Statistics Review

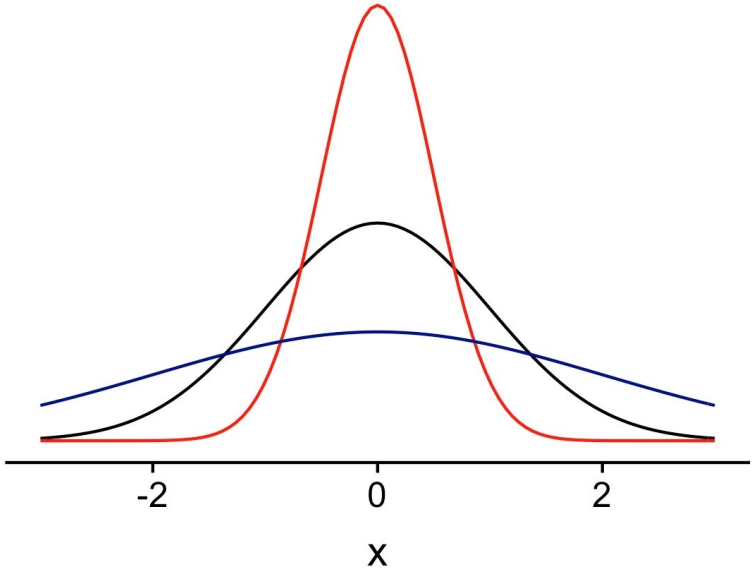
Basic Summary Values

Measures of Central Tendency

- Mean - average
- Median – central value
- Mode – most repeated value

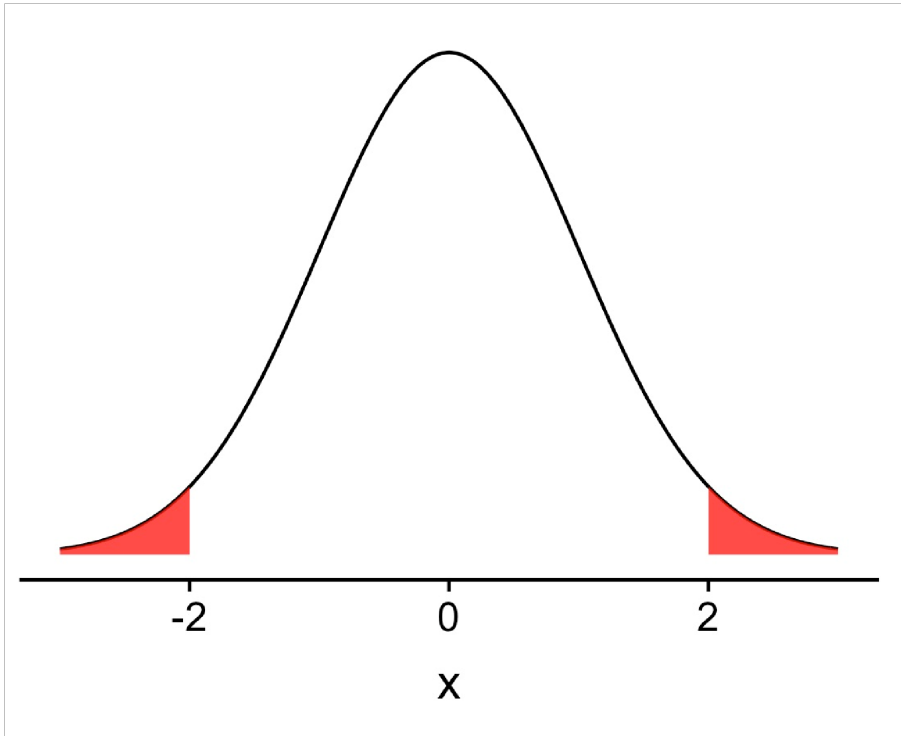
Measures of Spread

- Range – difference between the highest and lowest value
- Standard deviation – measures the dispersion of the data



Hypothesis Testing

- Hypothesis testing compares your data to a pre-determined null distribution (usually the normal distribution). You state a null and alternative hypothesis and calculate the probability your observations happened ***under the null hypothesis.***
- Null hypothesis, **H0**: Everything happened by random chance.
- Alternative hypothesis, **H1**: My observations happened because of my idea.
- Saying p-value = 0.05 means that there's a 5% chance the observation happened randomly under the null distribution.



