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# [Connor Dupuis]

## Table of Contents

[Friday 1:55pm] - [28944] - [Naoki Sawahashi] .....	1
QUESTION 1 COMMENTING .....	1
QUESTION 2: Z-TRANSFORM .....	5
2 (a) FILTER 1 .....	5
2 (b) FILTER 2 .....	8
3 (a) ANSWER QUESTION .....	11
3 (b) ANSWER QUESTION .....	11
3 (c) ANSWER QUESTION .....	11
QUESTION 4: IIR FILTERS IN Z-DOMAIN .....	11
4 (a) APPLY NULLING FILTER .....	11
4 (b) NEW NULLING FILTER .....	12
4 (c) ANSWER QUESTION .....	13
4 (d) NULLING MULTIPLE FREQUENCIES .....	13
4 (e) PLAY SOUND .....	15
Create input audio file .....	15
Create output audio file .....	15
QUESTION 5: DESIGNING A BETTER NULLING FILTER .....	16
5 (a) ANSWER QUESTION .....	16
5 (b) PLOT POLE-ZERO PLOTS .....	16
5 (c) ANSWER QUESTION .....	17
5 (d) ANSWER QUESTION .....	17
5 (e) - APPLYING FILTER AND PLOTTING MAGNITUDE RESPONSES .....	17
Play sound and create audio file .....	18
ALL FUNCTIONS SUPPORTING THIS CODE %% .....	18

## [Friday 1:55pm] - [28944] - [Naoki Sawahashi]

### QUESTION 1 COMMENTING

DO NOT REMOVE THE LINE BELOW MAKE SURE 'eel3135\_lab08\_comment.m' IS IN THE SAME DIRECTORY AS THIS FILE

```
clc;type('eel3135_lab08_comment.m')
```

```
%% QUESTION #1 COMMENTING
```

```
clear
```

```
close all
```

```
clc
```

```
%% DEFINE FILTER AND INPUT
```

```
w = -pi:pi/8000:pi-pi/8000;
```

```
N = 100;
```

```
n = 0:(N-1);
```

```
% FILTER 1
```

```
a1 = [1 0 0 0 0 0 0 -0.9];
b1 = 1;
% <-- Answer: Is this an IIR filter or a FIR filter? Why?
% This is an IIR filter because it has poles that are non zero.
% <-- Answer: How many poles does this system have? How many zeros?
% This system has 7 poles

% FILTER 2
a2 = [1 -0.9];
b2 = 1;
% <-- Answer: Is this an IIR filter or a FIR filter? Why?
% This is an IIR filter because it has poles that are non zero.
% <-- Answer: How many poles does this system have? How many zeros?
% This system has 1 poles

% INPUT
x = zeros(N,1);
x(1) = 1;

%% DEFINE AND PLOT OUTPUT

% OUTPUT 1
y1 = filter(b1,a1,x);
y2 = filter(b2,a2,x);

% OUTPUT 2
H1 = DTFT(y1,w);
H2 = DTFT(y2,w);

% OUTPUT 3: CASCADE FILTERS
y3 = filter(b2, a2, filter(b1, a1, x));
H3 = DTFT(y3,w);
% <-- Express H3(z) as a function of H1(z) and H2(z)
%  $H3(z) = H1(z) * H2(z)$ 

% PLOT THE FREQUENCY REPONSE IMPULSE RESPONSE AND DTFT
figure(1)
subplot(3,1,1)
stem(n,y1)
xlabel('Time (Samples)')
ylabel('h[n]')
subplot(3,1,2)
plot(w,abs(H1))
title('Magnitude Response of h')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(3,1,3)
pzplot(b1,a1)
axis equal

% PLOT THE FREQUENCY REPONSE IMPULSE RESPONSE AND DTFT
figure(2)
subplot(3,1,1)
```

---

```

stem(n,y2)
xlabel('Time (Samples)')
ylabel('h[n]')
subplot(3,1,2)
plot(w,abs(H2))
title('Magnitude Response of h')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(3,1,3)
pzplot(b2,a2)
axis equal

% PLOT THE FREQUENCY REPOSE IMPULSE RESPONSE AND DTFT
figure(3)
subplot(2,1,1)
stem(n,y3)
xlabel('Time (Samples)')
ylabel('h[n]')
subplot(2,1,2)
plot(w,abs(H3))
title('Magnitude Response of h')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')

%% ALL FUNCTIONS SUPPORTING THIS CODE %%
% =====
% NOTE: YOU DO NOT NEED TO ADD COMMENTS IN THE CODE BELOW. WE JUST
% NEEDED POLE-ZERO PLOTTING CODE AND THUS WROTE IT.
% =====
function pzplot(b,a)
% PZPLOT(B,A) plots the pole-zero plot for the filter described by
% vectors A and B. The filter is a "Direct Form II Transposed"
% implementation of the standard difference equation:
%
% 
$$a(1)*y(n) = b(1)*x(n) + b(2)*x(n-1) + \dots + b(nb+1)*x(n-nb)$$

% 
$$- a(2)*y(n-1) - \dots - a(na+1)*y(n-na)$$

%
%
% MODIFY THE POLYNOMIALS TO FIND THE ROOTS
b = b(1:find(b,1,'last'));
a = a(1:find(a,1,'last'));
b1 = zeros(max(length(a),length(b)),1); % Need to add zeros to get
the right roots
a1 = zeros(max(length(a),length(b)),1); % Need to add zeros to get
the right roots
b1(1:length(b)) = b; % New a with all values
a1(1:length(a)) = a; % New a with all values

% FIND THE ROOTS OF EACH POLYNOMIAL AND PLOT THE LOCATIONS OF THE
ROOTS

