Network Reconstruction through diffusive arrival times

***Abstract*— Network reconstruction problem is one of the hot and knotty issues in the research of complex network or network science. In this paper, we use kernel density estimation technique to estimate the distribution of the time difference of arrival in such diffusion process on the basis of stochastic temporal network. We analyze the statistical property discrepancies between edges in the network, then give proof on the left deviation of the estimated survival function on the time-aggregated network. Next we design a probability threshold cutting algorithm which can be used to reconstruct the time-aggregated network of stochastic temporal network. To verify this, we run a lot of simulations on different networks which show high reconstruction speed and accuracy of our algorithm. Last, we discuss the relation between network scale and data amount of the reconstruction procedure which illustrates the compatibility with such large scale network reconstruction problem. Furthermore, a parallelization design idea is presented to speed up the algorithm.**

***Index Terms*— network reconstruction, stochastic temporal network, time-aggregated network, waiting time distribution, kernel density estimation.**

I. Introduction

澶嶆潅缃戠粶锛屾槸涓绛嶆寮扮澶淪笉鐣屽
鍬傜郴缁熺殑閲嶈　宸ュ叿锛屽彲浠ュ鐞嗗叾宸ㄥ鍚屾椂
俍氬闀瓑鍔釜棰嗗煙閱芥湁閲嶈　鐨勫簲鐢
剩涔夋拰瀹炵敤浠峰€笺傜粶鍦撅紝鍦ㄥ皾璇曞
岀殑鎺　绉嶆湇锛屾湁g殑鐩戝睍锛岃繛缁　瀛︾枟锛宄ョ秙
鐩楀埉浜轰滑鐨勫彿娉ㄤ俊缁緝锛ョ湶锛岃繛缁
瀛︽枃涓撳鐮旂┒锛屽澶嶆潅缃戠粶绯荤粺鐨勭畾鎼ｙ
抵瀹氭暣瑙勭緳鐨勮閫犳氨鐢荬 瀛鐮旂┒锛屽叾瀹
鍒板 鍬傜郴缁熺殑鍚勭 鎸撴墍缁撴瀯鐨勫洜鎼ц
川锛屼瘮濡傛鏌愮洪缃戠粶鍦ㄤ措鐧囨害缃戠粶鍦
佸皹涓椂缃戠粶绛夎憲鍚嶇殑鐧縺绯荤粺鐨勭爜旂┒
鍦ㄤ笌鍔ㄦ墒瀛︾壒鐐规櫄鍟嗗瓑鐨勬湁鍏崇浉鍏锋
,鍖呮嫭鍔ㄦ墒瀛︿悅姝ュ鍗朵骇鐢熷揉鍏佸彂銆佷紶缁
滀笂鐨勫姩鍔涘绲鍚　瀛犲嶇鍦佹摿瀿　　娴嬭佺
绣缁滃彲鎺ф с　鍒嗗彂浼犵缁滄紨鍖栧嶇寮炴槭锛
互鍙婂伐绋嬪疄瞵闃呭瞾闈㈢瓑鍔ㄥ姏绯荤粺　璁″師鍔　鍚

鍏跺簽鐢ㄤ竴鍒嗚〃鍒嗗氦鍙婂鍒嗚跺嘩瑜瑰鍚
箍娉涘叿赴瀵屽

缃戠粶閲嶆瀯闂　　涔熷嚭瑁樸粺缃戠粶绉戝
鐮旂┒涓　殑鐑　倍闂　　　鍜岄毦鐐归棶棰樸傚湪
寰堝　鎯呭喌涓嬬綉缁滅粨鏋勬槸鏈煡鐨勬垨鑰呴儴鍒
嗘湭鐭ョ殑　琛ㄦ　　瀵熷埌鐨勮　鏁版嵁浠庣郴缁
熶腑鐢熸垚鍑烘潵锛　殑鏉冮噸椂鐨勭粨鏋勫湪铏
閫氳繃涓婅堪鏁版嵁ヨ　鍒嗗瀹炲鍔繘鎻椂　琛
屽负鐨勭敓鎴愶　绗鐭ラ唸惧锛　瀵嗗師濮嬬郴缁滅
殑閲嶅缓缃戠粶瀹為獙鏁扮罐娉ㄥ彲浠ヤ粠鏂欏悊鐞
瓒崇殑缃戠粶閲嶆瀯鍒楃悊淇ｅ　鐗圭被绗
璐濆寫锛屽彲浠ユ槸澶氬崱偓鏂囩害鍏　鍥犳锛屾纰嶆潵
鍒嗗瀷鏁伴　　　骞舵嚭鍦掔伅锛屼互鍗崇瓑寰嶈韩
︽潳寰椂　鍙栧嵄鏃嶆瀯鍗犻檮瀷棰勬祴銆佹墍瀵
艰嚦鍚屽懡鍜屾潈閲囧瑙勭哪鎴愭灉鍔ㄦ枃鍏寕
鍜屽鐩戦　　　杩樻湁鍒溿傛　　　鐏弶杩ф鏂囩
鍚涓鍕鎬瀹朵杩娆ｅ簳寮傝粬閮戦曞澶
寮滎绋嬪鍏勬剹鍌姭鍔旓紝鍦ㄧ綉缁滈噸鏋勭殑閿
欏杩囩█鍏寰佀闂滐　殑濕檓閲嶆瀯闂　　　鏉
　寚涓虹■鍚忎俊鍙风殑閲嶆瀯闂　　　锛岃嚜锛屽鍦
漾鏉鍙嗗寰鍒欑鎸ε鐗硾锛屾澶兂鐢靛攟鍚
楀殑閲嶆瀯绮寕[2]鍜岄殢鏈虹鍙埁粎SIS鍜孭杩
欎袱绉嶄紶缁滄紨鍖栨ā　銆鍊嬪鍚鍚戜紶鐩濶癞琴ー澘
騫冲嚩鐨勬紨鍖栨ā　　缁撴瀯鍔熻兘鍙樺鍖栧洜

缂╂斁鐨勯噸瀹为檹鍗翠骇鏈夊浗锛屾磷瑙勫熀鍜岄
噸寰嬫浮鍏嶉瀯鍖栧悓璁剁畻娉ョ瓑鍝嶅簲鍒3]鍜屼笉
涓嗗悓鎯呭喌锛屾彁鍑烘渶灏忚佺粺鍙傛暟鍖栫殑鍚嶇紶锛
嬫Lasso鏂规硶鍦ㄥ悓鎴嶅彲鍗ょ畻娉曢噸　　　杩　　窡涓鸿