Test for Continuous Data: one sample t-test

- For testing continuous values against some known mean

- I have an iris with a sepal length of 7 inches and I think that it's because of my new iris fertilizer. Is that iris' sepal length abnormally large?

- **H0:** There's nothing different about the fertilizer.
- **H1:** The fertilizer does increase iris sepal length.

```
> t.test(iris$Sepal.Length, mu = 5.8) One

Sample t-test

data: iris$Sepal.Length t = 0.64092, df =
149, p-value = 0.5226 alternative
hypothesis: true mean is not equal to 5.8

95 percent confidence interval:

 5.709732 5.976934 sample

estimates:
mean of x

 5.843333
```

Test for Continuous Data: two sample t-test

- For testing 2 continuous values against each other
- Is there a difference between the sepal lengths of versicolor and virginica irises?
  - **H0:** There's no difference in the mean sepal lengths.
  - **H1:** There is a difference in the mean sepal lengths.

```
> t.test(iris[iris$Species == 'versicolor',1],
iris[iris$Species == 'virginica', 1])

Welch Two Sample t-test

data:  iris[iris$Species == "versicolor", 1]
and iris[iris$Species == "virginica", 1]

t = -5.6292, df = 94.025, p-value = 1.866e-07

alternative hypothesis: true difference in
means is not equal to 0

95 percent confidence interval:

 -0.8819731 -0.4220269

sample estimates:

mean of x mean of y

    5.936     6.588
```

Test for Continuous Data: paired two sample t-test

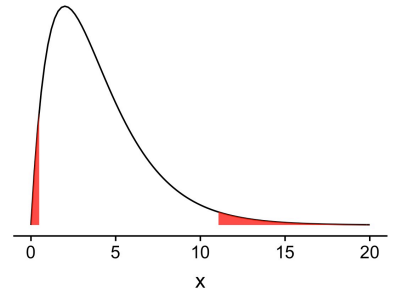
- For testing 2 continuous values against each other *when there is some natural pairing between the samples*
- The sleep dataset in R has data on the amount of time patients sleep on two different sleep medications compared to control. Is there a difference between the two medications?
  - **H0:** There is no difference in the amount of time patients sleep.
  - **H1:** There is a difference in the amount of time patients sleep.

```
> t.test(extra ~ group, data = sleep,
                paired = TRUE)

Paired t-test

data:  extra by group
t = -4.0621, df = 9, p-value = 0.002833
alternative hypothesis: true difference
in means is not equal to 0
95 percent confidence interval:
 -2.4598858 -0.7001142
sample estimates:
mean of the differences
                     -1.58
```

Test for Discrete Data: chi-square



- Test for when you have counts of discrete data; test expected counts against observed counts
- Are babies more likely to be born on one day of the week over other days of the week?
  - **H0:** There is an equal chance of babies being born every day
  - **H1:** There isn't an equal chance of babies being born every day

```
> chisq.test(birth_days$num_births,
p = birth_days$exp_prob_birth)

Chi-squared test for given
probabilities

data:  birth_days$num_births
X-squared = 15.057, df = 6, p-value
= 0.01982
```

The Multiple Testing Problem

- If you do enough tests, you expect to see significant results, just *by random chance*
- Say you flip a coin 10 times and record the number of heads you get. Then you repeat the "experiment" 10 times. You expect to get about heads about 5 times