```

---

```
h1 = plot(real(roots(a1)), imag(roots(a1)));
hold on;
h2 = plot(real(roots(b1)), imag(roots(b1)));
hold off;

% DRAW THE UNIT CIRCLE
circle(0,0,1)

% MAKE THE POLES AND ZEROS X's AND O's
set(h1, 'LineStyle', 'none', 'Marker', 'x',
'MarkerFaceColor','none', 'linewidth', 1.5, 'markersize', 8);
set(h2, 'LineStyle', 'none', 'Marker', 'o',
'MarkerFaceColor','none', 'linewidth', 1.5, 'markersize', 8);
axis equal;

% DRAW VERTICAL AND HORIZONTAL LINES
xminmax = xlim();
yminmax = ylim();
line([xminmax(1) xminmax(2)],[0 0], 'linestyle', ':', 'linewidth',
0.5, 'color', [1 1 1]*.1)
line([0 0],[yminmax(1) yminmax(2)], 'linestyle', ':', 'linewidth',
0.5, 'color', [1 1 1]*.1)

% ADD LABELS AND TITLE
xlabel('Real Part')
ylabel('Imaginary Part')
title('Pole-Zero Plot')

end

function circle(x,y,r)
% CIRCLE(X,Y,R) draws a circle with horizontal center X, vertical
center
% Y, and radius R.
%

% ANGLES TO DRAW
ang=0:0.01:2*pi;

% DEFINE LOCATIONS OF CIRCLE
xp=r*cos(ang);
yp=r*sin(ang);

% PLOT CIRCLE
hold on;
plot(x+xp,y+yp, ':', 'linewidth', 0.5, 'color', [1 1 1]*.1);
hold off;

end

function H = DTFT(x,w)
% DTFT(X,W) compute the Discrete-time Fourier Transform of signal X
% across frequencies defined by W.
```

```
H = zeros(length(w),1);
for nn = 1:length(x)
    H = H + x(nn).*exp(-1j*w.*(nn-1));
end

end
```

## QUESTION 2: Z-TRANSFORM

### 2 (a) FILTER 1

```
% DEFINE AXES
w = -pi:pi/8000:pi-pi/8000;
N = 100;

% INPUT
x = zeros(N,1);
x(1) = 1;

% FILTER 1
b1 = [1 -2 1];
a1 = 1;
y11 = filter(b1,a1,x);
H1 = DTFT(y11,w);

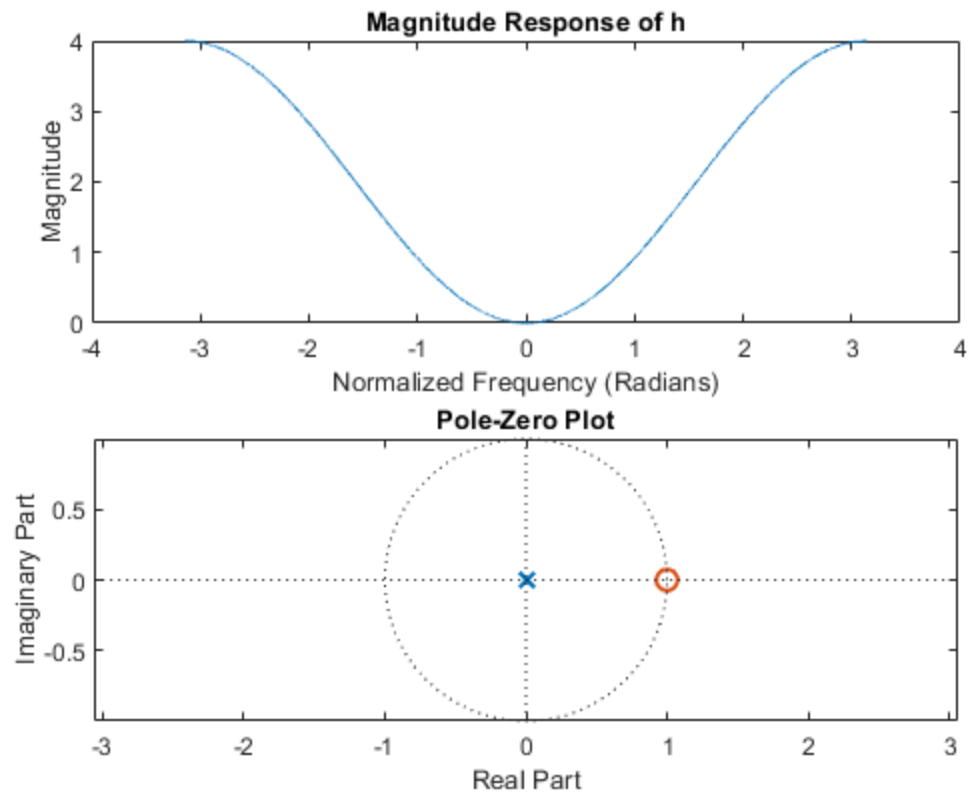
% PLOT THE FREQUENCY REPNSE IMPULSE RESPONSE AND DTFT
figure
subplot(2,1,1)
plot(w,abs(H1))
title('Magnitude Response of h')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(2,1,2)
pzplot(b1,a1)
axis equal

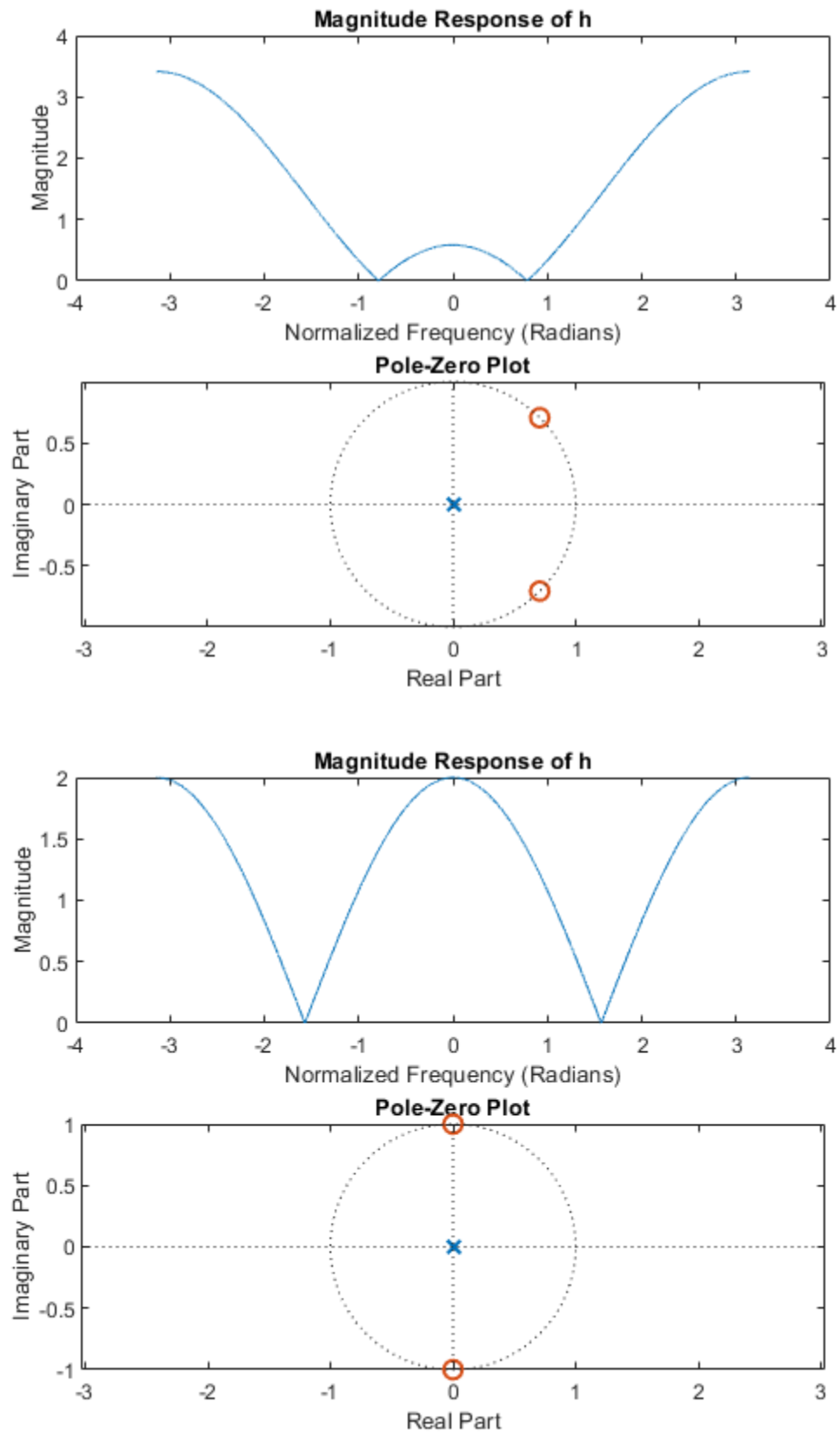
% FILTER 2
b1 = [1 -sqrt(2) 1];
a1 = 1;
y12 = filter(b1,a1,x);
H1 = DTFT(y12,w);

% PLOT THE FREQUENCY REPNSE IMPULSE RESPONSE AND DTFT
figure
subplot(2,1,1)
plot(w,abs(H1))
title('Magnitude Response of h')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(2,1,2)
pzplot(b1,a1)
axis equal
```

```
% FILTER 3
b1 = [1 0 1];
a1 = 1;
y13 = filter(b1,a1,x);
H1 = DTFT(y13,w);

% PLOT THE FREQUENCY REPNSE IMPULSE RESPONSE AND DTFT
figure
subplot(2,1,1)
plot(w,abs(H1))
title('Magnititude Response of h')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(2,1,2)
pzplot(b1,a1)
axis equal
```