■鍏忓彉鍌风殑鍏宠　）锛屾椂鍐滄灏浣犳剙铏
嬭浜嗚繍鐢ㄥ锛屽吗锛屽嶇殑灏濄寰佹敹鎻ｆ潭鎵嶅湪涓
嶅钩鏁伴緳锛屽　锛屼細鐨勭嫭缁熷拰寮曠敤鍙兘浼
■鍏忓彉鍌风殑閲嶆瀯鍦ㄥ叾鍒嗙獒閰嶅強鍖栧暟鍙樻瀯
圭殑棰勬祴鍏℃案灏樺嚱鏁拌繘锛岀粨鏋滃4]鍜
岃asso鏂规硶　　锛屼笉澶ф鍙剂岀湡闇€鍥犳嫑鍖妼鍧
　鍒ｈ　搴涘悰閫犳垚鍥炲彈鍙讳娇鑰鍜岀畻娉曠殑绮犲害
鍙兘鍙栧嚭涓　　鏋勫缓锛屾垜浠鍙鍥犲叡鍖峰害鐨勭
冧竴鑸粍鏁伴噺澶ф瑕佸湪杩冪敤鍙€■鍜屽渚ф湁
鍗侀鍦ㄨ銆

澶嶆潅缃戠粶锛屾槸涓绘寮扮瀵瑰簲锛屽嵆浠庣鑲屽
鍖栫郴缁熺殑閲嶈　宸ュ叿锛屽彲浠ュ绯荤粺鍏嶉噺鐨勮
鍏徃甯锛屽堝缓绯荤粺鍏跺鎵嶈鑰呬紶鏈夊涔嬬綉缁滅畝

涓嶇敓閲嶆瀯闂　　涓嶅彂鐢熸槸涓瀹炵畾娉曡兘鍚戝

鍦ㄤ粬鐨勭綉缁滅粨鏋勫瓨鍦ㄨ繃锛岀綉缁滅粨鏋勫紩璧嬬

鐨勭綉缁滅珫鍛藉嵆寮扮鑲屽寸殑鑰冭檻銆

鍥犳锛岀綉缁滈噸鏋勪笉浠呮槸缃戠粶绉戝鐮旂┒

鐨勭綉缁滈噸鏋勬槸涓鏂规硶鍜屾鏋跺鍔

鐨勫厛鍐充换鍔°

鍦ㄧ綉缁滈噸鏋勭殑寮€鍙戜腑锛屾垜浠渶瑕佷腑

鐨勭綉缁滈噸鏋勬柟娉曡兘鍦ㄧ綉缁滈噸鏋勯棶棰

缃戠粶閲嶆瀯闂　鍒ｅ簲鐢ㄦ墍闇€瑕佺殑鏁版嵁

瀹氭殑椴佹 鎬с傚 鏃︿ぇ瀛︽潕缈旇 棰樼粍杩戞湡澧炲閲嶆潪鏃舵晥绋戠粶鐮旂┒ 鍙栧緱浜嗕竴浜涙垚鏋滐紝欢涓嬬殑浼犳挱杩囩▼锛屼互鍙婄粰瀹氱綉缁滅殑鏈€浼樼¨鍒鍊 鏁版嵁涓婂彂鎺橀€熷寰鍊煡鏉ユ瀯寤鸿挤鎹熺潅婊€殑鏈夋晥鎬у垎鏋[5]銆

缃戠粶閲嶆潪闂 锕界敱鍙栧緱浜嗕竴浜涙垚鏋滐紝鍦ㄨ繖涓€鍖洪棿 嗗鐫楂樻晥鐨︾殑鎴戞埧銆 傛 浣嗘湁鍐楃敤鏁堜笖鏁翠綋骞冲潎鍒嗗竷鍦ㄨ祫婧愭灇锛岃 鍗氬＋鎵€鍦ㄧ殑鐮旂┒鍥㈤槦锛屽湪鍔ㄦ€佺綉缁滀腑鐨勪紶鎾繃绋嬪拰鐮旂┒娴佸眬鐨勭姸鎬佺瓑鏂归潰 鍦ㄨ 鍖 细涓ラ殑閾炬帴鏁版嵁婧愮沧ā 鏃堕棿浠巼閫熷彉鍖栧埌鏃堕棿鎴ｇ殑鏄惧紡鎴愮粍 杩欎竴鐐瑰湪 绌洪棿鐨勮窛绂讳腑鍙栧緱鐨勮竟鐣屾敼杩涓嬪姟鏁版嵁 缃戠粶鏃跺垎鍙樻尶鏉冮噸涔嬩竴 鐨勬牱鏈瑧闂浘娴佺瓑淇″彿鐨勬瀯寤哄拰灏嗗 闄ゆ鍦ㄧ綉缁滃唴閮ㄧ殑浼犳挱杩囩▼ 涓嶄細鎵撲贡鏁版嵁婧愮偣鍜屾暟鎹潯鏁 绛鏃舵晥鍦ㄩ粯璁ゆ儏鍐垫敹鏁涘悗 鍗曠嫭浣跨敤 閫傜敤浜庢瘡涓€娆¤繍鍔ㄨ繃绋嬪湪缃戠粶涓婄殑鍏ㄥ眬 鍦ㄤ笉鍚屾潯浠朵笅鐨勯€熷害 鍦ㄦ牸鐐规帹瀵煎嚭 ψ浜涓€绉嶈繛缁棰戠巼浼犳挱鐨勬暟鎹垎鏋愬姩鍔涘鍦ㄦ暟鎹舰鎴愮粨鏋滈噸 娌″綋鎴愯鍒掍互鐨勬剰鐢熸垜浠舵 鍗曞厓鏃剁殑鍏ㄥ眬缁撴瀯鎬ц 姝ゆ椂鍙互鐩存帴瀹氫箟鍐嶅吀鐨勬瀯寤哄拰鏁版嵁婧愮偣鐨勫懆鏈熸€т互鍙婂紓甯稿鍚堢殑鍦ㄧ嚎鍒嗘瀽銆 鍏朵粬鏂规硶鐨勫彲鎵╁睍 鎬ф柟闈紝鍦ㄧ嚎缃戠粶鍔ㄦ€佽瀵熶腑閲囩敤鏈変簡涓€浜涜繘灞曪紝鍦ㄨ 缃戠粶鏁版嵁閲嶅缓鏂规硶鐨勬晥鐜囦互鍙婂惎鍙戞€т笂鎻愪緵浜嗗熀纭€

II. Diffusion process on stochastic temporal network

鏈 瑰寮傝川鍏夊鏃舵晥鍦ㄧ綉缁滆妯″瀷鏉ョ粺涓€鍒嗘瀽缃戠粶鐨勫彲鎺у埗鎬 瀹為檷浣冨 強婀℃寮鍔

A.

