40.305 Advanced topics in Stochastic Modeling	Mar 25, 2020
Mini-project exercise 1	
	Due date: Apr 7, 2020

Instructions.

1) Computer simulations can be carried out, for example, in Matlab, R, Python or a similar platform.
2) This exercise can be done in groups of size not exceeding two.
3) Please submit i) a consice written report (about 2 pages) specifying the choice of regeneration times, the outline of simulation scheme and the procedure for constructing interval estimates; ii) the code; and iii) snapshots of your output simulation estimates.
4) The submission amounts to 10% of the final grade.

An inventory distribution model

We consider a model of an inventory distribution system for a single item that was actually in use: The inventory system fills orders for two types of customers: retail stores, ordering in large quantities; and direct customers, ordering at most a single unit at a time. Table I below shows the ordering statistics for each of the three retail stores.

TABLE I

RETAIL-STORE ORDERING STATISTICS

Store s	Lot size	Ordering probabilities $p_{s,j}$					
		$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	40	0	1/8	1/4	1/4	1/4	1/8
2	30	0	1/5	1/5	1/5	1/5	1/5
3	50	0	0	1/4	1/2	1/4	0

In addition to the lot size, the table shows p_{sj}, the probability that j weeks transpire between orders of store s. For example, if store 3 orders this week, the probability is 1/4 that it will order again in 3 weeks, 1/2 in 4 weeks, and 1/4 in 5 weeks. Given that it orders, the quantity ordered will be 50.

In addition to the retail stores, there are 160 small customers who order directly from the warehouse. Each of the customers has a probability 1/4 of ordering in any given week, regardless of the length of the time since the last order. The order quantity for each direct customer is one.

The simulation is to be performed for upto 1280 weeks. The simulation outputs to be observed are: q_{ns}, the number of units ordered by store s in the n^{th} week ($s = 1, 2, 3,$), and q_{n4}, the number of orders received from direct customers in the n^{th} week. Let $\mathbf{q}_n = (q_{n1}, q_{n2}, q_{n3}, q_{n4})$.

i) Identify the suitable renewal times where the process $(\mathbf{q}_n : n \geq 1)$ regenerates.

ii) Use regenerative simulation to construct point estimates and confidence intervals for the following *steady-state* expectations*:

**Hint:* Identify suitable functions f that allow you to compute these expectations. For example, for Part a), take $f_1(\mathbf{q}) = g(\mathbf{q}) + q_4$, where $g(\mathbf{q})$ is the number of components among q_1, q_2, q_3 that are positive; similary, for Part b), take $f_2(\mathbf{q}) = q_1 + q_2 + q_3 + q_4$. The steady-state expectation of interest is then simply $\lim_{n\to\infty} E[f_i(\mathbf{q}_n)]$.

a) the expected number of orders in a week in steady-state

b) the expected number of units ordered in a week in steady-state ordered in a week in steady-state

c) the probability that more than 75 units are ordered in a week in steady-state

d) the probability that no store orders in a week in steady-state

e) the steady-state expected value of the cost function of the warehouse operations, where the cost function is given by

$$c(\mathbf{q}) = 5 + 0.1(g(\mathbf{q}) + q_4) + 0.1(q_1 + q_2 + q_3 + q_4) + 100(q_1 + q_2 + q_3 + q_4)^{1/4}$$

for $\mathbf{q} = (q_1, q_2, q_3, q_4) \in \mathbb{R}^4$ denoting the number of inventory orders and $g(\mathbf{q})$ denoting the number of components among q_1, q_2, q_3 that are positive. The cost function may be interpreted as follows: a setup cost of 5 per week, a requisition cost of 0.1 per order received, a material handling cost of 0.1 per unit ordered, and a transportation cost per unit ordered of 100 (number of units)$^{-3/4}$. The transportation cost reflects economies of scale.