Micro-Ondes

Rappel électromagnétisme

- $\vec{rot}\,\vec{E}(M,t) = -\dfrac{\partial \vec{B}(M,t)}{\partial t}$
- $\vec{rot}\,\vec{H}(M,t) = \vec{J}(M,t) + \dfrac{\partial \vec{D}(M,t)}{\partial t}$
- $div\,\vec{D}(M,t) = \rho(M,t)$
- $div\,\vec{B}(M,t) = 0$

$$\vec{D} = \varepsilon \vec{E}$$
$$\vec{B} = \mu \vec{H}$$
$$\vec{J} = \sigma \vec{E}$$

En regime sinusoidale.

$\vec{E}(M,t) \rightarrow \vec{E}(M)\, e^{j\omega t}$;

$\vec{E} = Re(\vec{E}(M)\, e^{j\omega t})$

$\vec{H} = \vec{H}(M)\, e^{j\omega t}$

$\vec{B} = \vec{B}(M)\, e^{j\omega t}$

$\vec{D} = \vec{D}(M)\, e^{j\omega t}$

Sol° des eq de Maxwell pour les prob de rayonnement :

$div\,\vec{B} = 0$

$div\,\vec{H} = 0 \Rightarrow \exists A \rightarrow \boxed{\vec{H} = \dfrac{1}{\mu}\vec{rot}\,\vec{A}}$

$\vec{A}$: potentiel vecteur magnétique

RQ
$$\boxed{\begin{aligned} div\,\vec{rot}(\vec{x}) &= 0 \\ \vec{rot}\,grad(\varphi) &= 0 \end{aligned}}$$

$\vec{A}_1 = \vec{A} + grad\,\lambda$

$\Rightarrow \vec{rot}\,\vec{A}_1 = \vec{rot}\,\vec{A} + \underbrace{\vec{rot}\,grad\,\lambda}_{0}$

Eb $\vec{rot}\,\vec{E} = -j\omega\mu\vec{H}$

$= -j\omega\mu\dfrac{1}{\mu}\vec{rot}\,\vec{A}$

Formule
$$\boxed{\alpha\,\vec{rot}\,\vec{A} + \beta\,\vec{rot}\,\vec{B} = \vec{rot}(\alpha\vec{A} + \beta\vec{B})}$$

$\vec{rot}(\vec{E} + j\omega\vec{A}) = \vec{0} \rightarrow \exists V$ quivenfie

$\vec{E} + j\omega\vec{A} = -\vec{grad}\,V$ (car $\vec{rot}\,grad = 0$)

$$\boxed{\vec{E} = -\vec{grad}\,V - j\omega\vec{A}}$$

V n'est pas Unique $\exists\ \infty$ de V possible

Eb Formule
$$\boxed{\vec{rot}\,\vec{rot}(\vec{x}) = \vec{grad}\,div(\vec{x}) - \Delta\vec{x}}$$

$\vec{rot}\,\vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$

et comme $\vec{H} = \dfrac{1}{\mu}\vec{rot}\,A$

$\Rightarrow \vec{rot}\left(\dfrac{1}{\mu}\vec{rot}\,A\right) = \vec{J} + j\omega\varepsilon\vec{E}$

$\Rightarrow \vec{rot}\,\vec{rot}\,A = \mu(\vec{J} + j\omega\varepsilon\vec{E})$

et comme $\vec{E} = -\vec{grad}\,V - j\omega\vec{A}$

$\Rightarrow \vec{rot}\,\vec{rot}\,\vec{A} = \mu(\vec{J} + j\omega\varepsilon(-\vec{grad}\,V - j\omega\vec{A}))$

$\Rightarrow \vec{grad}\,div(\vec{A}) - \Delta\vec{A} = \mu\vec{J} - j\omega\varepsilon\mu\,\vec{grad}\,V + \underline{\omega^2\varepsilon\vec{A}\mu}_{\beta^2\vec{A}}$

$\beta^2 = \omega^2\varepsilon\mu$

$\Rightarrow \Delta\vec{A} + \beta^2\vec{A} = -\mu\vec{J} + \underline{j\omega\varepsilon\mu\,\vec{grad}\,V + \vec{grad}\,div(\vec{A})}$

