EX1:

$X = 1 - U^2$, $U \rightsquigarrow \mathcal{U}_{[0,1]}$

-1- $G(x) = \begin{cases} 0 & \text{si } x<0 ; \\ x & \text{si } x \in [0,1] ; \\ 1 & \text{si } x>1 ; \end{cases}$

$E(U) = \frac{1}{2}$; $V(U) = \frac{1}{12}$ 1,5

-2- $E(U^4) = \int_0^1 x^4 \, dx = \left[\frac{x^5}{5}\right]_0^1 = \frac{1}{5}$

$E(X) = 1 - E(U^2) = 1 - \frac{1}{3} = \frac{2}{3}$ 1,5

$V(X) = V(U^2) = E(U^4) - (E(U^2))^2 = \frac{1}{5} - \frac{1}{9} = \frac{4}{45}$

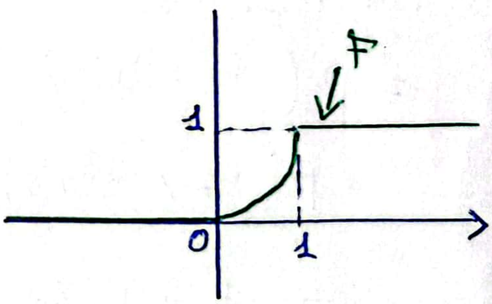
$F(x) = P(X<x) = P(1-U^2<x) = P(-U^2<x-1)$

$= P(U^2>1-x) = 1 - P(U^2<1-x)$

$= 1 - P(|U| < \sqrt{1-x}) = 1 - P(-\sqrt{1-x} < U < \sqrt{1-x})$

$= 1 - G(\sqrt{1-x})$

$F(x) = \begin{cases} 0 & \text{si } x<0 \\ 1-\sqrt{1-x} & \text{si } x \in [0,1] ; \\ 1 & \text{si } x>1. \end{cases}$ 1



EX2:

- Indépendant
- loi exponentiel

1°/ $E(X_1) = \frac{1}{\theta_1} = 1200 \Rightarrow \boxed{\theta_1 = \frac{1}{1200}}$ 1

2°/ $P(X_2 < 1700) = P(X_2 \geq 1700) = \frac{1}{2}$

$P(X_2 < 1700) = \int_0^{1700} \theta_2 e^{-\theta_2 t} dt = 1 - e^{-1700\theta_2} = \frac{1}{2}$

$\Rightarrow \boxed{\theta_2 = \frac{\ln(2)}{1700}}$ 1

1

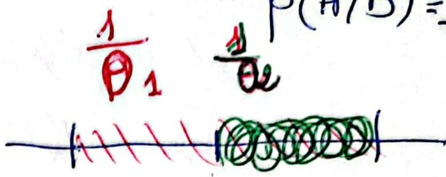
-3- $P(T>x) = P(\{X_1>x\}\cap\{X_2>x\}) = P(X_1>x)\,P(X_2>x)$ (1)

-4- $P(T\leq x) = 1 - P(T>x) = 1 - \left[1-P(X_1\leq x)\right]\left(1-P(X_2\leq x)\right]$

$= 1 - e^{-\theta_1 x} e^{-\theta_2 x} = 1 - e^{-(\theta_1+\theta_2)x}$ (1)

-5- $P\left(T>\frac{1}{\theta_2} \middle| T>\frac{1}{\theta_1}\right)$ $\qquad P(A/B) = \frac{P(A\cap B)}{P(B)}$

$= \frac{P\left(\{T>\frac{1}{\theta_2}\}\cap\{T>\frac{1}{\theta_2}\}\right)}{P\left(T>\frac{1}{\theta_1}\right)}$



(2) $= \frac{P\left(T>\frac{1}{\theta_2}\right)}{P\left(T>\frac{1}{\theta_1}\right)} = \frac{e^{-\frac{\theta_1}{\theta_2}}\,e^{-1}}{e^{-\frac{\theta_2}{\theta_1}}\,e^{-1}} = e^{\frac{\theta_2}{\theta_1}-\frac{\theta_1}{\theta_2}}$ avec $\theta_1 = \frac{1}{1200}$; $\theta_2 = \frac{Ln 2}{1700}$

EX3:

-1- $X_1 \sim B(30, 0,5)$

$E(X_1) = 15$, $V(X_1) = 30\times0,5\times0,5 = 7,5$ (2)

-2- $P(X_1>15) = 1 - P(X_1\leq 15) = 1 - F(15) = 1 - 0,5722 = 0,42$ (2)

$P\left(\left|\frac{X_1}{n_1} - 0,5\right| > 0,05\right) = ?$ $\qquad$ ($\frac{X_1}{n_1} = F_1$) $\qquad |a|>b \Rightarrow a>b$ ou $a<b$

$= P(F_1 - 0,5 > 0,05) + P(F_1 - 0,5 < 0,05) = P(F_1 > 0,55) + P(F_1 < 0,45)$

$= P(X_1 > 16,5) + P(X_1 < 13,5) = P(X_1 \geq 17) + P(X_1 \leq 13)$

$= 1 - P(X_1 \leq 17) + P(X_1 \leq 13) = 1 - F(17) + F(13)$

$= 1 - 0,7077 + 0,2923 = 0,5846$ (2)

$* n_1 = 125$; $\qquad X_1 \sim B(125 ; 0,5)$

$E(X_1) = 62,5$; $V(X_1) = 31,25$; on peut approcher la lois Binomiale par $N(62,5 ; 5,6) \sim T = \frac{X_1 - 62,5}{5,6} \sim N(0,1)$ (2)

$P(X_1\geq 63) = 1 - P(X_1<63) = 1 - P(T<0,1) = 1 - F(0,1)$

$= 1 - 0,5398 = 0,46$.

- $F_1 \longrightarrow N(0,5, 0,044)$
- $F_2 \longrightarrow N(0,5, 0,022)$

$F_1 - F_2 \longrightarrow N(0, 0,05)$

$E(F_1 - F_2) = 0$

$V(F_1 - F_2) = V(F_1) + V(F_2) = 0,0024$ (2)

- $P(|F_1 - F_2| > 0,05) = P(|T| > 1)$

$= 2(1 - F_0(1)) = 2(1 - 0,8413)$

$= 0,3174$