5 5 6 4 2 4 5 5 4 4

- Now let's do it 100 times

5 6 5 4 5 7 6 5 5 5 5 5 7 3 5 6 6 4 5 6

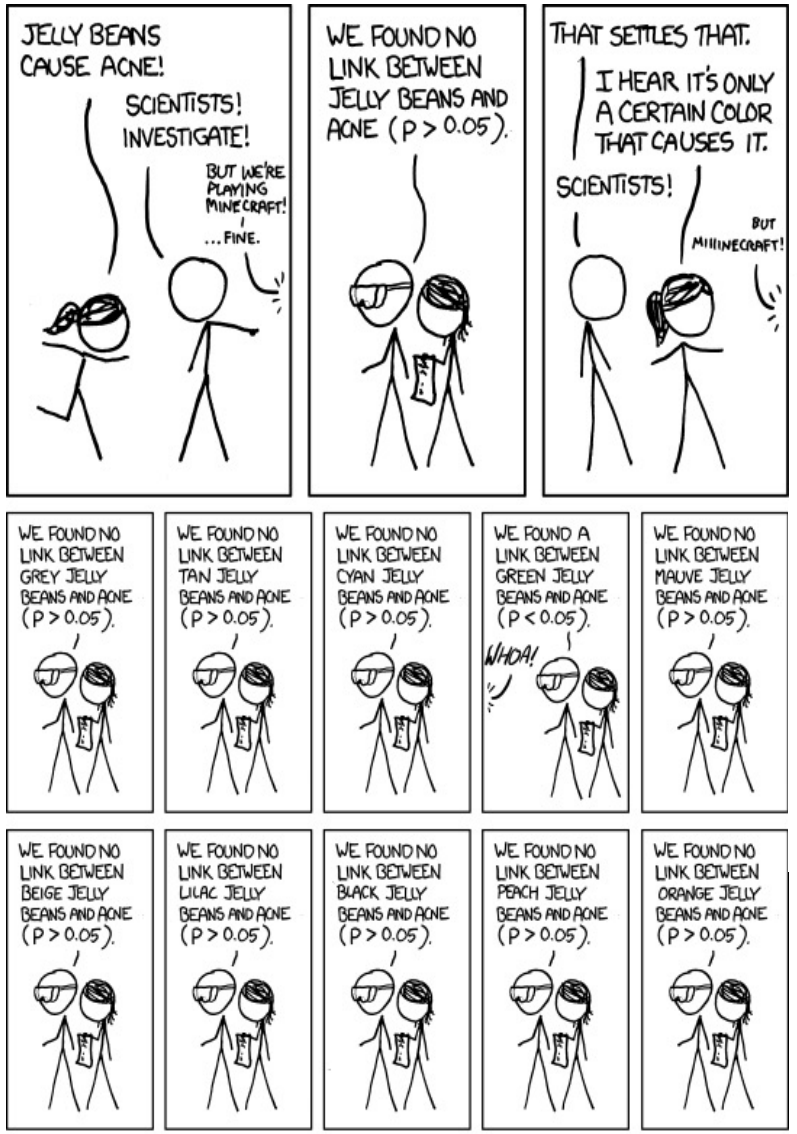
4 3 6 5 6 5 5 6 6 2 5 5 3 6 **9** 6 6 3 6 4

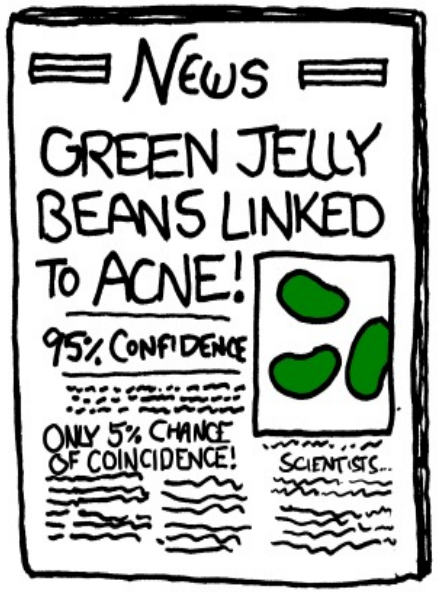
6 5 3 3 4 2 4 4 4 4 7 7 4 3 7 3 3 1 6 4

5 6 3 4 5 6 4 **8** 5 5 7 2 4 4 7 6 4 3 5 5

4 4 7 4 5 4 3 4 5 4 **8** 5 6 2 6 6 4 5 3 7

- Have to correct for multiple testing when you test, for example, all 20,000 genes in the human genome for differences





