TD N° 1 : TRANSFORMEE DE FOURIER DISCRETE

Exercice 01

Déterminer la TFTD de séquences suivantes :

a) $x(k) = 0.5^k u(k)$

b) $x(k) = 0.5^{|k|}$

c) $x(k) = 0.5^k u(-k)$

Exercice 02

On considère les deux séquences $x_1(k)$ et $x_2(k)$ de durée finie $N = 4$

$$x_1(k) = \cos\left(\frac{\pi}{2}k\right),\ k = 0,1,2,3,4$$

$$x_2(k) = \left(\frac{1}{2}\right)^k,\ k = 0,1,2,3,4$$

1. Calculer $y(k) = x_1(k) \otimes x_2(k)$ utilisant la convolution circulaire
2. Calculer $y(k)$ utilisant la TFD

Exercice 03

On considère la séquence :

$$x(k) = \begin{cases} e^{j2\pi f_0 k} & 0 \le k \le N-1 \\ 0 & Si\ non \end{cases}$$

1. Trouver la transformée de Fourier $X(f)$ de $x(k)$
2. Trouver la TFD $X(n)$ sur N points de la séquence $x(k)$

Exercice 04

Mettre l'équation de la TFD sous la forme $X_N = W_N X_k$ où W_N est la matrice de la TFD. Noter que la matrice W_N est symétrique.

- Montrer que $W_N^{-1} = \frac{1}{N} W_N^*$
- Trouver explicitement W_4 et W_4^{-1}
- Si $x(k) = \{0,1,2,3\}$, trouver la TFD $X(n)$ et la TFDI de $X(n)$

Exercice 05

La TFD d'une séquence $x(k), k = 0, \dots, N-1$ est $(n), n = 0, \dots, N-1$. on considère les deux séquences :

$s = [x(0), \dots, x(N-1), x(0), \dots, x(N-1)]$

$$y = [x(0), \dots, x(N-1), \underbrace{0, \dots, 0}]$$

1. Montrer que $S(2m) = 2X(m)$ et $S(2m+1) = 0$, pour $m = 0, \dots, N-1$
2. Montrer que $Y(2m) = X(m)$ pour $m = 0, \dots, N-1$

Exercice 06

On considère la séquence : $x(k) = \{1,1,-1,-1,-1,1,1,-1\}$

Déterminer $X(n)$ la TFD de x(k) utilisant par l'algorithme FFT (décimation temporelle).

Exo 1

* $x(k) = 0{,}5^{k}\, u(k)$

$$X(f) = \sum_{k=-\infty}^{\infty} x(k) e^{-j2\pi fk} = \sum_{k=0}^{\infty} 0{,}5^{k} e^{-j2\pi fk} = \sum_{k=0}^{\infty} \left(0{,}5\, e^{-j2\pi f}\right)^{k}.$$

$$X(f) = \frac{1 - r^{N}}{1 - r}, \quad r = 0{,}5\, e^{-j2\pi f}.$$

$$X(f) = \frac{1 - 0{,}5^{N} e^{-j2\pi fN}}{1 - 0{,}5\, e^{-j2\pi f}}, \quad N \to \infty, \quad X(f) = \frac{1}{1 - 0{,}5\, e^{-j2\pi f}}.$$

* $x(k) = 0{,}5^{|k|}$,

$$X(f) = \sum_{-\infty}^{-1} 0{,}5^{-k} e^{-j2\pi f} + \sum_{k=0}^{\infty} 0{,}5^{k} e^{-j2\pi f}$$

$$= \sum_{1}^{\infty} 0{,}5^{k} e^{+j2\pi fk} + \sum_{k=0}^{\infty} 0{,}5^{k} e^{-j2\pi fk}$$

$$= \sum_{k=0}^{\infty} 0{,}5^{k} e^{+j2\pi fk} - 1 + \sum_{k=0}^{\infty} 0{,}5^{k} e^{-j2\pi fk}$$

$$\frac{1}{1 - 0{,}5\, e^{j2\pi f}} + \frac{1}{1 - 0{,}5\, e^{-j2\pi f}} - 1$$

* $x(k) = 0{,}5^{k}\, u(-k)$, $X(f) = \sum_{-\infty}^{0} 0{,}5^{k} e^{-j2\pi fk} = \sum_{0}^{\infty} 0{,}5^{-k} e^{j2\pi fk}$

