Neuroprothetik Exercise
Exercise 3

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1 Solve Function

To solve the differential equation in1, three different solvers are implemented and used:

- Explicit Euler
- Heun Method
- Exponential Euler

$$\frac{dV}{dt} = 1 - V - t = f(V,t) \tag{1}$$

With $V_0 = V(t_0) = -4$ and $t_0 = 4.5$. All three algorithms were evaluated using four different stepsizes, $\Delta t \in \{1\,\mathrm{s}, 0.5\,\mathrm{s}, 0.1\,\mathrm{s}, 0.012\,\mathrm{s}\}$.

1.0.1 Explicit Euler

The Explicit Euler Method estimates the next point V_{n+1} using the value of $f(V,t)$ using the current values V_n and t_n:

$$V_{n+1} = V_n + f(V_n, t_n) \cdot \Delta t) \tag{2}$$

1.0.2 Heun Method

The Heun Method uses a similar idea to the Explicit Euler Method to approximate V_{n+1} but instead of only using the slope at V_n, it also takes the slope at the next point returned by the Explicit Euler Method:

$$V_{n+1} = V_n + \frac{f(V_n, t_n) + f(V_n + f(V_n, t_n) \cdot \Delta t)}{2} \cdot \Delta t \tag{3}$$

1.0.3 Exponential Euler

For the Exponential Euler Method, the exponential equation 1 must be representable in the following form:

$$\frac{dV}{dt} = A(t)V(t) + B(V, t) \tag{4}$$

Then, the next step of V_n can be calculated using:

$$V_{n+1} = V_n \cdot exp(A(t_n) \cdot \Delta t) + \frac{B(V_n, t_n)}{A(t_n)} \cdot [exp(A(t_n)\Delta t) - 1] \tag{5}$$

For function 1, function 4 can be described by:

- $A(t) = -1$
- $B(V, t) = 1 - t$

Figure 1 shows the approximation of function 1 by all three solvers and different timesteps Δt.

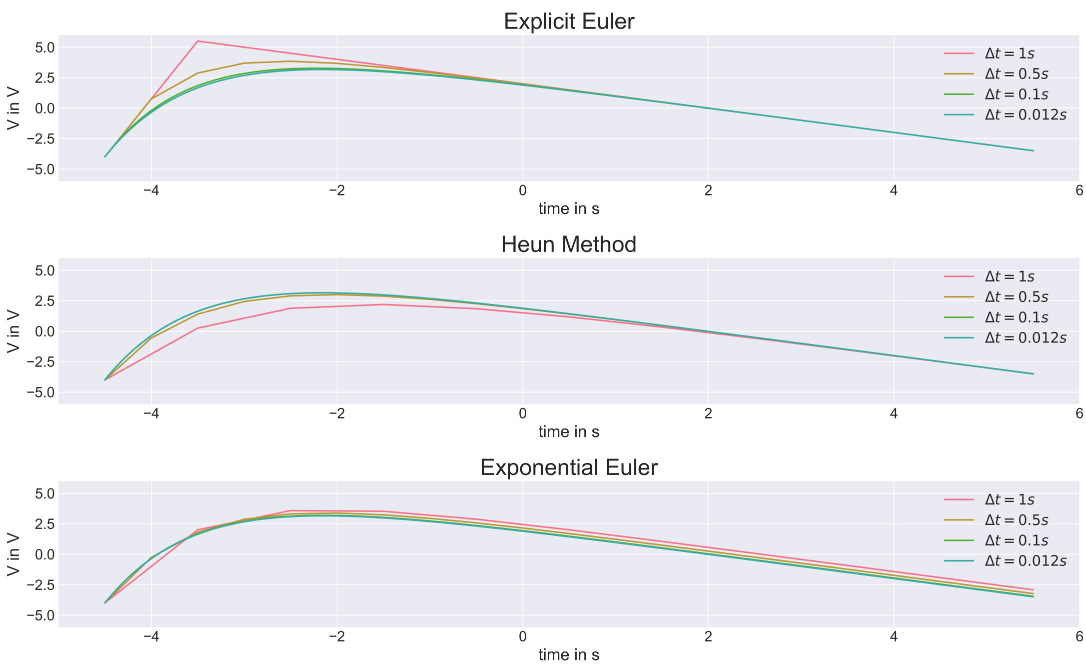

Figure 1: The approximation of 1 using three different solvers and four different timesteps

As we can see, the impact of a smaller timestep differs for the three solvers. The Explicit Euler Method benefits a lot from a smaller stepsize, as the initial error is rather large. Looking at the error-change in respect to the actual error, the Heun Method benefits more, as a reduction of the stepsize by half will reduce the error by three quater, while it will onyl reduce the error of the Explicit Euler by half as well. The Exponential Euler on the otherhand does not benefit from a higher stepsize (except for an increase in resolution of course). It returns the exact value of the described function for every stepsize.

From this analysis, it seems only logical to use an very small stepsize to approximate the unknown function as good as possible. This has two problems. First, a smaller stepsize means more steps to calculate. So using very small steps, the computational costs increase. Besides that, the stepsize also influences the round-off error. Therefore, just decreasing the stepsize and using the rather simple Explicit Euler Method is not recommended.

