Neuroprothetik Exercise 6
Electric Stimulation

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1 Calculate the Potential Field

1.1 Potential Field

The potential field from a current-powered point-source can be calculated by equation 1 with *rho* being the specific electrical resistance of the medium of the surrounding area, I the driving current and r the distance from the point source. Figure 1 shows the resulting potential in a 50 µmx50 µm 2D-plane, 10 µm away from the point-source. The potential is in a medium with $\rho_{medium} = 3.0\,\Omega\mathrm{m}$ and the source is driven with $I = 0.001\,\mathrm{A}$.

$$\Phi(r) = \frac{\rho}{4\pi} \cdot \frac{I}{r} \tag{1}$$

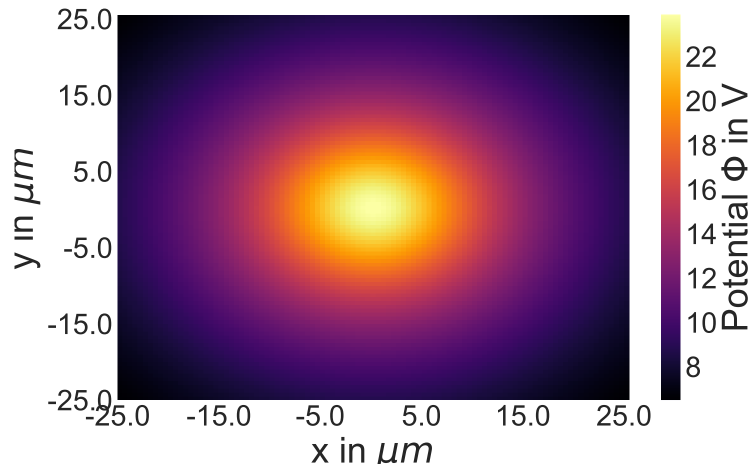

Figure 1: The potential of a point source, 10 µm away from the plane.

1.2 Activation Function

If we recall the differential equation for the membrane potential from Exercise 5, equation 2, we assumed the external potential $V_{m,n}$ is equal at all compartments n. Now, lets have a look at how an external stimulation will affect the neuron. For this we can replace the membrane capacitance C_m and the axon resistance R_a with equations 3 and 4 respectively. We therefore get the so called activation function 5.

$$\frac{\delta}{\delta t} V_{m,n} = \frac{1}{C_m} \left(-\frac{V_{m,n}}{R_m} + \frac{V_{m,n-1} - 2V_{m,n} + V_{m,n+1}}{R_a} + \frac{V_{e,n-1} - 2V_{e,n} + V_{e,n+1}}{R_a} \right) \tag{2}$$

$$C_m = \frac{\epsilon_r \epsilon_0 \cdot 2\pi \cdot \Delta l}{d_m} = c_m \cdot \Delta l \tag{3}$$

$$R_a = \frac{\rho_a \cdot \Delta l}{\pi r^2} = r_a \cdot \Delta l \tag{4}$$

$$f_a = \frac{1}{c_m r_a} \frac{V_{m,n-1} - 2V_{m,n} + V_{m,n+1}}{R_a} \Delta l^2 \tag{5}$$

If we now look at a infinitely small compartment-piece, we get

$$\lim_{\Delta l \to 0} f_a = \frac{1}{c_m r_a} \frac{\delta 2}{\delta t^2} V_e \tag{6}$$

Now consider a axon as a one-dimensional line, we get

$$r = \sqrt{d^2 + (x - x_0)^2} \tag{7}$$

for the distance from the point-source located at x_0. For simplicity, we will consider the point-source as the origin or $x_0 = 0$. From this, we get the electrical field E at the axon:

$$E(x) = -\nabla \Phi = \frac{\rho I}{4\pi} \cdot \frac{x}{(x^2 + d^2)^{\frac{3}{2}}} \tag{8}$$

and the activation function f_a:

$$f_a(x) = \frac{\delta^2}{\delta x^2} \frac{1}{c_m r_a} \Phi(x) = -\frac{\rho I}{4\pi r_a c_m} \cdot \left(\frac{1}{(x^2 + d^2)^{\frac{3}{2}}} - \frac{3x^2}{(x^2 + d^2)^{\frac{5}{2}}} \right) \tag{9}$$

Figures 2 and 3 show the potential, electric field and activation function for a 50 µm long axon positioned 10 µm from the current point source which is driven with 1 mA and −1 mA respectively.