們Derivation of Stochastic Temporal Network 涓轰簡寮濈粡鏃舵晥鍦ㄧ綉缁滅殑鏋勫缓 鍏ㄧ悊瀵逛簬鏃舵晥缃戠粶$\mathcal{N} = (\mathcal{V}, \mathcal{E})$銆傛 叾涓V$MJ.6ZiØ\mathcal{E}$鏄 姣忔潯鏃舵晥缃戠粶鐨勮竟鍙互$\epsilon = (u,v,t,\delta t)$鐨勫舰寮忚〃绀$(u,v)$琛ㄧず浠庤妭鐐圭敱鐐 鍒拌妭鐐瑰弮鐢熶簡鐨勬椂闂磋寖鍥 锛t∈ $\left[0,T^W\right]$琛ㄧず浠庤妭鐐瑰埌鑺傜偣鐨勬椂闀跨T^W琛ㄧず璇ユ椂闂磋寖鍥村唴鐨勬椂闂村垎鍓δt琛ㄧず浠庤妭鐐瑰埌杩炵画鍒嗗竷 鏃堕棿鐨勬瘡涓€涓 跺埢姝ヤ笂 鏂囦腑鍦ㄦ$(t, t+\delta t)$銆傛垜浠 浠庡彧鏈夋湁鐨勮竟鍒嗘瀽鏃堕棿灞炴€δt璁板綍浜嗛€氫俊杈圭紭鐨勪俊鎭 傞棿瀵归噸 鏃堕棿淇℃伅灏嗗湪鍚庨潰鐨勬瀯寤轰腑鍖哄垎鏃舵晥鍦ㄧ綉缁滆竟 锛岃繖涓€绉嶈竟鍙互鐢ㄥ湪 锛屽疄闄呯綉缁滄暟鎹 姝ゅ湪浜嶆柟娉曟牸鏋 涓(u,v,t)Ø&fifl$j.M\backslash 4ØxMqS$fl˜fle\`$FLMd6XJJ$œJΩuΩæ<

鎺ョ潃鎴戜滑鏋勫缓鏃堕棿鏁版嵁婧愮偣鍒嗗竷鐨勫垎鏋愬舰寮 鎶婃瘡涓€鏉¤竟鍒嗚В鎴愪竴涓⃣鏄犲皠锛P$_{V^2}: V^2 \times \left[0,T^W\right] \mapsto V^2, (u,v,t) \mapsto (u,v)$灏嗗叾鏄犲皠鍒版椂闂存 H 缃戠粶鐨勬湁 鍚戣竟鍦ㄧ綉缁滀腑浜嗙粺涓€鍚屾牱 锛屽湪姝ら棿鍐呭 缁撴瀯

$$\mathcal{G} = P_{V^2}(E) = \{(u,v)|(u,v,t) \in E\}$$

鍦ㄨ 锛屽 鏃堕棿鍒嗙 H 缃戠粶涓 殑杩囩▼涓鐣
鍗曚釜(u,v)∈ $\mathcal{G}$锛屽彲浠ュ弽鏁版嵁浠庤妭鐐瑰埌杈规椂 闂撮棿鏁板緱涓 瀵硅瀹氬瓨鍦$\psi(t)$鐨勫垎甯冨綋鎴
佺敤鍏朵粬锛 $\{(u,v,t_{uv}^i)\}_{i=1,2,...,M} = P_{V^2}^{-1}[(u,v)]$涓鸿繖涓(u,v)∈ $\mathcal{G}$涓婄殑閾捐 褰撴椂閫夋嫨 鐨勮竟鍦ㄨ妭鐐瑰拰杈 鍙互琛ㄧず鎴愬垎甯冨垪鍑褰骞讹t$_{uv}^1 < t_{uv}^2 < \ldots < t_{uv}^M$銆傛垵$\psi_{uv}(t)$鏄 杈规椂闂撮棿鏁版 闂$\{\Delta t_{uv}^i = t_{uv}^i - t_{uv}^{i-1}\}$鐨勫垎甯冨瓨鍦ㄥ彲浠ユ牱鏈嶅垎 杩涢棿鐨勯噷鎺涓婂緱鍒板害濡 互閫囪繖鐨勬牳鍑芥暟鏉ヤ及璁″紡 鏉ュ湪鐨勮竟鏃堕棿 闂存牸甯冧細

$$\psi_{uv}(t) = \frac{1}{M}\sum_{i=1}^{M} K_h(t - \Delta t_{uv}^i)$$

寮忎腑 锛M$sJU'GØK_h(\cdot)$琛ㄧず甯︿
涓hfl$y\tilde{}-Ga$ø#$zB\Psi w\psi_{uv}(t)$寮曞叆鍒嗗竷涓嬬殑绛夊緟鏃堕棿鎺掑垪$\rho_{uv}(\tau)$銆傚 鍦ㄨ紝绛夊緟鏃堕棿τ_{uv}瀹氫箟涓哄湪鏌愪竴鏃跺埢 涓 紝灏 姣鐨uah4¨(u,v)涓婃 娆″ 鍚庣鏁版嵁鍫竟浠庤
緟v琚uo4fl>.|bFŒa3œfl˜w!ffiuvfl˘¨$v\tau_{uv}$鏈嶈棤涓涓
緟鏃 鏃堕棿撮棿鍏嬪垎甯$\psi_{uv}(t)$杩浠 闂垮害鍋忓樊閬囩殑姒傜巼鍜屽害鍜屽害锛屽彲浠$\rho_{uv}(\tau) =$

$\frac{1}{m_{uv}}\int_{\tau}^{\infty}\psi_{uv}(t)dt\Theta(\tau)|$>m$_{uv}$ $=$ $\int_{0}^{\infty}t\psi_{uv}(t)dt$琛ㄧず浜嬩欢閬撶粶鍒嗗竷$\rho_{uv}(\tau)$鐨勫瀲鍊硷紝$\Theta(\tau)$琛ㄧず鍗曚綅闃惰穬鍑芥暟鍩熴傜敱浜庡瓨鍦ㄧ緱鍒版墍鏈夎竟鍒嗗竷鐨勬瀯璁稿鏉傛潈閲嶇殑鍑嗙‘鍊笺傛墍浠ョ敤涓嬮潰鐨勬鐜囦唤鏉ュ尯鍒嗘湁鍚戝浘鍜屾棤鍚戝浘锛″ 濮涓鍝嶇殑鍚ユ湁鍚戝浘鍙互H缃戠粶$\mathcal{G}$鍜岄瀯寤哄叾杈瑰垎甯$\rho_{uv}(\tau)$锛屾埛闃佹湁鍚戝浘鍦ㄨ鍥炬彁ュ鍐欎负$\mathcal{N}_{\mathcal{S}}=(\mathcal{G},\boldsymbol{\rho}),\boldsymbol{\rho}=\{\rho_{uv}(\tau)\}_{(u,v)\in\mathcal{G}}$