$$= \sum_{k=0}^{\infty} 2^{k} e^{j2\pi fk}$$

ne converge pas

pas de TFTD

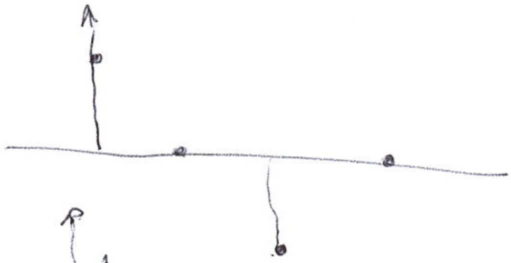
Ex2 $x_1(k) = \cos\left(\frac{\pi}{2}k\right) \quad k=0, \ldots 4$

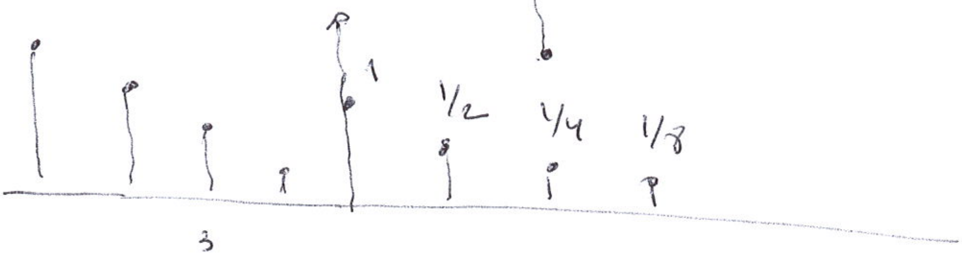
$$x_2(k) = \left(\frac{1}{2}\right)^k \quad k=0 — 4$$

$y(k) = x_1(k) \circledast x_2(k)$ $\quad x_1(k) = \{1, 0, -1, 0\}$

$$x_2(k) = \{1, 1/2, 1/4, 1/8\}$$

$x_1(k)$:





$$y(k) = \sum_{n=0}^{3} x_1(n)\, x_2(k-n)$$

$$y(0) = x_1(0)\,x_2(0) + x_1(1)\,x_2(-1) + x_1(2)\,x_2(-2) + x_1(3)\,x_2(-3)$$

$$1 + 0 - 1/4 + 0 = 3/4$$

$$y(1) = x_1(0)\,x_2(1) + x_1(1)\,x_2(0) + x_1(2)\,x_2(-1) + x_1(3)\,x_2(-2)$$

$$= 1/2 - 1/8 = \frac{4-1}{8} = 3/8$$

$y(2) = -3/4$ $\qquad [3/4 \quad 3/8 \quad -3/4 \quad -3/8]$

$y(3) = -3/8$

$$X_1(n) = \sum_{n=0}^{3} x_1(k)\, W_4^{nk} = 1 - W_4^{2n}, \quad n = 0, \ldots 3$$

$$X_2(n) = \sum_{k=0}^{3} x_2(k)\, W_4^{nk} = 1 + 1/2\, W_4^{n} + 1/4\, W_4^{2n} + 1/8\, W_4^{3n}$$

$$Y(n) = X_1(n) \odot X_2(n) = \left(1 - W_4^{2n}\right)\left(1 + 1/2\, W_4^{n} + 1/4\, W^{2n} + 1/8\, W^{3n}\right)$$

$$= 1 + 1/2\, W^{n} + 1/4\, W^{2n} + 1/8\, W^{3n} - W^{2n} - 1/2\, W^{3n} - 1/4\, W^{4n} - 1/8\, W^{5n}$$

$$= 1 + 1/2\, W^{n} - 3/4\, W^{2n} - 3/8\, W^{3n} - 1/4\, W^{4n} - 1/8\, W^{5n}$$