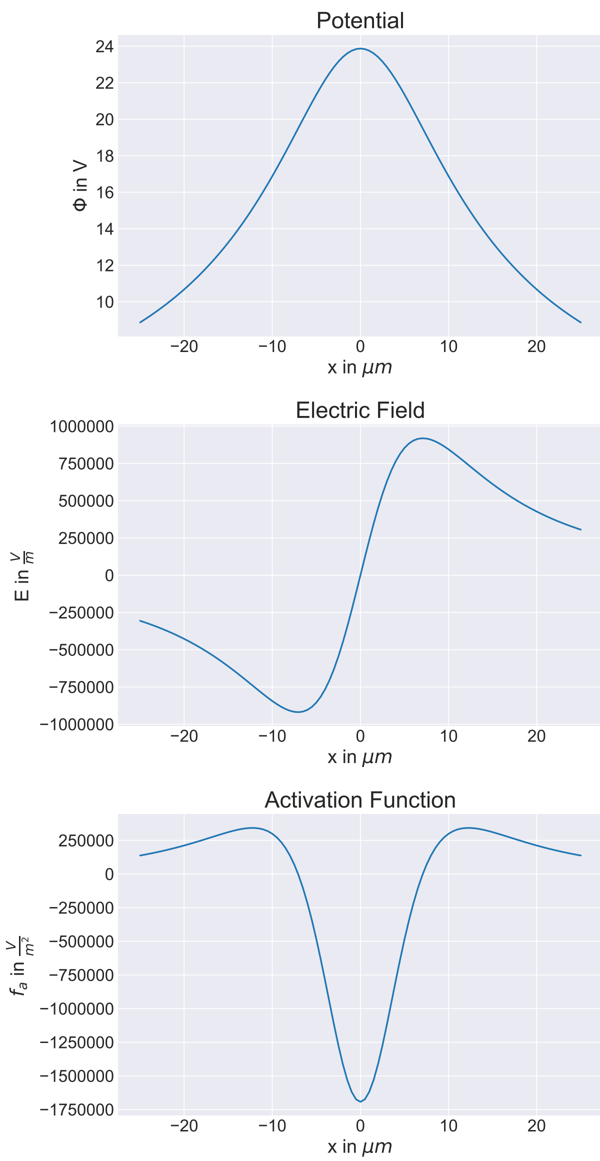

Figure 2: The potential (top), electric field (middle) and activation function (bottom) of an 50 µm long axon 10 µm away from an electrode driven by 1 mA.

This shows an interesting pattern: If we want so stimulate as close as possible to the electrode and which as little current as possible, it seems much more feasible to use a negative current to drive the electrode instead of a positive one.

