

# Static stability calculation of Małgosia II aircraft with use of MSC.Nastran software

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## 1. Abstract

This paper deals with static stability analysis of Małgosia II aircraft. MSC.Nastran 2013.1 is used to calculate aerodynamic coefficients which describe dynamic behavior of an aircraft. Also the method for calculating correction coefficients used in static aeroelastic solution will be discussed.

Presented method of calculation correction coefficients can be used to calculate aerodynamic coefficients in the earliest step of conceptual design without use of time-consuming CFD calculations and wind tunnel tests.

The calculated results will also be presented.

## 2. Introduction

Małgosia II is a prototype of light, propeller driven, two person plane with rare wings configuration of canard type designed by Edward Margański. This airplane configuration will be used to present the analysis methodology.

The speed of aircraft ranges from 160 to 220 km/h. The reference wing area is equal to 11,42m<sup>2</sup>, the reference wing span is 8,967m, and mean aerodynamic chord is equal to 1,513m. The empty weight is 475 kg and maximal take off weight is 794 kg.

The NACA641412 is used as main wing profile and FX71-L-150 for a front wing profile.

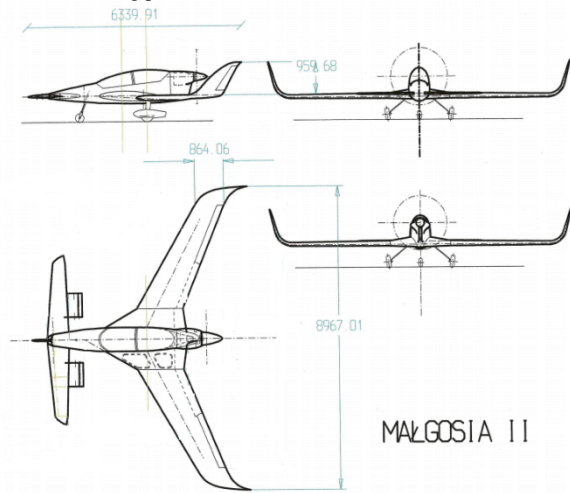


Fig. 1 Views of Małgosia II aircraft

The paper is organized as follows. Section 3 describes goal of this paper. Section 4 contains introduction and basic data concerning MSC CAERO1 elements and used aerodynamic theory. Section 5 interprets the meaning of the correction coefficients. Section 6 describes the FEM aerodynamic model with calculation of “w2gj” correction coefficient. Section 7 shows way of calculating the rest of correction coefficients. Section 8 presents the results. Finally, section 9 presents our conclusions.

## 3. Goal of the paper

The goal of this paper is to prove that analyzed plane is statically stable. Additionally stability coefficients needed for dynamic stability analysis will be obtained. All this calculations will be done without wind tunnel tests or CFD calculations.

## 4. MSC CAERO1 elements

The aerodynamics is computed by the use of Double-Lattice subsonic lifting surface theory (DLM) in which the linearized potential flow theory is used to compute the interference between CAERO1 elements. The theory is presented in [1] and [2] and is not reproduced here.

The lifting force generated is calculated as:

$$P_k = q S_{kj} (W_{kk} A_{jj}^{-1} w_j + f_j^e / q) \quad (1)$$

Where:

$P_k$ - force generated,

$q$ - dynamic pressure,

$S_{kj}$ - Integration matrix which integrates aerodynamic pressures to lifting force and pitching moment,

$W_{kk}$ - empirical correction factors to adjust each theoretical lift and moment to agree with experimental data for incidence changes,

$A_{jj}$ - aerodynamic influence coefficient matrix,

$f_j^e / q$ - initial pressure correction coefficient,

$w_j$ - downwash, the flow is deflected by this angle on each element.

$$w_j = (D_{jk}^{Re} + D_{jk}^{Im} i) u_k + w_j^g \quad (2)$$

$D_{jk}^{Re}$ ,  $D_{jk}^{Im}$ - real and imaginary parts of substantial differentiation matrix,

$u_k$ - displacements of aerodynamic DOFs,

$w_j^g$ - initial downwash.