$$W_4^{4k} = e^{-j2\pi\frac{4k}{4}} = e^{j2\pi k} = 1$$

$$W_4^{5k} = \cancel{W_4^{4k}}^{1} \cdot W_4^{k} = W_4^{k}$$

$$y(n) = 3/4 + 1/8 W_4^{k} - 3/4 W^{2k} - 3/8 W^{3k}$$

$$\Rightarrow \; y(k) = \{3/4 , 1/8 , -3/4 , -3/8\}$$

ex3

$$x(k) = e^{j2\pi f_0 k} \qquad 0 \le k \le N-1$$

$$X(f) = \sum_{k=0}^{N-1} x(k)\, e^{-j2\pi f k} = \sum_{k=0}^{N-1} e^{j2\pi f_0 k} e^{-j2\pi f k}$$

$$= \sum_{k=0}^{N-1} e^{j2\pi k(f-f_0)} = \frac{1 - e^{j2\pi f N(f-f_0)}}{1 - e^{j2\pi(f-f_0)}}$$

$$= \frac{e^{-j2\pi(f-f_0)N/2}}{e^{-j2\pi(f-f_0)1/2}} \cdot \frac{e^{j2\pi(f-f_0)N/2} - e^{-j2\pi(f-f_0)N/2}}{e^{j2\pi(f-f_0)1/2} - e^{-j2\pi(f-f_0)1/2}}$$

$$= e^{j2\pi(f-f_0)\left(\frac{N-1}{2}\right)} \cdot \frac{\sin\left(2\pi(f-f_0)N/2\right)}{\sin\left(2\pi(f-f_0)/2\right)}$$

$$X(n) = X(f)\Big|_{f=\frac{n}{N}} = e^{j2\pi\left(\frac{n}{N}-f_0\right)\left(\frac{N-1}{2}\right)} \left[\frac{\sin\left(2\pi\left(\frac{n}{N}-f_0\right)\frac{N}{2}\right)}{\sin\left(2\pi\left(\frac{n}{N}-f_0\right)/2\right)}\right.$$

ΕΡΩ 4

$$X(n) = \sum_{k=0}^{N-1} x(k)\, W_N^{nk}$$

$$\begin{bmatrix} X(0) \\ \vdots \\ X(N-1) \end{bmatrix} = \begin{bmatrix} & & \\ & W & \\ & & \end{bmatrix} \begin{bmatrix} x(0) \\ \vdots \\ x(N-1) \end{bmatrix}$$

$$X = W x \Longrightarrow x = W^{-1} X$$

$$x(k) = \frac{1}{N} \sum_{n=0}^{N-1} X(n)\, W_N^{-nk}, \quad x = \frac{1}{N} W_N^{*} X$$

$$\Longrightarrow W^{-1} = \frac{1}{N} W_N^{*}$$

$* \; W^4 \quad W_4^{-1}$

$$W_4^{nk} = e^{\frac{-j2\pi nk}{4}} = e^{-j\frac{\pi}{2} nk} = (-j)^{nk}$$

$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \qquad W_4^{-1} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$x(k) = \{0, 1, 2, 3\}$$

$$X = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$x = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

4/

exo1 $x(k),\ k=0,\ N-1$

$$S = [x(0) \text{———} x(N-1)\ x(0) \text{———} x(N-1)]$$

$$S(n) = \sum_{k=0}^{N-1} x(k) W_{2N}^{nk} + \sum_{k=0}^{N-1} x(k) W_{2N}^{(n+N)k}$$

$$* \ W_{2N}^{Nk} = e^{-j\frac{2\pi Nk}{2N}} = e^{-j\pi k} = (-1)^k$$

$$S(n) = \sum_{k=0}^{N-1} x(k) W_{2N}^{nk} + \sum_{k=0}^{N-1} x(k) W_{2N}^{nk} \cdot \underbrace{W_{2N}^{Nk}}_{(-1)^k}$$

$$S(n) = \sum_{k=0}^{2N-1} x(k) W_{2N}^{nk}$$

$$= \sum_{k=0}^{N-1} x(k) W_{2N}^{nk} + \sum_{k=N}^{2N-1} x(k) W_{2N}^{nk}$$

$$S(n) = \underbrace{x(0) W^0 + x(1) W^n + \dots + x(N-1) W^{n(N-1)}} + x(N) W^{nN} + \dots$$