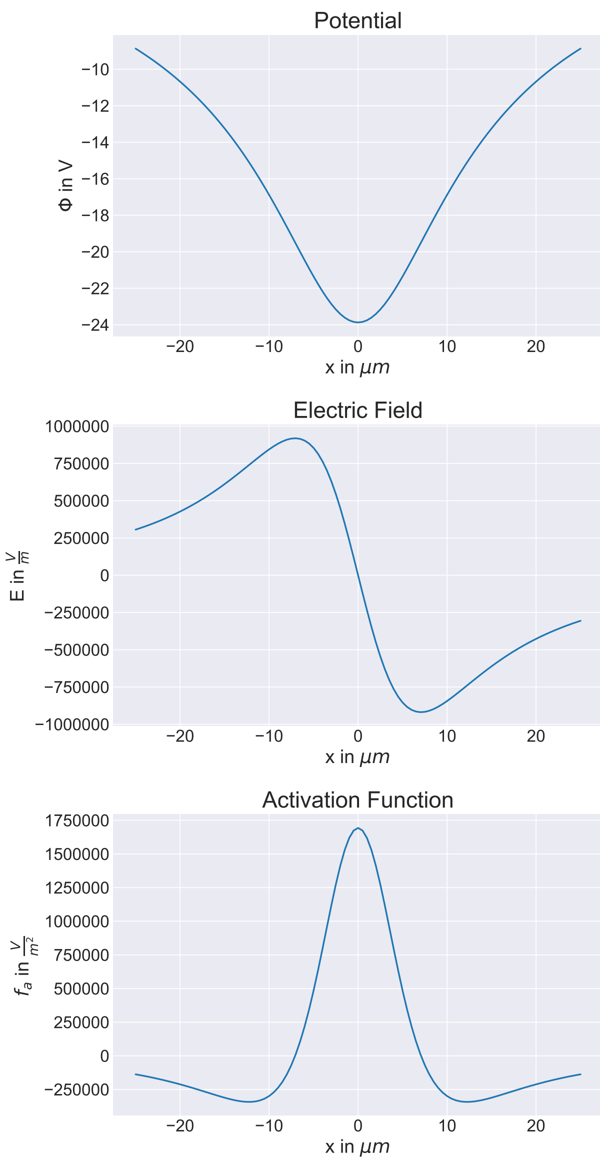

Figure 3: The potential (top), electric field (middle) and activation function (bottom) of an 50 µm long axon 10 µm away from an electrode driven by −1 mA.

2 Stimulate the Axon

We now take the multicompartment model from last exercise and stimulate the axon with an electrode 10 mm away. The simulation runs for 30 ms with a timestep of $\Delta t = 0.025\,\text{ms}$ and $N = 100$ compartments. The physical parameters for the simulation are:

$$\begin{aligned} \rho_{medium} &= 3\,\Omega\,\text{m} \\ \rho_{axon} &= 0.1\,\Omega\,\text{m} \\ r_{axon} &= 1.5\,\mu\text{m} \\ l_{comp} &= 0.5\,\mu\text{m} \end{aligned}$$

The following stimulation patterns are used:

1. Stimulation by a mono-phasic current pulse, with a phase duration of 1 ms and a driving current of $-0.25\,\text{mA}$
2. Stimulation by a mono-phasic current pulse, with a phase duration of 1 ms and a driving current of $-1.0\,\text{mA}$
3. Stimulation by a bi-phasic current pulse, with a phase duration of 1 ms and a driving current of $0.5\,\text{mA}$
4. Stimulation by a bi-phasic current pulse, with a phase duration of 1 ms and a driving current of $2.0\,\text{mA}$
5. Stimulation by a mono-phasic current pulse, with a phase duration of 1 ms and a driving current of $0.25\,\text{mA}$
6. Stimulation by a mono-phasic current pulse, with a phase duration of 1 ms and a driving current of $5.0\,\text{mA}$

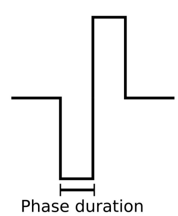

Figure 4: The shape of a biphasic pulse.

Figure 4 shows the shape of a biphasic current pulse, where the stimulation first occurs with a negative amplitude for the phase duration and then for the same time with the positive amplitude. This is used to prevent the buildup of chloride or hydrogen due to electrolysis. Figures 5 to 10 show the resulting potentials of the compartments. For each pattern, the stimulation occurs at 5 ms.

Those patterns show the effect of the shape of the activation function derived in section 1.2. A pulse with a negative amplitude (Figures 5 and 6), the

