Full Name: ______________________________

EEL 3135 (Spring 2021) – Lab #08 **Due: 11:59 PM ET, Mar. 24 - Mar. 29 (On Lab Day)**

Question #1: *(Z-transform, Filtering, and Magnitude Response)*

Download `EEL3135_lab08_comment.m` from Canvas, replace each of the corresponding comments with the corresponding descriptions. This is designed to show you how to visualize the IIR filters in the z-domain in MATLAB.

Note: You should run the code to help you understand how it works and help you write your comments. You will use elements of this MATLAB code for the rest of the lab assignment.

Question #2: *(Design Nulling Filters in Z-Domain)*

In Lab 6, you used the nulling filters to remove noise. In this question, you will study how to design the nulling filters in z-domain. For the following transfer functions, plot the magnitude repsonses and pole-zero plots for $\omega_0 = 0, \pi/4, \text{and } \pi/2$.

(a) $H_a(z) = (1 - e^{j\omega_0}z^{-1})(1 - e^{-j\omega_0}z^{-1})$

(b) $H_b(z) = \frac{1}{(1-(2/3)e^{j\omega_0}z^{-1})(1-(2/3)e^{-j\omega_0}z^{-1})}$

Question #3: *(More Z-transform, Filtering, and Magnitude Response)*

Answer in your comments: Based on the previous results, answer the following questions.

(a) What kind of filters are $H_a(z)$ and $H_b(z)$? Are they a low-pass filter, high-pass filter, band-pass filter, band-stop (or notch) filter, all-pass filter, or none-of-the-above?

(b) State whether each filter is an FIR filter or an IIR filter.

(c) How does changing ω_0 affect the poles and zeros?

Question #4: *(IIR Filters in Z-Domain)*

Cascading simple filters, for example,

$$H(z) = H_0(z)H_1(z)\cdots H_{M-1}(z)$$

or equivalently in time

$$x[n] = x_0[n] * x_1[n] * \ldots * x_{M-1}[n]$$

is a powerful tool because each simple filter dictates the location of each pole and zero. In this problem, we will use this concept to design complex IIR filters. Consider the signal defined by

```
N = 100; n = 0:(N-1);
x = cos( (pi/8)*n ) + cos( (3*pi/8)*n ) + cos( (pi/4)*n ) + ...
cos( (5*pi/8)*n ) + cos( (pi/2)*n ) + cos( (3*pi/4)*n );
```

(a) Use filter $H_a(z)$ from Question #2 to remove the $\pi/2$ frequency from $x[n]$. Plot the magnitude of the DTFT of $x[n]$ before and after applying the nulling filter.

(b) Cascade the filter from (a) with two $H_b(z)$ filters from Question #2: one with $\omega_0 = 3\pi/8$ and one with $\omega_0 = 5\pi/8$. Cascade the three filters and apply to $x[n]$. Use Question #1 as a guide. Plot the magnitude of the DTFT of $x[n]$ before and after applying the nulling filter.

(c) **Answer in your comments:** Does the filter magnitude response improve compared with (a)? Why / how?

(d) Now use cascading to remove all but one frequency from $x[n]$. That is, keep $\widehat{\omega} = \pi/2$. Plot the magnitude of the DTFT of $x[n]$ before and after applying the new filter.

(e) Change your input to

```
N = 1000; n = 0:N-1;
x = [cos( (pi/8)*n ) cos( (3*pi/8)*n ) cos( (pi/4)*n ) ...
 cos( (5*pi/8)*n ) cos( (pi/2)*n ) cos( (3*pi/4)*n )];
```

With your filter from (d), use `soundsc` to play $x[n]$ and $y[n]$ with a sampling rate of `fs = 2000`. **Turn in a .wav file of this audio output along with your PDF.**

Question #5: *(Designing a better Nulling Filter)*

Consider the following transfer function:

$$H(z) = \frac{(1 - e^{j\widehat{\omega}_0} z^{-1})(1 - e^{-j\widehat{\omega}_0} z^{-1})}{(1 - \alpha e^{j\widehat{\omega}_0} z^{-1})(1 - \alpha e^{-j\widehat{\omega}_0} z^{-1})},$$

This filter is called a **notch filter**, where $\widehat{\omega}_0$ represents the normalized radian frequency of the notch and α to be a positive real number that determines the "sharpness" of the filter.

(a) **Answer in your comments:** State the range for α for which the filter above is stable.

(b) Plot two pole-zero plots for the $H(z)$. Choose $\widehat{\omega}_0$ to be the frequency that you determined in Lab 6, question 4. For the first plot, choose a value of α that makes the filter stable, and for the second, choose a value of α that makes the filter unstable.

(c) **Answer in your comments:** Deduce a general rule to describe the "sharpness" of the notch as a function of α.

(d) Determine the difference equation of the filter using the transfer function above.

(e) Using this difference equation, create feed-forward vector `b` and feed-foward vector `a` by choosing `alpha=0.99` and `w0` to be the normalized radian frequency that you chose in (b). Apply this filter to `Noisy.wav`, and plot the magnitude response of the filtered signal. **Submit this .wav file along with your PDF.**