$$+ \dots + x(2N-1) W^{n(2N-1)}$$

$$\sum_{k=0}^{N-1} x(k) W_{2N}^{nk} + \sum_{k=0}^{N-1} x(k) W^{n(k+N)}$$

$$= \sum_{k=0}^{N-1} x(k) W_{2N}^{nk} + \sum_{k=0}^{N-1} x(k) W^{nk} W_{2N}^{nN} \leftarrow (-1)^n$$

$$S(n) = \sum_{k=0}^{N-1} x(k) W_{2N}^{nk} \left[1 + (-1)^n\right] = \begin{cases} n \text{ Pair} & 2X(n) \\ n \text{ imp} & 0. \end{cases}$$

$$y(n) = \sum_{k=0}^{N-1} x(k)\, W_{2N}^{nk}$$

$$W_{2N}^{nk} = e^{-j\frac{2\pi nk/2}{2N/2}} = W_N^{nk/2} \quad , \quad W_{2N}^{nk} = e^{-j\frac{\pi nk}{N}}$$

$$y(2n) = \sum_{k=0}^{N-1} x(k)\, W_{2N}^{2nk} = \sum_{k=0}^{N-1} x(k) \left(W_{2N}^{nk}\right)^2$$

$$W_{2N}^{2} = e^{-j\frac{2\pi \cdot 2}{2N}} = W_N \Rightarrow y(2n) = \sum_{k=0}^{N-1} x(k)\, W_N^{nk} = X(n)$$

$$n = 0, N-1$$

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<u>EX 06</u>

TFD 8 Points

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ \vdots\\ X(7)\end{bmatrix} = \begin{bmatrix} W_8^0 & \cdots & & \\ W_8^0 & W_8^1 & \cdots & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 \cdots & W_8^{14}\\ \vdots & & & \\ W_8^0 & W_8^7 & \cdots & W_8^{7.7}\end{bmatrix}\begin{bmatrix} x(0)\\ x(1)\\ \vdots\\ x(7)\end{bmatrix}$$

$$\begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\end{bmatrix} = \underbrace{\begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0\\ W_8^0 & W_8^2 & W_8^4 & W_8^8\\ W_8^0 & W_8^4 & W_8^8 & W_8^{16}\\ W_8^0 & W_8^6 & W_8^{12} & W_8^{24}\end{bmatrix}\begin{bmatrix} x(0)\\ x(2)\\ x(4)\\ x(6)\end{bmatrix}}_{\text{TFD 4 points}} + \begin{bmatrix} W_8^0 & 0 & 0 & 0\\ 0 & W_8^1 & 0 & 0\\ 0 & 0 & W_8^2 & 0\\ 0 & 0 & 0 & W_8^3\end{bmatrix}\underbrace{\begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0\\ W_8^0 & W_8^2 & W_8^4 & W_8^8\\ W_8^0 & W_8^4 & W_8^8 & W_8^{16}\\ W_8^0 & W_8^6 & W_8^{12} & W_8^{24}\end{bmatrix}\begin{bmatrix} x(1)\\ x(3)\\ x(5)\\ x(7)\end{bmatrix}}_{\text{TFD 4 pts}}$$

$$\begin{bmatrix} X'(0)\\ X'(1)\\ X'(2)\\ X'(3)\end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0\\ W_4^0 & W_4^1 & W_4^2 & W_4^4\\ W_4^0 & W_4^2 & W_4^4 & W_4^8\\ W_4^0 & W_4^3 & W_4^6 & W_4^{12}\end{bmatrix}\begin{bmatrix} x(0)\\ x(2)\\ x(4)\\ x(6)\end{bmatrix}$$

I, II, III, IV : TFD 2pts

$$\begin{bmatrix} X'(0)\\ X'(1)\end{bmatrix} = \underbrace{\begin{bmatrix} W_4^0 & W_4^0\\ W_4^0 & W_4^2\end{bmatrix}\begin{bmatrix} x(0)\\ x(4)\end{bmatrix}}_{\text{I}} + \underbrace{\begin{bmatrix} W_4^0 & 0\\ 0 & W_4^1\end{bmatrix}\begin{bmatrix} W_4^0 & W_4^0\\ W_4^0 & W_4^2\end{bmatrix}}_{\text{II}}\begin{bmatrix} x(2)\\ x(6)\end{bmatrix}$$