Four experimental values needs to be provided by user for each aerodynamic element. This is:  $w_j^g$ ,  $f_j^e / q$  and two  $W_{kk}$  coefficients (separately for lifting force and pitching moment).

## 5. Corrections coefficients

The value of  $w_j^g$  which is represented in MSC.Nastran by “w2gj” matrix, basically can be interpreted as the angle between the direction of flow and direction of deflected flow by flat plate at some incidence.

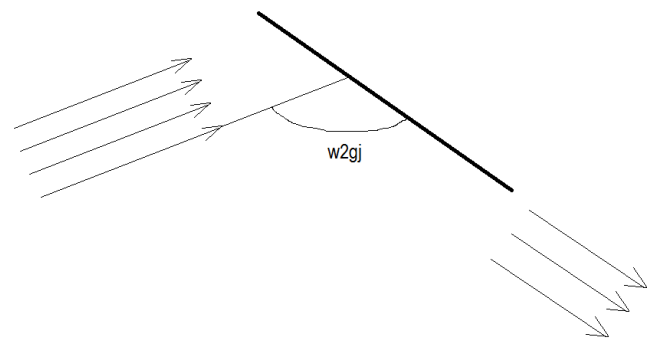


Fig. 2 The definition of „w2gj” correction coefficient for flat plate

In this way twist of a wing can be modeled (in this case the “w2gj” changes with span of a wing). It is also used to model camber of the airfoil which results in more accurate pressure distribution on airfoil and downwash generated behind the wing (in this case the changes of “w2gj” are a result of camber line changes of wing cross section).

The value of  $f_j^e/q$  estimate a lift coefficient at zero incidence angle. This value is represented in MSC.Nastran by “f2gj” matrix.

The first value of  $W_{kk}$  coefficients for an element represents the correction of slope of lift characteristics, while the second one represents the correction the slope of moment characteristics.

All of this values can be obtained from time-consuming and difficult CFD calculations or from wind tunnel results [5], but author uses other less expensive way based on aerodynamic characteristics of profiles used in airplane.

## 6. FEM model

Authors build a FEM aerodynamic model by using CQUAD4 element common in structural analyses [6] but with assigned property of PAERO1 entities. In this way the initial incidence of each element can be represented by its position on 3D mid-surface of a wing.

Additionally author did mesh the leading edge with 1D CROD elements (again with PAERO1 property) which contain information about the “f2gj” and “wkk” coefficients for all elements which lie behind current CROD element in the direction of flow. The numbering of CRODs elements is continuous in order to allow interpolation of coefficients provided by user.

The lift generated by the fuselage elements is assumed to be a portion of lift generated by wings inside the fuselage. In this way the influence of fuselage on lift generated by wings can also be modeled without using MSC slender body elements [4].