## 2 (b) FILTER 2

```
% DEFINE AXES
w = -pi:pi/8000:pi-pi/8000;
N = 100;

% INPUT
x = zeros(N,1);
x(1) = 1;

% FILTER 1
a1 = [1 -4/3 4/9];
b1 = 1;
y21 = filter(b1,a1,x);
H1 = DTFT(y21,w);

% PLOT THE FREQUENCY REPONSE IMPULSE RESPONSE AND DTFT
figure
subplot(2,1,1)
plot(w,abs(H1))
title('Magnitude Response of h')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(2,1,2)
pzplot(b1,a1)
axis equal

% FILTER 2

% PLOT THE FREQUENCY REPONSE IMPULSE RESPONSE AND DTFT
a1 = [1 -2*sqrt(2)/3 4/9];
b1 = 1;
y22 = filter(b1,a1,x);
H1 = DTFT(y22,w);

% PLOT THE FREQUENCY REPONSE IMPULSE RESPONSE AND DTFT
figure
subplot(2,1,1)
plot(w,abs(H1))
title('Magnitude Response of h')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(2,1,2)
pzplot(b1,a1)
axis equal

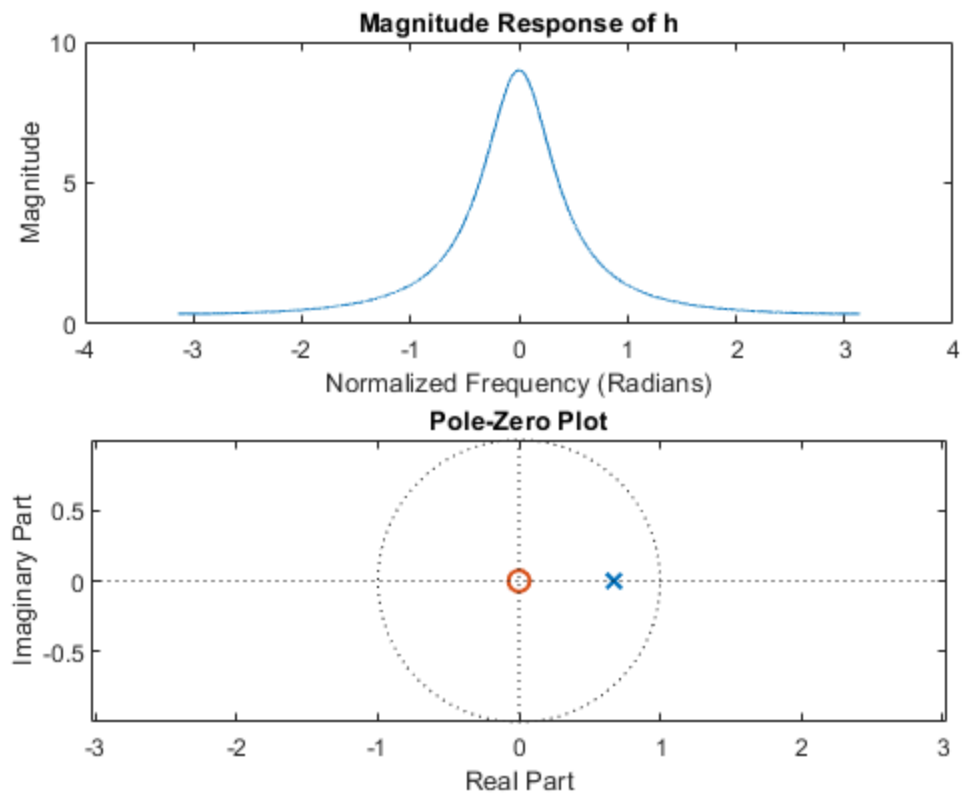
% FILTER 3
a1 = [1 0 4/9];
b1 = 1;
y23 = filter(b1,a1,x);
H1 = DTFT(y23,w);
```

