$$\hat{\theta}(4) = \begin{bmatrix} 0,61 & -0,15 \\ -0,15 & 0,37 \end{bmatrix} \begin{bmatrix} -1,9 \\ 0,73 \end{bmatrix}$$

$$= \begin{bmatrix} -1,26 \\ 0,55 \end{bmatrix} = \begin{bmatrix} \hat{a}(4) \\ \hat{b}(4) \end{bmatrix}$$

5) MCR $\rightarrow \hat{\theta}(5) = ?$

$$\begin{cases} \hat{\theta}(5) = \hat{\theta}(4) + P(5)\, \varphi^T(5)\, \varepsilon^0(5) \\ \varepsilon^0(5) = y(5) - \hat{\theta}^T(4)\, \varphi(5) \\ P(5) = P(4) - \dfrac{P(4)\, \varphi^T(4)\, \varphi(5)\, P(4)}{1 + \varphi(5) P(4)\, \varphi^T(5)} \end{cases}$$

6) Test de validation

test de Blancheur

Théoriquement

$$\begin{cases} RN(0) = 1 \quad \text{avec } R(0) = \sum_{k=1}^{N} \varepsilon^2(k) \\ R(i) = \frac{1}{N} \sum_{k=1}^{N} \varepsilon(k)\, \varepsilon(k-i) \\ RN(i) = \frac{R(i)}{R(0)} = 0 \geq 1 \end{cases}$$

Pratiquement $\begin{cases} RN(0) = 1. \\ |RN(i)| < 0,15 \;(\text{ou } 0,17)\; i \geq 1 \end{cases}$

nbre de ($N = 3$ observation. Φ [

$$R(0) = \frac{1}{3} \sum_{k=2}^{4} \varepsilon^2(k).$$

$$= \frac{1}{3} \left[\varepsilon^2(2) + \varepsilon^2(3) + \varepsilon^2(4) \right].$$

$\varepsilon(2) = y(2) - \hat{y}(2) = y(2) - \hat{\theta}^T(4)\, \varphi^T(2) = -0,05$

$\varepsilon(3) = y(3) - \hat{y}(3) = y(3) - \hat{\theta}^T(4)\, \varphi^T(3)$

$\varepsilon(4) = y(4) - \hat{y}(4) = y(4) - \hat{\theta}(4)\, \varphi(4)$

$\varepsilon(2) = -0,5 - [-1,26 \;\; 0,55] \begin{bmatrix} 0 \\ -1 \end{bmatrix} = 0,05.$

$\varepsilon(3) = -1,85 \; [-1,26 \;\; 0,55] \begin{bmatrix} 0,5 \\ -1 \end{bmatrix} = 0,07.$

$\varepsilon(4) = -1,02 - [-1,26 \;\; 0,55] \begin{bmatrix} 1,25 \\ 1 \end{bmatrix} = 0,005$

$$R(0) = \frac{1}{3} \left[(0,05)^2 + (-0,07)^2 + (0,005)^2 \right]$$

$$= 0,0024.$$

$$R(1) = \frac{1}{3} \sum_{k=2}^{4} \varepsilon(k)\, \varepsilon(k-1)$$

$$= \frac{1}{3} \left[\varepsilon(2)\varepsilon(1) + \varepsilon(3)\varepsilon(2) + \varepsilon(4)\varepsilon(3) \right] \quad (\varepsilon(1) = 0)$$

$$= \frac{1}{3} \left[(-0,07 \times 0,05) + (0,005) + (-0,07) \right] = -1,2 \cdot 10^{-3}.$$

$$R(2) = \frac{1}{3} \sum_{k=2}^{4} \varepsilon(k)\, \varepsilon(k-2)$$

$$= \frac{1}{3} \left[\varepsilon(2)\varepsilon(0) + \varepsilon(3)\varepsilon(1) + \varepsilon(4)\varepsilon(2) \right] \quad (\varepsilon(0) = 0,\ \varepsilon(1) = 0)$$

$$= \frac{1}{3} \left[0,005 \times (-0,07) \right] = -1,16 \cdot 10^{-4}$$

$$R(3) = \frac{1}{3} \left[\sum_{k=2}^{4} \varepsilon(k)\, \varepsilon(k-3) \right] = 0.$$

$$RN(0) = \frac{R(0)}{R(0)} = 1.$$

$$RN(1) = \frac{R(1)}{R(0)} = \frac{-1,2 \cdot 10^{-3}}{0,0024} = -0,5$$

$$RN(2) = \frac{R(2)}{R(0)} = \frac{-1,16 \cdot 10^{-4}}{0,0024} = -0,048.$$

$$RN(3) = \frac{R(3)}{R(0)} = 0.$$

$$RN(4) = \frac{R(4)}{R(0)} = 0$$

$\Rightarrow$ le modèle n'est pas valide car $0,5 > 0,15$ ou $0,17$.

7) $n_A = 2 \quad n_B = 1$

$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2}$

$B(q^{-1}) = b_1 q^{-1}$

$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b_1 u(k-1) + v(k).$

$\theta^T = [a_1 \;\; a_2 \;\; b_1].$

$\varphi(k) \;\; [-y(k-1) \;\; -y(k-2) \;\; u(k-1)].$

$\hat{\theta}(4) = [\Phi^T(4)\, \Phi(4)]^{-1}\, \Phi^T(4)\, y(4))$