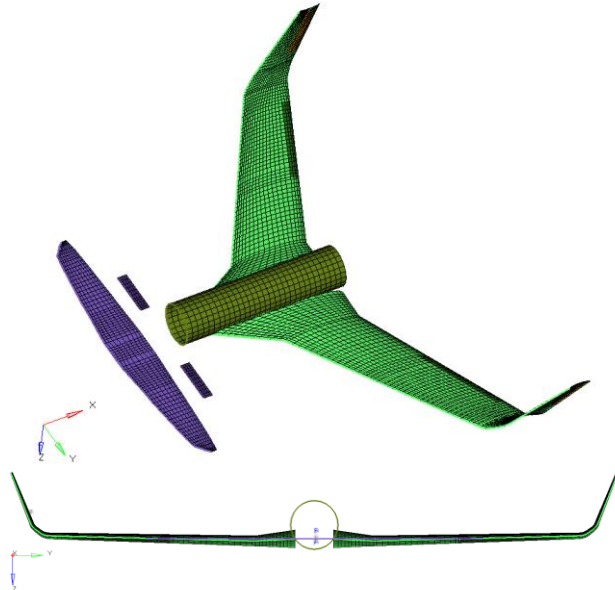


Fig. 3 The aerodynamic model build by CQUAD4 and CROD elements

The program written by author projects the CQUAD4 grid points to lines defined by the flow direction and grid points of the closest CROD element with same property ID. During the projection the “w2gj” matrix is calculated from position of each element in space. Additionally the “wkk” and “f2gj” values are taken from same CROD for each projected element. The final step of the program is converting all projected CQUAD4 elements into CAERO1 elements and deleting all CROD elements with PAERO1 property.

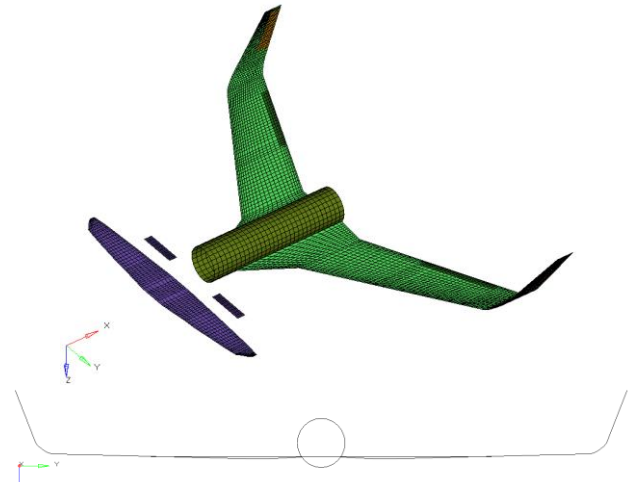


Fig. 4 The Aerodynamic model of CAERO1 elements generated by program

As the result of running the program four files are generated: new “.bdf” file, “.w2gj” file, “.f2gj” file and “.wkk” file. The last three files contain appropriate matrixes with experimental coefficients for all CAERO1 elements.

In this way “w2gj” matrix can be calculated form geometrical position of elements but the user still needs to know values of “f2gj” and “wkk” correction coefficients for all airfoils used.

Table 1 Calculated correction parapeters for used airfoils

	NACA641412	FX71-L-150
$f2gj$	-1.50991	0.000
$wkk1$	1.43116	1.24449
$wkk2$	-0.51663	-0.58200

