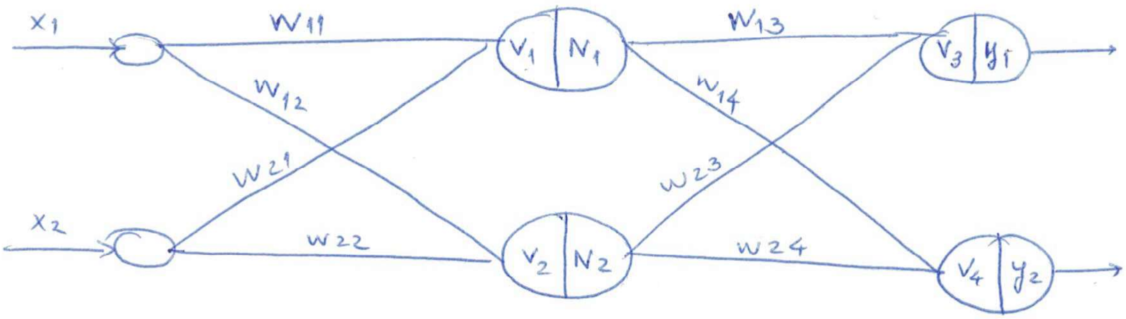
Problem 2



$w_{11} = 0.1$

$w_{12} = 0.25$

$w_{21} = 0.1$

$w_{22} = 0.7$

$w_{13} = 0.4$

$w_{14} = 0.5$

$w_{23} = 0.6$

$w_{24} = 0.3$

Activation function: $\phi(x) = \dfrac{1}{1 + e^{-x}}$

$$\phi'(x) = \phi(x) \cdot (1 - \phi(x))$$

Input / Target point: $x = [1, 1]^T$

$d = [1, 0]^T$

1st Forward-Pass:

$$V_1 = w_{11} \cdot x_1 + w_{21} \cdot x_2 = 0.1 \times 1 + 0.1 \times 1 = 0.2$$

$$N_1 = \phi(V_1) \cong 0.55$$

$$V_2 = w_{12} x_1 + w_{22} \cdot x_2 = 0.25 \times 1 + 0.7 \times 1 = 0.95$$

$$N_2 = \phi(V_2) \simeq 0.72$$

$$V_3 = w_{13} \cdot N_1 + w_{23} \cdot N_2 = 0.4 \times 0.55 + 0.60 \times 0.72 \simeq 0.65$$

$$y_1 = \phi(V_3) \simeq 0.66$$

$v_4 = w_{14} \cdot N_1 + w_{24} \cdot N_2 = 0.5 \times 0.55 + 0.3 \times 0.72 \simeq 0.49$

$y_2 = \phi(v_4) \simeq 0.62$

$e_1 = d_1 - y_1 = 1 - 0.66 = 0.34$

$e_2 = d_2 - y_2 = 0 - 0.62 = -0.62$

Let's consider the cost function $J(w) = \frac{1}{2} \sum_{i=1}^{N} e_i^2$.

$$J = \frac{1}{2} \cdot \sum_{i=1}^{2} e_i^2 = \frac{1}{2}\left((0.34)^2 + (-0.62)^2\right) \simeq 0.25$$

Backward Pass

① From output to hidden layer: $\Delta w_{ij} = e_j\,(-1) \cdot \phi'(v_j) \cdot x_i$

ⓐ $w_{13} = w_{13} - \eta \cdot \Delta w_{13}$, $\Delta w_{13} = \frac{\partial J}{\partial w_{13}}$, $\eta = 0.1$

$= 0.4 - 0.1 \times \left[e_1 \cdot (-1) \cdot \phi'(v_3) \cdot N_1\right]$

$= 0.4 + 0.1 \times 0.34 \times \phi'(0.65) \times 0.55$

$\simeq 0.40$

ⓑ $w_{14} = w_{14} - \eta \cdot \Delta w_{14}$

$= 0.5 - 0.1 \times \left[e_2 \times (-1) \cdot \phi'(v_4) \cdot N_1\right]$

$= 0.5 - 0.1 \times 0.62 \times \phi'(0.49) \times 0.55$

$\simeq 0.49$

(c) $w_{23} = w_{23} - \eta \cdot \Delta w_{23}$

$$= 0.6 - 0.1 \times [e_1 \times (-1) \times \phi'(v_3) \times N_2]$$

$$\simeq 0.61$$

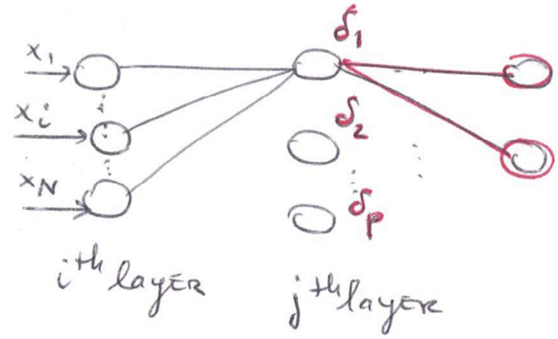
(d) $w_{24} = w_{24} - \eta \, \Delta w_{24}$

$$= 0.3 - 0.1 \times [e_2 \times (-1) \times \phi'(v_4) \times N_2]$$

$$\simeq 0.29$$

(2) From hidden layer to input layer:

$$\Delta w_{ij} = \eta \cdot \delta_j \, x_i$$



where

$$\delta_j = -\frac{\partial J}{\partial y_j} \cdot \phi'(v_j)$$

$$= -\left[\sum_{l} e_l \times (-\phi'(v_l)) \cdot w_{kj}\right] \cdot \phi'(v_j)$$

$$\delta_1 = -\left[(e_1 \times (-\phi'(v_3)) \cdot w_{13}) + (e_2 \times (-\phi'(v_4)) \cdot w_{14})\right] \times \phi'(v_1)$$

$$= -\left[-0.34 \times \phi'(0.65) \times 0.4 + 0.62 \times \phi'(0.49) \times 0.5\right] \times \phi'(0.2)$$

$$\simeq -0.01049$$

$$\delta_2 = -\left[(e_1 \times (-\phi'(v_3)) \times w_{23}) + (e_2 \times (-\phi'(v_4)) \times w_{24}\right] \times \phi'(v_2)$$

$$\simeq 0.0004332$$

$$w_{11} = w_{11} + \eta \cdot \delta_1 \, x_1$$

$$= 0.1 + 0.1 \times (-0.01049) \times 1$$

$$\simeq 0.099$$

$$w_{12} = w_{12} + \eta \, \delta_2 \cdot x_1 \simeq 0.25$$

$$w_{21} = w_{21} + \eta \cdot \delta_1 \cdot x_2 \simeq 0.099$$

$$w_{22} = w_{22} + \eta \cdot \delta_2 \cdot x_2 \simeq 0.70$$