LES LIGNES DE TRANSMISSION

$$\frac{\partial \underline{U}}{\partial z} = -R'\underline{I} - L'\frac{\partial \underline{I}}{\partial t} \ ; \ \frac{\partial \underline{I}}{\partial z} = -G'\underline{U} - C'\frac{\partial \underline{U}}{\partial t}$$

Equations des télégraphistes

$$\frac{\partial^2 \underline{U}}{\partial z^2} - \frac{\partial^2 \underline{U}}{\partial t^2}L'C' - \frac{\partial \underline{U}}{\partial t}\left[R'C' + L'G'\right] - R'G'\underline{U} = 0$$

Régime sinusoïdal

$$\underline{U}(z,t) = \underline{u}(z)e^{j\omega t} \ ; \frac{\partial \underline{u}}{\partial z} = -(R'+j\omega L')\underline{i} \ ;$$

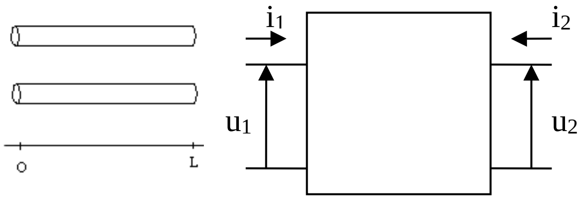
$$\frac{\partial \underline{i}}{\partial z} = -(G'+j\omega C')\underline{u} \ ; \frac{\partial^2 \underline{u}}{\partial z^2} = (R'+j\omega L')(G'+j\omega C')\underline{u}$$

$$\underline{\gamma} = \sqrt{(R'+j\omega L')(G'+j\omega C')} = \alpha + j\beta$$

$$\underline{u}(z) = \underline{u}_+ e^{-\underline{\gamma} z} + \underline{u}_- e^{+\underline{\gamma} z} \ ; \underline{i}(z) = \underline{Y}_c(\underline{u}_+ e^{-\underline{\gamma} z} - \underline{u}_- e^{+\underline{\gamma} z})$$

$$\underline{Z}_c = \frac{\underline{u}_+}{\underline{i}_+} = -\frac{\underline{u}_-}{\underline{i}_-} = \sqrt{\frac{(R'+j\omega L')}{(G'+j\omega C')}}$$

Une ligne de transmission de longueur L est équivalente à un quadripôle :



$$\begin{pmatrix} \underline{u}_2 \\ \underline{i}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} \cosh(\underline{\gamma}L) & -\underline{Z}_c\sinh(\underline{\gamma}L) \\ -\underline{Y}_c\sinh(\underline{\gamma}L) & \cosh(\underline{\gamma}L) \end{pmatrix}}_{\text{matrice de chaine}} \begin{pmatrix} \underline{u}_1 \\ \underline{i}_1 \end{pmatrix}$$

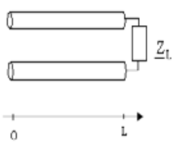
Coefficient de reflection :

$$\underline{\rho}(z) = \frac{\underline{u}_-}{\underline{u}_+}e^{2\underline{\gamma} z} \ ; \underline{u}(z) = \underline{u}_+ e^{-\underline{\gamma} z}[1+\underline{\rho}(z)] \ ;$$

$$\underline{i}(z) = \underline{Y}_c \underline{u}_+ e^{-\underline{\gamma} z}[1-\underline{\rho}(z)]$$

$$\underline{Z}(z) = Z_c\frac{1+\underline{\rho}(z)}{1-\underline{\rho}(z)} \ ; \underline{\rho}(z) = \frac{\underline{Z}(z) - \underline{Z}_c}{\underline{Z}(z) + \underline{Z}_c}$$