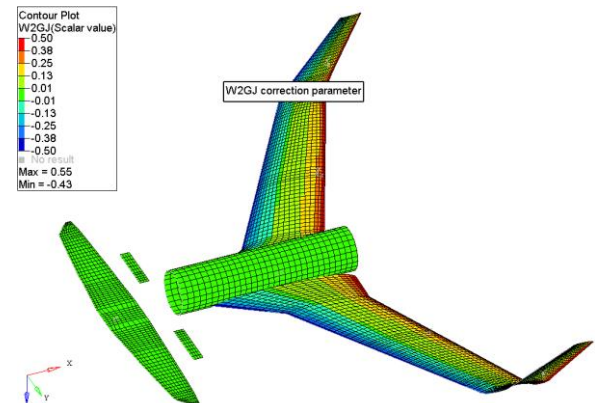


Fig. 5 Contour plot showing the distribution of „w2gj” correction parameter

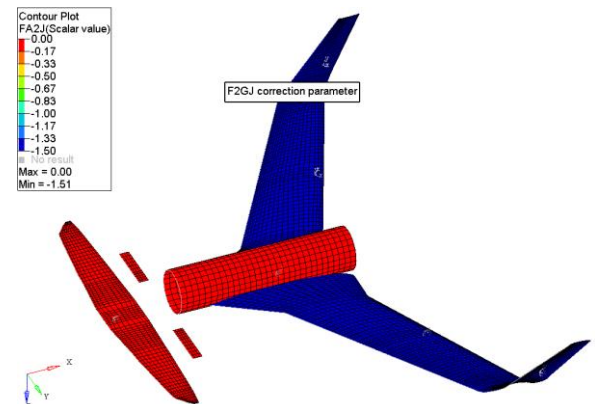


Fig. 6 Contour plot showing the distribution of „f2gj” correction parameter

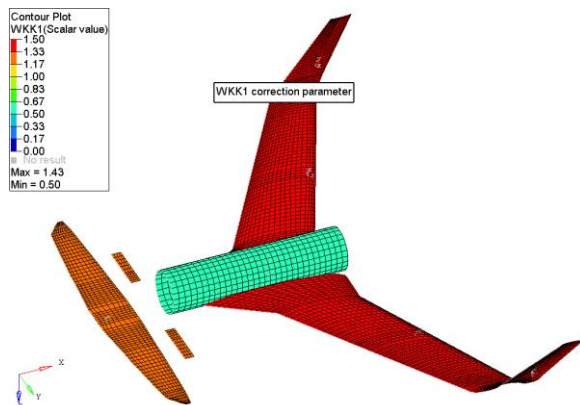


Fig. 7 Contour plot showing the distribution of „WKK1” correction parameter

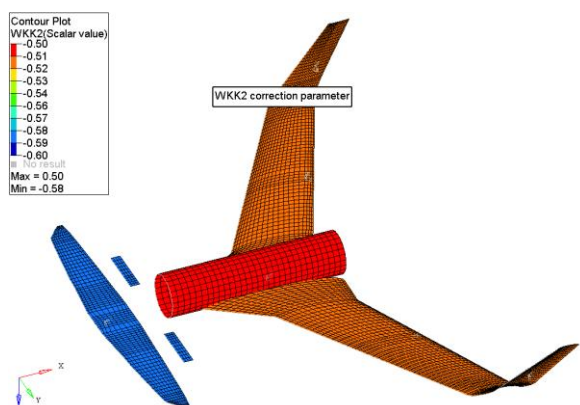


Fig. 8 Contour plot showing the distribution of „WKK2” correction parameter

Calculations are run for straight fly with three different dynamic pressures: 1.154 kPa, 1.627 kPa and 2.181 kPa (it corresponds to 160 km/h, 190 km/h and 220 km/h respectively).

Only the rigid structure will be analyzed due to not enough data to build proper structural model.

The analysis coordinate system x axis is defined in the direction of flow and z axis is directed downward. The Origin is located at center of gravity of aircraft.

## 7. Results

The angle of attack is defined as rotation about y axis. In this case positive value implies nose-down rotation in reference to wind. Side angle is defined as rotation about z axis.

The calculated aerodynamic DOFs which describe the parameters of flight to maintain quasi-static equilibrium are listed below.

Table 2 Calculated aerodynamic DOFs

	160 km/h	190 km/h	220 km/h
Angle of attack ( $\alpha$ ) [deg]	-5.453	-3.940	-3.002
Side angle ( $\beta$ ) [deg]	0.015	0.013	0.012
Horizontal stabilizer [deg]	-0.002	0.002	0.005
Trim tabs [deg]	0.000	0.000	-0.001
Left aileron [deg]	-0.011	-0.005	-0.002
Right aileron [deg]	-0.011	-0.005	-0.002
Left rudder [deg]	-0.002	-0.001	0.000
Right rudder [deg]	-0.002	-0.001	0.000

The very small deflections of horizontal stabilizer required to compensate pitching moment of aircraft show that the plane will be very sensitive to this control input.

The non-zero deflections of ailerons and rudders together with non-zero value of side angle are a result of placing center of gravity of aircraft not exactly on plane of symmetry and not modeling the effects of propeller.

Calculated results show that the airplane while flying with minimal speed will need 5.453 deg. to generate lift force which will balance the gravity. Together with wing angle of incidence total angle of attack seen by wing equals 8.953 deg. This is less than stall angle of attack taken from the aerodynamic characteristics of profile which is equal to 10.00 deg. This implies that airplane can possibly fly at constant altitude with speeds a bit lower than 160 km/h.