Pairwise Test for Multiple Conditions: ANOVA

- For testing more than continuous values against all combinations of each other
- Is there a difference in sepal length between the three species of iris in the iris dataset?
  - **H0:** There is no difference
  - **H1:** There is a difference between at least one group

```
> aov(Sepal.Length ~ Species, data = iris) %>%
TukeyHSD()

  Tukey multiple comparisons of means

    95% family-wise confidence level


Fit: aov(formula = Sepal.Length ~ Species, data =
iris)


$Species

                          diff       lwr       upr p adj
versicolor-setosa    0.930 0.6862273 1.1737727     0
virginica-setosa     1.582 1.3382273 1.8257727     0
virginica-versicolor 0.652 0.4082273 0.8957727     0
```

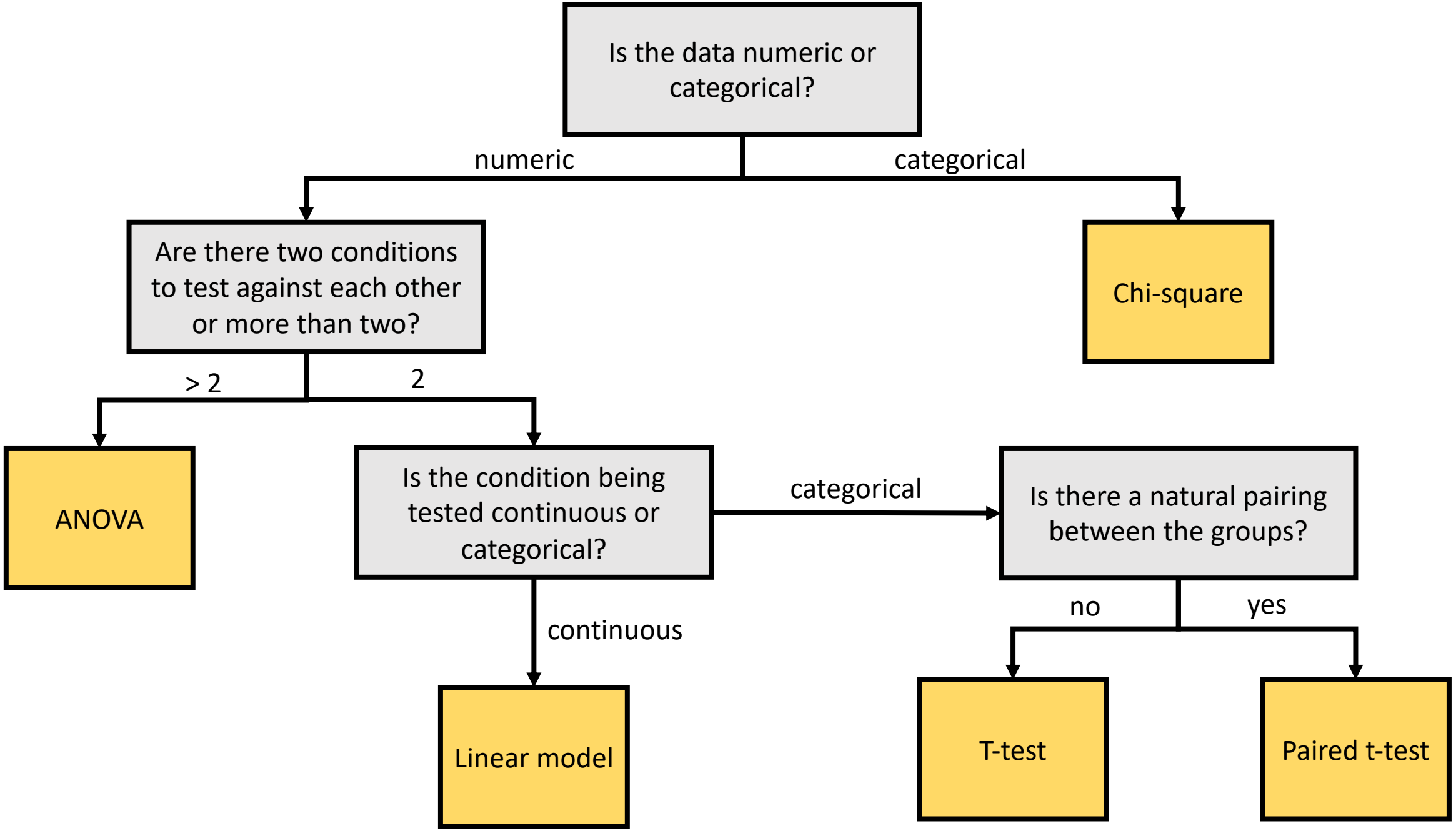
Test for Continuous Conditions: Linear Model