2 Leaky Integrate and Fire Model

Here, a model of the leaky integrate and fire model is implemented and simulated using four different stimulations. The model can be described by equation 6.

$$u(x) = \begin{cases} V_n + \frac{\Delta t}{C_m}(-g_{leak}(V_n - V_{rest}) + I_{input}(t_n)) & \text{if } V_n < V_{th} \\ V_{spike} & \text{if } V_n = V_{th} \\ V_{rest} & \text{if } V_n = V_{spike} \end{cases} \tag{6}$$

With the following parameters:

- $g_{leak} = 100\,\mathrm{mS}$
- $C_m = 1\,\mu\mathrm{F}$
- $V_{rest} = -60\,\mathrm{mV}$
- $V_{th} = -20\,\mathrm{mV}$
- $V_{spike} = 20\,\mathrm{mV}$

Four different stimulation currents I_{input} were simulated:

- constant 10 mA
- constant 20 mA
- rectified 50 Hz sinus with 10 mA amplitude
- rectified 50 Hz sinus with 30 mA amplitude

Figure 2 shows the voltage of the leaky integrate and fire neuron for 50 ms and a timestep of $\Delta t = 25\,\mathrm{ms}$. As one can see, a higher input current directly corresponds to a higher spike-frequency, as the model does not include a refractory period (except the one timestep for resetting the voltage to V_{rest}). Usually, after firing one spike, a neuron has a so called refractory period, where it is not able to fire (absolute refractory period)

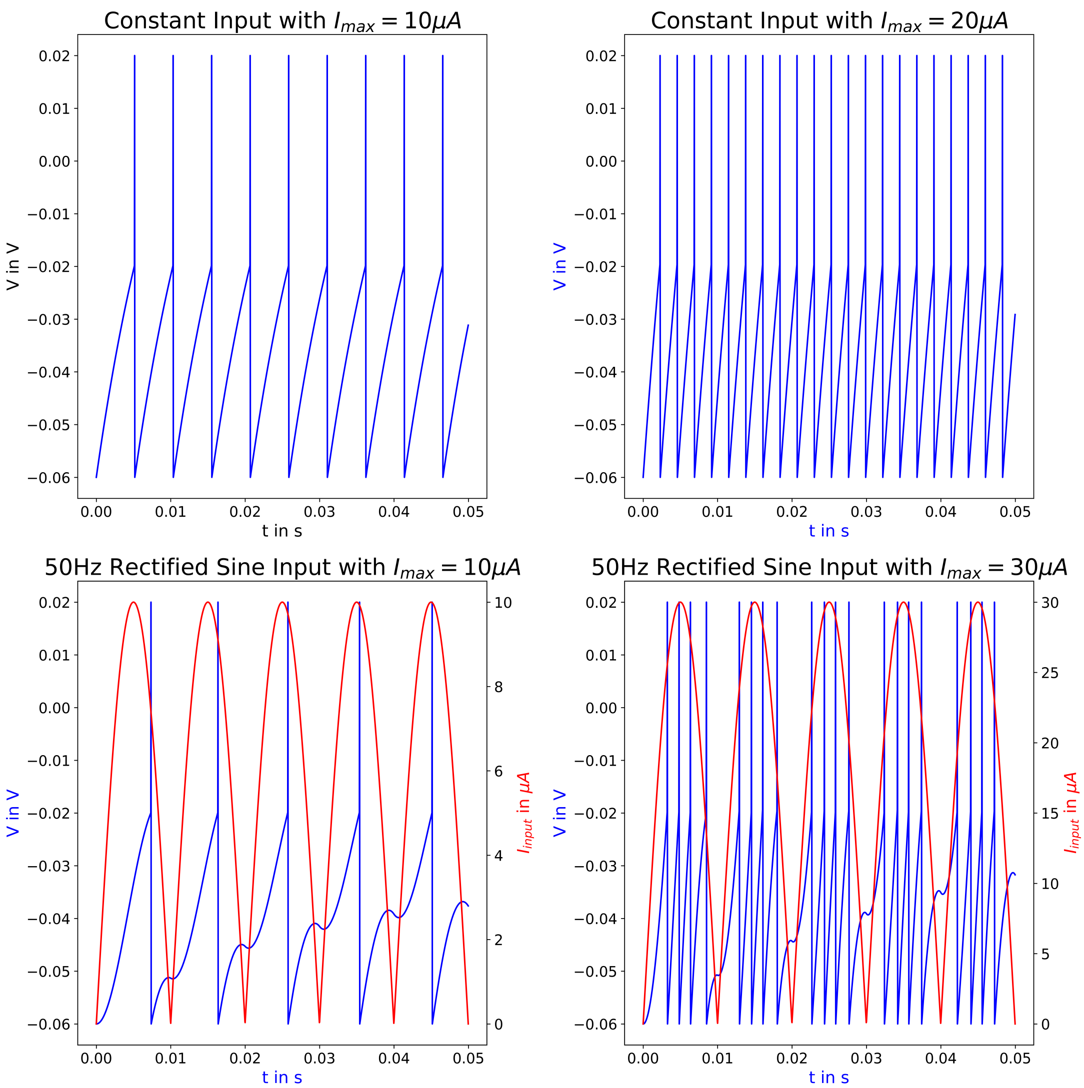

Figure 2: The membrane potential (blue) of the leaky integrate and fire model of a neuron with different stimulations (red)

or the membrane pontential is still hyperpolarized, so it is less likely to fire (relative refractory period). Here, we do not account for such a effect. Therefore increasing the stimulus from 10 mA to 30 mA will increase the spiking frequency. This also can be seen at the lower plots in figure 2, which shows the results of a rectified sinus-stimulation. Here, one can also observe the relation between higher input current and higher spike frequency. In the lower-left plot, one can observe the 'leaky'-part of the leaky integrate and fire model. If no or very little stimulation occures, the voltage will slowly start to decrease again until it reaches V_{rest}.