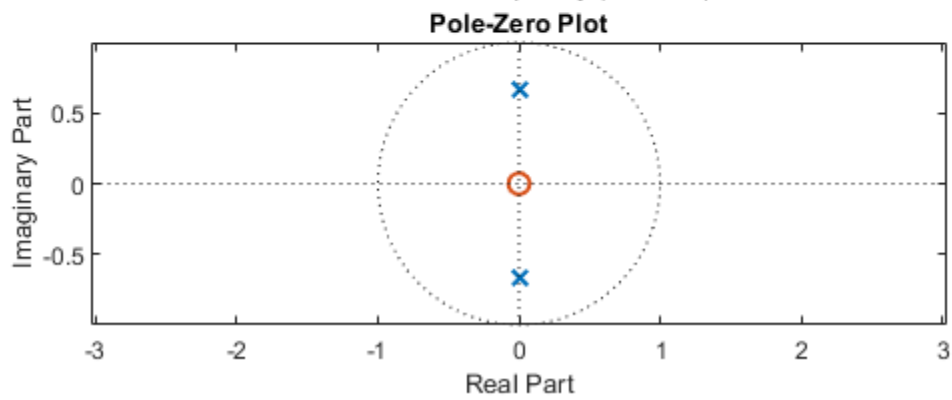
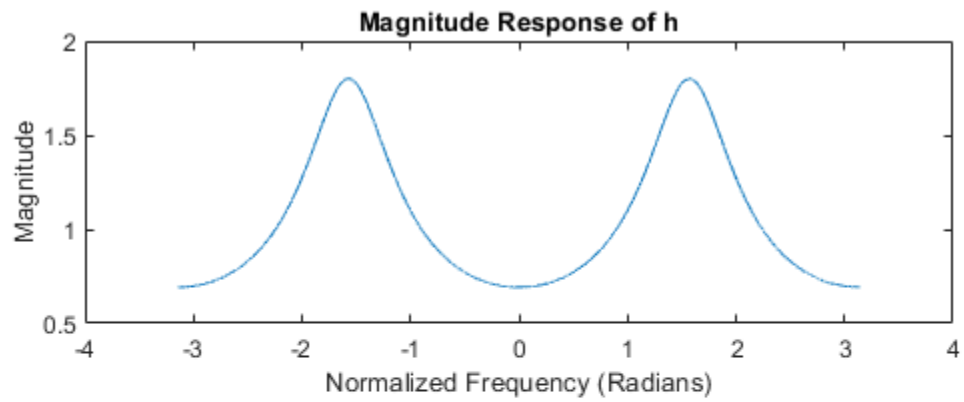
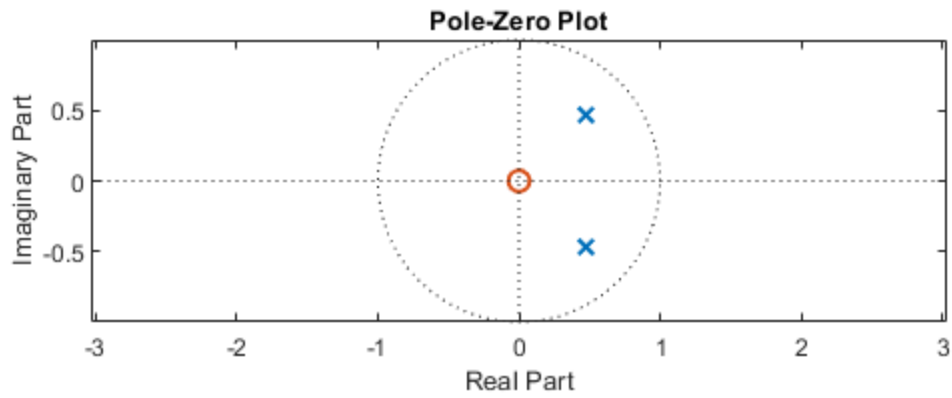
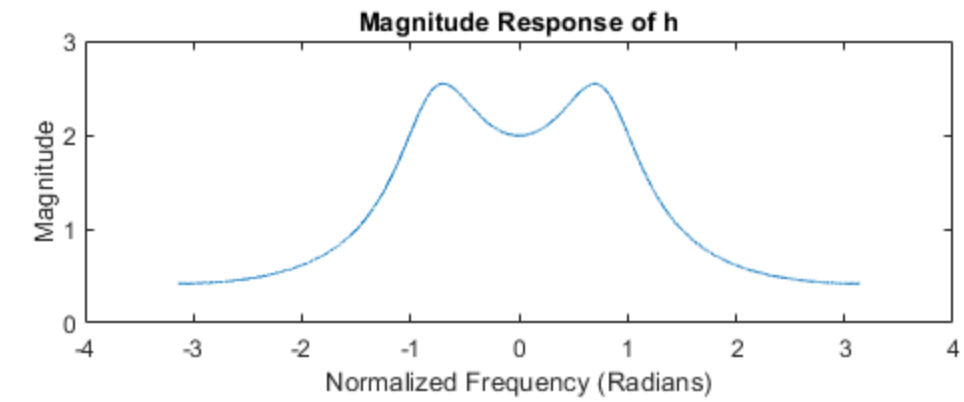


```

% PLOT THE FREQUENCY REPNSE IMPULSE RESPONSE AND DTFT
figure
subplot(2,1,1)
plot(w,abs(H1))
title('Magnitude Response of h')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(2,1,2)
pzplot(b1,a1)
axis equal