$\vec{grad}(div\,\vec{A} + j\omega\varepsilon\mu V)$

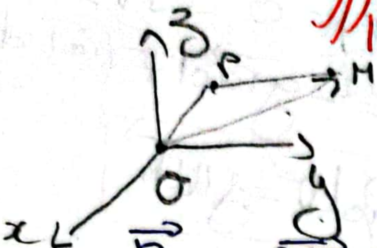
on pose $div\,\vec{A} + j\omega\varepsilon\mu\vec{V} = 0$

$$\Rightarrow \boxed{\Delta\vec{A} + \beta^2 A = -\mu\vec{J}}\ *$$

β : cte de propagat° ds le milieu illimité (ε,

$$\boxed{\beta = \omega\sqrt{\varepsilon\mu}}$$

$$\Rightarrow \vec{A}(M) = \frac{\mu}{4\pi}\iiint_{(V)} \frac{J(P)\,e^{-j\beta R}}{R}\, dV$$



$$\vec{R} = \vec{PM} = \vec{OM} - \vec{OP}$$

Pour antennes filaires

Source volumique	Source filaire
$\iiint \frac{J(P)e^{-j\beta R}}{R}$	$\int_{fil} \frac{I(P)e^{-j\beta R}}{R}\, d\vec{P}_P$

Ex

Le vecteur de Poynting

$$\vec{\Pi}(r,t) = \vec{\mathcal{E}}(M,t)\wedge \vec{\mathcal{H}}(M,t)$$
$$= \frac{1}{2}\vec{E}(M)\wedge \vec{H}^*(M)$$

RQ

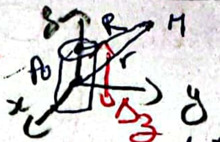
$$\boxed{\begin{array}{l} \vec{\mathcal{E}}(M,t) = Re(\vec{E}(M)e^{j\omega t}) \\ \bullet\ z\in\mathbb{C},\ z^* \text{ son conjugué} \\ z+z^* = 2Re(z) \\ \bullet\ f(t) \text{ périodique de période } T \\ \langle f(t)\rangle = \frac{1}{T}\int_t^{t+T} f(t)\,dt \end{array}}$$

$$\langle \Pi(M,t)\rangle_t = \left\langle \frac{\vec{E}(M)e^{j\omega t} + \vec{E}^* e^{-j\omega t}}{2} \wedge \frac{H(M)e^{j\omega t} + H^* e^{-j\omega t}}{2}\right\rangle$$

$$= \frac{1}{4}\langle \vec{E}(M)\wedge\vec{H}(M)e^{2j\omega t}\rangle + \langle \vec{E}(M)\wedge H^*(M)\rangle + \langle$$

$$= Re\left(\frac{1}{2}\big(\underbrace{\vec{E}\wedge\vec{H}^*}_{\Pi(M)}\big)\right)$$

Dipôle idéale



↳ dipôle infinie.

↳ c'est un morceau d'un conducteur droit de petite dimension et mince parcouru par un courant $I_0 e^{j\omega t}$ ou I_0 est une cte

• Pour le dipôle idéale on suppose q.

$$\left.\begin{array}{l}\Delta z \ll r \\ \Delta z \ll \lambda\end{array}\right\} \Rightarrow R \simeq r \text{ donc}$$

terme de phase

$$\frac{e^{-j\beta R}}{R} \simeq \frac{e^{-j\beta r}}{r}$$

terme d'amplitude

1/ $\vec{A}(M)$?

$$\vec{A}(M) = \frac{\mu}{4\pi}\int_{dipole} \frac{I(P)e^{-j\beta R}}{R}\, d\vec{P}_P \qquad R = r,\quad d\vec{P}_P = dz_P\vec{k}$$

$$A(M) = \frac{\mu}{4\pi}\int_{-\frac{\Delta z}{2}}^{\frac{\Delta z}{2}} \frac{I_0 e^{-j\beta r}}{r}\, dz_P\vec{k}$$

