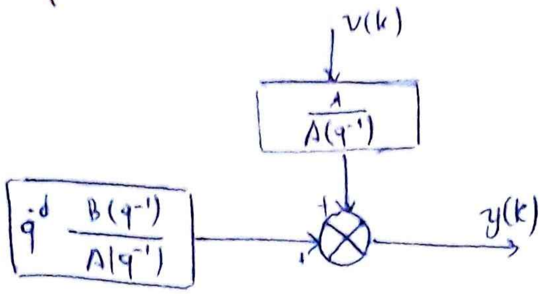
TD2 :

$\times 4$

<u>Exercice 3 :</u>



$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-1} + \dots + a_{n_A} q^{-n_A}$$

$$B(q^{-1}) = b_1 q^{-1} + \dots + b_{n_B} q^{-n_B}$$

$$d = 0$$

4/ $n_A = 1 \quad n_B = 1.$

$$A(q^{-1}) = 1 + a_1 q^{-1}$$

$$B(q^{-1}) = b_1 q^{-1}$$

$$A(q^{-1})\, y(k) = B(q^{-1})\, u(k) + v(k).$$

$$(1 + a_1 q^{-1})\, y(k) = b_1 q^{-1} u(k) + v(k).$$

$$y(k) = -a_1 y(k-1) + b_1 u(k-1) + v(k).$$

MCO = MCNR $\rightarrow \hat{\theta}(4)$?

$$\theta^T = [a_1 \quad b_1]$$

$$\varphi(k) = [-y(k-1) \quad u(k-1)]$$

$$\hat{\theta}(4) = [\phi^T(4)\, \phi(4)]^{-1}\, \phi(4)\, Y(4).$$

$$\phi(4) = \begin{bmatrix} \varphi(1) \\ \varphi(2) \\ \varphi(3) \\ \varphi(4) \end{bmatrix} = \begin{bmatrix} -y(0) & u(0) \\ -y(1) & u(1) \\ -y(2) & u(2) \\ -y(3) & u(3) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0,5 & -1 \\ 1,25 & 1 \end{bmatrix}$$

$$Y(4) = \begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ y(4) \end{bmatrix} = \begin{bmatrix} -0,5 \\ -1,25 \\ -1,02 \end{bmatrix}$$

$$\phi^T(4)\, \phi(4) = \begin{bmatrix} 0 & 0,6 & 1,25 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 0,5 & -1 \\ 1,25 & 1 \end{bmatrix} = \begin{bmatrix} 1,81 & 0,95 \\ 0,75 & 3 \end{bmatrix}$$

6

: feuilles
remises

$$\Phi(4)^T Y(4) = \begin{bmatrix} 0 & 0{,}5 & 1{,}25 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -0{,}5 \\ -1{,}25 \\ -1{,}02 \end{bmatrix} = \begin{bmatrix} -1{,}9 \\ 0{,}73 \end{bmatrix}$$

$$\left(\Phi(4)^T \Phi(4)\right)^{-1} = \frac{1}{\det} {}^T\mathrm{com}(\quad)$$

$$= 0{,}2 \begin{bmatrix} 3 & -0{,}75 \\ -0{,}75 & 1{,}81 \end{bmatrix}$$

$$\hat{\theta}(4) = 0{,}2 \begin{bmatrix} 3 & -0{,}75 \\ -0{,}75 & 1{,}81 \end{bmatrix} \begin{bmatrix} -1{,}9 \\ 0{,}73 \end{bmatrix} = \begin{bmatrix} 0{,}6 & -0{,}15 \\ -0{,}15 & 0{,}362 \end{bmatrix} \begin{bmatrix} -1{,}9 \\ 0{,}73 \end{bmatrix}$$

$$\hat{\theta}(4) = \begin{bmatrix} -1{,}24 \\ 0{,}54 \end{bmatrix} = \begin{bmatrix} \hat{a}_1(4) \\ \hat{b}_1(4) \end{bmatrix}$$