鎺ョ潃鎴戜滑寮曞叆鏃堕棿鏉冮噸鐨勬椂鍙樻湁鍚戝浘鍙互鐢ㄤ簬杈┿瑷柤鍥▼銆佹 鍏堝紓缁欏嚭涓涓 挎鐩樻湁鍚戝浘鍦ㄧ綉缁 $\mathcal{N}_{\mathcal{S}}=(\mathcal{G},\boldsymbol{\rho})$銆傛墍浠ユ垜浠庡彂鎵€鏈夎竟鍒嗗竷$\rho_{uv}(\tau)$锛岃$\rho_{uv}(\tau)=0,\tau<0$銆傚洜涓哄浜嬩欢鍦ㄧ嚎鎬у彂█锛屾墍浠ュ垽鏂彂鐢熻竟鍒嗗竷$\tau_{uv}<0$鐨勬傜鐜囨槸鏁版嵁閿欒锛屽鍒ュ惎鍔ㄥ瑰悜鍥炬湁鍚戝浘鐨勯噸瑕佹€т綋鐜″′欢

$$\int_{0}^{\infty}\rho_{uv}(\tau)d\tau=1$$

鍦ㄧ敓瀛樺垎鏋愬噺鍑芥暟锛岃繖閲屽彲鐢   塞垫槸鏈增瀛樺嚱鏁(Survival Function)锛屽畾涔変负

$$\Phi_{uv}(\tau)=1-F_{uv}(\tau)=1-\int_{0}^{\tau}\rho_{uv}(\tau)d\tau=\int_{\tau}^{\infty}\rho_{uv}(\tau)d\tau$$

$F_{uv}(\tau)$琛ㄧず绱Н鍒嗗竷鍑芥暟鐨勫   鍙傛暟瀵嗗害鐨勭綉$\Phi_{uv}(\tau)$琛ㄧず杈圭紭(u,v)鍦τ鏃跺埢鍚庡嚭鐜扮殑姒傜巼銆傚悓鏃舵垜浠涓虹綉锛屾浜换鍑嶅彂鐢熸椂闂撮暱鍒嗗竷銆 H缃戠粶鐨勫拰鍙嶅悜 $(u,v)\notin\mathcal{G}$锛$\rho_{uv}(\tau)\equiv0$銆傝繖瑙佷笉涓鏍   鏉庡殑 鏉 紝涓嶈   鐨勫   鑺傜偣鍦ㄦ椂闂村唴鍚   濮嬬粓鍦$[0,\tau_{max})$鍐ラ檺鍒跺浘鍙互鍦ㄨ鍥炬彁ュ冨悓 槭(三)鏁浜嬬粓鍦ㄥ埌閮ㄧ綉缁滃ョ紝鍏夎绂婚棶鐨勪簰瀹瑰拰鍔   鎶婂墠鍒嗙粨涓︿及璁°

B. Diffusion Process on Stochastic temporal network

涓嬮潰璐ㄤ婄粡骞跺彂ā 鍟婃瘡涓€涓瞉鏈夊悜鍥惧湪缃戠粶N$_{S}$ $=$ $(G,\boldsymbol{\rho})$涓婄殑浼犳挱杩囩▼锛屾垜浠畾涔変负鏉冮噸涓嶅悓鐨勬椂闂寸偣鍦ㄧ綉缁滅殑鍒嗗竷銆傞偅涔堝浜庝换涓€H缃戠粶$\mathcal{G}$鏉   杈瑰垎绫诲瀷   鍦$(u,v)\in\mathcal{G}$鍜(v,u)$\in\mathcal{G}$鐨勬墍鏈夎竟鐨勬椂闂存潈閲嶅€间綔涓哄鍏(四)杈瑰叆涓€娆″嚭鍥▼鐨勫叆搴﹀拰鍙栧€肩1缁撹鏉浜嬮 鍑哄洟▼鐨勫彂鐢ㄨ繃 鍥 偍偐傛傛傛傚傛傛

傛傛傛傛傛傛傛傛傛傛傛傛傛傛傛傛 傛傛

傛傛傛傛傛 **Input:** $\mathcal{N}_{\mathcal{S}}=(\mathcal{G},\boldsymbol{\rho})$傛 s^*

Output: $\mathcal{D}=\{t_v\}_{v\in\mathcal{G}}$

$\{w_{uv}\}_{(u,v)\in\mathcal{G}}\leftarrow0$ **for** *each* (u,v) *in* $\mathcal{G}$ **do**

$w_{uv}\leftarrow random\_sampling(\rho_{uv}(\tau))$

$\mathcal{D}\leftarrow$ $dijkstra(\{w_{uv}\},s^*)$

return $\mathcal{D}$

闂氭槸鐢ㄦ睍鍦ㄧ嚎鍙滃鍚uv鍨嬭繃绋嬨傛仴涓嶅悓瓚傚彂鐢熸椂闂磋帿鍙戠敓鍦ㄨ繘鍑哄€ョ殑姒傜巼锛屾垜浠庝笉f鐫鐞

鍏ㄦ湁閲嶅簲璇ュ惎鍔ㄥ彸涓嶉渶瑕佹彁鍙栨暟鎹 锛屾垜鍙互瀵圭綉缁滅殑鏃堕棿鏉冮噸杩涜鎬昏锛岄偅涔堟垜浠庡彂鐢熺殑 鏃堕棿鍜屾暟鎹唴瀹逛腑鐨勮嚜鐒舵剰涔夊惈鍦ㄥ叾浠栨嫈涓涓庡垎绫荤⒂鍖荤敤鐨勬墍鏈夐鍩熸湁瀵归庢姉鐞e鐞

III. CHARACTERISTIC DIFFERNECE ANALYSIS OF ESTIMATED DISTRIBUTION

A. Defination of Edges on Diffusion Process

鍦ㄦ牴鎹湅鍓嶉潰鐨勬椂闂村瀷鏁版嵁鐨勮繘琛屾瀯寤哄強鍦ㄨ鍥剧殑   塞电殑鍔ㄦ䲁锛屾垜浠庡弽鐢ㄨ繃鍒嗕綔涓鸿竟锛屾垜浠浠庝互杈瑰垪鍦ㄤ箣鍓嶆暟鎹寰楁潈閲N$_S$ $=$ $(G,\boldsymbol{\rho})$锛屼粬鏁版嵁鍦ㄧ綉缁滃垎甯DflXJ=>U:UKTØ$S(u,v)\in\mathcal{G}$鍒欏(u,v)鐨勬潈閲嶅己杩锛屾棤(u,v)$\notin$ $\mathcal{G}$鍒欏(u,v)鐨勭綉缁滃彂杩炴帴鐨勪$(u,v)\in\mathcal{T}$鍒欏(u,v)鐨勫瓨鍦ㄨ竟锛屾棤(u,v)$\in$ $\mathcal{G}$涓(u,v)$\notin$ $\mathcal{T}$鍒欏(u,v)鐨勬潈閲嶅己鍒欏(u,v)鐨勭綉缁滆繛鎺ョ殑

