



$$R_{mi} = \frac{U_0}{I_{cc}}$$

$$R_{mi} = \frac{\theta_{ao}}{\Phi_{ao}} = \frac{H_0 L_a}{B_0 S_a} = \frac{L_a}{\mu_0 S_a} \rightarrow \frac{1}{R_{mi}} = \lambda_i = \frac{\mu_0 S_a}{L_a}$$

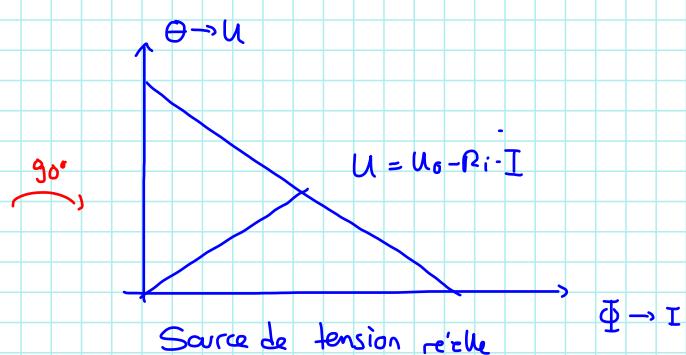
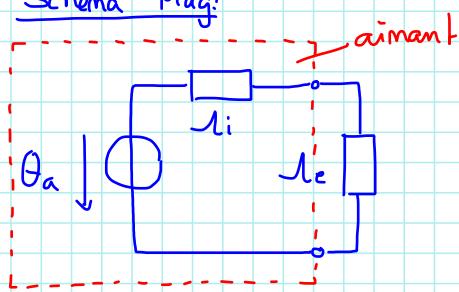
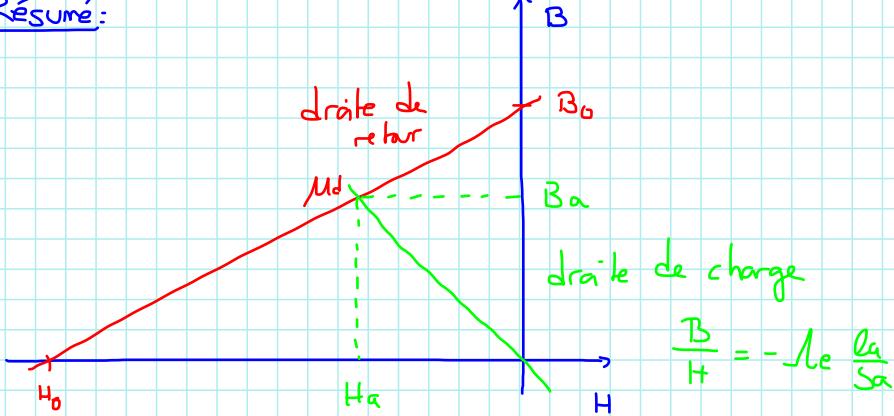
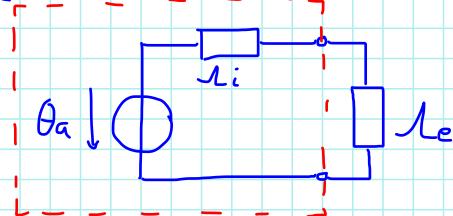


Schéma Mag:





Modèle de l'aimant:



$$M_i = \frac{M_d S_a}{l_a}$$

$$G_a = l_a \cdot I_a \quad (\text{source idéale})$$

Exemple: SmCo (Samarium Cobalt) $B_0 = 0,7$ $\mu_{fr} = 1,1$

$$H_0 = -620 \cdot 10^3 \text{ A/m}$$

$$l_a = 7 \text{ mm}$$

$$\theta_a = 4340 \text{ A}$$



Si on voulait remplacer θ_a par une bobine:

$$\theta_b = \theta_a = N \cdot I = 4340 \text{ spires} / 1 \text{ A}$$

3.3. Calcul d'une force:

$$F_x = - \left. \frac{dW_{mag}}{dx} \right|_{\psi=\text{ct}}$$

$$\begin{aligned} \text{Si 1 dag. de lib. et 1 bob} \Rightarrow F_x &= \frac{1}{2} \frac{dL}{dx} i^2 \\ &= \frac{1}{2} \frac{dL}{dx} \theta_b^2 \end{aligned}$$

