Dissertation: Hybrid Methods for Finite Element Meshing

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Contents

1 Introduction

The overall aim of this project is to perform the task of refining a Finite Element Mesh (FEM) using a stress based method as is typical of FEM refinement in conjunction with an approach that uses techniques from Artificial Intelligence and Machine Learning. It should be possible to reason about the quality of a mesh produced using both methods analytically in order to evaluate the success of the approach and potential for continued use.

Finite Element Analysis (FEA) is a method widely used across different engineering domains to simulate structures under certain conditions. The method works by taking a geometry defined in continuous terms, discretize it into a mesh system before calculating the property values for each of the discrete elements. This allows engineers to observe the effect the conditions have on the entire structure, see figure 1.

The success of the project will be determined by implementation of a system that is able to combine the two approaches described above in order to refine a mesh to of a quality comparable to that of a successful stress based method.

2 Motivation and Background

Over the past forty years FEA has emerged as a prominent technology for simulating complex real world engineering problems [1, 5]. FEA works by solving a system of differential equations with each equation representing a single element in a geometric mesh. By doing this FEA is able to generate highly accurate approximations for the properties of complex physical systems [5] [18]. The method can also be highly computationally expensive with the complexity typically increasing exponentially with the model size [5]. Analysis therefore proves to be highly time consuming and costly for individuals and organisations to conduct [7]. [16]

2.1 Properties of Finite Element Models

Although this is a computer science as opposed to a mechanical engineering dissertation it's still important to briefly outline the general principals and properties that underpin the FE method in order to have a general appreciation for how the design and evaluation of the final software system was conducted.

Finite Element Models have several key properties that need to be specified by the engineers who create them, the configuration of these properties greatly determine the results obtained from the models execution. The first of these properties that an FE model posesses is the mesh. The mesh is constructed out of nodes, points which act as intersections between the second component- elements which are either a polygon or a polyhedron between the nodes. Nodes and elements are important concepts as they provide the theoretical framework for reasoning about the other properties of a mesh and hence the overall quality of the model [18].

In addition to the nodes and elements FE models also contain inputs and constraints. Inputs can be thought of as the phenomena from the outside world which is acting on the structure and consequently inducing some kind of physical effect on it. Inputs are used by engineers to model the conditions under which the structure will be expected to perform under when it is manufactured and enters operation.

Constraints are another fundamental concept that describe where the model is attached to the outside world. When computing stresses induced through the model there needs to be an area through which the stresses are assumed to leave. FEA is only able to calculate the path of the stresses through the model and thus the overall stress at given points using the law of conservation of energy i.e. energy cannot be created or destroyed meaning that any energy provided to the system as input through for example force needs a means by which to leave it, the constraint.

For example in figure 6 showing a suspension bridge model, the simulation is to be run with the forces are induced upon the cables and the towers in the negative x direction as represented by the green vectors. The corresponding constraint area through which the force must leave is specified as the base of each bridge pillar and represented by multiple red arrows on each corresponding corner.

The final piece of information needed in order to calculate the stresses through the model is material data. Material data is associated with the models elements and is usually defined using two main parameters which are:

- Youngs Modulus - The ratio of stress over strain for a given material, i.e. for an amount of internal force endured by a material how much does it deform, a material such as rubber therefore has a low value for Young's modulus while diamond has a high value [36].
- Poissons ratio - Amount of deformation that occurs perpendicular to the force that is applied to the material.

For the sake of simplicity all structures used to evaluate the final software solution have assumed steel as their material property. Steel is a pretty common material used within manufacturing of many mechanical

components and does not exhibit any abnormal properties. This is beneficial for evaluation as it removes variability in the results that could arise from selecting a more complex material.

2.2 Limitations and general considerations

An important consideration when conducting FEA is the trade-off of a models accuracy against the time in which it can be solved. A major variable determining this trade-off is the models mesh structure which discretizes the problem so that simulation can be run on it. A mesh that is finer is more computationally expensive but also produces results of greater accuracy. It is therefore desirable to generate a mesh which is fine where accuracy is most needed but coarse where it is not [17].

In every type of analysis that the FE method is used for (thermal, structural, fluid flow, electrical) there is a specific differential equation associated with each of the elements. In order to achieve overall convergence of the model the equations must be solved simultaneously to achieve a value for each of the discrete elements [18]. For this dissertation project attention will be given specifically to the problem of FEA meshing in the context of static structural analysis where the value calculated across each element is its stress. It makes sense to work on hybrid mesh refinement in the context of static structural analysis as it is likely the most common engineering application of the method and as such has the largest body of research that is relevant [5][18].