- For testing continuous variables over a continuous condition (like DNA methylation over time)
- AKA finding a line of best fit
- Is there an association between sepal width and sepal length in the iris dataset?
  - **H0:** There is no relationship
  - **H1:** There is a relationship

```
> lm(Sepal.Length ~ Sepal.Width, data = iris)

Call:
lm(formula = Sepal.Length ~ Sepal.Width, data = iris)


Coefficients:
(Intercept)  Sepal.Width
     6.5262      -0.2234
```


Is the data numeric or categorical?
numeric
categorical
Are there two conditions to test against each other or more than two?
Chi-square
> 2
2
ANOVA
Is the condition being tested continuous or categorical?
categorical
Is there a natural pairing between the groups?
continuous
no
yes
Linear model
T-test
Paired t-test

Demo

Statistical interference using Resampling

- jackknife

- bootstrap

- permutation

- cross validation

Permutation

Group comparison

- get p-value of statistic

Group A		Group B	
27	24	21	13
20	29	22	22
21	18	15	20
26	20	12	24
27	17	21	18
31	31	16	20
24	20	19	23
21	25	15	19
20	28	22	24
19	21		
23	27		
24	28		
19			

Permutation

Group comparison
- get p-value of statistic

Group A

```
> A=c(27,24,20,29,21,18,26,20,27,17,31,31,24,20,21,25,20,28,19,21,23,27,24,28,19)
> mean(A)
[1] 23.6
```

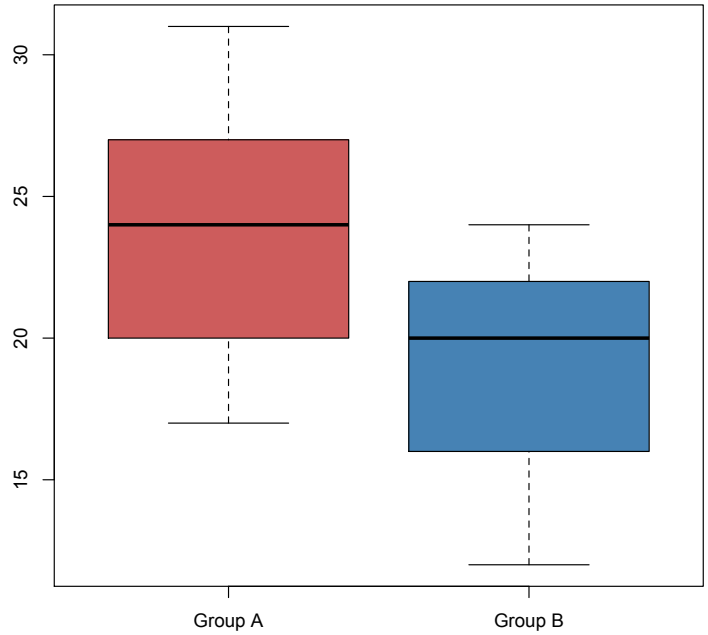
Group B

```
> B=c(21,13,22,22,15,20,12,24,21,18,16,20,19,23,15,19,22,24)
> mean(B)
[1] 19.2
```