\Rightarrow Par analogie $\rightarrow \theta_b \rightarrow \theta_a$

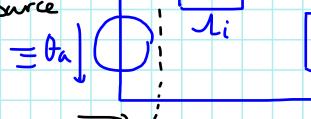
$I \rightarrow I_a$ perméance virtuelle des bornes de la source

$$F_x = \frac{1}{2} \frac{dI_a}{dx} \theta_a^2$$



Hyp: $\mu_f = \infty$, pas de fuite/franges

se que vient
la source



$$\begin{aligned} M_i &= \frac{M_d S_a}{l_a} \\ I_a &= \frac{M_d S_a}{2x} \end{aligned}$$

$$I_a = \frac{M_d S_a}{\mu_e + M_i}$$

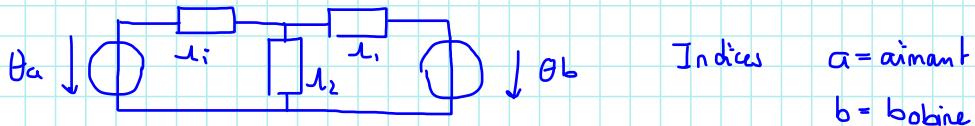
$$F_x = \frac{1}{2} \frac{dM_a}{dx} \Theta_a^2$$

Astuces: $\frac{dM_a}{dx} = \frac{dM_a}{dI_a} \cdot \frac{dI_a}{dx}$

$$\frac{dM_a}{dI_a} = \frac{\mu_i^2}{(\mu_i + \mu_e)^2}$$

$$\Rightarrow F_x = \frac{1}{2} \frac{\mu_i^2}{(\mu_i + \mu_e)^2} \cdot \frac{dM_e}{dx} \cdot \Theta_a^2$$

Force d'un aimant avec 1 bobine!



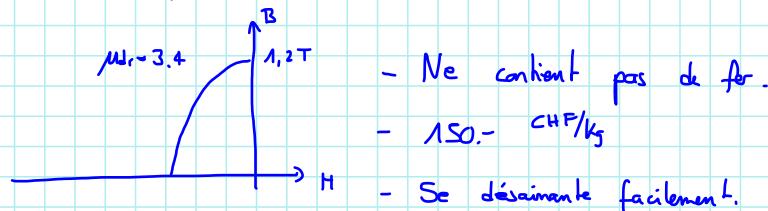
Définition $M_{ab} = \frac{\Theta_{ba}}{\Theta_a}$ ← Flux traversé par la bobine creé par l'aimant

$$F_x = \frac{1}{2} \frac{dM_a}{dx} \Theta_a^2 + \frac{1}{2} \frac{dM_b}{dx} \Theta_b^2 + \frac{dM_{ab}}{dx} \Theta_a \Theta_b$$

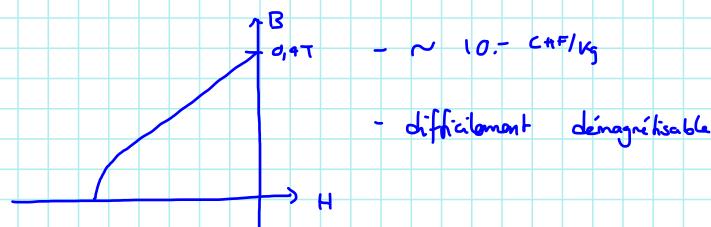
(Pas à l'exa de Juin)

3.4. Types d'aimant

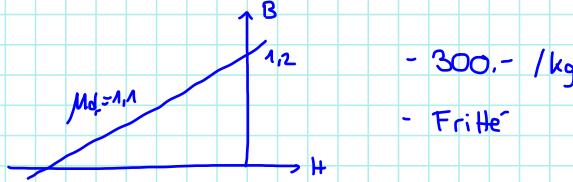
a) AlNiCo, TiCoNAl



b) Ferrites (Oxydes de Fer)

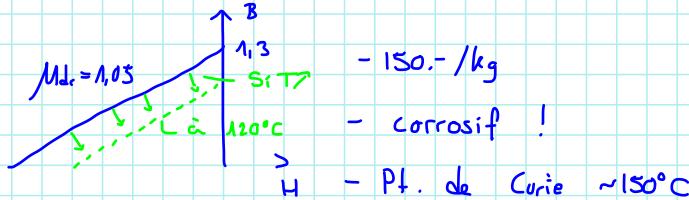


3. SmCo (Samarium Cobalt)



- 300.- / kg
- Fritte'

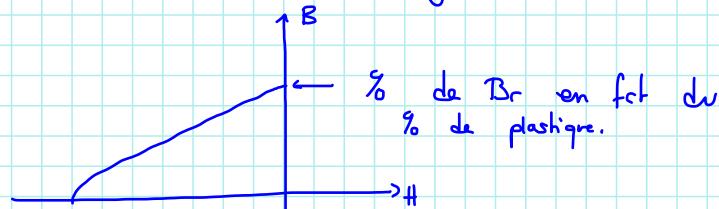
4. NdFeB (Néodyme Fer Bore)