Δz : longueur dipôle

$$A(M) = \frac{\mu I_0 e^{-j\beta r}}{4\pi r}\int_{-\frac{\Delta z}{2}}^{\frac{\Delta z}{2}} dz_P\vec{k}$$

$$\boxed{\vec{A}(M) = \frac{\mu I_0 e^{-j\beta r}}{4\pi r}\Delta z\vec{k}}$$

2/ $\vec{H}(M) = ?$

$$\vec{H}(M) = \frac{1}{\mu}\, rot(\vec{A}(M))$$

on utilise coordonné sphérique (car $\frac{e^{-j\beta r}}{r}$

RQ

$$\vec{rot}(f\vec{u}) = \vec{grad}\, f\wedge\vec{u} + f\,\vec{rot}\,\vec{u}$$

$$\vec{rot}\,\vec{k} = \vec{0} \text{ car } \vec{k} \text{ cte vect}$$

$$\vec{grad}\, f = \frac{\partial f}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial f}{\partial\theta}\vec{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}\vec{u}_\varphi$$

f dépend que de r

$f = \frac{\mu I_0 \Delta z}{4\pi r} e^{-j\beta r}$ dépend de r

$\Rightarrow \overrightarrow{grad} f = \frac{\partial f}{\partial r} \vec{u}_r$ $\quad \left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

$= \frac{\mu I_0 \Delta z}{4\pi} \frac{\partial}{\partial r}\left(\frac{e^{-j\beta r}}{r}\right) \vec{u}_r$

$= \frac{\mu I_0 \Delta z}{4\pi} \left(\frac{-j\beta r e^{-j\beta r}}{r^2} - \frac{1}{r^2} e^{-j\beta r}\right) \vec{u}_r$

$= -\frac{\mu I_0 \Delta z \, e^{-j\beta r}}{4\pi r^2} (1 + j\beta r) \vec{u}_r$

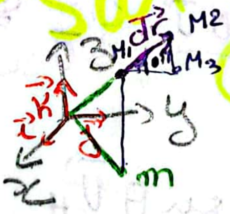
$\vec{H}(M) = \frac{1}{\mu} \overrightarrow{rot}(\overset{f\vec{K}}{\vec{A}(M)})$

$= \frac{1}{\mu} \overrightarrow{grad}(f) \wedge \vec{U} + f \underbrace{\overrightarrow{rot}(\vec{U})}_{=0}$ $\quad \overrightarrow{rot}\,\vec{K} = 0$

$= \frac{1}{\mu} \cdot \frac{-\mu I_0 \Delta z \, e^{-j\beta r}}{4\pi r^2} (\vec{U}_r \wedge \vec{K})(1 + j\beta r)$

⚠ Rappel sur sys coordonné

1 - Système cartésien



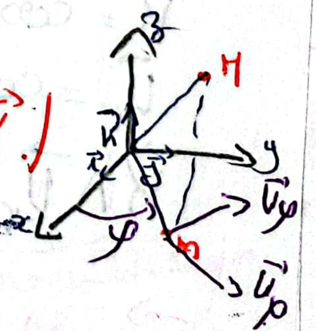
$\overrightarrow{OM} = \overrightarrow{Om} + \overrightarrow{mM}$

$= x\vec{i} + y\vec{j} + z\vec{k}$

$\vec{dr} = dx\vec{i} + dy\vec{j} + dz\vec{K}$

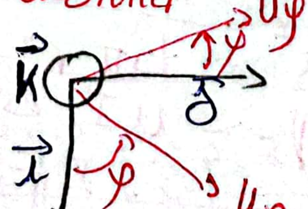
Volume
$dV = dx\,dy\,dz$

2 - Système cylindrique $(O, \vec{u}_\rho, \vec{u}_\varphi, \vec{K})$



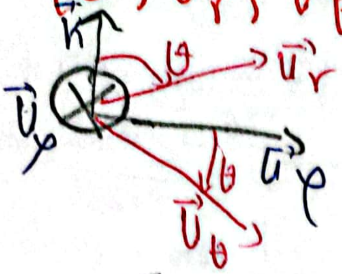
	$\vec{u}_\rho$	$\vec{u}_\varphi$	$\vec{K}$
$\vec{i}$	$\cos\varphi$	$-\sin\varphi$	0
$\vec{j}$	$\sin\varphi$	$\cos\varphi$	0
$\vec{K}$	0	0	1