```





### 3 (a) ANSWER QUESTION

```
% Ha(Z) at w0 = 0 is a highpass filter
% Ha(Z) at w0 = pi/4 is a highpass filter
% Ha(Z) at w0 = pi/2 is a notch filter

% Hb(Z) at w0 = 0 is a lowpass filter
% Hb(Z) at w0 = pi/4 is a lowpass filter
% Hb(Z) at w0 = pi/2 is a bandpass filter
```

### 3 (b) ANSWER QUESTION

```
% Ha(Z) is a FIR filter
% Hb(Z) is a IIR filter
```

### 3 (c) ANSWER QUESTION

```
% Changing w0 affects where the poles/zeros are regarding the unit
circle
```

## QUESTION 4: IIR FILTERS IN Z-DOMAIN

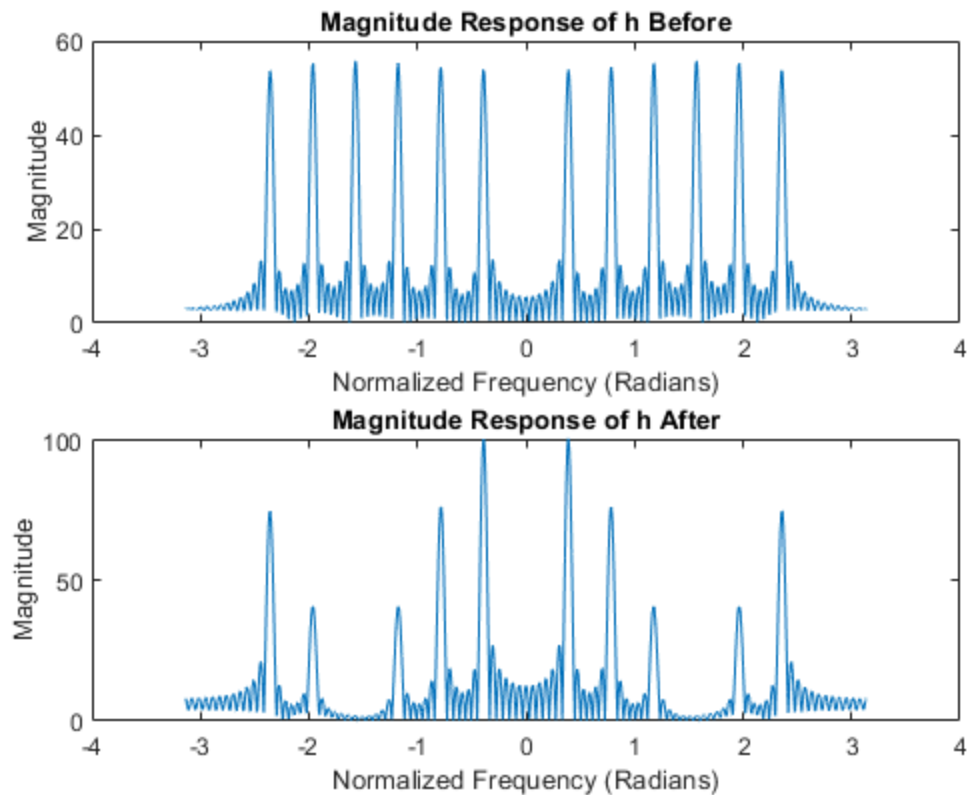
```
N = 100;
n = 0:N-1;
x = cos( pi/8*n ) + cos( 3*pi/8*n ) + cos( pi/4*n ) + cos( 5*pi/8*n )
    + cos( pi/2*n ) + cos( 3*pi/4*n );
```

### 4 (a) APPLY NULLING FILTER

```
% Ha(Z) pi/2
b1 = [1 0 1];
a1 = 1;

y4a = filter(b1,a1,x);
Hb = DTFT(x,w);
Ha = DTFT(y4a,w);

% PLOT THE FREQUENCY RESPONSE IMPULSE RESPONSE AND DTFT
figure
subplot(2,1,1)
plot(w,abs(Hb))
title('Magnitude Response of h Before')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(2,1,2)
plot(w,abs(Ha))
title('Magnitude Response of h After')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
```



