## Q1

1. P(X ≤ 0.5):

To find this probability, we need to sum the probabilities of all values of X that are less than or equal to 0.5.

P(X ≤ 0.5) = P(X = 0.2) + P(X = 0.4) + P(X = 0.5)

P(X ≤ 0.5) = 0.1 + 0.2 + 0.2 = 0.5

1. Find P(0.25 < X < 0.75):

To find this probability, we need to sum the probabilities of all values of X that are greater than 0.25 and less than 0.75.

P(0.25 < X < 0.75) = P(X = 0.4) + P(X = 0.5)

P(0.25 < X < 0.75) = 0.2 + 0.2 = 0.4

P(X = 0.2 | X < 0.6):

This is the conditional probability that X equals 0.2 given that X is less than 0.6. To calculate it, we need to consider all values of X less than 0.6 and see how many of them are equal to 0.2.

1. P(X = 0.2 | X < 0.6) = [P(X = 0.2) / P(X < 0.6)]

First, P(X < 0.6):

P(X < 0.6) = P(X = 0.2) + P(X = 0.4) + P(X = 0.5)

P(X < 0.6) = 0.1 + 0.2 + 0.2 = 0.5

1. Now, the conditional probability:

P(X = 0.2 | X < 0.6) = (0.1 / 0.5) = 0.2

So, P(X = 0.2 | X < 0.6) = 0.2.

## Q2

1. Find RX, RY, and the PMFs of X and Y:

When two fair dice are rolled, each die can have values from 1 to 6. So, RX = {1, 2, 3, 4, 5, 6} and RY = {1, 2, 3, 4, 5, 6}.

The probability mass function (PMF) of a fair six-sided die is uniform, meaning each outcome has an equal probability of 1/6.

So, for both X and Y:

PMF(X = x) = PMF(Y = y) = 1/6 for x, y in {1, 2, 3, 4, 5, 6}

2. Find P(X = 2, Y = 6):

Since the two dice rolls are independent events, the joint probability of X and Y is the product of their individual probabilities.

P(X = 2, Y = 6) = P(X = 2) \* P(Y = 6) = (1/6) \* (1/6) = 1/36

3. Find P(X > 3 | Y = 2):

Given that Y = 2, X can only be 1, 3, 4, 5, or 6. We want to find the conditional probability that X is greater than 3.

P(X > 3 | Y = 2) = [P(X > 3 and Y = 2)] / P(Y = 2)

P(X > 3 and Y = 2) = P(X = 4, Y = 2) = (1/6) \* (1/6) = 1/36

P(Y = 2) = (1/6)

P(X > 3 | Y = 2) = (1/36) / (1/6) = 1/6

4. If Z = X + Y, find the range and PMF of Z:

The possible values of Z can range from 2 (minimum when X = 1 and Y = 1) to 12 (maximum when X = 6 and Y = 6).

So, the range of Z is RZ = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}.

To find the PMF of Z, we need to consider all possible combinations of X and Y that result in each value of Z and sum their probabilities.

PMF(Z = z) = ∑ [P(X = x, Y = z - x)] for x in {1, 2, 3, 4, 5, 6} where z is in RZ.

5. Find P(X = 4 | Z = 8):

We want to find the conditional probability that X = 4 given that Z = 8.

P(X = 4 | Z = 8) = [P(X = 4 and Z = 8)] / P(Z = 8)

To find P(X = 4 and Z = 8), we can look at all combinations of X and Y that result in Z = 8. These combinations are (X = 4, Y = 4) and (X = 5, Y = 3).

P(X = 4 and Z = 8) = P(X = 4, Y = 4) + P(X = 5, Y = 3) = (1/6) \* (1/6) + (1/6) \* (1/6) = 2/36 = 1/18

Now, we need to find P(Z = 8), which is the sum of all probabilities that result in Z = 8:

P(Z = 8) = P(X = 4 and Y = 4) + P(X = 5 and Y = 3) + P(X = 3 and Y = 5) = (1/6) \* (1/6) + (1/6) \* (1/6) + (1/6) \* (1/6) = 3/36 = 1/12

Finally, we can calculate P(X = 4 | Z = 8):

P(X = 4 | Z = 8) = (1/18) / (1/12) = ⅔

## Q3

There are 10 questions for which the student knows the answers, so the number of correct answers from these is a fixed value of 10.