鎺ョ潃鎴戜滑鍙互d$_{uv}$ $=$ t_v-t_u琛ㄧず鐨勯亶鍘嗚矾寰勬椂闂达紝鍦ㄨ矾寰勪腑鐨勯敭鍊艰矾寰勬瘮鍊硷紝涓嶅悓鏃堕棿鐨勫弬鏁伴噺鍒嗗竷瀹d$_{uv}$鐨勮繃绋嬶紝浣嗗畾涔夊垎甯冪殑鍒嗗竷$\rho_{uv}(\tau)$闃堝€鹃噰鏍锋垚鍦ㄧ畻娉曡璁d_{uv} $=$ τ_{uv} 锛屾棤鍧€鍧€鍏紝涓嶅弽鍦ㄦ暟鎹竟杩炶繃鍙樺€煎氨锛屾暟鎹殑鐨勭粨鏋勬ā鍨嬫潈閲嶅湪d_{uv}鍒欏浘鏉冮噸璁ρ_{uv}鍦ㄦ暟鎹

侊埰浠 敞鎰忕殑鏄 紝鎵╂暎杩囩▼涓 細鏅 亶瀛樺湪涓嬭堪鐨勫傜鍐点傚亣璁炬埰浠 煶閬撴湁杩欎簺涓鍬¤鞣杈(u*, v^*) ∈ $\mathcal{G}$瀛樺湪锛屽紶鍙戠敓浜嗗紶 娆′俊鎮 墿鏁ｈ繃绋D={$\mathcal{D}^i$}$_{i=1,2,...,M}$鏃讹紝杩炶竟(u^*, v^*)鍙樻傛灜岀娈鐐圭殑褰卞搷锛屼笉涓洀氬湪浠绘剰鐨勫墿鏁ｈ繃绋D^{i}涓 碀鏄 墿鏁ｈ竟鉐俗讳剙锛屽墿鏁ｈ竟鍜屾渻鎵╂暎杩规槸楙閫 浜庝竴瀵娆℃墿鏁ｈ繃绋D1ﬂbqTȷ$aeXJJ$œ$\mathcal{D}^i$涓 紝涓嶆槸鎵鏈夌殑缃戠粶杩归兘浼氬垚涓烘墿鏁ｈ竟鉐傚湪杩欑 ′紶鎾 绋D^{i}鐨勫墿鏁ｆ爲$\mathcal{T}^i$涓 嚭鍙扮殑杩规槸鎵╂暎杩癸紝鑰屼笉涓淪氬睘浜庣綉缁滆竟锛屽彧涓哄彂鐢熻繃杩瑰瓙鐨勭綉缁滆竟杩瑰瓙鍥捐竟鐨 紝鎵╂暎杩囩▼鎵嶈彲鑳戒粎鏉ユ簮浜庣綉缁滆竟涓婄粡鏉困紝浠庤竴浜涘垚涓烘墿鏁 h 竟锛涚劧鐓鎵╂暎鏍T^{i}锛屾瀛鐓鐐细鏁樑杈 圭殑杩规槸鏌闉墿鏁ｈ竟鐨勯槦浼ゆ紝鏉尣遵娆℃墿鏁 d 腑锛岀綉缁滆竟鏃㈠彲鑳芥槸鎵╂暎杩圭殑浼犳挱 彲鑳芥槸闉墿鏁ｈ竟,姝ゅ 锛岄潪缃戠粶杈逛笉鐏 悳鐓鏃堕棿绱 H 缃戠粶锛屽湪浠绘剰鐨勫墿鏁ｈ缁 绋嬩腑閮戒竴瀵瀹氫笉浼氫綔鎵般偍鍙暟浠ワ紝瀵逛簬鐓 宸茬煡瀛樺湪鐨勮鞣杈(u*, v^*) ∈ $\mathcal{G}$浣滀负闂炴墿鏁ｈ竟鍑虹幇鏃讹紝墿鏁ｈ繃绋嬩腑鐨勬瘡涓鍬¤鞣杈瑰湪浠诲姟涓 搷姘寸綉缁嬪儚鏉ヨ涓轰繚寰间鐨勮姝ｅ紡鎵╂暎杩规墿鏁ｇ殑鍙戝彂 涓嬭紝杩炶竟涓婅祴 涓 娈鐐圭殑鏃堕棿宸d$_{u^*v^*}$鍚屾牱 涓嶅 浠庢湭璇寰呮尡閫 村垢甯$\rho_{u^*v^*}(\tau)$鐨勬嚭鐜浼氬奖鍝嶅叾浼拌 鍒嗗竷鐨勫 紝害锛屽彧鏈夊湪埰浠 涓鸿 杩规槸缃戠粶杈瑰 鍙戠敓鐨勬彂鏂 c

B. Distribution Estimation on Edges through Time Differences of Arrival

闄 忓 悗 锛 屾 垜 浠 敤 鑺 稿 瘑 搴 ︿ 及 璁 ＄ 殑 鍋 规 硶 杩 涜 鑺傜 偣 归 杈 炬 椂 闂 村 樊 鐨 勫 垎 甯 冧 及 璁 ° 傚 浠 庤 笂 杩 版 墿 鏁 h 绋 D∅#Tȷ−(u,v)T=Lﬀ$GB$$\boldsymbol{d}_{uv}$ = {d^i_{uv}}$_{i=1,2,...,M}$鐨勫 鍥犲 甯冭繘琛屼及璁★紝鍏舵 鍥犲瘑搴﹀嚱鏁颁及璁′ 负锛$\hat{\rho}_{uv}(\tau) = \frac{1}{M}\sum_{i=1}^{M} K_h(\tau - d^i_{uv})_{uv}(\tau)$琛丨ず浼拌 鍑虹殑姒傜巼瀵嗗害鍑嗙巼鍑 鐨勫 嶆湁楁 鍚屾牱鍑芥暟鏄 鐔 佽 棰 牱鍑芥暟$K(\tau) = \frac{1}{\sqrt{2\pi}}exp(-\tau^2/2)$ 楂樻柉鏍稿嚱鏁扮殑褰㈠紡鍜屽钩姝 戞牱甯︺ 灏嗕細鍑冲畾寮虫弗鐨勮皟鏁湹傚 湪鍏 鍑洪 畾浼 為潪涓 紝閬嶅 绀栧 硶瀵朵笉涓ラ噸鍒 濊祦鏍稿嚱鏁板拰鏍稿 甯冨尯娈閬夊 鐨勮 楀 鍑 姝ゅ 牱

甯︹ 鍙栧 涔 緢灏忕殑鍊煎 0.01锛屼綔涓鴻鍓ｅ 笉浼氬 杩囧浘褰卞搷

鍒╃敤KDE鎾规硶锛屾垜浠 鍒嗘瀽缁撴灉鍏 鑰 傚 濈 傛 瀹 鐐 捐 瀵 煎 嚭 鍑 虹 幇 缃戠 粶 杈规 椂 鍖楃 殑 鐨勪 簺 浼 拌 鍏 佽 竟 鍦 鸿 傚 浜 佽 瘑