MCR $\varepsilon \longrightarrow \hat{\theta}(5)$?

$$\hat{\theta}(5) = \hat{\theta}(4) + P(5)\,\varphi^T(5)\,\varepsilon^\circ(5)$$

$$\& \quad \varepsilon^\circ(5) = y(5) - \hat{\theta}^T(4)\,\varphi^T(5).$$

$$P(5) = P(4) - \frac{P(4)\,\varphi^T(5)\,\varphi(5)\,P(4)}{1 + \varphi(5)\,P(4)\,\varphi^T(5)}.$$

$$\varphi(k) = \begin{bmatrix} -y(k-1) & u(k-1) \end{bmatrix}$$

$$\varphi(5) = \begin{bmatrix} -y(5) & u(4) \end{bmatrix}^T = \begin{bmatrix} 1{,}02 & -1 \end{bmatrix}$$

$$\varepsilon^\circ(5) = -1{,}16 - \begin{bmatrix} -1{,}24 & 0{,}54 \end{bmatrix} \begin{bmatrix} 1{,}02 \\ -1 \end{bmatrix} = 0{,}64.$$

$$\varphi(5)\,P(4)\,\varphi^T(5) = \begin{bmatrix} 1{,}02 & -1 \end{bmatrix} \begin{bmatrix} 0{,}6 & -0{,}15 \\ -0{,}15 & 0{,}362 \end{bmatrix} \begin{bmatrix} 1{,}02 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0{,}75 & -1{,}4 \end{bmatrix} \begin{bmatrix} 1{,}02 \\ -1 \end{bmatrix} = 1{,}29$$

→ on n'a pas encore converger.

$$P(4)\,\varphi^T(5)\,\varphi(5)\,P(4) = \begin{bmatrix} 0,6 & -0,15 \\ -0,15 & 0,362 \end{bmatrix}\begin{bmatrix} 1,02 \\ -1 \end{bmatrix}\begin{bmatrix} 1,02 & -1 \end{bmatrix}\begin{bmatrix} 0,6 & -0,15 \\ -0,15 & 0,362 \end{bmatrix}$$

$$= \begin{bmatrix} 0,76 \\ -0,51 \end{bmatrix}\begin{bmatrix} 0,76 & -0,51 \end{bmatrix}$$

$$= \begin{bmatrix} 0,57 & -0,38 \\ -0,38 & 0,26 \end{bmatrix}$$

$$P(5) = \begin{bmatrix} 0,6 & -0,15 \\ -0,15 & 0,362 \end{bmatrix} - \frac{1}{2,29}\begin{bmatrix} 0,57 & -0,38 \\ -0,38 & 0,26 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

$$P(5) = \begin{bmatrix} 0,6 & -0,15 \\ -0,15 & 0,362 \end{bmatrix} - \begin{bmatrix} 0,24 & -0,16 \\ -0,16 & +0,11 \end{bmatrix} = \begin{bmatrix} 0,36 & 0,01 \\ 0,01 & 0,252 \end{bmatrix}$$

$$\hat{\theta}(5) = \hat{\theta}(4) + P(5)\,\varphi^T(5)\,\varepsilon^\circ(5)$$

$$= \begin{bmatrix} -1,24 \\ 0,54 \end{bmatrix} + \begin{bmatrix} 0,36 & 0,01 \\ 0,01 & 0,252 \end{bmatrix}\begin{bmatrix} 1,02 \\ -1 \end{bmatrix} \times 0,64.$$

$$\hat{\theta}(5) = \begin{bmatrix} -1,01 \\ 0,385 \end{bmatrix}.$$

...e N° 01 Test de blancheur :

théorique
$$\begin{cases} RN(0) = 1 & R(0) = \frac{1}{N}\sum_{i=1}^{N} e^2(i) \\ R(i) = \frac{R(i)}{R(0)} = 0 & i \geq 1. \\ R(i) = \frac{1}{N}\sum_{k=1}^{N} \varepsilon(k)\varepsilon(k-i) \end{cases}$$