$$\begin{bmatrix} X'(2)\\ X'(3)\end{bmatrix} = \underbrace{\begin{bmatrix} W_4^0 & W_4^0\\ W_4^0 & W_4^2\end{bmatrix}\begin{bmatrix} x(0)\\ x(4)\end{bmatrix}}_{\text{IV}} - \begin{bmatrix} W_4^0 & 0\\ 0 & W_4^1\end{bmatrix}\underbrace{\begin{bmatrix} W_4^0 & W_4^0\\ W_4^0 & W_4^2\end{bmatrix}\begin{bmatrix} x(2)\\ x(6)\end{bmatrix}}_{\text{III}}$$

$$\begin{bmatrix} X_1'(0)\\ X_1'(1)\end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0\\ W_4^0 & W_4^2\end{bmatrix}\begin{bmatrix} x(0)\\ x(4)\end{bmatrix} \Rightarrow \begin{bmatrix} X_2'(0)\\ X_2'(1)\end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0\\ W_4^0 & W_4^2\end{bmatrix}\begin{bmatrix} x(2)\\ x(6)\end{bmatrix}$$

$$\begin{bmatrix} X_1'(2)\\ X_1'(3)\end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0\\ W_4^0 & W_4^2\end{bmatrix}\begin{bmatrix} x(0)\\ x(4)\end{bmatrix}, \quad \begin{bmatrix} X_2'(2)\\ X_2'(3)\end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0\\ W_4^0 & W_4^2\end{bmatrix}\begin{bmatrix} x(2)\\ x(6)\end{bmatrix}$$

$$\left.\begin{array}{l} W_4^0 = W_2^0\\ W_4^2 = W_2^1\end{array}\right\} \quad \begin{array}{l} \begin{bmatrix} X'(0)\\ X'(1)\end{bmatrix} = \begin{bmatrix} X_1'(0)\\ X_1'(1)\end{bmatrix} + \begin{bmatrix} 1 & 0\\ 0 & W_4^1\end{bmatrix}\begin{bmatrix} X_2'(2)\\ X_2'(3)\end{bmatrix}\\ \begin{bmatrix} X'(2)\\ X'(3)\end{bmatrix} = \begin{bmatrix} X_1'(0)\\ X_1'(1)\end{bmatrix} - \begin{bmatrix} 1 & 0\\ 0 & W_4^1\end{bmatrix}\begin{bmatrix} X_2'(2)\\ X_2'(3)\end{bmatrix}\end{array}$$

$$\begin{bmatrix} X'(4) \\ X'(5) \\ X'(6) \\ X'(7) \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 \\ W_8^0 & W_8^4 & W_8^8 & W_8^{16} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{24} \end{bmatrix} \begin{bmatrix} x(1) \\ x(3) \\ x(5) \\ x(7) \end{bmatrix}$$

$$\begin{bmatrix} X'(4) \\ X'(5) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 \\ W_4^0 & W_4^2 \end{bmatrix} \begin{bmatrix} x(1) \\ x(5) \end{bmatrix} + \begin{bmatrix} W_4^0 & 0 \\ 0 & W_4^1 \end{bmatrix} \begin{bmatrix} W_4^0 & W_4^0 \\ W_4^0 & W_4^2 \end{bmatrix} \begin{bmatrix} x(3) \\ x(7) \end{bmatrix}$$

$$\begin{bmatrix} X'(6) \\ X'(7) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 \\ W_4^0 & W_4^2 \end{bmatrix} \begin{bmatrix} x(1) \\ x(5) \end{bmatrix} - \begin{bmatrix} W_4^0 & 0 \\ 0 & W_4^1 \end{bmatrix} \begin{bmatrix} W_4^0 & W_4^0 \\ W_4^0 & W_4^2 \end{bmatrix} \begin{bmatrix} x(3) \\ x(7) \end{bmatrix}$$

$$\begin{bmatrix} X_1'(4) \\ X_1'(5) \end{bmatrix} = \underbrace{\begin{bmatrix} W_2^0 & W_2^0 \\ W_2^0 & W_2^1 \end{bmatrix}}_{\text{TFD 2 points}} \begin{bmatrix} x(1) \\ x(5) \end{bmatrix} , \quad \begin{bmatrix} X_1'(6) \\ X_1'(7) \end{bmatrix} = \begin{bmatrix} W_2^0 & W_2^0 \\ W_2^0 & W_2^1 \end{bmatrix} \begin{bmatrix} x(3) \\ x(7) \end{bmatrix}$$