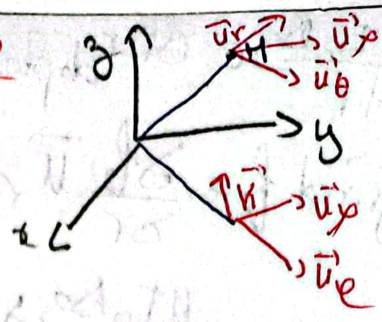
← matrice rotationnel



3 - Cordonné sphèrique

$(O, \vec{U}_r, \vec{U}_\theta, \vec{U}_\varphi)$



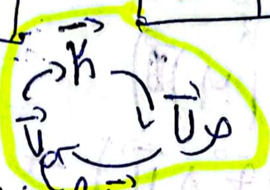


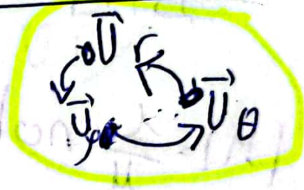
T1

$\downarrow\theta$	$\vec{U}_r$	$\vec{U}_\theta$	$\vec{U}_\varphi$
$\vec{K}$	$\cos\theta$	$-\sin\theta$	0
$\vec{U}_\rho$	$\sin\theta$	$\cos\theta$	0
$\vec{U}_\varphi$	0	0	1

T2

φ	$\vec{U}_\rho$	$\vec{U}_\varphi$	$\vec{K}$
$\vec{i}$	$\cos\varphi$	$-\sin\varphi$	0
$\vec{j}$	$\sin\varphi$	$\cos\varphi$	0
$\vec{K}$	0	0	1





Par exemple

$T2 \rightarrow \vec{i} = \cos\varphi\,\vec{U}_\rho - \sin\varphi\,\vec{U}_\varphi$

$T_1 \rightarrow \quad = \cos\varphi\,(\sin\theta\,\vec{U}_r + \cos\theta\,\vec{U}_\theta) - \sin\varphi\,\vec{U}_\varphi$

$\vec{i} \wedge \vec{U}_r = \cos\varphi \cdot \cos\theta \cdot \vec{U}_\theta \wedge \vec{U}_r - \sin\varphi\,\vec{U}_\varphi \wedge \vec{U}_r$

$= \cos\theta\,\vec{U}_\varphi ? - \sin\varphi\,\vec{U}_\theta$

$K \wedge \vec{U}_r = ?$

$\vec{K} = \cos\theta\,\vec{U}_r - \sin\theta\,\vec{U}_\theta$

$\vec{U}_r = \cos\theta\,\vec{K} + \sin\theta\,\vec{U}_\rho$

$\vec{K} \wedge \vec{U}_r = \cos\theta\,\cancel{\vec{K}} \cdot \cos\theta\,\vec{U}_r \wedge \vec{K} + \cos^2\theta\,\cancel{\vec{K}_r}\,\sin\theta\,\overbrace{\vec{U}_r \wedge \vec{U}_\rho}^{\vec{U}_\theta}$

$- \sin\theta\,\cancel{\vec{K}}\cos\theta\,\vec{U}_\theta \wedge \vec{K} - \sin^2\theta\,\underbrace{\vec{U}_\theta \wedge \vec{U}_\rho}_{\vec{U}_r}$

$\vec{H}(M) = \dfrac{-\mu I_0 \Delta z\, e^{-j\beta r}}{4\pi r^2}\,|\vec{U}_r \wedge \vec{K}|\,(1 + j\beta r)$

$$\boxed{\vec{H}(M) = -\frac{\mu I_0 \Delta z\, e^{-j\beta r}}{4\pi r^2}(1 + j\beta r)\sin\theta\,\vec{U}_\varphi}$$

$$\boxed{= H_r\,\vec{U}_\varphi}$$

Ex Mq $\vec{E}(M) = \frac{1}{j\varepsilon w} \frac{I_0 \Delta z\, e^{-j\beta r}}{4\pi r^3} \left\{ 2\cos\theta\,(1+j\beta r)\vec{U}_r + (1 + j\beta r - (\beta r)^2)\sin\theta \right\}$

on a $\vec{rot}\,\vec{H}(M) = jw\varepsilon\,\vec{E}(M)$

et $\vec{H}(M) = \frac{I_0 \Delta z\, e^{-j\beta r}}{4\pi r^2}(1 + j\beta r)\sin\theta\,\vec{U}_\varphi$