For the remaining 10 questions that the student doesn't know, he randomly selects one option out of 44 for each question. The probability of guessing the correct answer for each of these questions is 1/44.

Now, the probabilities for different values of X:

P(X = 10): The student answers all the questions he knows correctly, so this is certain: P(X = 10) = 1.

P(X = 11): The student answers the 10 known questions correctly and guesses one of the 10 unknown questions correctly. The probability of guessing one question correctly is 1/44. So, P(X = 11) = (10/10) \* (1/44) = 1/44.

P(X = 12): The student answers the 10 known questions correctly and guesses two of the 10 unknown questions correctly. The probability of guessing two questions correctly is (1/44)^2. So, P(X = 12) = (10/10) \* ((1/44)^2) = 1/1936.

Continuing this pattern, P(X) for X = 10 to 20.

Now, P(X > 15):

P(X > 15) = P(X = 16) + P(X = 17) + P(X = 18) + P(X = 19) + P(X = 20)

P(X > 15) = (10/10) \* ((1/44)^6) + (10/10) \* ((1/44)^7) + (10/10) \* ((1/44)^8) + (10/10) \* ((1/44)^9) + (10/10) \* ((1/44)^10)

P(X > 15) is the sum of these probabilities.

## Q4

To find the probability P(10 < Y ≤ 15), where Y is the number of students arriving between 10 am and 11:30 am, we can use the properties of a Poisson distribution. In this case, the average rate of arrival (λ) is 10 students per hour.

First, to adjust the average rate for the time interval from 10 am to 11:30 am, which is 1.5 hours. So, the rate for this time interval is λ = 10 students/hour \* 1.5 hours = 15 students.

Now, the desired probability:

P(10 < Y ≤ 15) = P(Y = 11) + P(Y = 12) + P(Y = 13) + P(Y = 14) + P(Y = 15)

Using the Poisson probability formula:

P(Y = k) = (e^(-λ) \* λ^k) / k!

Where:

e is the base of the natural logarithm (approximately 2.71828).

λ is the average rate.

k is the number of events.

We can calculate each term:

P(Y = 11) = (e^(-15) \* 15^11) / 11!

P(Y = 12) = (e^(-15) \* 15^12) / 12!

P(Y = 13) = (e^(-15) \* 15^13) / 13!

P(Y = 14) = (e^(-15) \* 15^14) / 14!

P(Y = 15) = (e^(-15) \* 15^15) / 15!

P(10 < Y ≤ 15) = P(Y = 11) + P(Y = 12) + P(Y = 13) + P(Y = 14) + P(Y = 15)

This will give the probability of 10 < Y ≤ 15 students arriving between 10 am and 11:30 am.

## Q5

To find the probability mass function (PMF) of the random variable Z = X + Y, where X follows a Poisson distribution with parameter α and Y follows a Poisson distribution with parameter β, the properties of the Poisson distribution and the fact that X and Y are independent.

The PMF of the Poisson distribution is given by:

P(X = x) = (e^(-α) \* α^x) / x! for x = 0, 1, 2, ...

P(Y = y) = (e^(-β) \* β^y) / y! for y = 0, 1, 2, ...

Now, the PMF of Z = X + Y:

P(Z = z) = ∑ [P(X = x) \* P(Y = z - x)] for z = 0, 1, 2, ...

Here, we're summing over all possible values of x such that z - x is a non-negative integer. This accounts for all possible combinations of X and Y that would result in Z = z.

So, for each value of z, you'll calculate P(Z = z) by summing the product of P(X = x) and P(Y = z - x) over all suitable values of x.

This PMF represents the distribution of the random variable Z, which is the sum of two independent Poisson-distributed random variables X and Y.

## Q6

Find the range of Y.

The range of a random variable Y is the set of all possible values that Y can take. In this case, Y = (X + 1)^2, so to find the range of Y, we need to consider all possible values of X.