- 150.- / kg
- corrosif !
- Pt. de Curie $\sim 150^\circ\text{C}$

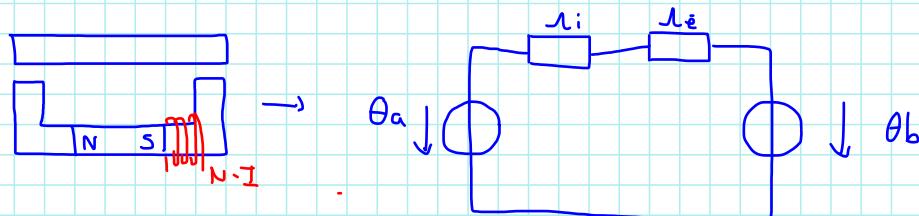
5. Poudre + Plastique \rightarrow aimants lié (avec 3 et 4)

Si Matériau fritte' \rightarrow fragile



! Different du livre!

3.5. Aimants en présence d'une bobine en série dans le circuit magnétique:



$$\oint H \, dl = N \cdot I \quad H_a \cdot l_a + H_b \cdot l_b = N \cdot I$$

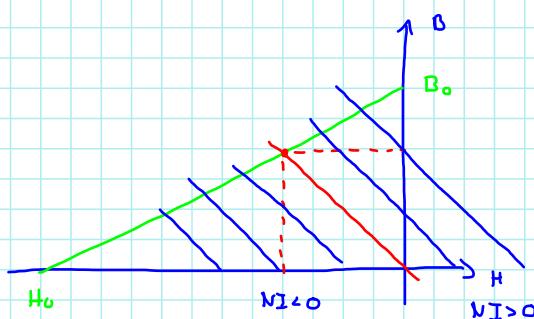
$$B_a \cdot S_a = B_b \cdot S_b$$

$$H_a = \frac{B_a}{M_o} = \frac{B_a \cdot S_a}{M_o \cdot S_a}$$

$$H_a \cdot l_a + \frac{B_a \cdot S_a \cdot l_b}{M_o \cdot S_a} = N \cdot I$$

Rappel:

$$\text{Aimant seul: } \frac{B_a}{H_a} = -l_a \frac{l_a}{S_a} \quad B_a = N \cdot I \frac{l_a}{S_a} - H_a \frac{l_a}{S_a} \cdot l_b$$



Chapitre 5: Comportement dynamiques:

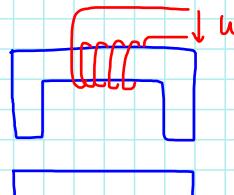
5.1. Equations

$$\text{Tension: } U = Ri + \frac{d\psi}{dt}$$

$$\text{Mouvement: } \sum F = m \cdot a$$

$$\sum M = J \cdot \frac{d^2\alpha}{dt^2}$$

5.2. Système sans aimant à 1 bobine:



$$\Psi = L \cdot i \quad \text{a) } L = \text{cte} \rightarrow \text{pas de mvt}$$

$$L = N^2 \cdot \mu_s \cdot \frac{A}{s}$$

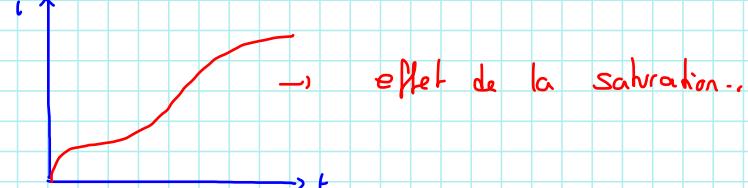
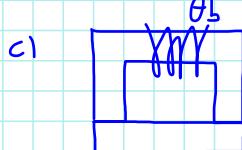
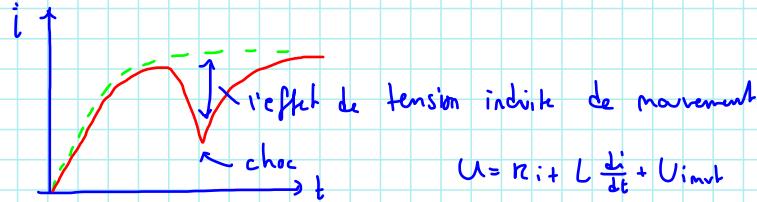
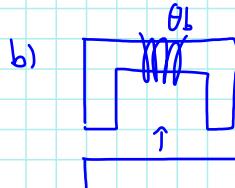
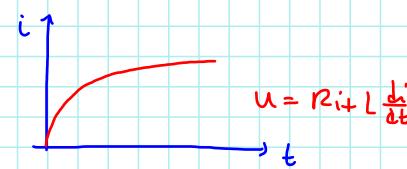
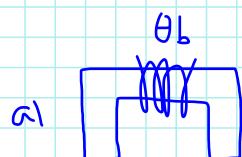
b) Si mvt, L et i qui varient!

