Neuroprothetik Exercise 5
Multicompartment-Model

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1 Passive and Active Multicompartment Model

In this Exercise, an active multicompartment model is simulated. Till this point, the neuron is simulated as on complete circuit. Now, the morphology, or the biological structure of the neuron, is considered as well. For this reason, the axon of the neuron is represented of multiple compartments, each divided by a so called *Node of Ranvier* and insulated by a *myelin sheath*. Each node can be represented as a RC-Element while the area between two nodes functions as an restistor. Figure 1 shows the corresponding equivalent circuit.

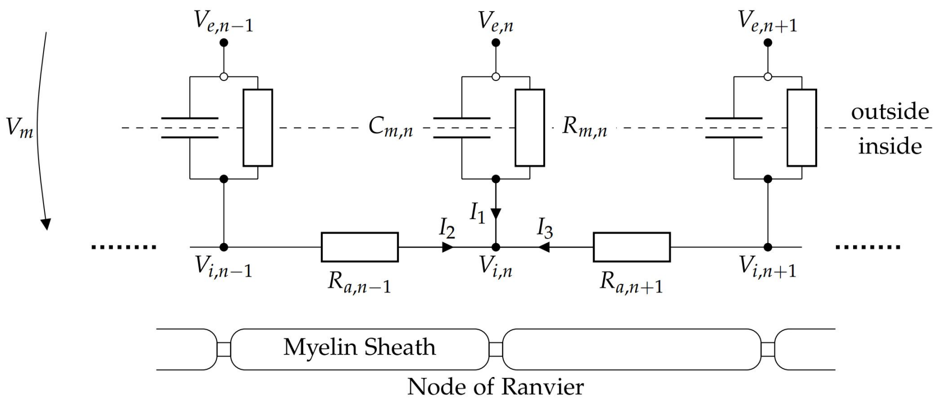

Figure 1: The equivalent circuit for an axon. The myelin sheath functions as an resistor while each Node of Ranvier can be represented as an RC-Element.

1.1 Passive Multicompartment Model

Using basic circuit theorey, one can derive a differential equation for the membrane potential V_m under the assumption that the axon resistance, membrane resistance and capacity do not depend on the location:

$$R_{a,n-1} = R_{a,n} = R_{a,n+1} = R_a$$
$$R_{m,n-1} = R_{m,n} = R_{m,n+1} = R_m$$
$$C_{m,n-1} = C_{m,n} = C_{m,n+1} = C_m$$

$$I_1 = \frac{V_{i,n} - V_{e,n}}{R_m} + C_m \frac{\delta(V_{i,n} - V_{e,n})}{\delta t} \tag{1}$$

$$I_2 = \frac{V_{i,n} - V_{i,n-1}}{R_a} \tag{2}$$

$$I_3 = \frac{V_{i,n} - V_{i,n+1}}{R_a} \tag{3}$$

$$I_1 + I_2 + I_3 = 0 \tag{4}$$

Combining equations 1 to 4 and replacing $V_{i,n} - V_{e,n}$ with $V_{m,n}$ as well as the unknown internal potential V_i with $V_m - V_e$ and assuming a constant external potential along the neuron, one gets:

$$\frac{\delta V_{m,n}}{\delta t} = \frac{1}{C_m}\left(- \frac{V_{m,n}}{R_m} + \frac{V_{m,n-1} - 2V_{m,n} + V_{m,n+1}}{R_a} \right) \tag{5}$$

1.2 Active Multicompartment Model

Equation 5 describes the passive multicompartment model. Using the previos work on the Hodgkin & Huxley model, we can adapt this model to an active multicompartment model. This can be done by replacing the passive $\frac{V_{m,n}}{R_m}$ with the net-current from the Hodgkin & Huxley model $I_{m,n} = I_{Na,n} + I_{K,n} + Ileak, n$ resulting in:

$$\frac{\delta V_{m,n}}{\delta t} = \frac{1}{C_m}\left(- I_{m,n} + \frac{V_{m,n-1} - 2V_{m,n} + V_{m,n+1}}{R_a} \right) \tag{6}$$

The parameters R_a and C_m can be calculated by equation 7 and 8:

$$R_a = \frac{\rho_a \cdot \Delta l}{\pi a^2} \tag{7}$$

$$C_m = \frac{\epsilon_r \epsilon_0 \cdot 2\pi a \cdot \Delta l}{d_m} = c_m \Delta l \tag{8}$$