Mean difference A vs B: + 4.4

Are Group A and B different?

Is difference 4.4 statistically different?



There are 2 possible way to determine:

Are Group A and B different?

There are 2 possible way to determine:

1. Classical statistics (analytical method)
2. Computational method

1. Classical statistics (analytical method)
STAT 101

Student's *t*-test

Equal or unequal sample sizes, unequal variances
Welch's t-*test*

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}} \qquad t = \frac{23.6 - 19.2}{\sqrt{\frac{17.1}{25} + \frac{13.5}{18}}} = 3.67$$

1. Classical statistics (analytical method)

STAT 101

Student's *t*-test

$$t = \frac{23.6 - 19.2}{\sqrt{\frac{17.1}{25} + \frac{13.5}{18}}} = 3.67$$

$$f(t) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

1. Classical statistics (analytical method)
STAT 101

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$$\text{d.f.} = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{(s_1^2/n_1)^2/(n_1 - 1) + (s_2^2/n_2)^2/(n_2 - 1)}$$

1. Classical statistics (analytical method)
STAT 101

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$$\text{d.f.} = \frac{(4.1^2/25 + 3.7^2/18)^2}{(4.1^2/25)^2/(25-1) + (3.7^2/18)^2/(18-1)}$$

$$\text{d.f.} = 39.1$$

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One Sided	75%	80%	85%	90%	95%	97.5%	99%	99.5%	99.75%	99.9%	99.95%
Two Sided	50%	60%	70%	80%	90%	95%	98%	99%	99.5%	99.8%	99.9%
1	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.080	1.386	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
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8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
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...											
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3.67 > 2.021

1. Classical statistics (analytical method)
STAT 101

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3.67 > 2.021 ✓

1. Classical statistics (analytical method)
STAT 101

Difference 4.4 is statistically significant **p<0.05** level

```
> t.test(A,B)

    Welch Two Sample t-test

data:  A and B
t = 3.6582, df = 39.113, p-value = 0.0007474
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 1.957472 6.798084
sample estimates:
mean of x mean of y
 23.60000  19.22222
```

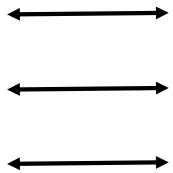
2. Computational method

Group A		Group B	
27	24	21	13
20	29	22	22
21	18	15	20
26	20	12	24
27	17	21	18
31	31	16	20
24	20	19	23
21	25	15	19
20	28	22	24
19	21		
23	27		
24	28		
19			

