Série N°2

Exercice 1 :

-1- a- $A_1 = a \downarrow (b+c)$ et $B_1 = (a \downarrow b) + (a \downarrow c)$

$A_1 = \overline{a + (b+c)}$

$= \bar{a}\,(\overline{b+c})$

$= \bar{a}(\bar{b}\cdot\bar{c})$

$B_1 = \overline{a+b} + \overline{a+c}$

$B_1 = \bar{a}\bar{b} + \bar{a}\bar{c}$

$= \bar{a}\,(\bar{b} + \bar{c})$

$\Rightarrow A_1 \neq B_1$

-b- $A_2 = a \downarrow (b.c)$ et $B_2 = (a \downarrow b)(a \downarrow c)$

$= \overline{a + (b.c)}$

$= \bar{a}\cdot(\overline{b\cdot c})$

$= \bar{a}(\bar{b} + \bar{c})$

$B_2 = (\overline{a+b})(\overline{a+c})$

$= (\bar{a}\cdot\bar{b})(\bar{a}\cdot\bar{c}) = \bar{a}.\bar{b}.\bar{a}.\bar{c}$

$= \bar{a}\,\bar{b}\,\bar{c}$

$\Rightarrow A_2 \neq B_2$

-C- NOR est non distributive.

-2-a- $A_3 = a \uparrow (b+c)$ et $B_3 = (a \uparrow b) + (a \uparrow c)$

$= \overline{a\cdot(b+c)}$

$= \bar{a} + (\overline{b+c})$

$= \bar{a} + \bar{b}\cdot\bar{c}$

$B_3 = (\overline{a\,b}) + (\overline{a\,c})$

$= \bar{a} + \bar{b} + \bar{a} + \bar{c}$

$= \bar{a} + \bar{b} + \bar{c}$

$\Rightarrow A_3 \neq B_3$

-b- $A_4 = a \uparrow (b.c)$ et $B_4 = (a \uparrow b).(a \uparrow c)$

$= \overline{a\,(b\cdot c)}$

$= \bar{a} + (\overline{b\cdot c})$

$= \bar{a} + \bar{b} + \bar{c}$

$B_4 = (\overline{a\,b}).(\overline{a\,c})$

$= (\bar{a} + \bar{b})(\bar{a} + \bar{c})$

$= \bar{a} + \bar{a}\bar{c} + \bar{a}\bar{b} + \bar{b}\bar{c}$ $= \bar{a}(1+\bar{c}) + \bar{b}\{\bar{a} + \bar{c}\bar{b}$

$= \bar{a} + \bar{b}\bar{a} + \bar{b}\bar{c}$

$= \bar{a}(1+\bar{b}) + \bar{b}\bar{c}$

$= \bar{a} + \bar{b}\bar{c}$

1 $= \bar{a} + (\bar{b}\bar{c})$

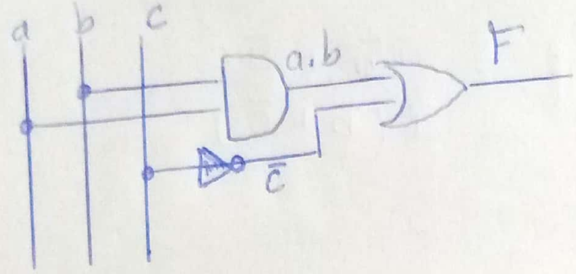
$\Rightarrow A_4 \neq B_4$

- c - NAND n'est pas distributive

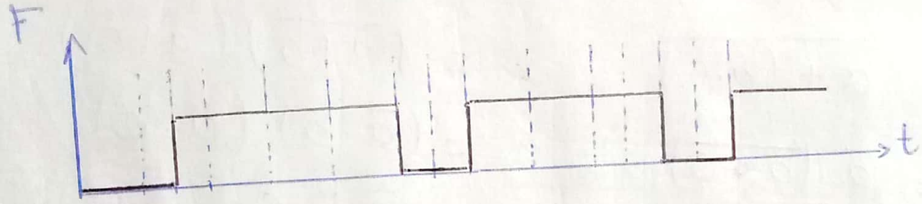
Exercice 2 :

$F = a.b + \bar{c}$

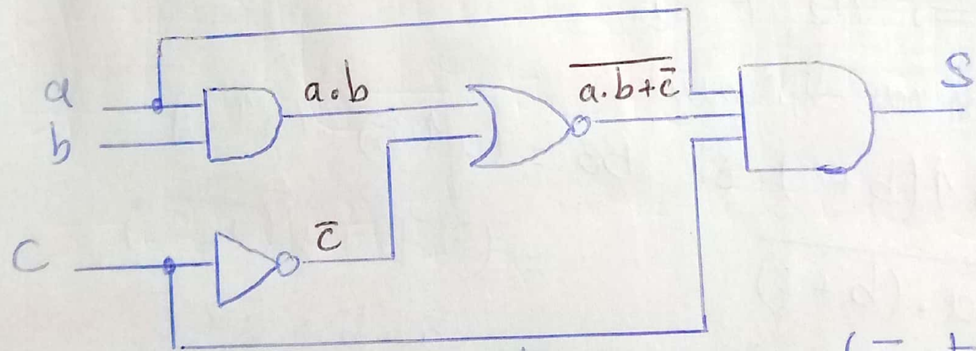
-1-



a	b	c	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



Exercice 3 :



$S = \overline{(a.b)+\bar{c}} \, . \, a \, . \, c = \overline{ab} \, . \, c \, . \, a \, . \, c = (\bar{a}+\bar{b}) \, ac = a\bar{a}c + a\bar{b}c$

$= a\bar{b}c$

Exercice 4 :

$F = a + bc$

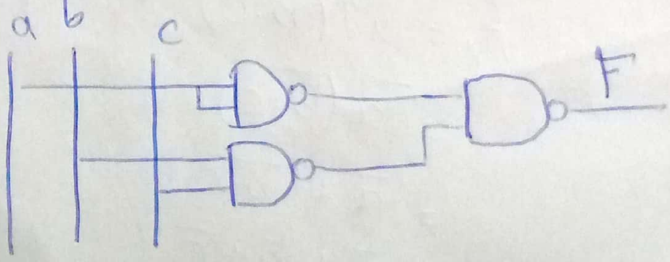
-1- Avec des NAND à 2 entrées.

