40.305 Advanced topics in Stochastic Modeling Apr 9, 2020

Mini-project exercise 2: Choice II (a theoretical exercise)

Due date: Apr 28, 2020

Note.

1) You are required to do only ONE of the three project choices provided. This pdf contains only a description of Project choice II which is theoretical in nature. Please see Choice I if you prefer a concretely defined simulation exercise of staffing call centers (or) Choice III if you prefer to do a more open-ended simulation exercise of your choice.
2) In line with the directives of the Government of Singapore and SUTD, you are NOT required to venture outdoors to collect data for any of the project choices. Our purpose in this mini-project exercise is to merely help you perfect the use of the theoretical concepts learnt (such as Little's law, PASTA, the inspection paradox, etc.) towards analysing models of considerable importance.
3) Relevant information about the scheduling schemes and the respective waiting time derivations can be found in the supplementary material attached.

Choice II: Scheduling in M/G/1 queues (a theoretical exercise)

Consider an M/G/1 single-server queueing model in which the service times of jobs are i.i.d. random variables with mean $1/\mu$ and the jobs arrive according to a Poisson process with rate λ. In the lectures, we examined how to use the concepts such as Little's law, PASTA, inspection paradox, etc. to derive expressions for steady-state wait time in M/G/1 queues in which the jobs are scheduled according to i) First-Come-First-Serve (FCFS) basis, and ii) non-preemptive priority. Here, the term "scheduling" stands for "determining the order in which jobs are served".

You may realize that there are numerous other ways in which the jobs can be scheduled. This theoretical exercise is aimed at making you familiar with deriving performance of systems with slightly different scheduling schemes than the ones we studied and obtain insights by comparing the resulting performances of different scheduling schemes.

a) If the jobs are scheduled according to *Shortest-Jobs First (SJF)* non-preemptively, show that the average time spent in the queue by a job of size x is given by,

$$E[T_Q(x)]^{SJF} = \frac{\rho E[S^2]}{2E[S]} \frac{1}{(1-\rho_x)^2},$$

where S is the random variable denoting the service time and ρ_x denotes the load composed of jobs of size 0 to x. We know that the wait time in FCFS queue is large if the coefficient of variation of S is high. Is the shortest-job-first effective in mitigating this effect of high coefficient of variation? When would you recommend using SJF over FCFS?

b) Consider the priority queue model considered in Lecture 6. Suppose that we allow the jobs to be preempted: that is, a job may be stopped partway through its execution and then resumed at a later point in time from the same point where it was stopped. Derive an expression for the mean time spent by a job (of any given priority) in the system in steady-state. Do you see any qualitative difference in the performance of the preemptive priority queue compared to the non-preemptive priority queue we examined in the class?

c) If the jobs are scheduled according to *Shortest-Jobs First* preemptively (PSJF), then show that the average time spent in the system by a job of size x is given by,

$$E[T_Q(x)]^{PSJF} = \frac{x}{1-\rho_x} + \frac{\lambda \int_0^x t^2 f(t) dt}{2} \frac{1}{(1-\rho_x)^2},$$

where $f(\cdot)$ is the probability density function of the service time S. Is this performance better than that of the non-preemptive case in Part a) above? What are your thoughts on when preemption is beneficial?

Some guidelines

1) This exercise is to be done in groups of size upto 2. If a group of 3 members prefers to do this exercise, it is recommended that such a group additionally considers the SRPT scheduling scheme in the supplementary material to make the workload proportional.
2) It is easier for groups doing this project to make a group presentation rather than submitting a report. Please email me to schedule a skype/zoom presentation at least a couple of days before the day you wish to make the presentation. There is no need to submit an additional report.
3) Please compare the performance of different scheduling schemes and try to develop insights on the circumstances under which one is preferable to the other.
4) As mentioned earlier, relevant information about the scheduling schemes and the respective wait time derivations can be found in the supplementary material attached. Please feel free to approach me with questions if you encounter difficulties.