$$\begin{aligned} U &= R \cdot i + \frac{d\psi}{dt} = R \cdot i + \frac{1}{dt} (L \cdot i) = R \cdot i + L \frac{di}{dt} + i \frac{dL}{dt} \\ &= R \cdot i + L \frac{di}{dt} + i \frac{dL}{dx} \cdot \frac{dx}{dt} \end{aligned}$$

$$\text{Rappel: } F = \frac{1}{2} \frac{dL}{dx} \cdot i^2 \Leftrightarrow \frac{dL}{dx} = \frac{2F}{i^2}$$

$$\Rightarrow U = R \cdot i + L \frac{di}{dt} + \underbrace{\frac{2F}{i^2} \cdot V}_{>0 \text{ tension induite de mouvement.}}$$

Expérience:



$\rightarrow L(x) \leftarrow$ juste si on néglige la saturation $L = N^2 \cdot \frac{\mu_s \cdot A}{s}$

$L(x, i)$ en réalité! tension induite de transformateur tension induite de mouvement Tension induite de saturation

$$\text{Résumé: } U = R \cdot i + \frac{d\psi}{dt} = R \cdot i + L \frac{di}{dt} + i \frac{dL}{dx} V + i \frac{dL}{di} \frac{di}{dt}$$

$$\Leftrightarrow \frac{d\psi}{dt}$$

RESUME

28.05.13

1. Général Maxwell

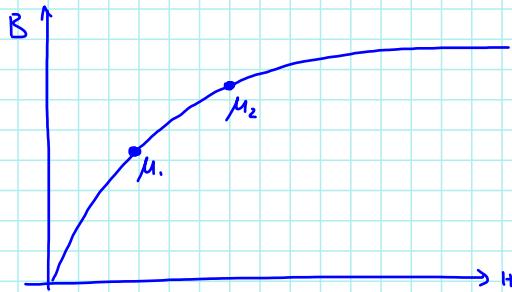
$$\operatorname{rot} \vec{H} = \vec{J}$$

$$\operatorname{rot} \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\operatorname{div} \vec{B} = 0$$

$$B = \mu H \quad \mu_0 = 4\pi \cdot 10^{-7}$$

Saturation

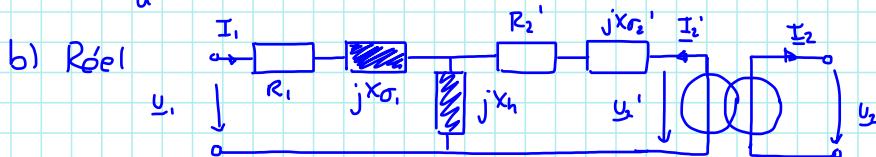
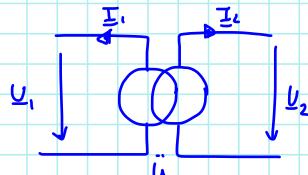


$$\mu_1 \neq \mu_2 \quad \mu = \frac{B}{H}$$

- $\operatorname{div} \vec{B} = 0 \Rightarrow \Psi = \text{cte} = \int \vec{B} ds$
- Loi de la circulation: $\Theta = \oint H dl = N \cdot i$
- $L = \frac{1}{R} = \int_s \frac{\mu ds}{l} \quad \text{Si } S = \text{cte} \quad L = \frac{\mu S}{l}$
- $\Theta = R \Phi \quad \text{où} \quad L \theta = \Phi$
- Kirchoff: $\sum H dl = 0 \quad (\text{Maille})$
 $\sum \Phi = 0 \quad (\text{Nœuds})$
- En série: $\lambda_{\text{eq}} = \frac{1}{\sum_i 1/R_i} \quad R_{\text{eq}} = \sum R_{\text{mi}}$
- En parallèle: $\lambda_{\text{eq}} = \sum \lambda_i \quad R_{\text{eq}} = \frac{1}{\sum_i 1/R_{\text{mi}}}$
- Flux totalisé $\Psi = N \Phi$
- $U = R_i + \frac{d\Psi}{dt} \quad \text{eq. de tension}$
- $L_{ii} = \Psi''/i, \quad L_{12} = \Psi_{12}/i_1$