$F = \overline{\overline{a+bc}}$

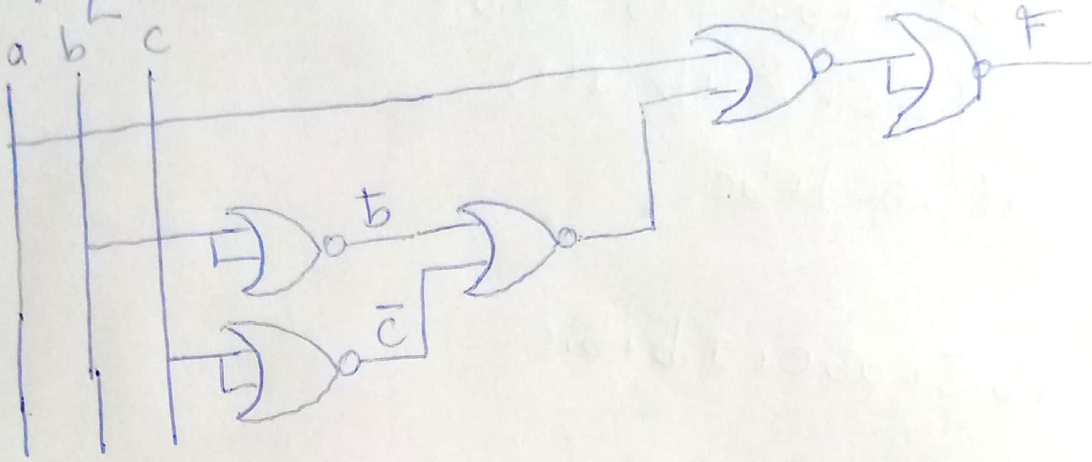
$= \overline{\bar{a} \, . \, \overline{b.c}}$

$= \bar{a} \uparrow (b \uparrow c)$

$= (a \uparrow a) \uparrow (b \uparrow c)$



$$F = \overline{a+bc} = (a \downarrow bc) \downarrow (a \downarrow bc)$$
$$= (a \downarrow (\bar b \downarrow \bar c)) \downarrow (a \downarrow (\bar b \downarrow \bar c))$$
$$= a \downarrow \left[((b \downarrow b) \downarrow (c \downarrow c))\right] \downarrow \left[a \downarrow ((b \downarrow b) \downarrow (c \downarrow c))\right]$$



Exercice 5 :

1) $F_1 = (\bar a + b)(a + \bar b)$

$$\overline{F_1} = \overline{(\bar a + b)(a + \bar b)} = (\overline{\bar a + b}) + (\overline{a + \bar b}) = a . \bar b + \bar a . b = a \oplus b$$

principe de dualité :

$$F_1 = (a . \bar b) + (\bar a . b) = a\bar b + \bar a b = a \oplus b$$

2) $F_2 = \bar a . \bar b + a . b . (c + \bar d)$; principe de dualité.

$$\overline{F_2} = (a+b)[(\bar a + \bar b) + (\bar c . d)]$$

$$\overline{F_2} = \overline{\bar a \bar b + ab(c + \bar d)} = \overline{\bar a . \bar b} . \overline{ab(c + \bar d)} = (a+b) . \overline{ab} + \overline{c + \bar d}$$
$$= (a+b) . [\bar a + \bar b + \bar c d]$$

Exercice 6 :

- $S_1 = \bar a (a+b) = \bar a a + \bar a b = ab$
- $S_2 = \overline{a + \bar b + \bar a . \bar b} = \overline{a + \bar b} . \overline{\bar a . \bar b} = (\bar a b) a . b = 0$
- $S_3 = \overline{(\bar a + \bar c)(b + \bar d)} = (\overline{\bar a + \bar c}) + (\overline{b + \bar d}) = (\bar{\bar a} . \bar{\bar c}) + \bar b \bar{\bar d}$
  $= ac + \bar b d$
- $S_4 = a.b.c + a.b + a.\bar b + a\bar c + a\bar b.c + c$
  $= a(bc + \underbrace{b + \bar b}_{1} + \bar c + \bar b c) + c = a + c$

$$\bullet\ S_5 = (a+a\cdot b)(a+b)+b(a+bc)$$
$$= a(a+b)+ba+bc$$
$$= a+ab+ab+bc = a+bc$$

$$\bullet\ S_6 = \bar a bc + a\bar b\cdot c + ab\bar c + a\cdot b\cdot c$$
$$= a(\bar b c + b\bar c) + bc(\bar a + a)$$
$$= a(b\oplus c)+bc$$

$$\bullet\ S_7 = a\cdot b\cdot \bar c + a\bar b c + \bar a\cdot b + b\cdot c$$

Exercice 7 :

S_1

dc \ ba	00	01	11	10
00	1	0	1	1
01	0	0	1	0
11	1	1	0	1
10	1	1	0	1

$$S_1 = \bar a\bar c + d\bar b + ba\bar d + \bar a d$$

S_2

dc \ ba	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$$S_2 = \bar a\bar b\bar c + ac + ad$$

Exercice 8 :