$$X_1'(4) = [W_2^0]\, x(1) + W_2^0 [W_2^0]\, x(5)$$

$$X_1(5) = [W_2^0]\, x(1) - W_2^0 [W_2^0]\, x(5)$$

$$X_1'(5) = W_2^0 x(3) + W_2^0 \cdot W_2^0 x(7)$$

$$X_1'(6) = W_2^0 x(3) - W_2^0 \; W_2^0 x(7)$$

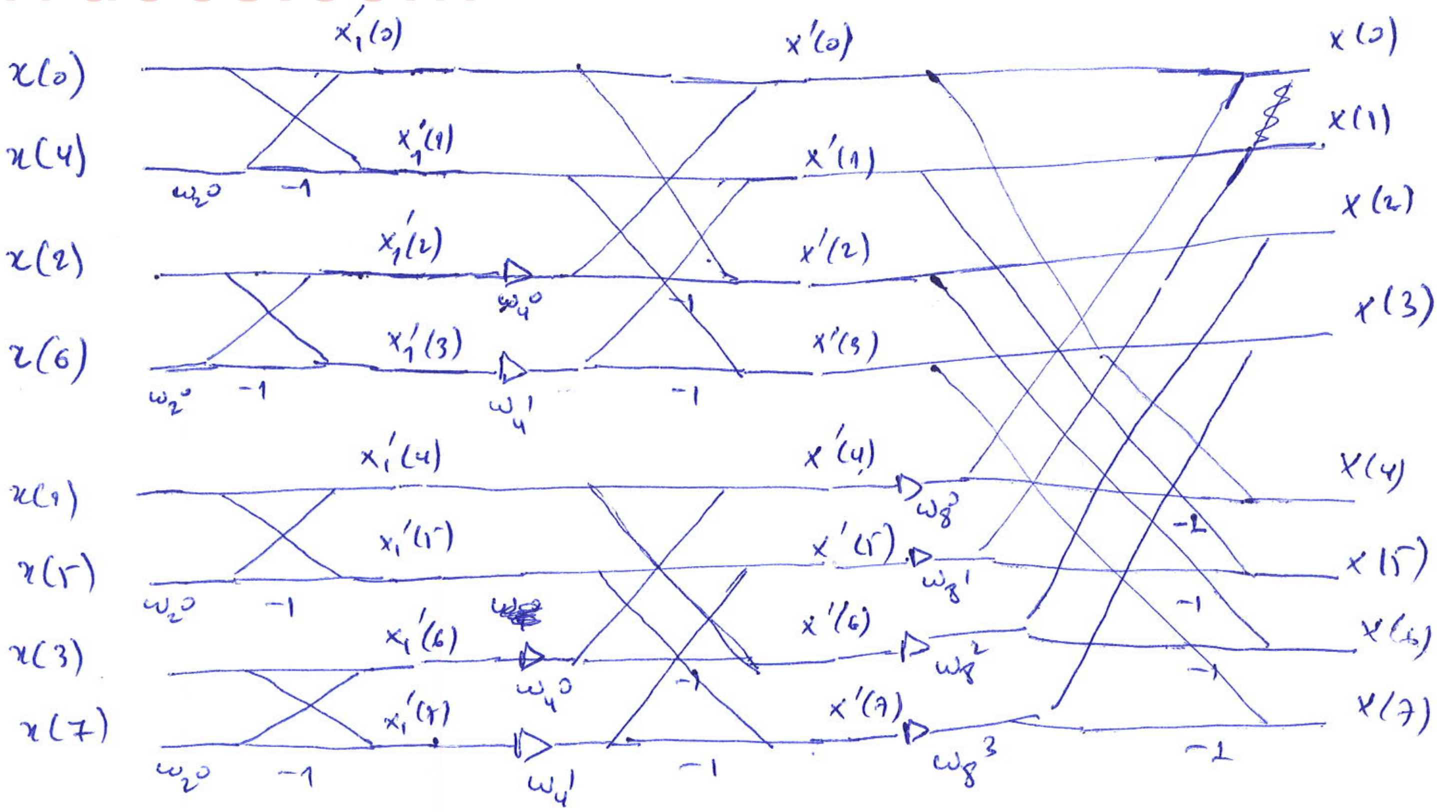
$$\begin{bmatrix} X'(4) \\ X'(5) \end{bmatrix} = \cancel{X_1'(4) + W_4^0 \, X_1'(5)}$$