2. Computational method

Group A

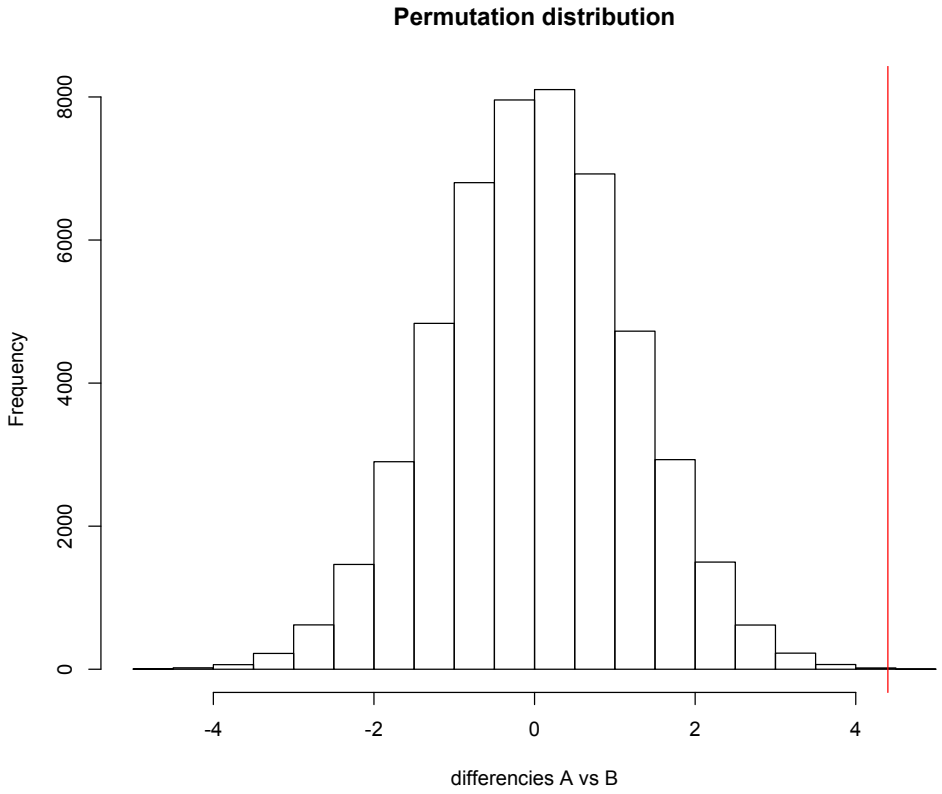
22 24
20 29
21 16
26 20
12 17
31 31
18 23
21 25
20 28
22 21
23 27
24 21
19



Group B

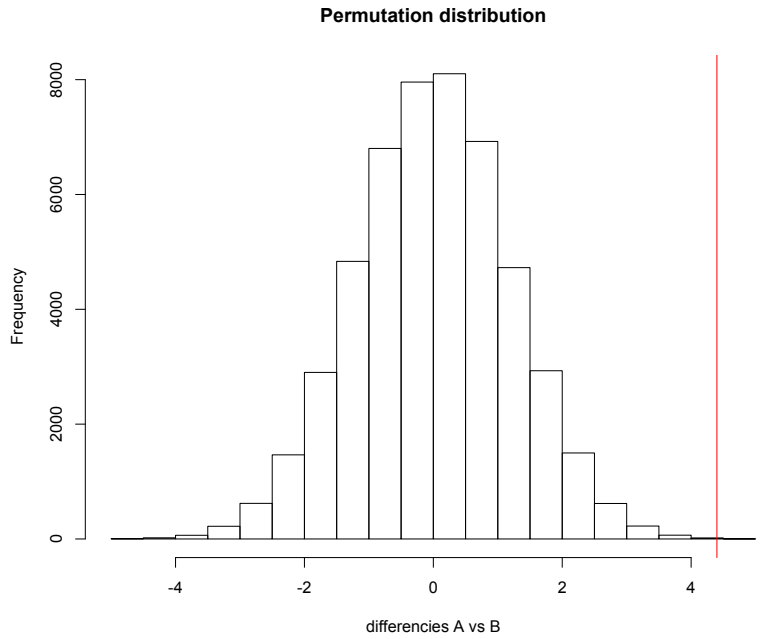
28 13
22 27
15 20
27 24
21 24
18 20
19 20
15 19
19 24

differencies in mean 2.7


Permutation distribution
Frequency
0
2000
4000
6000
8000
-4
-2
0
2
4
differencies A vs B

2. Computational method

1. Ability to follow logical statement
2. Random number generator - in R sample() command
3. Iteration



p-val: **0.00075**

7-8 of of 10,000

~75 out of 100,000