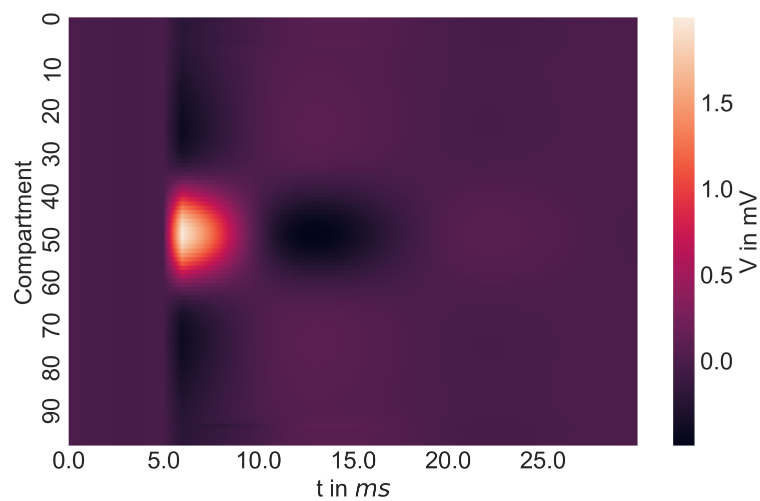

Figure 5: The simulation using stimulation pattern 1

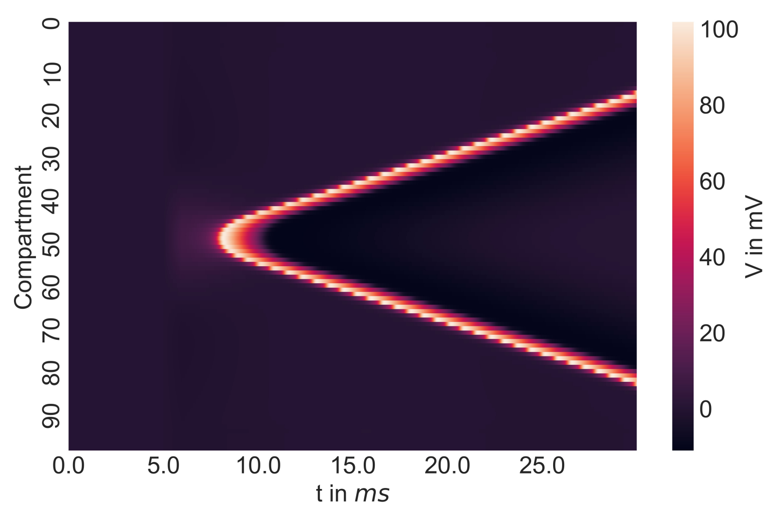

Figure 6: The simulation using stimulation pattern 2

maxima of the stimulation happens at the compartments closest to the electrode (compartment 50). If the used current is high enough, an action potential is triggered (Figure 6), which then propagates in both directions. For the pattern 1, the used current was not high enough to excite an action potential, but the

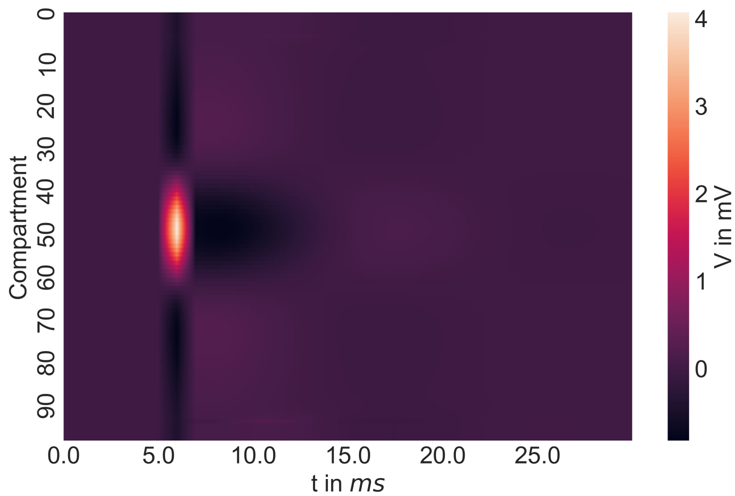

Figure 7: The simulation using stimulation pattern 3

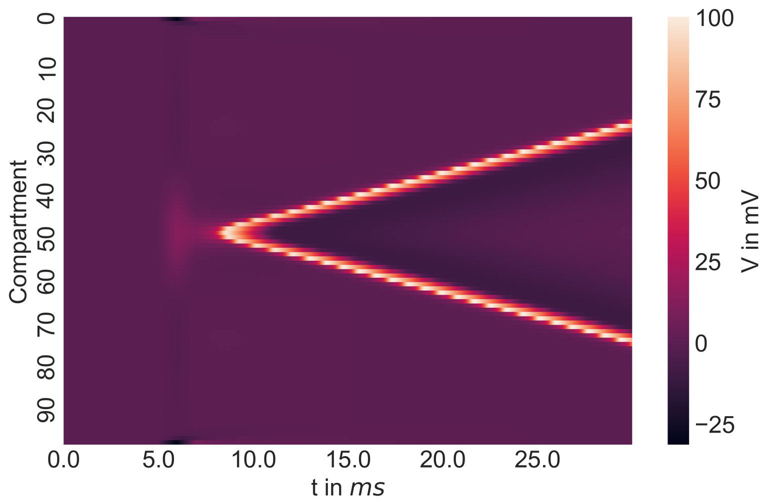

Figure 8: The simulation using stimulation pattern 4

shape of the activation function from Figure 3 is also visible in Figure 5.