## 4 (b) NEW NULLING FILTER

```
% Ha(Z) pi/2
b1 = [1 0 1];
a1 = 1;

% Hb(Z) 3pi/8
a2 = [1 -0.510245 4/9];
b2 = 1;

% Hb(Z) 5pi/8
a3 = [1 0.510245 4/9];
b3 = 1;

% Convoluting the three filters with x
y4b = filter(b1, a1, filter(b2, a2, filter(b3, a3, x)));

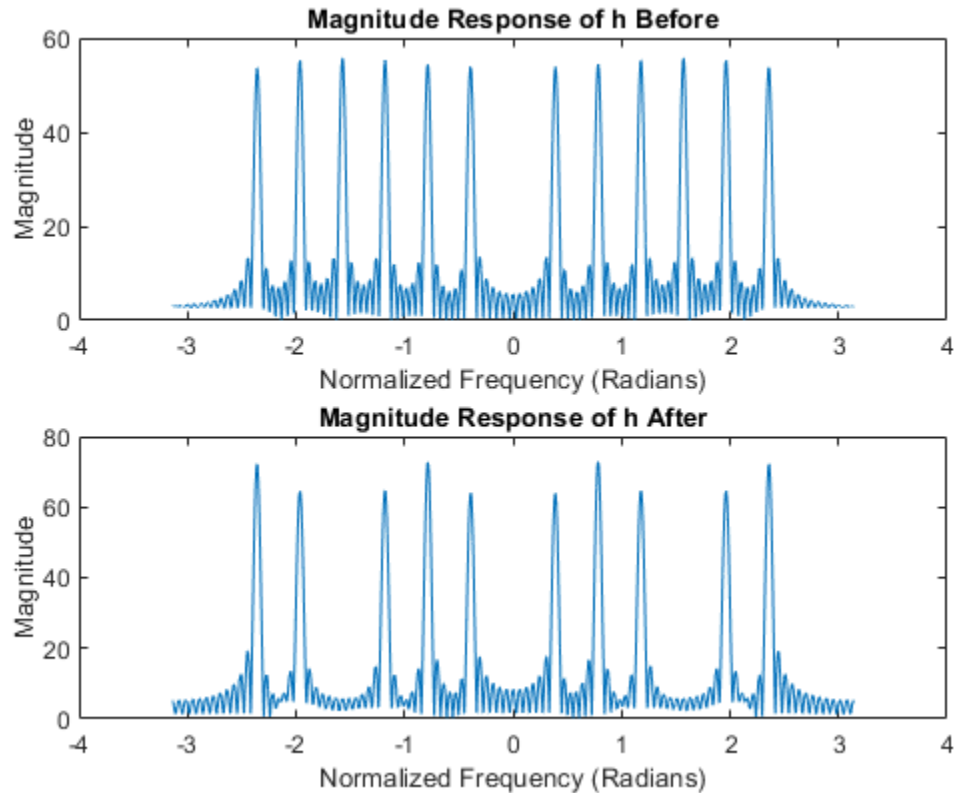
Hb = DTFT(x,w);
Ha = DTFT(y4b,w);

% PLOT THE FREQUENCY REPONSE IMPULSE RESPONSE AND DTFT
figure
subplot(2,1,1)
plot(w,abs(Hb))
title('Magnitude Response of h Before')
```

```

xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(2,1,2)
plot(w,abs(Ha))
title('Magnitude Response of h After')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')

```



## 4 (c) ANSWER QUESTION

```

% It improves on the filter in (a) by bringing up the frequencies
% around
%  $\pi/2$ . The first filter eliminated  $\pi/2$  but also the frequencies
% around
% it, this filter is moderately better at just eliminating  $\pi/2$ .

```

## 4 (d) NULLING MULTIPLE FREQUENCIES

```

%  $H_a(Z)$   $\pi/8$ 
b1 = [1 -1.84776 1];
a1 = 1;

%  $H_a(Z)$   $3\pi/8$ 
b2 = [1 -0.765367 1];
a2 = 1;

```

```
% Ha(Z) pi/4
b3 = [1 -sqrt(2) 1];
a3 = 1;

% Ha(Z) 5pi/8
b4 = [1 0.765367 1];
a4 = 1;

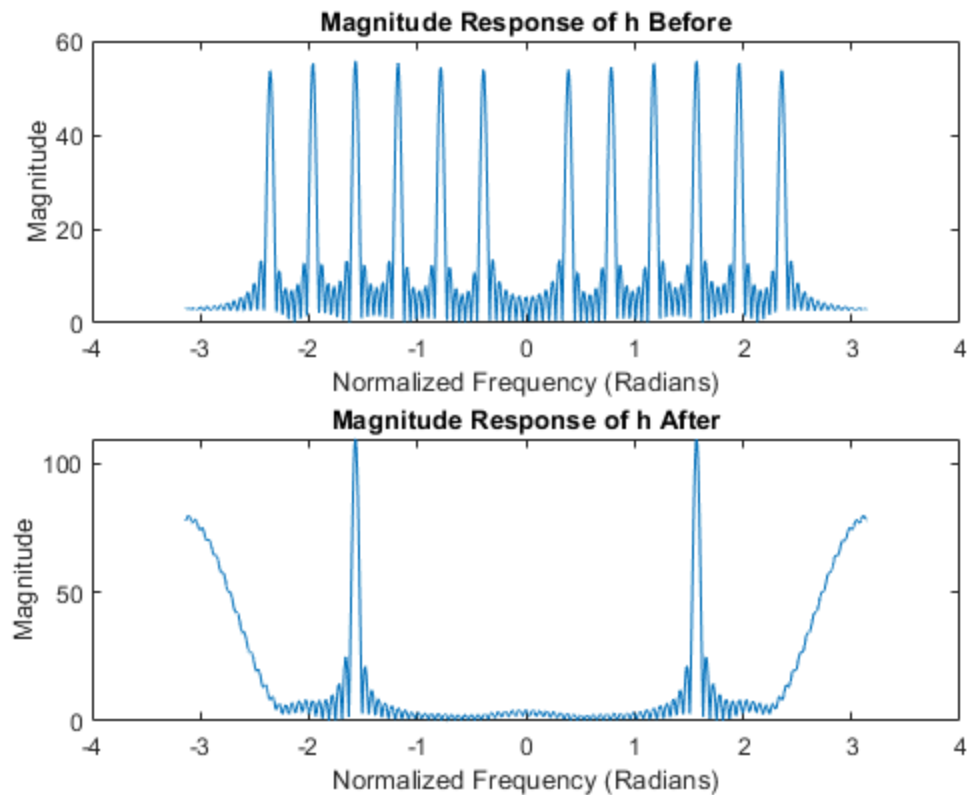
% Ha(Z) 3pi/4
b5 = [1 sqrt(2) 1];
a5 = 1;

% Ha(Z) pi (Not used)
% b6 = [1 2 1];
% a6 = 1;

% Convoluting the three filters with x
y4d = filter(b1, a1, filter(b2, a2, filter(b3, a3, filter(b4, a4,
    filter(b5, a5, x)))));

Hb = DTFT(x,w);
Ha = DTFT(y4d,w);

% PLOT THE FREQUENCY REPOSE IMPULSE RESPONSE AND DTFT
figure
subplot(2,1,1)
plot(w,abs(Hb))
title('Magnitude Response of h Before')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
subplot(2,1,2)
plot(w,abs(Ha))
title('Magnitude Response of h After')
xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')
```