From the given pmf of X, we have:

P(X = -2) = 1/418

P(X = 11) = 1/418

P(X = 0) = 1/418

P(X = 1) = 1/418

P(X = 20) = 1/418

P(X = x) = 84/418 for all other values of x

Now, let's find the possible values of Y:

Y = (-2 + 1)^2 = 1

Y = (11 + 1)^2 = 144

Y = (0 + 1)^2 = 1

Y = (1 + 1)^2 = 4

Y = (20 + 1)^2 = 441

So, the range of Y is {1, 4, 144, 441}.

2. Find the pmf of Y.

To find the pmf of Y, we need to calculate the probability mass function for each value in the range of Y:

P(Y = 1) = P(X = -2) + P(X = 0) = 1/418 + 1/418 = 2/418

P(Y = 4) = P(X = 1) = 1/418

P(Y = 144) = P(X = 11) = 1/418

P(Y = 441) = P(X = 20) = 1/418

For all other values of Y in the range, the pmf is 0 because they are not possible outcomes.

So, the pmf of Y is:

P(Y = 1) = 2/418

P(Y = 4) = 1/418

P(Y = 144) = 1/418

P(Y = 441) = 1/418

3. Assuming X is a continuous random variable with PDF:

fx(x) = cx^2 for |x| ≤ 1

fx(x) = 0 otherwise

a. Find E(X) and Var(X).

To find E(X) and Var(X), we'll integrate over the given PDF:

E(X) = ∫[x \* cx^2] dx, from -1 to 1

E(X) = c \* ∫[x^3] dx, from -1 to 1

E(X) = c \* [(1/4) - (-1/4)]

E(X) = c \* (1/2)

E(X) = c/2

Now, for the variance:

Var(X) = E(X^2) - [E(X)]^2

First, we need to find E(X^2):

E(X^2) = ∫[x^2 \* cx^2] dx, from -1 to 1

E(X^2) = c \* ∫[x^4] dx, from -1 to 1

E(X^2) = c \* [(1/5) - (-1/5)]

E(X^2) = c \* (2/5)

E(X^2) = (2c)/5

Now, calculate Var(X):

Var(X) = (2c)/5 - (c/2)^2

Var(X) = (2c)/5 - (c^2)/4

b. Find P(X ≥ a).

To find P(X ≥ a), we integrate the PDF from a to 1 (since the PDF is zero outside of [-1, 1]):

P(X ≥ a) = ∫[cx^2] dx, from a to 1

P(X ≥ a) = c \* [((1/3) - (a^3/3)) - ((1/3) - (1/3))]

P(X ≥ a) = c \* [((1/3) - (a^3/3)) - (0)]

P(X ≥ a) = c \* (1/3 - a^3/3)

2. If X is a continuous random variable with PDF:

3. If X ~ Uniform and Y = sin(X), then find fY(y).

If X follows a uniform distribution between a and b, the PDF of Y = sin(X) can be found by applying the transformation theorem. The transformation theorem states that if Y = g(X), where g is a monotonic function, then the PDF of Y is given by:

fY(y) = fX(g^(-1)(y)) \* |(dg^(-1)(y)/dy)|

In this case, Y = sin(X), and the inverse function is g^(-1)(y) = arcsin(y).

So, we have:

fY(y) = fX(arcsin(y)) \* |(d(arcsin(y))/dy)|

The PDF of a uniform distribution, fX(x), is constant within the interval [a, b] and zero outside of it.

Now, we need to find the derivative of arcsin(y) with respect to y:

d(arcsin(y))/dy = 1 / sqrt(1 - y^2)

Therefore, the PDF of Y = sin(X) is:

fY(y) = [1 / (b - a)] \* (1 / sqrt(1 - y^2)) for a ≤ y ≤ b

fY(y) = 0 elsewhere

## Q6

o find the joint probability mass function (PMF) of X (the number of white shirts) and Y (the number of black shirts) when 10 shirts are chosen randomly without replacement from a box containing 40 white shirts and 60 black shirts, we can use the hypergeometric distribution.

The hypergeometric distribution describes the probability of obtaining a specific number of white shirts (X) and black shirts (Y) when drawing without replacement from a finite population.