The calculated angle of attack for course speed should be equal to 0.000 in order of obtaining lowest possible drag force on fuselage. This could be done by increasing wing angle of incidence by around 3.50 deg.

The stability derivative coefficients which describe dynamic and static behavior of rigid airplane are listed below with respect to coordinate system axes described earlier.

Table 3 Calculated stability derivative coefficients

	$T_y$	$T_z$	$M_x$	$M_y$	$M_z$
Ref.	5.0930 $10^{-7}$	5.1882 $10^{-3}$	1.6116 $10^{-7}$	-2.5046 $10^{-2}$	2.8662 $10^{-8}$
Angle of attack ( $\alpha$ )	1.020 $10^{-5}$	6.401 $10^0$	-2.653 $10^{-4}$	-6.590 $10^{-2}$	9.130 $10^{-7}$
Side angle ( $\beta$ )	-3.492 $10^{-1}$	1.122 $10^{-5}$	-1.116 $10^{-1}$	1.802 $10^{-6}$	-7.443 $10^{-2}$
Roll speed ( $\gamma$ )	-3.673 $10^{-1}$	5.497 $10^{-4}$	-5.831 $10^{-1}$	2.321 $10^{-7}$	-9.634 $10^{-2}$
Pitch speed ( $\delta$ )	-2.583 $10^{-6}$	8.156 $10^0$	-3.907 $10^{-4}$	-1.429 $10^{-1}$	3.843 $10^{-8}$
Yaw speed ( $\epsilon$ )	-1.997 $10^{-2}$	-3.315 $10^{-6}$	-7.364 $10^{-2}$	-2.752 $10^{-7}$	-5.646 $10^{-2}$
Horizontal stabilizer	-7.866 $10^{-6}$	5.159 $10^{-1}$	-5.011 $10^{-5}$	2.099 $10^0$	-6.759 $10^{-7}$
Trim tabs	1.279 $10^{-5}$	7.694 $10^{-2}$	3.877 $10^{-5}$	2.397 $10^{-1}$	1.095 $10^{-6}$
Left aileron	3.054 $10^{-2}$	3.366 $10^{-1}$	9.738 $10^{-2}$	-3.313 $10^{-1}$	7.699 $10^{-3}$
Right aileron	3.054 $10^{-2}$	-3.367 $10^{-1}$	9.741 $10^{-2}$	3.313 $10^{-1}$	7.699 $10^{-3}$
Left rudder	5.831 $10^{-2}$	3.243 $10^{-2}$	1.708 $10^{-2}$	-5.120 $10^{-2}$	1.810 $10^{-2}$
Right rudder	5.830 $10^{-2}$	-3.243 $10^{-2}$	1.708 $10^{-2}$	5.120 $10^{-2}$	1.810 $10^{-2}$

### Longitudinal stability

The most important stability derivative is the change of  $M_y$  with angle of attack as it describes the ability of aircraft to align with the wind direction after some disturbance in vertical plane which basically is a static stability condition.

Static longitudinal stability margin is calculated as:

$$S_\alpha = -\frac{M_{y\alpha}}{T_{za}} = 1.03\% \quad (3)$$

The positive value of  $S_\alpha$  shows that unit increase in angle of attack results in creating pitching moment which tends to correct that increase in angle of attack without the pilot input.

The small value of  $S_\alpha$  shows that the aircraft phugoid mode will have long period and can be easily corrected by pilot input.

### Directional stability

The change of  $M_z$  with side angle is known as static directional stability coefficient. It describes the ability of aircraft rotate its longitudinal axis to the wind direction in horizontal plane.