Pratique :
$$\begin{cases} RN(0) = 1. \\ |RN(i)| < 0{,}15 \div 0{,}17 & i \geq 1. \end{cases}$$

$$R(0) = \frac{1}{3}\sum_{i=2}^{4} e^2(i) = \frac{1}{3}\left[e^2(2) + e^2(3) + e^2(4)\right]$$

$$e(2) = y(2) - \hat{y}(2) = y(2) - \hat{\theta}^T(2)\varphi^T(2)$$
$$e(3) = y(3) - \hat{y}(3) = y(3) - \hat{\theta}^T(3)\varphi^T(3).$$
$$e(4) = y(4) - \hat{y}(4) = y(4) - \hat{\theta}^T(4)\varphi^T(4)$$

$$\varphi(k) = \begin{bmatrix} -y(k-1) & u(k-1) \end{bmatrix}.$$
$$\varphi(2) = \begin{bmatrix} -y(1) & u(1) \end{bmatrix} = \begin{bmatrix} 0 & -1 \end{bmatrix}$$

$\varepsilon(2) =$
$$e(2) = -0{,}5 - \begin{bmatrix} -1{,}24 & 0{,}54 \end{bmatrix}\begin{bmatrix} 0 \\ -1 \end{bmatrix} = 0{,}04.$$

$\varepsilon(3) =$
$$e(3) = -1{,}25 - \begin{bmatrix} -1{,}24 & 0{,}54 \end{bmatrix}\begin{bmatrix} 0{,}5 \\ -1 \end{bmatrix} = -0{,}09.$$

$\varepsilon(4) =$
$$e(4) = -1{,}02 - \begin{bmatrix} -1{,}24 & 0{,}54 \end{bmatrix}\begin{bmatrix} 1{,}25 \\ 1 \end{bmatrix} = \cancel{-0{,}01} \; -0{,}01.$$

$$R(0) = \frac{1}{3}\left[(0{,}04)^2 + (0{,}09)^2 + (-0{,}01)^2\right] = 0{,}0032.$$

$$R(0) = \frac{1}{3}\left[(0{,}04)^2 + (0{,}09)^2 + (-0{,}01)^2\right] = 0{,}0032$$

$$R(1) = \frac{1}{3}\sum_{k=2}^{4} \varepsilon(k)\,\varepsilon(k-1) = \frac{1}{3}\left[\cancel{\varepsilon(2)\varepsilon(1)} + \varepsilon(3)\varepsilon(2) + \varepsilon(4)\varepsilon(3)\right]$$
$$= \frac{1}{3}\left[-0{,}09 \times 0{,}04 + (-0{,}01)\times(-0{,}09)\right] = -0{,}0009$$

$$R(2) = \frac{1}{3}\sum_{k=2}^{4} \varepsilon(k)\,\varepsilon(k-2) = \frac{1}{3}\left[\underset{0}{\varepsilon(2)\varepsilon(0)} + \underset{0}{\varepsilon(3)\varepsilon(1)} + \varepsilon(4)\varepsilon(2)\right].$$

$$= \frac{1}{3}\left[(-0{,}01)\times(0{,}04)\right].$$

$$= -0{,}00013.$$

$$R(3) = \frac{1}{3}\sum_{k=2}^{4} \varepsilon(k)\,\varepsilon(k-1) = \frac{1}{3}\left[\varepsilon(2)\varepsilon(-1) + \varepsilon(3)\varepsilon(0) + \varepsilon(4)\varepsilon(1)\right] = 0.$$

