Full Name: ______________________________

EEL 3135 (Spring 2021) – Lab #09 Due: 11:59 PM EST, Mar. 30 - Apr. 5 (On Lab Day)

Question #1: *(Comment Code)*

There is no comment code this week! Note that there are useful functions in the skeleton code: `pzplot`, `DTFT`, and `shift`.

Question #2: *(IIR Filters in Difference Equations)*

In this question, we will implement IIR filters in the time domain without the `filter` command. Note that we assume initial rest (i.e., for every $n < 0$, $y[n] = 0$) for all parts of this question.

(a) Consider the transfer function:

$$H_1(z) = 1 - 0.6z^{-1}$$

Answer in your comments: Express this as a difference equation.

(b) Use a `for`-loop (i.e., **do not** use the `filter` function) to apply the derived difference equation onto the input $x_1[n]$ provided as `x1` in the skeleton code (**Hint:** we have done similar in previous labs). Use `stem` to plot the input $x_1[n]$ and output $y_1[n]$ in subplots. Make sure you label your plots appropriately to receive full credit.

(c) Consider another transfer function:

$$H_2(z) = \frac{1}{1 + 0.6z^{-1}}$$

Answer in your comments: Express this as a difference equation.

(d) Use a `for`-loop (i.e., **do not** use the `filter` function) to apply the derived difference equation onto the input $x_2[n]$ provided as `x2` in the skeleton code. Use `stem` to plot the input $x_2[n]$ and output $y_2[n]$ in subplots. Label your plots appropriately to receive full credit.

(e) Now consider the transfer function:

$$H_3(z) = \frac{(1 - 0.3e^{j\pi/2}z^{-1})(1 - 0.3e^{-j\pi/2}z^{-1})}{(1 - 0.5e^{j\pi/3}z^{-1})(1 - 0.5e^{-j\pi/3}z^{-1})}$$

Answer in your comments: Express this as a difference equation with all real coefficients (i.e., no imaginary numbers).

(f) Use a `for`-loop (i.e., **do not** use the `filter` function) to apply the derived difference equation onto the input $x_3[n]$ provided as `x3` in the skeleton code. Use `stem` to plot the input $x_3[n]$ and output $y_3[n]$ in subplots. Label your plots appropriately to receive full credit.

Question #3: *(High-Order IIR Filters)* In general, the more poles and/or zeros we use, the better we can build our filter. "Better" often means:

- sharper cut-offs (between the retained and removed frequencies)
- flatter passbands (flatter magnitudes for the retained frequencies)
- deeper stopbands (lower magnitudes for the removed frequencies)

In this problem, we will examine these properties/improvements with a well-known *Butterworth* filter.

(a) Consider the following input signal $x[n]$ (this is known as a chirp signal):

```
x = cos((pi/30000) * n.^2)
```

Let $0 \le n \le 9999$ and a sampling rate be `fs=4000` (for the rest of the problem). Listen to this input signal using `soundsc`. **Answer in your comments:** Describe the signal (**Hint:** Pay attention to changes in the frequency).

(b) Plot the magnitude of the frequency response of the chirp signal $|X(e^{j\widehat{\omega}})|$.

(c) Consider the simple low-pass filter defined by

$$H_1(z) = \frac{1}{4}\left(1 + z^{-1} + z^{-2} + z^{-3}\right)$$

Use the `filter` function to compute the output of this system $y_1[n]$ with the input $x[n]$. Plot the pole-zero plot of the system and magnitude of output in the frequency domain $|Y_1(e^{j\widehat{\omega}})|$.

(d) Listen to this output signal using `soundsc`. **Answer in your comments:** Describe the output and explain how the system alters what it sounds like based on your previous plots.

(e) Find the transfer function coefficients for a 3rd-order Butterworth filter with a cut-off frequency of $\pi/4$ by using the MATLAB command:

```
[b, a] = butter(N, Wn)
```

Use `help butter` to understand how this function operates. Use the `filter` function to compute the output of this system $y_2[n]$ with the chirp input $x[n]$. Plot the pole-zero plot of this system and magnitude of output in the frequency domain $|Y_2(e^{j\widehat{\omega}})|$.

(f) Listen to this output signal using `soundsc`. **Answer in your comments:** Describe how and why it sounds different from the previous filter.

(g) Find the transfer function coefficients for a 25th-order Butterworth filter. Use the `filter` function to compute the output of this system $y_3[n]$ with the chirp input $x[n]$. Plot the pole-zero plot of this system and magnitude of output in the frequency domain $|Y_3(e^{j\widehat{\omega}})|$.

(h) Listen to this output signal using `soundsc`. **Answer in your comments:** Describe how and why it sounds different from the previous filters.

(i) **Answer in your comments:** Which filter does provide the most desirable effects on the input signal (i.e., which filter is the "best")? Discuss the possible cons of higher-order filters in the real-world applications.