2 Experiments

Now the above described active multicompartment model is simulated. The differential equation 6 is solved using the backward Euler method. All simulations occure at $T = 6.3\,°\mathrm{C}$, run for $100\,\mathrm{ms}$ with a timestep $\Delta t = 25\,\mathrm{\mu s}$. At the start of the experiments, the axon-parameters are fixed at:

$$\begin{aligned} c &= 1\,\mathrm{\mu F\,cm^{-2}} \\ \rho_{Axon} &= 0.1\,\mathrm{k\Omega\,cm} \\ r_{Axon} &= 2 \cdot 10^{-4}\mathrm{cm} \\ l_{Comp} &= 0.1 \cdot 10^{-4}\mathrm{cm} \end{aligned}$$

Two stimulation patterns were used: a $5\,\mathrm{ms}$ long rectanglular puls with $10\,\mathrm{\mu A\,cm^{-2}}$ at the first compartment, and two similar pulses at compartments 20 and 80. Figure 2 shows the resulting membrane potentials for the first stimulation pattern while figure 3 shows the membrane potential for the second stimulation.

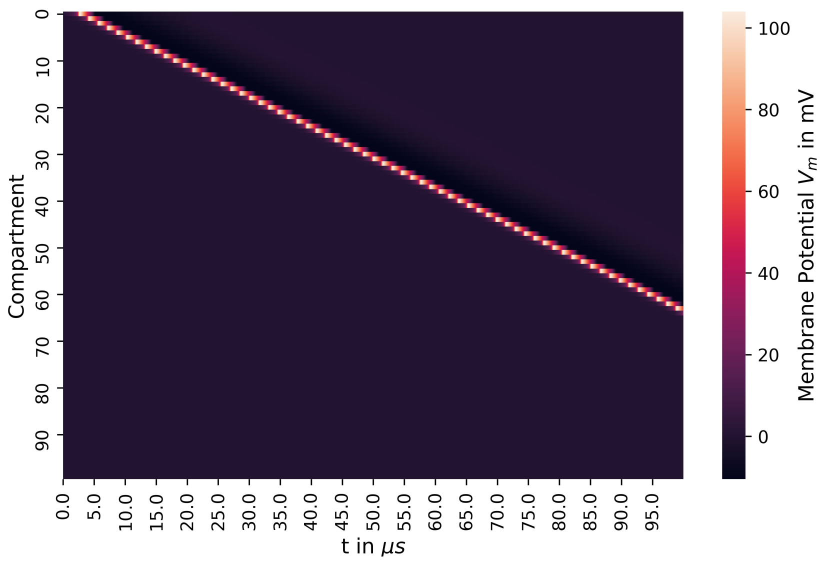

Figure 2: The action potential (color) of each compartment after stimulation with one rectangular pulse at the first compartment

Figure 2 shows the propagation along the compartments with a small delay after each action potential (bright color) as well as the hyperpolarization after an action potential (dark 'shadow'). If no stimulation occurse, the membrane potential stays constant. In figure 3 a different pattern visible. Starting at compartments 20 and 80, the action potentials first propagate both forward and backward, because those compartments

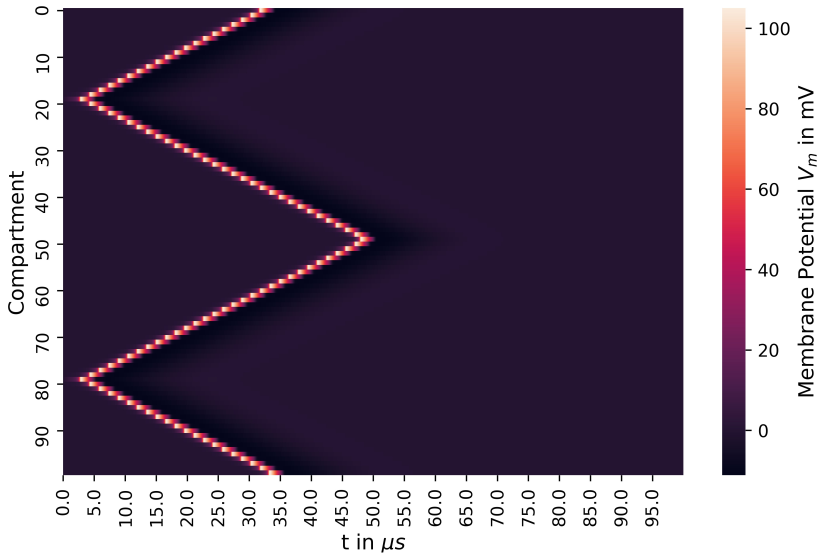

Figure 3: The action potential (color) of each compartment after stimulation with two similar rectangular pulses at the compartments 20 and 80.