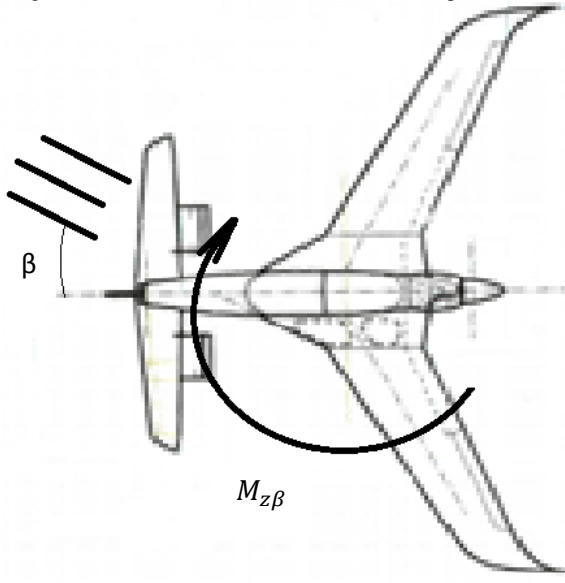


Fig. 9 Explanation of directional stability

Static directional stability margin is calculated as:

$$S_{z\beta} = \frac{M_{z\beta}}{T_{y\beta}} = 22.31\% \quad (4)$$

The positive value of  $S_{z\beta}$  shows that the aircraft tends to align its course with the wind direction without pilot input. In oppose the negative value of  $S_{z\beta}$  would not allow to fly with constant direction without pilot corrections.

The high value of  $S_{z\beta}$  shows that the airplane will probably have problems with rapid changing its course with only rudder deflections.

### Lateral stability

The side angle usually results also in creating roll moment. This effect is described by the change of  $M_x$  with side angle. The sign of this derivative describes in which direction aircraft tends to roll after some side angle disturbance.

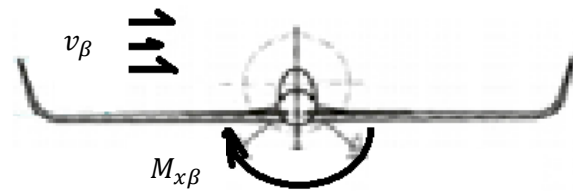


Fig. 10 Explanation of lateral stability

Static roll stability margin is calculated as:

$$S_{x\beta} = \frac{M_{x\beta}}{T_{y\beta}} = 31.96\% \quad (5)$$

The positive value of  $S_{x\beta}$  shows that the aircraft tends to align its wings perpendicularly to the wind direction.

The higher value of  $S_{x\beta}$  than  $S_{z\beta}$  shows that the airplane in order of correcting the side angle will first try to roll then yaw. Also the difference between this two margins shows that the Dutch roll can possibly be seen in the dynamic behavior of aircraft.

The negative values of  $M_{xy}$ ,  $M_{y\delta}$  and  $M_{z\epsilon}$  show that roll, pitch and yaw rotational speeds will be limited to maximal value.

### Hinge moments derivatives

Hinge moments derivatives are listed below.