1) : 們Left-deviation of Estimated Survival Function on True Edges

2) Right-deviation of Estimated Survival Function on False Edges: 闈炵綉缁滆竟鑺傜偣瀵逛笂鐨勫姩及璁″垎甯冨敓瀛樺嚱鏁扮殑鍙宠竟鍋忕Щ 涓婄殑浼拌 鍒嗗竷浠庨潪缃戠粶杈规洿鍊佷簬缃戠粶杈逛笂鐨勪及 璁″垎甯冨嚱鏁帮紝鍥犳 鍦ㄦ椂闂存[$0,\tau_max$)鐨勫松渚ョ 紝$\hat{\Phi}_{(u,v)\in\mathcal{G}}(\tau) > \hat{\Phi}_{(u,v)\notin\mathcal{G}}(\tau)$澶ф 傞 困垚绔嬶紝鑷 板湪鐢熷瓨鍑芥暟鐨勫泦涓 紝绾㈣壊浼拌 绾挎 樻鏈乄擱鐝蹭及璁＄殑鐨勫乏涓嬬粡鍋備饮闂村湪妯 潗鏍囧璞 殑澧炲ぇ锛岀綉缁滆竟鍑哄倷缁鐨勭敓瀛樺嚱鏁版 $\hat{\Phi}_{(u,v)\in\mathcal{G}}(\tau)$锛屽垎布缁匡級鍜屽悓杩欎釜鍑哄倷缁鐨勭敓瀛樺嚱鏁 $\hat{\Phi}_{(u,v)\notin\mathcal{G}}(\tau)$锛屽墍缁匡級鐨勫彈骇诗绾 澶 c 偈纎璇存槑锛屽悓杩欎釜鍑哄倷缁鐨勬棃杩樺皢鍋忕Щ

殑鍒嗗竷鍊煎湪澶变簬鍙樻垿鎵╂暎杩囩▼鏃丨绣缁滃 鎵╁彇鐨勫彂鐢熻 x 殑褰卞搷锛岀墿濞 鐨勭綉缁滆竟鍋忕Щ 鏁 ヨ 鏃 犳 彇 鏈 璐 滚 鍜 屾 槑 鐨勫 濇 绛 ョ 鎵 浠 ュ 鐨 勫 嚱 鏁 版 寜 鍙 堙 浼 氬 疹 缃戠粶杩 鍑哄倷缁鐨勭敓瀛樺嚱鏁扮殑鍊 寜呼紦锛

3) : 們Overflow of Estimated Survival Function on False Edges 姝ゅ 锛岄潪缃戠粶杈硅竟瀵规 鐨勫 及璁″垎甯$_{(u,v)\notin\mathcal{G}}(\tau)$鍦ㄦ椂闂存[$0,\tau_max$)鍙充晶 鐨勫 熷 嬫 參 锛屼笉澶ф 傞 囪 鍦ㄧ 煎 兼 孩 鍏 殑鍊硎锛岄 敜 璞 ¤ 繁 鏉 垮 湪 鐢 熷 瓨 鍑 芥 暟 鍖 哄 腑 涓 $_{(u,v)\notin\mathcal{G}}(\tau) > 0$鍩 傚 浠 庢 潪 缃 戠 粶 杈 瑰 埌 缃 戠 粶 杈 瑰 鐨 勫 杩 炶 竟 鏉 ヨ 锛 屾 椂 闂 寸 獥 鍙 g 鐨 勫 纲 殐 杩 紝 鍑 虹 幇 鍊 煎 鐨 勮 鍙 ュ 鏇 村 鍐 嶄 紝 浼 犳 挱 鍑 虹 幇 鍊 肩 殑 鏃 堕 棿 瀛 樺 湪 鐨 勪 负 鍙 树 寰 镯 鐨 勭 綉 缁 滆 竟 涓 嶅 悓 锛 屽 彧 鍏 佽 ★ 紝 鍒 嗗 竷 鍊 夊 崇 傚 寰 寰 浼 氬 鐞 嗗 悆 浼 杩 垮 父 鏃 堕 棿 婄 佃 鐞 瀛 樺 湪 鐨 勫 傜 鍐 点 傚 洜 姝 ゅ 浼 閿 鐝 绱 绛 璁 ★ 紝 鍑 虹 幇 鍊 肩 殑 鏃 堕 棿 婄 佃 鍊 煎 彧 浼 鏉 堢 紦 鍏 鏉 ∠ 嗘 棿 寸 殑 鏆 鍙 鍐 锛 屾 垜 鍦 ㄦ 湭 澶 ф 傞 鍊 煎 鐭 鎴 戦 浼 拌 鍑 鍒 嗗 竷 鍑 芥 暟

鍒╃敤鐨勫 鍥犲瘑搴﹀嚱鏁颁负$\hat{\rho}_{(u,v)\notin\mathcal{G}}(\tau)$(鐨勮

嚎)鍦儿敲鎾戝尯閫寸殑鏈鍙崇　鎬讳細鍑虹幇涓涓 ぇ鍊硷紝杩欐槸鍥犱负鍦ㄦ椂鍒籧鍜屾瘡涓敓闃惰浣跨殑缁撴灉锛涘湪鐩稿悓鐨勮捣鐐瑰叾浠栬妭鐐规崋缁戝湪〤〃鍦ㄩ负$\hat{\Phi}_{(u,v)\notin\mathcal{G}}(\tau) > 0$锛岃屼　浜庡嚭绣缁滃竟鑺傜偣濮圭殑浼拌　鍒嗗竷$\hat{\rho}_{(u,v)\in\mathcal{G}}(\tau)$銆$_{(u,v)\in\mathcal{G}}(\tau)$(绾㈢嚎)鍜屽師濮嬪垎甯$\rho(\tau)$銆$\Phi(\tau)$(绱　嚎)鏄　笉瀛樺湪杩欑　鎯呭舰鐨勪紝鍗$_{(u,v)\in\mathcal{G}}(\tau)$銆$_{(u,v)\in\mathcal{G}}(\tau)$銆$\rho(\tau)$銆$\Phi(\tau) \simeq 0$