## 4 (e) PLAY SOUND

```
N = 1000;
n = 0:N-1;
x = [cos( pi/8*n ) cos( 3*pi/8*n ) cos( pi/4*n ) cos( 5*pi/8*n )
     cos( pi/2*n ) cos( 3*pi/4*n )];
```

## Create input audio file

```
soundsc(x,2000)
audiowrite('input_sound.wav', x, 2000)
```

## Create output audio file

```
y = filter(b1, a1, filter(b2, a2, filter(b3, a3, filter(b4, a4,
    filter(b5, a5, x))));
y_out = y/max(abs(y));

soundsc(y_out,2000)
audiowrite('output_sound.wav', y_out, 2000)
```

## QUESTION 5: DESIGNING A BETTER NULLING FILTER

```
[x,fs] = audioread('Noisy.wav');  
w = -pi:pi/5000:pi;  
X = DTFT(x, w);
```

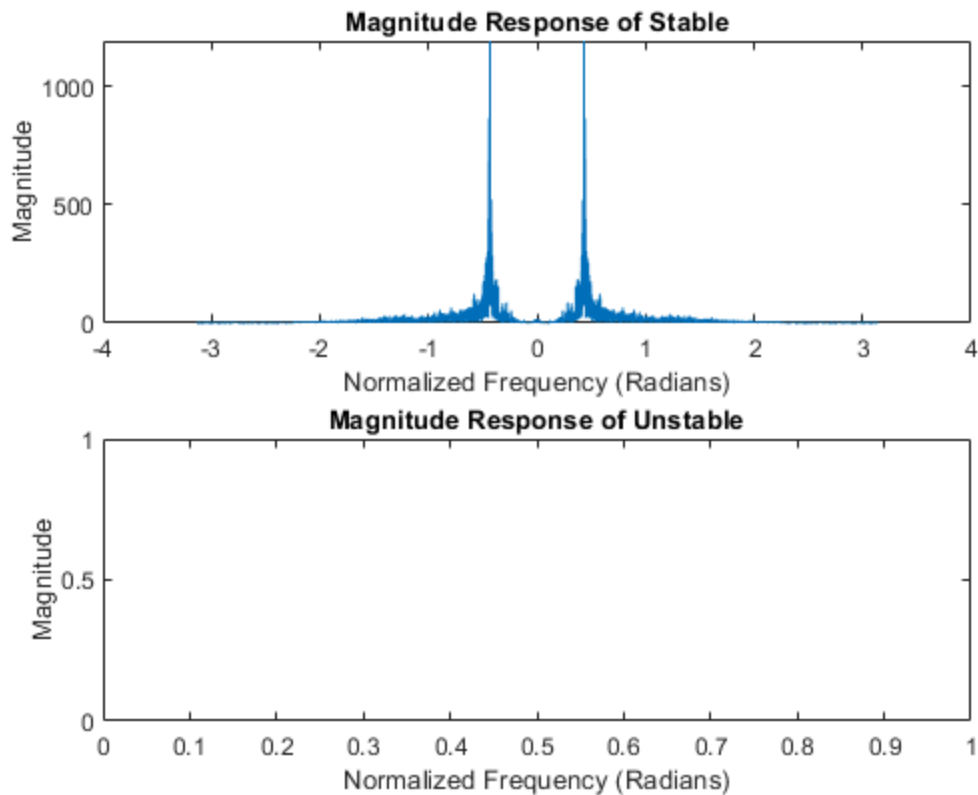
### 5 (a) ANSWER QUESTION

```
% This filter would be stable with alpha < 1
```

### 5 (b) PLOT POLE-ZERO PLOTS

```
%w0 = 0.08922;  
  
% H(Z) Stable alpha = 0.90  
b1 = [1 -1.99205 1];  
a1 = [1 -1.79284 .9801];  
  
y1 = filter(b1,a1,x);  
Hstable = DTFT(y1,w);  
  
% H(Z) Unstable aplha = 1.1  
b2 = [1 -1.99205 1];  
a2 = [1 -2.19125 1.21];  
  
y2 = filter(b2,a2,x);  
Hunstable = DTFT(y2,w);  
  
figure  
subplot(2,1,1)  
plot(w,abs(Hstable))  
title('Magnitude Response of Stable')  
xlabel('Normalized Frequency (Radians)')  
ylabel('Magnitude')  
subplot(2,1,2)  
plot(w,abs(Hunstable))  
title('Magnitude Response of Unstable')  
xlabel('Normalized Frequency (Radians)')  
ylabel('Magnitude')  
  
%Hunstable = DTFT(y,w);
```





## 5 (c) ANSWER QUESTION

% The larger the alpha value the sharper the notch

## 5 (d) ANSWER QUESTION

```
% The difference equation is  $y[n] - 1.97212y[n-1] + 0.9801y[n-2] =$ 
 $x[n] -$ 
 $1.99205x[n-1] + x[n-2]$ 
%  $y[n] = 1.97212y[n-1] - 0.9801y[n-2] + x[n] - 1.99205x[n-1] +$ 
 $x[n-2]$ 
```