$\vec{rot}\,\vec{H}(M) = \frac{\vec{U}_r}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(\frac{I_0 \Delta z\, e^{-j\beta r}}{4\pi r^2}(1+j\beta_r)\sin^2\theta\right)\right) + \frac{\vec{U}_\theta}{r}\left(\frac{\partial}{\partial r}\left(\frac{r I_0 \Delta z\, e}{}(1+j\beta_r)\right)\right)$

$$\boxed{\vec{E}(M) = \frac{1}{jw\varepsilon} \frac{I_0 \Delta z\, e^{-j\beta r}}{4\pi r^3}}$$

Ex si $r \gg \lambda \Rightarrow \beta r > 1$, $\beta = \frac{2\pi}{\lambda}$, $\beta = w\varepsilon\mu$

- $\vec{H}(M) = \frac{I_0 \Delta z\, e^{-j\beta r}}{4\pi r^2}(1 + j\beta r)\sin\theta\,\vec{U}_\varphi$

$\vec{H}(M) = j\beta \frac{I_0 \Delta z\, e^{-j\beta r}}{4\pi r^2}\sin\theta\,\vec{U}_\varphi$

- $\vec{E}(M) = \frac{1}{jw\varepsilon} \frac{I_0 \Delta z\, e^{-j\beta r}}{4\pi r^3}$

$\vec{E}(M) = jw\mu \frac{I_0 \Delta z\, e^{-j\beta r}}{4\pi r}\sin\theta\,\vec{U}_\theta$

On RQ

$\left.\begin{aligned} \vec{E} &= E_\theta \vec{U}_\theta \\ H &= H_\varphi \vec{U}_\varphi \end{aligned}\right\} \vec{E} \perp \vec{H}$

$\frac{E_\theta}{H_\varphi} = \xi = \frac{jw\mu}{j\beta} = \frac{w\mu}{w\sqrt{\varepsilon\mu}} = \boxed{\sqrt{\frac{\mu}{\varepsilon}} = \xi}$

impédance intrinsèque au milieu (ε, μ)

ds l'air $\xi = 120\pi\,\Omega$

Ex vect de Poynting

$\vec{\pi} = \frac{1}{2}\vec{E} \wedge \vec{H}^*$

$= \frac{1}{2}\left(jw\mu \frac{I_0 \Delta z\, e^{-j\beta r}}{4\pi r}\sin\theta\,\vec{U}_\theta\right) \wedge \left(j\beta \frac{I_0 \Delta z\, e^{j\beta r}}{4\pi r}\sin\theta\,\vec{U}_\varphi\right)$ conjg

$$= \frac{1}{2} |\gamma|^2 (w\mu)\beta \cdot \left|\frac{I_0 \Delta z}{4\pi r}\right|^2 \sin^2\theta (\vec{u}_\theta \wedge \vec{u}_\varphi)$$

$$\boxed{\vec{\Pi} = +\frac{1}{2}\, w\mu\beta \left(\frac{I_0 \Delta z}{4\pi r}\right)^2 \sin^2\theta\, \vec{u}_r}$$

$$P = Re \iint_{sphere} \vec{\Pi}\, d\vec{s}$$

Ex Puissance moyenne Rayonnée ?

$$d\vec{s} = r^2 \sin\theta\, d\theta\, d\varphi\, \vec{u}_r$$

$$P = Re \iint_{Sphere} \frac{1}{2} w\mu\beta \left(\frac{I_0\Delta z}{4\pi r}\right)^2 \sin^2\theta\, \vec{u}_r \cdot r^2 \sin\theta\, d\varphi\, \vec{u}_r$$

$$= Re \left(\frac{1}{2} w\mu\beta \left|\frac{I_0 \Delta z}{4\pi r}\right|^2 \int_0^{\pi} \sin^3\theta\, d\theta \int_0^{2\pi} d\varphi \right)$$

$$= Re \left| \frac{w\mu\beta}{2} \left|\frac{I_0 \Delta z}{4\pi}\right|^2 \left[\frac{4}{3}\right] [2\pi - 0]\right.$$

$$\frac{1}{12}\cos 3\pi - \frac{3}{4}\cos\pi - \frac{1}{12} - \frac{3}{4}$$

$$= Re \left(\mu w\beta \frac{1}{12\pi} (I_0 \Delta z)^2 \right)$$

$$= \frac{\mu w\beta}{12\pi} (I_0 \Delta z)^2$$