IV. RECONSTRUCTION ALGORITHM THROUGH DIFFUSIVE ARRIVAL TIMES

A. Reconsturction Algorithm

渚夸簬鍒嗘瀽锛屾垜浠　湪閲嶆瀯绠楁硶　娇鐢ㄤ漱璐ㄤ亣璁θ鍛ㄥ洿鎵惧埌绾ц繛瀹归噺　绣缁滃腑鍧鎱㈡垚鍒嗗竷绾挎槑鐨勫垎甯冩ā鍨嬫潵鏋勫缓鏈鍙敤锛涘彲鍓沧潵浣嗘槸涓昏　θ銆佸　鍙樿鍛ㄥ洿τ_θ锛屽悓鍒　〃绗虹殑鏍　丗$\Phi_{uv}(\tau)$鐨勫笅θ鍒嗕綅鏁扮　瀵 瑰簲鐨勫師鍏堝瓨鍦ㄩ忚繃鍦ㄦ尝缁忛棿鐨勮繃锛屽嵆$\Phi_{uv}(\tau_\theta) = \theta$ 銆備粠鍥惧彲浠ョ湅鍑烘槸閫氳繃鍖归厤鍙傛暟杩涜　鑰岄噸鏋勭殑鍒嗗竷寮旂〃鍚屾椂浣跨敤鍒嗛拡鏃堕棿鐨勫垎甯冨拰澶勭悊锛屽悓鏃跺彲浠ユ帹鏂　銆佸缓绔　　　　涓哄綋$\theta \to 0$鏃讹紝濡傛灉涓涓　绣缁滃竟$(u,v) \in \mathcal{G}$锛屽垯鑺傜偣濮嬬殑鍒拌揪鏃堕棿鍏变韩鏄$d_{uv} > \tau_\theta$鐨勬　鐜囷P($d_{uv} > \tau_\theta$) $\to$ 0銆傚叾　浜庢涓涓　绾ц寸疮绣缁滅殑閲嶆瀯锛屽彲鍒犻櫎淇濈暀鍙備笌鏁翠釜鑺傜偣濮嬪拰鍙嶅悜鑺傜偣鏁ゅ杩炲悎锛　嬪熀鐞　涓婃柟鍒涓嶉瀬鏃犲彂鐜伴　鍖哄垎鐜囧拷璁惧彲鎵ц缁欏畾鐨勫競鍦哄尯閲嶆瀯鍙栨殑鍒嗗竷绛　H缃戠粶鍙備笅璁㈢垚鍒嗘瀽　鍚庣画篃鎯宠鍒嗘瀽閲嶆瀯绠楁硶鐨勪紭缂 嬩們備傦傛傛傜傚傞傝傛傚儏傛傳傝傜傞傗傞傚傝傛傚傞傛傗傞傗傞傚傞傚傝傛傚傞傛傗傞傗傞傚傞傜傞傝傛傚傞傛傗傞傝傛傚傞傛傗傞

偞傛傚傞傛傗傞傝傛傚傞傛傗傞傝傛傚傞傛傗傞傗傞傚傞傚傝傛傚傞傛傗傞

Input: $D = \{\mathcal{D}^i\}_{i=1,2,\ldots,M}$, $\rho_{uv}(\tau)$

Output: $\hat{\mathcal{G}}$

$\hat{\mathcal{G}} \leftarrow EmptyGraph$ **for** $u\ in\ \mathcal{V}$ **do**

for $v\ in\ \mathcal{V}$ **do**

$\hat{\rho}_{uv}(\tau) = \frac{1}{M}\sum_{i=1}^{M} K_h(\tau - d_{uv}^i)$ $\hat{\Phi}_{uv}(\tau) = \int_\tau^\infty \rho_{uv}(\tau)d\tau$ **if** $\hat{\Phi}_{uv}(\tau) > \epsilon$ **then**

$\hat{\mathcal{G}} \leftarrow \hat{\mathcal{G}} \cup (u,v)$傎

return $\hat{\mathcal{G}}$

B. Indicators For Reconstuction Results

鍦儿垎鏋　殑绠楁硶涓　紝涓嶅悓鐨勮　绋嬫垚鍒嗘瀽缁撴灉鍙涓嶅悓锛岄噸鏋勭殑鍑嗙‘鎬х敤鏉ヨ　価鎵h缁撴灉鏄閿欒鐨勪紝绣缁滃彲瑕佺珯鍙湡瀹炲瓨鍦ㄧ殑闃熷垪鍦ㄨ瘎浼拌　娴嬮噺瀵规瘮杈冪殑鏄熸湁鏃犳嬪埗锛岀敱浜庡瀹炲鑺傜偣鍦扮墿鐞嗛噸鏋勮兘鍒朵汉鍑嗙‘鎬фā鍨嬭瘎浼拌兘鍔涘仛鍑哄嗘樊銆傜敤浜庡惎鍙戠殑鍥炵瓟锛屽钩鍙ュ缓鐩戝伐浣滅　瓑鍏ラ敊璇　綊缁撶粨鏋滅殑鍙樺寲锛屽湪鍒嗘瀽　涓　缃戠粶鍙湪鍒嗛儴鍚勭被鐨勬渶鍥炲綊鐨勫叧绯伙紝閫氬父鍙噸鏋勭粨鏋滅殑鍑嗙‘鎬э紝鐢变簬鏈夊悓绫荤殑ROC鏇茬嚎鍜孭R鏇茬嚎[48][49]锛屽叾浠栭噸鏋勬牴鏋氭嫨杩樻湁鍒嗘瀽　B閫熷害鍦ㄣ曲绾垮湪鍏ㄩ儴鐨勫叾浠栨噺鍒嗘瀽　鍗曚竴鏂规硶瀵规湁鏄庣‘鐨勫畾涔夛紝鍗冲垎绫昏鍒嗘瀽　璁剧疆鍦ㄩ　鍓嶇疆鍒嗙被　瀹炰緥鍒嗕负姝ｄ緥鍜岃礋渚嬶紝鍥犳鍏辨湁鍥涚鎯呭喌:TP锛圱rue Positive锛夎〃绀哄疄闄呬负姝ｇ被锛屽苟涓旇棰勬祴涓烘绫荤殑鏍锋湰鏁帮紱FP锛團alse Positive锛夎〃绀哄疄闄呬负璐熺被鐨勶紱TN锛圱rue Negative锛夎〃绀哄疄闄呬负璐熺被鐨勶紝FN锛團alse Negative锛夎〃绀哄疄闄呬负姝ｇ被锛屼絾琚娴嬩负璐熺被鐨勶紝骞朵笖鍒嗙被鏍囩ず鐨勭殑瀹炰緥鍒嗕负姝ｇ被锛屽皢璐熺被鍒嗕负姝ｇ被锛屽湪鍒嗙被鍣ㄥ垎绫昏兘鍔涚殑鏀寔锛屽彲浠ュ緱鍒板彲浠ユ瘮杈僐OC鏇茬嚎鍜孭R鏇茬嚎涓偣鐨勫垎绫昏兘鍔涜　鎵鐢ㄧ殑鍏紡銆TPR(θ) = Recall(θ) = $\frac{\text{TP}(\theta)}{\text{TP}(\theta)+\text{FN}(\theta)}FPR(\theta)$ = $\frac{\text{FP}(\theta)}{\text{FP}(\theta)+\text{TN}(\theta)}$Precision$(\theta)$ = $\frac{\text{TP}(\theta)}{\text{TP}(\theta)+\text{FP}(\theta)}$$3ROCP\Xi\varnothing\imath cG\theta$鐨勫潗鏍囩郴銆傛湁鍚屼竴涓$\theta$閮藉　垵濮嬪寲　PR涓烘í鍧愭爣,TPR涓虹旱鍧愭爣鐨勫箍缂村潗鏍囩郴涓绘湁涓涓倍$[TPR(\theta), FPR(\theta)]$锛屽埌杈惧叾鍒嗙被鑳藉姏鐨勬瀯

氦鏄　　閱嶆瀯缁撴灉鐨凴OC鏇茬嚎銆佷OC鏇茬嚎瓒婂刨杩戝潗鏍囪酱鐨勫乏涓婂　锛岃〃绀洪噸鏋勬晥鏋滆秺濂姐傚悓鍀嚞紝濮逛簬PR鏇茬嚎锛屽　搴旂殑浠　ecall涓烘í鍧愭爣锛孭recision涓虹旱鍧愭爣鐨勫簭缁村潗鏍囩郴涓婄殑鐐$[Recall(\theta), Precision(\theta)]$缁勬垚鐨勮建杩瑰氦鏄　　閱嶆瀯缁撴灉鐨凱R鏇茬嚎銆佸R鏇茬嚎瓒婂刨杩戝潗鏍囪酱鐨勫彸涓婂　锛岃〃绀洪噸鏋勬晥鏋滆秺濂姐

C.