LIGNE TERMINEE PAR UNE CHERGE



$$\underline{\rho}(z) = \underline{\rho}_L e^{2\underline{\gamma}(z-L)}$$

Cas d'une ligne adaptée :

$$\underline{Z}_L = \underline{Z}_c \Rightarrow \underline{\rho}_L = 0; \underline{u}_- = 0; \underline{\rho}(z) = 0;$$

$$\underline{Z}(z) = \underline{Z}_c$$

Cas d'une ligne terminée par un court-circuit(cc)

$$\underline{Z}_L = 0; \underline{u}_L = 0; \underline{\rho}_L = -1; \underline{\rho}(z) = -e^{2\underline{\gamma}(z-L)};$$

$$\underline{Z}(z) = -\underline{Z}_c \tanh[\underline{\gamma}(z-L)]$$

Cas d'une ligne terminée par un circuit ouvert(co)

$$\underline{Z}_L = \infty; \underline{i}_L = 0; \underline{\rho}_L = +1; \underline{\rho}(z) = +e^{2\underline{\gamma}(z-L)};$$

$$\underline{Z}(z) = -\underline{Z}_c \coth[\underline{\gamma}(z-L)]$$

LIGNES DE TRANSMISSION SANS PERTES

$$Z_c = \sqrt{\frac{L'}{C'}} \ ; R'=G'=0 \ ; \ \underline{\gamma} = j\beta \ ;$$

$$|\underline{u}(z)| = |\underline{u}_+|\left|1+\underline{\rho}(0)e^{2j\beta z}\right|$$

Le taux d'ondes stationnaires (TOS)

$$s = \frac{u_{max}}{u_{min}} = \frac{1+|\rho|}{1-|\rho|} \Rightarrow |\rho| = \frac{s-1}{s+1}$$

ABAQUE DE SMITH

C'est la transformation : $\underline{z}(z) = \frac{\underline{Z}(z)}{\underline{Z}_c} = r + jx \ \rightarrow \ \underline{\rho}(z) = a + jb$

	$\underline{z}(z)$	$\underline{\rho}(z)$	Commentaire
C.A	1	0	Point O
C.C	0	-1	Point O'
C.O	∞	+1	Point O''
Resistance pure	r	$1-\frac{2}{1+r}$	Portion de l'axe €[-1,1]
Réactance pure	jx	$\frac{jx-1}{jx+1}$	Cercle unité

GUIDES D'ONDES

$$\Delta\underline{\psi} = \underline{k}^2\underline{\psi} \ avec \ \underline{k}^2 = j\omega\mu(\sigma + j\omega\varepsilon) \quad et \ \underline{\psi} = \begin{pmatrix} \vec{\underline{E}} \\ \vec{\underline{H}} \end{pmatrix}$$

Pas de pertes $\Rightarrow \underline{k}^2 = -k_0^2 = -\omega^2\mu\varepsilon$

GUIDE RECTANGULAIRE :

Résolution : $\underline{\psi}(x,y,z) = \underline{X}(x)\underline{Y}(y)\underline{Z}(z)$

$$\Rightarrow \beta_g^2 = k_0^2 - k_c^2 ; avec \, k_c^2 = k_x^2 + k_y^2$$

$$\underline{X}(x) = \underline{A}\sin(k_x x) + \underline{B}\cos(k_x x)$$

$$\underline{Y}(y) = \underline{C}\sin(k_y y) + \underline{D}\cos(k_y y)$$

$$\underline{Z}(z) = \underline{F}.\exp(-j\beta_g z)$$

conditions aux limites : $\underline{E}_y = \underline{E}_z = 0 \ en \ x = 0 \ et \ a$

$$\underline{E}_x = \underline{E}_z = 0 \ en \ y = 0 \ et \ b$$

$$k_{cmn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \omega_c\sqrt{\mu\varepsilon}$$

modes $TM_{mn} \Rightarrow \underline{H}_z = 0$

$\underline{E}_z(x,y,z) = \underline{E}_{mnz}\sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)\exp(-j\beta_g z)$ m=1,2,3,…

n=1,2,3…

modes $TE_{mn} \Rightarrow \mathbf{\underline{E}_z} = 0$