haven't fired yet. In the middle of the neuron, the propagation stops where both potentials meet. This is due to the refractory period. When both action potentials meet, the action potential coming from compartment 20 wants to propagate to the following compartment, but that has already fired due to the action potential coming from compartment 80 and therefore is in its refractory period and cannot fire. The same is true for the action potential coming from compartment 80 which wants to propagate to the prior compartment. Due to this the propagation stops here.

Now, the axon parameters are changed to analyze their effect on the model. For this case, only the first stimulation pattern is used. First, we change the axon resistance $\hat{\rho}_{Axon} = \frac{1}{5}\rho_{Axon}$ for figure 4. As this plot indicates, reducing the axon resistance will increase the propagation speed of the action potential. Then the membrane capacity is changed $\hat{c}_m = 2 \cdot c_m$ for figure 5. This has the opposite effect, the propagation speed is drastically reduced.

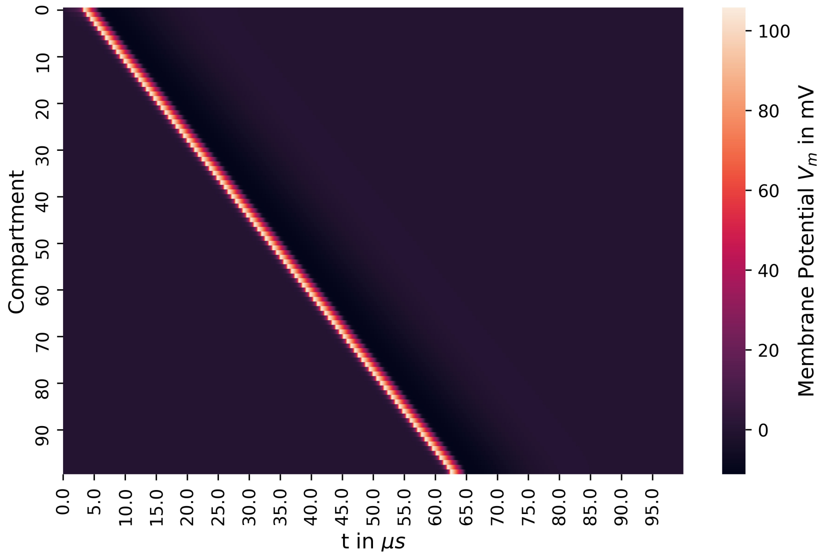

Figure 4: Using $\hat{\rho}_{Axon}$, the propagation speed is increased.

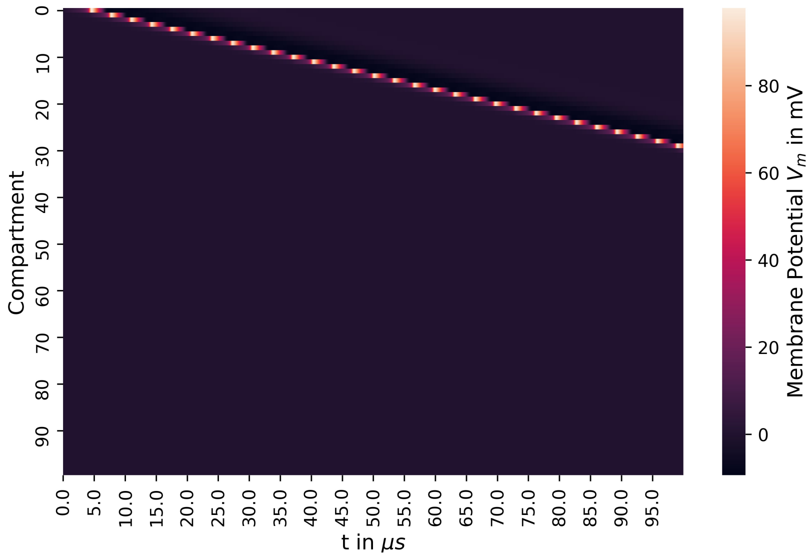

Figure 5: Using $\hat{c}_m$, the propagation speed is decreased.