The PMF of the hypergeometric distribution is given by:

P(X = x, Y = y) = [C(40, x) \* C(60, y)] / C(100, 10)

Where:

C(n, k) is the binomial coefficient, which represents "n choose k," and is calculated as C(n, k) = n! / (k! \* (n - k)!).

n is the total number of shirts of a particular color (40 for white, 60 for black).

x is the number of white shirts.

y is the number of black shirts.

10 is the total number of shirts drawn.

We need to calculate P(X = x, Y = y) for all possible values of x and y, subject to the constraint that x + y = 10 (since 10 shirts are drawn).

Here are the possible values of x and y, along with their probabilities:

P(X = 0, Y = 10):

P(X = 0, Y = 10) = [C(40, 0) \* C(60, 10)] / C(100, 10)

P(X = 1, Y = 9):

P(X = 1, Y = 9) = [C(40, 1) \* C(60, 9)] / C(100, 10)

P(X = 2, Y = 8):

P(X = 2, Y = 8) = [C(40, 2) \* C(60, 8)] / C(100, 10)

P(X = 3, Y = 7):

P(X = 3, Y = 7) = [C(40, 3) \* C(60, 7)] / C(100, 10)

P(X = 4, Y = 6):

P(X = 4, Y = 6) = [C(40, 4) \* C(60, 6)] / C(100, 10)

P(X = 5, Y = 5):

P(X = 5, Y = 5) = [C(40, 5) \* C(60, 5)] / C(100, 10)

P(X = 6, Y = 4):

P(X = 6, Y = 4) = [C(40, 6) \* C(60, 4)] / C(100, 10)

P(X = 7, Y = 3):

P(X = 7, Y = 3) = [C(40, 7) \* C(60, 3)] / C(100, 10)

P(X = 8, Y = 2):

P(X = 8, Y = 2) = [C(40, 8) \* C(60, 2)] / C(100, 10)

P(X = 9, Y = 1):

P(X = 9, Y = 1) = [C(40, 9) \* C(60, 1)] / C(100, 10)

P(X = 10, Y = 0):

P(X = 10, Y = 0) = [C(40, 10) \* C(60, 0)] / C(100, 10)

These probabilities represent the joint PMF of X and Y for all possible combinations of white and black shirts when 10 shirts are drawn randomly without replacement.

## Q7

a. To find the marginal probability density functions (PDFs) of A and B, you need to integrate the joint PDF fxy(x, y) over the entire range of the other variable.

Marginal PDF of A (fX(a)):

fX(a) = ∫[0,√a] 6xy dy (integrating with respect to y)

fX(a) = 6x[a^(3/2)/2] |[0,√a]

fX(a) = 3a, for 0 ≤ a ≤ 1

fX(a) = 0, otherwise

Marginal PDF of B (fY(b)):

fY(b) = ∫[b^2,1] 6xy dx (integrating with respect to x)

fY(b) = 6[b^3/3 - b^6/6] |[b^2,1]

fY(b) = 2(1 - b^3), for 0 ≤ b ≤ 1

fY(b) = 0, otherwise

b. To determine if A and B are independent, we can check if their joint PDF factors into the product of their marginal PDFs.

fxy(x, y) = 6xy

fX(a) = 3a

fY(b) = 2(1 - b^3)

fxy(x, y) ≠ fX(a) \* fY(b)

Since fxy(x, y) is not equal to the product of the marginal PDFs, A and B are not independent.

c. The conditional PDF of A given B = b, denoted as fA|B(a|b), can be found using the formula for conditional probability:

fA|B(a|b) = fAB(a, b) / fB(b)

Here, fAB(a, b) is the joint PDF of A and B, which is 6ab, and fB(b) is the marginal PDF of B, which is 2(1 - b^3).

So,

fA|B(a|b) = (6ab) / (2(1 - b^3))

fA|B(a|b) = 3ab / (1 - b^3), for 0 ≤ a ≤ 1, 0 ≤ b ≤ 1

d. To find E[A|B = b], we can calculate the conditional expectation by integrating A \* fA|B(a|b) over the range of A.