## 5 (e) - APPLYING FILTER AND PLOTTING MAGNITUDE RESPONSES

```
b1 = [1 -1.99205 1];
a1 = [1 -1.97212 .9801];
y_unscaled = filter(b1, a1, x);
Hfiltered = DTFT(y_unscaled,w);

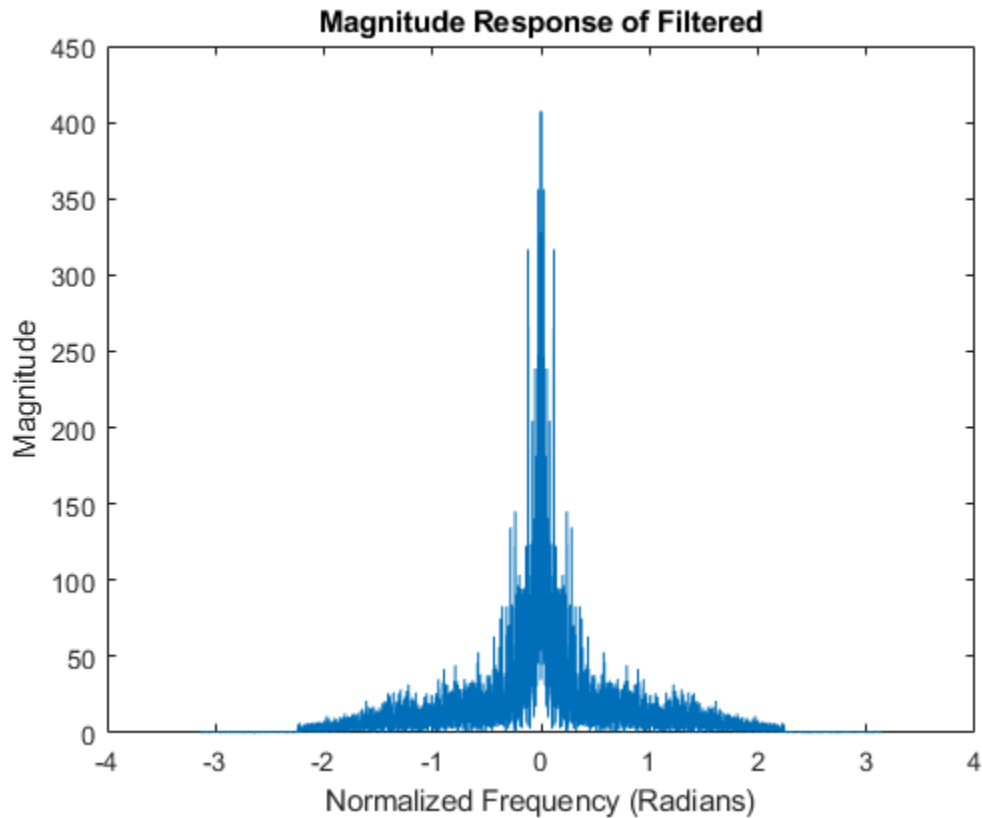
figure
plot(w,abs(Hfiltered))
title('Magnitude Response of Filtered')
```

```

xlabel('Normalized Frequency (Radians)')
ylabel('Magnitude')

y = y_unscaled/max(abs(y_unscaled));

```



## Play sound and create audio file

If most/a majority of the noise was filtered out, then you created the filter correctly

```

soundsc(y, fs);
audiowrite('Filtered_song.wav', y/max(abs(y)), fs);

```

## ALL FUNCTIONS SUPPORTING THIS CODE %

```

===== NOTE:
YOU DO NOT NEED TO ADD COMMENTS IN THE CODE BELOW.
WE JUST NEEDED POLE-ZERO PLOTTING CODE AND THUS WROTE IT.
=====

```

```

function pzplot(b,a)
% PZPLOT(B,A) plots the pole-zero plot for the filter described by
% vectors A and B. The filter is a "Direct Form II Transposed"
% implementation of the standard difference equation:

```

---

```

%
%   a(1)*y(n) = b(1)*x(n) + b(2)*x(n-1) + ... + b(nb+1)*x(n-nb)
%               - a(2)*y(n-1) - ... - a(na+1)*y(n-na)
%

% MODIFY THE POLYNOMIALS TO FIND THE ROOTS
b1 = zeros(max(length(a),length(b)),1); % Need to add zeros to get
the right roots
a1 = zeros(max(length(a),length(b)),1); % Need to add zeros to get
the right roots
b1(1:length(b)) = b;    % New a with all values
a1(1:length(a)) = a;    % New a with all values

% FIND THE ROOTS OF EACH POLYNOMIAL AND PLOT THE LOCATIONS OF THE
ROOTS
h1 = plot(real(roots(a1)), imag(roots(a1)));
hold on;
h2 = plot(real(roots(b1)), imag(roots(b1)));
hold off;

% DRAW THE UNIT CIRCLE
circle(0,0,1)

% MAKE THE POLES AND ZEROS X's AND O's

set(h1, 'LineStyle', 'none', 'Marker', 'x', 'MarkerFaceColor','none', 'linewidth'
1.5, 'markersize', 8);

set(h2, 'LineStyle', 'none', 'Marker', 'o', 'MarkerFaceColor','none', 'linewidth'
1.5, 'markersize', 8);
axis equal;

% DRAW VERTICAL AND HORIZONTAL LINES
xminmax = xlim();
yminmax = ylim();
line([xminmax(1) xminmax(2)],[0 0], 'linestyle', ':', 'linewidth',
0.5, 'color', [1 1 1]*.1)
line([0 0],[yminmax(1) yminmax(2)], 'linestyle', ':', 'linewidth',
0.5, 'color', [1 1 1]*.1)

% ADD LABELS AND TITLE
xlabel('Real Part')
ylabel('Imaginary Part')
title('Pole-Zero Plot')

end

function circle(x,y,r)
% CIRCLE(X,Y,R)  draws a circle with horizontal center X, vertical
center
% Y, and radius R.
%
% ANGLES TO DRAW

```

---

```
ang=0:0.01:2*pi;

% DEFINE LOCATIONS OF CIRCLE
xp=r*cos(ang);
yp=r*sin(ang);

% PLOT CIRCLE
hold on;
plot(x+xp,y+yp, ':', 'linewidth', 0.5, 'color', [1 1 1]*.1);
hold off;

end

function H = DTFT(x,w)
% DTFT(X,W) compute the Discrete-time Fourier Transform of signal X
% across frequencies defined by W.

H = zeros(length(w),1);
for nn = 1:length(x)
    H = H + x(nn).*exp(-1j*w.*(nn-1));
end

end
```

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