Math 313: Lab - Euler Numerical Method

September 30, 2020

1 Problem 1

1.1 a

$$N = \frac{t_f - t_0}{h} = \frac{2-(-1)}{0.1} = 30 \tag{1}$$

1.2 b

$$N = \frac{t_f - t_0}{h} = \frac{2-(-1)}{0.05} = 60 \tag{2}$$

1.3 c

$$h = \frac{t_f - t_0}{N} = \frac{2-(-1)}{100} = 0.03 \tag{3}$$

1.4 d

From the graph we see that simulation i part (c) fits the analytic solution the best since it is simulated with the most steps.

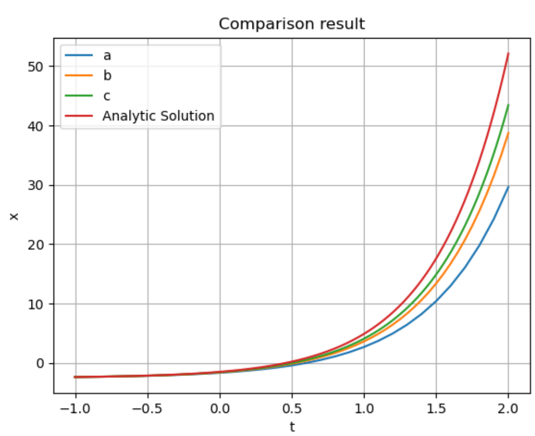

Figure 1: Comparison between Simulation and Analytic Solution

2 Problem 2

2.1 a,b

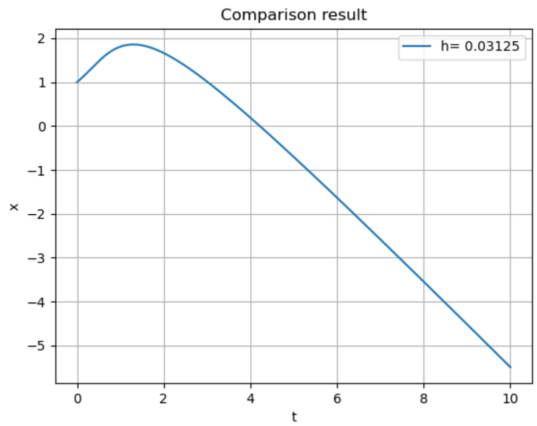

Figure 2: Convergence of h

2.2 c

- For $h = 0.5, N = \frac{10-0}{0.5} = 20$
- For $h = 0.25, N = 20 \times 2 = 40$
- For $h = 0.125, N = 40 \times 2 = 80$
- For $h = 0.00625, N = 80 \times 2 = 160$
- For $h = 0.003125, N = 160 \times 2 = 320$

3 Problem 3

3.1 a

By separation of variable,

$$\int xdx = \int cos(t)dt \tag{4}$$

$$\frac{1}{2}x^2 = sin(t) + C \tag{5}$$

$$\frac{1}{2}x_0^2 = C \tag{6}$$

$$x^2 = 2sin(t) + x_0 \tag{7}$$

The solution is real for all t if and only if $2sin(t) + x_0 \geq 0$, i.e., $x_0 \geq 2$ and

$$x(t) = \pm\sqrt{2sin(t) + x_0} \tag{8}$$

3.2 b,c,d,e

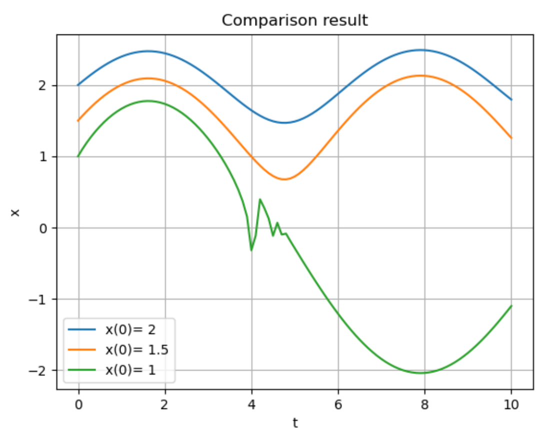

Figure 3: Simulation Result