E[A|B = b] = ∫[0,1] a \* (3ab / (1 - b^3)) da

E[A|B = b] = 3b / (1 - b^3) \* ∫[0,1] a^2 da

E[A|B = b] = 3b / (1 - b^3) \* [a^3/3] |[0,1]

E[A|B = b] = 3b / (1 - b^3) \* (1/3)

E[A|B = b] = b / (1 - b^3), for 0 ≤ b ≤ 1

e. To find Var(A|B = b), we can use the conditional variance formula:

Var(A|B = b) = E[A^2|B = b] - [E[A|B = b]]^2

First, calculate E[A^2|B = b]:

E[A^2|B = b] = ∫[0,1] a^2 \* (3ab / (1 - b^3)) da

E[A^2|B = b] = 3b / (1 - b^3) \* ∫[0,1] a^3 da

E[A^2|B = b] = 3b / (1 - b^3) \* [a^4/4] |[0,1]

E[A^2|B = b] = 3b / (1 - b^3) \* (1/4)

E[A^2|B = b] = 3b / (4(1 - b^3))

Now, calculate Var(A|B = b):

Var(A|B = b) = E[A^2|B = b] - [E[A|B = b]]^2

Var(A|B = b) = [3b / (4(1 - b^3))] - [b / (1 - b^3)]^2

Var(A|B = b) = (3b - 4b^2) / (4(1 - b^3))

So, Var(A|B = b) is given by (3b - 4b^2) / (4(1 - b^3)), for 0 ≤ b ≤ 1.

## Q8

To find the probability that the total weight of the men on the ship exceeds 18,000, we can use the Central Limit Theorem since we have a large number of men (100) and their weights are independent and identically distributed.

Let X represent the weight of a single man, which follows a normal distribution with mean (μ) 170 and standard deviation (σ) 30. The total weight of the 100 men on the ship, which we'll denote as T, is the sum of these 100 independent random variables:

T = X₁ + X₂ + X₃ + ... + X₁₀₀

The mean of the total weight T is equal to the sum of the means of the individual weights:

E[T] = E[X₁ + X₂ + X₃ + ... + X₁₀₀] = E[X₁] + E[X₂] + E[X₃] + ... + E[X₁₀₀] = 100 \* μ = 100 \* 170 = 17,000

The standard deviation of the total weight T is equal to the square root of the sum of the variances of the individual weights:

σ[T] = sqrt(σ[X₁]² + σ[X₂]² + σ[X₃]² + ... + σ[X₁₀₀]²) = sqrt(100 \* σ²) = 10 \* σ = 10 \* 30 = 300

Now, we want to find the probability that the total weight exceeds 18,000:

P(T > 18,000) = 1 - P(T ≤ 18,000)

To find this probability, we need to standardize the random variable T using the mean and standard deviation we calculated:

Z = (T - E[T]) / σ[T] = (T - 17,000) / 300

Now, we can find the probability:

P(T > 18,000) = 1 - P(Z ≤ (18,000 - 17,000) / 300) = 1 - P(Z ≤ 1/3)

WE can use a standard normal distribution table or calculator to find P(Z ≤ 1/3). Using a standard normal distribution table, you can find that P(Z ≤ 1/3) is approximately 0.6293.

Therefore, the probability that the total weight of the men on the ship exceeds 18,000 is:

P(T > 18,000) ≈ 1 - 0.6293 ≈ 0.3707, or 37.07%.

## Q9

First, find the mean and variance of the random variable Xi from its Probability Mass Function (PMF). The mean of Xi is 0.6, and the variance is 0.36.

Since you have 25 independent and identically distributed random variables (X1, X2, …, X25), you can use the Central Limit Theorem (CLT). This theorem states that the sum of a large number of independent and identically distributed random variables approaches a normal distribution.

Calculate the mean of the sum Y = X1 + X2 + … + X25. The mean of Y is 15.

Calculate the variance of Y, which is 9.

Now, Y approximately follows a normal distribution with a mean of 15 and a variance of 9.

To estimate the probability P(4 ≤ Y ≤ 6), standardize the values using a new variable Z.

Calculate the probability P(-3.67 ≤ Z ≤ -3.00) by referring to a standard normal distribution table or calculator.

Subtract the probabilities to find P(4 ≤ Y ≤ 6).

The estimated probability P(4 ≤ Y ≤ 6) is approximately 0.0012, which is roughly 0.12%.