Table 4 Calculated hinge moment derivatives coefficients

	Horizontal stabilizer with trim tabs	Left aileron	Right aileron	Left rudder	Right rudder
Ref. chord [m]	0.715	0.150	0.150	0.200	0.200
Ref. area [m <sup>2</sup> ]	2.500	0.250	0.250	0.150	0.150
Ref.	$-4.219 \cdot 10^{-2}$	$-1.573 \cdot 10^{-1}$	$1.573 \cdot 10^{-1}$	$5.065 \cdot 10^{-2}$	$-5.064 \cdot 10^{-2}$
Angle of attack ( $\alpha$ )	$-1.179 \cdot 10^0$	$-5.057 \cdot 10^{-1}$	$5.067 \cdot 10^{-1}$	$-5.150 \cdot 10^{-1}$	$5.149 \cdot 10^{-1}$
Side angle ( $\beta$ )	$-6.780 \cdot 10^{-6}$	$2.980 \cdot 10^{-2}$	$2.980 \cdot 10^{-2}$	$4.070 \cdot 10^{-1}$	$4.069 \cdot 10^{-1}$
Roll speed ( $\gamma$ )	$-1.130 \cdot 10^{-4}$	$2.390 \cdot 10^{-1}$	$2.391 \cdot 10^{-1}$	$4.070 \cdot 10^{-1}$	$4.069 \cdot 10^{-1}$
Pitch speed ( $\delta$ )	$1.782 \cdot 10^0$	$-1.536 \cdot 10^0$	$1.536 \cdot 10^0$	$-1.419 \cdot 10^0$	$-1.419 \cdot 10^0$
Yaw speed ( $\epsilon$ )	$1.840 \cdot 10^{-6}$	$2.005 \cdot 10^{-2}$	$2.005 \cdot 10^{-2}$	$3.161 \cdot 10^{-1}$	$3.161 \cdot 10^{-1}$
Horizontal stabilizer	$-2.651 \cdot 10^{-1}$	$-9.351 \cdot 10^{-3}$	$9.360 \cdot 10^{-3}$	$-2.118 \cdot 10^{-2}$	$2.118 \cdot 10^{-2}$
Trim tabs	$-5.772 \cdot 10^{-1}$	$-3.322 \cdot 10^{-3}$	$3.304 \cdot 10^{-3}$	$-3.041 \cdot 10^{-3}$	$3.032 \cdot 10^{-3}$
Left aileron	$-5.191 \cdot 10^{-3}$	$-1.113 \cdot 10^0$	$2.500 \cdot 10^{-3}$	$-5.260 \cdot 10^{-2}$	$2.840 \cdot 10^{-3}$
Right aileron	$5.191 \cdot 10^{-3}$	$2.501 \cdot 10^{-3}$	$-1.113 \cdot 10^0$	$2.084 \cdot 10^{-3}$	$-5.260 \cdot 10^{-2}$
Left rudder	$-3.059 \cdot 10^{-4}$	$-4.509 \cdot 10^{-3}$	$1.666 \cdot 10^{-4}$	$-5.441 \cdot 10^{-1}$	$2.380 \cdot 10^{-4}$
Right rudder	$3.059 \cdot 10^{-4}$	$1.666 \cdot 10^{-4}$	$-4.509 \cdot 10^{-3}$	$2.381 \cdot 10^{-4}$	$-5.440 \cdot 10^{-1}$

The generated pressure distribution on airplane is shown below.

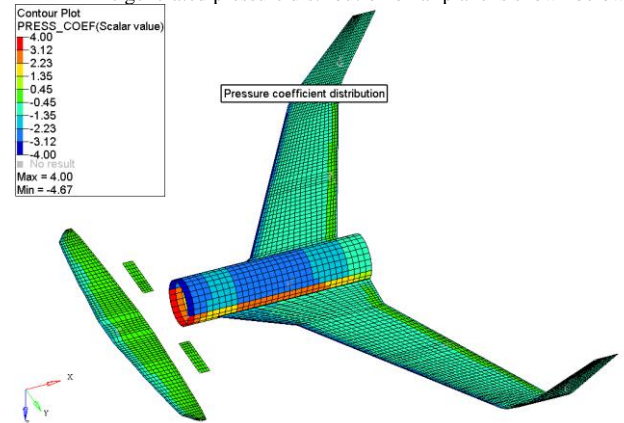


Fig. 11 Distribution of pressure coefficient for straight fly

## 8. Conclusions

The analysis of stability derivatives shows that the analyzed aircraft is statically stable. Equations of motion should be build (with use of stability derivatives) and solved in order to analyze dynamic stability of this aircraft.

During dynamic simulation control surfaces effectiveness should be calculated together with all aircraft frequencies of modes.

Presented method of calculation correction coefficients can be used to calculate aerodynamic coefficients in the earliest step of conceptual design without use of time-consuming CFD calculations and wind tunnel tests.

This calculations can be also used without modifications to calculate all aerodynamic coefficients for deformable aircraft in which case results will be more accurate.

## 9. References and bibliography

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