Full Name: ______________________________

EEL 3135 (Spring 2021) – Lab #07 **Due: Mar. 16 - Mar. 22 (On Lab Day)**

Question #1: *(Difference Equations and Pole-Zeros Plots)*

Download `EEL3135_lab07_comment.m` from Canvas, replace each of the corresponding comments with the corresponding descriptions. This is designed to show you how to visualize the output impulse response, filter output, pole-zero response for a given transfer function (in the Z-domain).

Note: You should run the code to help you understand how it works and help you write your comments. You will use elements of this MATLAB code for the rest of the lab assignment.

Question #2: *(Z-Transform)*

For the following Z-transforms, plot the corresponding impulse response and the pole-zero plot. Use Lab Question #1 as a guide.

(a) $H(z) = 1 - (0.2)z^{-1}$

(b) $H(z) = 1 - 1.5z^{-1}$

(c) $H(z) = 1 - 2z^{-1} + 0.5z^{-2}$

(d) $H(z) = (1 - e^{j3\pi/4}z^{-1})(1 - e^{-j3\pi/4}z^{-1})$

(e) $H(z) = \dfrac{1}{1 - (0.8)z^{-1}}$

(f) $H(z) = \dfrac{1}{1 - (1.4)z^{-1}}$

(g) $H(z) = \dfrac{1}{(1 - e^{j3\pi/4}z^{-1})(1 - e^{-j3\pi/4}z^{-1})}$

(h) $H(z) = \dfrac{1}{(1 - (0.8)e^{j\pi/4}z^{-1})(1 - (0.8)e^{-j\pi/4}z^{-1})}$

Question #3: *(More Z-Transform)*

Answer in your comments: Based on the previous results, answer the following questions.

(a) For what pole-zero conditions is the impulse response unstable (i.e., goes to ∞ as $n \to \infty$)?

(b) For what pole-zero conditions is the impulse response stable (i.e., goes to zero as $n \to \infty$)?

(c) For what pole-zero conditions is the impulse response critically stable (i.e., steady amplitude as $n \to \infty$)?

(d) For what pole-zero conditions is the impulse response finite in length?

(e) For what pole-zero conditions is the impulse response infinite in length?

(f) For what pole-zero conditions is the impulse response periodic (with a frequency > 0)?

Question #4: *(Loan Difference Equations)*

In this problem, we will study a simplified difference equation interest model. Hence, consider the following difference equation model for your student loan:

$$y[n] = (1+\alpha)y[n-1] - \beta y[n-R] - \gamma y[n-Q] + x[n]$$

where α is the yearly loan interest rate, β is the percentage of the loan you pay per year after graduation, R is the number of years till graduation, γ is the percentage of the loan you pay after you get rich from your awesome invention, and Q is the number of years until you create your invention. The output $y[n]$ is the loan amount. The time parameter n is in years. When positive, the input $x[n]$ is the initial loan amount.

(a) Assume you never pay your loan ($\beta = 0$, $\gamma = 0$ and R, Q does not matter). Assume we start with a loan \$120,000 at time $n = 0$, i.e., $x[n] = 120000\,\delta[n]$. Let $\alpha = 0.09$. Plot y[n] for 50 years, $0 \leq n \leq 50$, and the pole-zero plot for the system. Use Lab Question #1 as a guide.

(b) **Answer in your comments:** For the preceding parameters, how many years will it be before you owe 1 million dollars?

(c) Now assume you begin to pay $\beta = 0.07$ after $R = 6$ years (that is, you pay 7% of the loan each year). Plot $y[n]$ for 50 years, $0 \leq n \leq 50$, and the pole-zero plot for the system.

(d) **Answer in your comments:** For the preceding parameters, does the loan amount go zero, level-out, or go toward infinity? If the first, when is the loan paid (i.e., $y[n] < 1$)? If the middle, what is the level value? If the last, when will you owe 1 million dollars?

(e) Now assume you begin to pay $\beta = 0.07$ after $R = 6$ years and an additional $\gamma = 0.08$ (that is, you pay 15% of the loan each year) after $Q = 12$ years. Plot $y[n]$ for 50 years, $0 \leq n \leq 50$, and the pole-zero plot for the system.

(f) **Answer in your comments:** For the preceding parameters, does the loan amount go zero, level-out, or go toward infinity? If the first, when is the loan paid (i.e., $y[n] < 1$)? If the middle, what is the level value? If the last, when will you owe 1 million dollars?