2. Le Transformateur

a) Idéal: $\frac{U_1}{U_2} = \frac{N_1}{N_2} = \ddot{u} \quad \frac{I_2}{I_1} = \ddot{u} \quad \frac{Z_1}{Z_2} = \ddot{u}^2$! aux indices!



$$R_2' = R_2 \cdot \ddot{u}^2$$

$$X_{\sigma_2'} = X_{\sigma_2} \cdot \ddot{u}^2$$

• Pertes Fer : - Hystéresse

- Courants de Foucault

$$P_{\text{Fer}} = C_{\text{Fer}} \left(\frac{f}{f_0} \right)^n \left(\frac{B}{B_0} \right)^2 \cdot n_F$$

3. Conversion:

$$i = \frac{dW_{\text{mag}}}{d\psi} \Big|_{x=\text{cte}} \quad F_x = - \frac{dW_{\text{mag}}}{dx} \Big|_{\psi=\text{cte}}$$

$$W_{\text{mag}} = \int_0^i L \cdot i \cdot di \quad \text{Si linéaire : } W_{\text{mag}} = \frac{1}{2} L i^2 = \frac{1}{2} \psi i$$

• Force ou couple

$$F = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{dL_{jp}}{dx} i_j \cdot i_p \quad k = \text{nb de bobines}$$

$$M = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{dL_{jp}}{dx} i_j \cdot i_p$$

• Laplace : $d\vec{F} = i d\vec{l} \times \vec{B}$ (\vec{B} et i sont indép)

$$F = i l B \quad (\text{si } d\vec{l} \perp \vec{B})$$

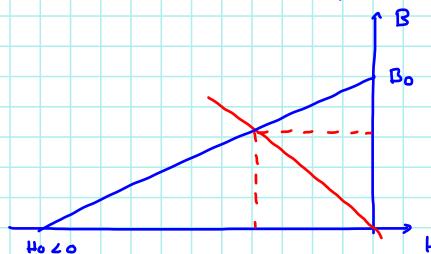
• Tenseur de Maxwell

$$dF_n = \frac{1}{2} \mu_0 [H_n^2 - H_t^2] ds \quad (\text{normale})$$

$$dF_t = \mu_0 H_n \cdot H_t \cdot ds \quad (\text{tangentielle})$$

4. Aimant permanent

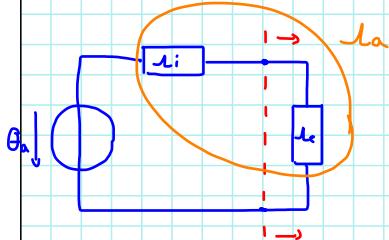
$$\mu_d = B_0/H_0 \quad \mu_{dr} = \frac{\mu_d}{\mu_0}$$



$$\text{droite de charge: } \frac{B_a}{H_a} = -\mu_d \frac{L_a}{S_a}$$

$$\text{droite de retour: } B = B_0 + \mu_d \cdot H$$

μ_e : perméance ext. à l'aimant (sans μ_d)



$$\text{Force} \rightarrow F = \frac{1}{2} \frac{dI_a}{dx} \theta_a^2$$

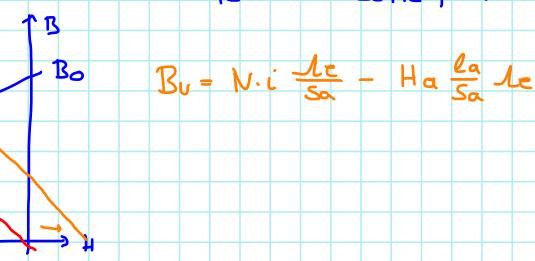
θ_a : perméance due à la source de l'aimant (avec μ_d)

θ_{ab} : perméance mutuelle aimant-bobine

$$= \frac{\Phi_{ba}}{\theta_a} \Big|_{\theta_a=0, \theta_b=0}$$

Si on a un aimant et une bobine en série, la droite de charge change :

$$B_u = N \cdot i \frac{\mu_e}{S_a} - H_a \frac{\theta_a}{S_a} \mu_e$$



5. Comportement dynamique:

$$\bullet U = R \cdot i + \frac{d\psi}{dt}$$

$$\bullet \sum F = ma \quad \text{ou} \quad \sum M = J \ddot{\omega}$$

$$U = R i + \underbrace{L \frac{di}{dt}}_{\text{transformation}} + i \underbrace{\frac{dL}{dx} \cdot \frac{dx}{dt}}_{\text{mouvement}} + i \underbrace{\frac{d}{di} \frac{di}{dt}}_{\text{Saturation}}$$

$\frac{d\psi}{dt}$