Using a biphasic pulse with an negative amplitude first (Figure 7 and 8) has a similar effect. The positive phase then abruptly ends the polarisation when

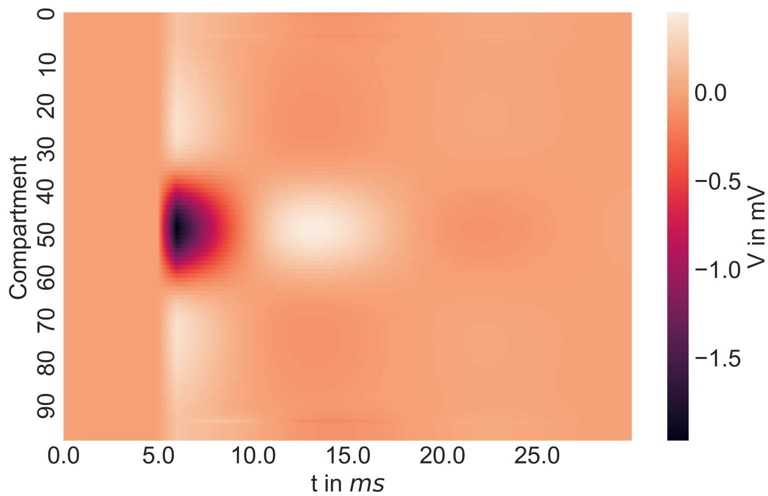

Figure 9: The simulation using stimulation pattern 5

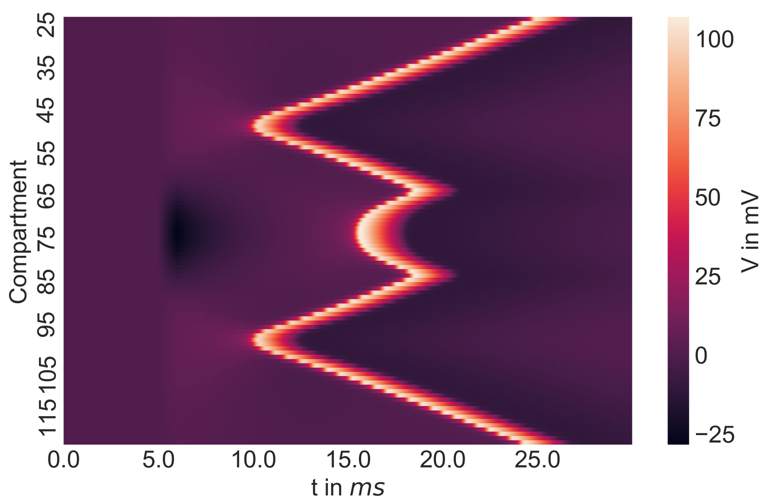

Figure 10: The simulation using stimulation pattern 6

no action potential is triggered (Figure 7 compared to 5), but when an action potential is triggered, the positive pulse does not inhibit it (Figure 8; Note that the difference in color is due to some numerical errors at the boundaries, but the resting potential is actually the same through all plots).

Using a positive pulse leads to some interesting patterns. If the stimulating current is to small (Figure 9), the membrane potential is depolarized not at the point closest to the electrode, but at two points further away, as Figure 2 implies. But when the current is high enough to excite an action potential (Figure 10), this leads to two simultaneous action potentials at those positions and only later a third one at the compartment closest to the electrode. This effect alone, while being curious, seems to be rather unimportant, as the action potentials travelling towards the middle compartment later inhibit themselves due to the refractory period of the neurons. Comparing the last few ms of Figure 10 to 6 and 8, the difference seems rather small, as in all three simulations an action potential is travelling towards the outer regions of the model.

The big difference between the stimulation patterns come into place when looking at the used current. When using a monophasic negative pulse only 1 mA are required to trigger an action potential, while a biphasic pulse requires 2 mA and a positive monophasic positive pulse requires 5 mA. This also corresponds to the shape of the activation function in Figures 2 and 3: the function has its peak at the point closest to the electrode, and that by a margin. Therefore, when using a negative pulse, we can use much less current to achieve the same depolarization at the axon. As a biphasic pulse is required due to safety reasons, a biphasic pulse with a negative amplitude first seems to be the best stimulation pattern so far.