$$\begin{bmatrix} X'(4) \\ X'(5) \end{bmatrix} = \begin{bmatrix} X_1'(4) \\ X_1'(5) \end{bmatrix} + \begin{bmatrix} W_4^0 & 0 \\ 0 & W_4^1 \end{bmatrix} \begin{bmatrix} X_1'(6) \\ X_1'(7) \end{bmatrix}$$

$$\begin{bmatrix} X'(6) \\ X'(7) \end{bmatrix} = \begin{bmatrix} X_1'(4) \\ X_1'(5) \end{bmatrix} - \begin{bmatrix} W_4^0 & 0 \\ 0 & W_4^1 \end{bmatrix} \begin{bmatrix} X_1'(6) \\ X_1'(7) \end{bmatrix}$$

$$
\begin{bmatrix} X(3) \\ X(4) \\ X(5) \\ X(6) \end{bmatrix}
=
\underbrace{\begin{bmatrix} \omega_8^0 & \omega_8^0 & \omega_8^0 & \omega_8^0 \\ \omega_8^0 & \omega_8^2 & \omega_8^4 & \omega_8^8 \\ \omega_8^0 & \omega_8^4 & \omega_8^8 & \omega_8^{16} \\ \omega_8^0 & \omega_8^6 & \omega_8^{12} & \omega_8^{24} \end{bmatrix}}_{}
\begin{bmatrix} x(0) \\ x(2) \\ x(4) \\ x(6) \end{bmatrix}
-
\begin{bmatrix} \omega_8^0 & & & \\ & \omega_8^1 & & \\ & & \omega_8^2 & \\ & & & \omega_8^2 \end{bmatrix}
\begin{bmatrix} \omega_8^0 & \omega_8^0 & \omega_8^0 & \omega_8^0 \\ \omega_8^0 & - & - & \\ \omega_8^0 & - & - & \\ \omega_8^0 & - & - & \omega_8^4 \end{bmatrix}
\begin{bmatrix} x(1) \\ x(3) \\ x(5) \\ x(7) \end{bmatrix}
$$

$$
\begin{bmatrix} X'(4) \\ X'(5) \\ X'(6) \\ X'(7) \end{bmatrix}
=
\begin{bmatrix} \omega_4^0 & \omega_4^0 & \omega_4^0 & \omega_4^0 \\ \omega_4^0 & \omega_4^1 & \omega_4^2 & \omega_4^4 \\ \omega_4^0 & \omega_4^2 & \omega_4^4 & \omega_4^8 \\ \omega_4^0 & \omega_4^3 & \omega_4^6 & \omega_4^{12} \end{bmatrix}
\begin{bmatrix} x(0) \\ x(2) \\ x(4) \\ x(6) \end{bmatrix}
$$

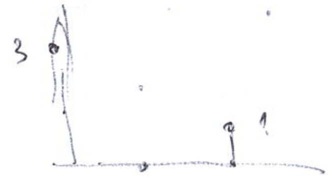


<u>multiplication</u> $\frac{N}{2} \log_2 N$. $4 \cdot 3 = 12$ multiplication

$x(n) = \{1, 2\}$

$$\begin{cases} X(0) = 1 + 2 = 3. \\ X(1) = 1 + 2e^{-2\pi j/2} = 1 - 2 = -1 \end{cases}$$

$x(n) = \{1, 2, 0, 0\}$



$X(0) = 3$

$X(1) = 1 + 2e^{-\frac{2\pi}{4}} = 1 + 2e^{-j\frac{\pi}{2}} = 1 - 2j.$ $\quad \sqrt{1+4}. \quad \sqrt{5}$

$X(2) = 1 + 2e^{-2\pi j\,2/4} = -1$