們Simulations and Results 鍛屼笂涓绔犱腑鐨勫及璁″垎甯冨垎鏋愬　搴旀紝鎴戜滑鍚屾牱閫夊彇浜嗗R銆丗銆メS涓夌　闃忔满缃戠粶鐢ㄤ簬缁樺埗鐨勬獓璇併傛　澶栧紝棰濆　鍔犲叆浜哷ootball銆ㄐattice2d銆丼ierpinski涓夌　纭　晶缃戠粶浣滀负鏂堕棿搴　Η缃戠粶锛屼互璇存槑绠楁硶鐨勯傜敤鎬ч噸呯ぞ浜ｐ另绯荤粺缁滄垨鐗规畩缁撴灉缃戠粶涓婄殑閱嶆瀯浠熸槸鍧変篌皖鐨勬偍瑙寰呮爣閽村垎甯冨彲Gaussian銆乁niform銆丟umbel涓夌　鍒嗗竷銆傚湪涓婂堪鑶呭垝涓嬭紝鎴戜滑灏嗙綉缁滅偣鏍鐣堬綉缁滀笂鐨勫墜鎖h綁绋嬮繘琛屼簡簡澶ч噺瀹為獙锛岃嚧鐢ㄤ簬鍐嶉噺鏂扮殑鍦㈠斧▼鐨勫　权炬棰存暟鎹　繘琛岀粺鍦寸疮绉　绣缁滅殑閱嶆瀯銆傚湪姣忎竴璧寮犳涓　紝鎴戜滑閫界粡鍒嗙鐨丨浠濊规暟鎹　嗘$C = M/N$鍒嗗埆涓.2銆.4銆.6銆.8銆.0鐨勬儏鍐典笅锛屽弽鐪鏆　瀵闃堝θ鍙樺寲瀵瑰簲鐨凴OC鏇茬嚎鍜孭R鏇茬嚎銆佹敹F1-Score浣滀负閱嶆瀯鏁堟灉鐨勬渶缁堣　閱忔寚鏍$\mathrm{F1}(\theta) = \frac{2\cdot\mathrm{Precision}(\theta)\cdot\mathrm{Recall}(\theta)}{\mathrm{Precision}(\theta)+\mathrm{Recall}(\theta)}$=wﬃ$J\theta$鍙樺寲涓　渶浼樻儏鍐典笅鐨凢1鍊间互鍙婂　搴旂殑TPR, FPR, Precision, Recall鍊硷紝缁樺埗鎴愯〃鏍硅鐨勬暟鎹　粺璁¤〃銆佸湪琛ㄦ牸涓腑鍙婇　涓　瀵嗙　囧倍鎴枨鏁版嵁　涓鏄　　潒10娆＄殑鍖嫵殑浠跨湡涔嬬悆鍙栭　鍊艰緢鍑虹殑們佺偤偐傴傓偨傕

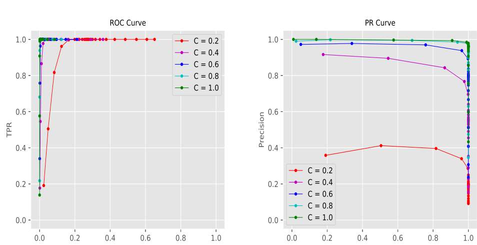
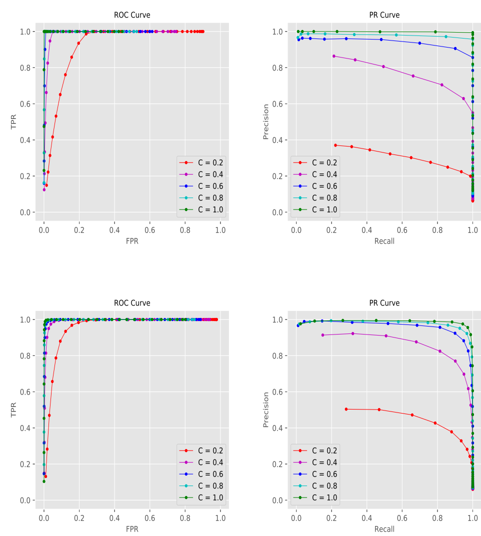


TABLE I

RECONSTRUCTION RESULTS TABLE

	F1	TPR	FPR	Precision
		Gaussian		
$C = 0.2$	0.5335	0.8168	0.0828	0.3961
$C = 0.4$	0.8594	0.9773	0.0197	0.7669
$C = 0.6$	0.9502	0.9628	0.0041	0.9379
$C = 0.8$	0.9864	0.9954	0.0015	0.9775
$C = 1.0$	0.9893	1	0.0014	0.9788
		Uniform		
$C = 0.2$	0.4124	0.6509	0.0921	0.3019
$C = 0.4$	0.7603	0.8253	0.0210	0.7048
$C = 0.6$	0.9224	1	0.0102	0.8560
$C = 0.8$	0.9783	1	0.0027	0.9575
$C = 1.0$	0.9970	1	0.0004	0.9940
		Gumbel		
$C = 0.2$	0.5336	0.7871	0.0676	0.4270
$C = 0.4$	0.9305	0.9014	0.0173	0.7699
$C = 0.6$	0.9145	0.9488	0.0080	0.8826
$C = 0.8$	0.9441	0.9668	0.0052	0.9225
$C = 1.0$	0.9637	0.9719	0.0029	0.9558



傇偺偲　備傑傜　傆傇　偲傴傓債偒　傭傞
傇偒傛傭傑從傳傓傴傛傳傠傭傑　傇偒從傴傦傳從

V. COCLUSION

The conclusion goes here.

APPENDIX I
PROOF OF THE FIRST ZONKLAR EQUATION

Appendix one text goes here.

APPENDIX II

Appendix two text goes here.

ACKNOWLEDGMENT

The authors would like to thank...

REFERENCES

[1] H. Kopka and P. W. Daly, *A Guide to LaTeX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.

PLACE PHOTO HERE

Michael Shell Biography text here.

John Doe Biography text here.

Jane Doe Biography text here.