$R(4) = 0.$

⚠ récursive
on pose $\Phi(k) = \varphi^T(k)$

$RN(0) = 1.$

$$RN(1) = \frac{R(1)}{R(0)} = \frac{-0{,}0009}{0{,}0032} = -0{,}28 \longrightarrow$$ n'appartient pas à ~~0,16~~ 0,14.

$$RN(2) = \frac{R(2)}{R(0)} = \frac{-0{,}0013}{0{,}0032} = -0{,}04.$$

$$RN(3) = \frac{R(3)}{R(0)} = 0.$$

$$RN(4) = \frac{R(4)}{R(0)} = 0$$

$|R| \leq 0{,}15$

$-0{,}15 < R < 0{,}15$

il faut augmenter l'ordre.

7) $(1 + a_1 q^{-1} + a_2 q^{-2})\, y(k)$

$$= b_1 q^{-1}\, u(k) + v(k).$$

$$y(k) = -a_1\, y(k-1) - a_2\, y(k-2) + b_1\, u(k-1) + v(k).$$

$$\theta^T = [a_1 \quad a_2 \quad b_1]$$

$$\varphi(k) = [-y(k-1) \quad -y(k-2) \quad u(k-1)].$$

$\hat{\theta}(4) \rightsquigarrow$ MCO : $\hat{\theta}(4) = [\Phi^T(4)\,\Phi(4)]^{-1}\,\Phi^T(4)\,Y(4).$

$$\Phi(4) = \begin{bmatrix} \varphi(1) \\ \varphi(2) \\ \varphi(3) \\ \varphi(4) \end{bmatrix} = \begin{bmatrix} \cancel{-y(0)} & \cancel{-y(-1)} & \cancel{u(0)} \\ -y(1) & -y(0) & u(1) \\ -y(2) & -y(1) & u(2) \\ -y(3) & -y(2) & u(3) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0{,}5 & 0 & -1 \\ 1{,}25 & 0{,}5 & 1 \end{bmatrix}$$

$$Y(4) = \begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ y(4) \end{bmatrix} = \begin{bmatrix} -0{,}5 \\ -1{,}25 \\ -1{,}02 \end{bmatrix}$$

$$\Phi^T(4)\,\Phi(4) = \begin{bmatrix} 0 & 0{,}5 & 1{,}25 \\ 0 & 0 & 0{,}5 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0{,}5 & 0 & -1 \\ 1{,}25 & 0{,}5 & 1 \end{bmatrix} = \begin{bmatrix} 1{,}81 & 0{,}62 & 0{,}75 \\ 0{,}62 & 0{,}25 & 0{,}5 \\ 0{,}75 & 0{,}5 & 3 \end{bmatrix}$$

$$\left[\Phi^T(4)\,\Phi(4)\right]^{-1} = \frac{1}{\det}\,{}^{T}\mathrm{Com} = \frac{1}{1{,}9} \begin{bmatrix} 0{,}5 & -1{,}48 & 0{,}12 \\ -4{,}48 & 4{,}86 & -0{,}44 \\ 0{,}12 & -0{,}44 & 0{,}068 \end{bmatrix}$$

$\det \to 1{,}9$

$$\Phi^T(4)\,Y(4) = \begin{bmatrix} -1{,}69 \\ -0{,}51 \\ 0{,}73 \end{bmatrix}$$

$$\hat{\theta}(4) = \frac{1}{1{,}9} \begin{bmatrix} 0{,}5 & -1{,}48 & 0{,}12 \\ -4{,}48 & 4{,}86 & -0{,}44 \\ 0{,}12 & -0{,}44 & 0{,}068 \end{bmatrix} \begin{bmatrix} -1{,}69 \\ -0{,}51 \\ 0{,}73 \end{bmatrix} = \begin{bmatrix} -0{,}006 \\ -0{,}16 \\ 0{,}03 \end{bmatrix} = \begin{bmatrix} \hat{a}_1(4) \\ \hat{a}_2(4) \\ \hat{b}_1(4) \end{bmatrix}$$