For engineers the value obtained through computing the stresses under a particular set of conditions is feedback on the quality of their design. Ideally the results from an analysis will provide a good understanding of where the design is weak and how concentrated this weakness is. This information is used to either help verify the designs' quality or alternatively inform changes to its geometry or material properties so as to reduce stress on subsequent analysis [20].

To understand the gradient of stress within a part of the model the mesh needs to be designed carefully. As each element can only display values calculated from its edge nodes a smooth gradient requires a higher concentration of elements in areas under higher stress. A high quality mesh is therefore considered to have a higher concentration of elements in areas of predicted high stress while retaining lower concentration elsewhere, thereby revealing weaknesses in the design while minimising the models runtime.

Traditionally the automated mesh refinement process consists of computing stresses for a model with an initial coarse mesh and low computation cost, once rough stresses have been computed the elements in areas of higher stress can be divided recursively into additional elements in order to achieve smoother gradients on further executions [16]. Figure 1 shows a mesh which has been refined in an area of higher stress thus providing a clearer indication of a components weakness.

Unfortunately for many large models this method for refining a mesh is still excessively costly [4]. It is therefore worth investigating use of alternative approaches posed by the field of computer science and artificial intelligence that could support the traditional high stress meshing approach.

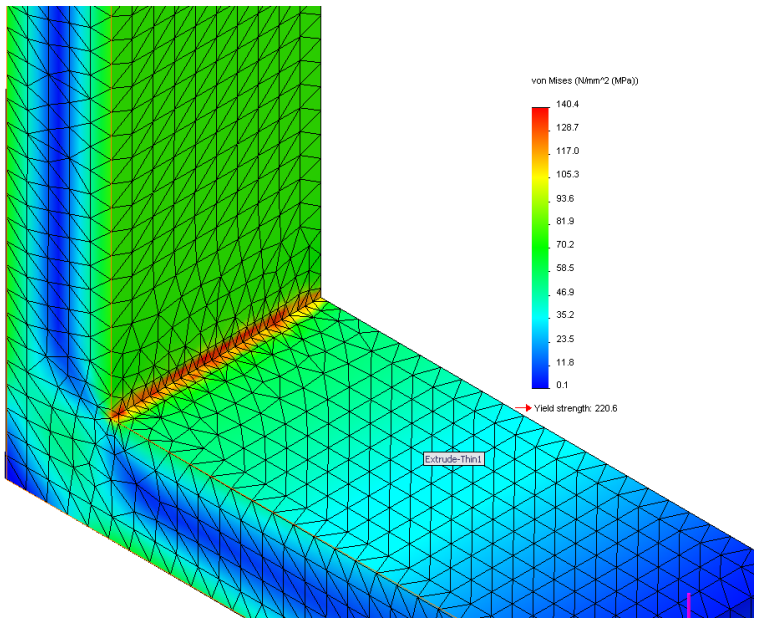


Figure 1: Mesh refinement in corner under high stress image source: ([32])

3 Related Work

Many approaches have so far been taken in an attempt to improve a computer's ability to perform the task of finite element meshing. The following subsections present an overview of the research conducted for the various aspects of the project which are more explicitly outlined under "Aims and Objectives"

3.1 Traditional Subdivision Approaches

Multiple approaches exist for subdividing different types of meshing with the most common being higherarchical refinementent also known as h-refinement and relocation refinement or r-refinement[2] [3]

h-refinement: H-refinement is the process of recursively refining a mesh by splitting elements into additional sub elements. This process can be performed for elements with both both triangular and quadrilateral shapes. [2].

r-refinement: R-refinement is a method which attempts to improve the quality of the stress gradient without the alteration of the mesh element count and thus the computational cost. This is achieved by the relocation of elements within the mesh which effectively increases the size of elements in areas of low stress, while reducing the size in areas where stress is high [3].

3.2 Stress Refinement

Refining a mesh based on some model parameter is consistent across all types of FE modelling. Execution of a model with a generic mesh is first required so as to obtain a set of results by which further refinement can be targeted. Using unique ids for nodes allows results from the previous iteration to be compared against the mesh and refinement to be focused on those nodes exhibiting a high amount of a specified property [25].

3.3 Uses of Artificial Intelligence and Machine Learning

Within the domains of AI and machine learning methods such as neural networks [9], case based reasoning [10] and inductive logic programming [5] have all been adopted to facilitate generation of meshes comparable to that of human experts. Similarly there has also been effort to combine multiple numerical methods simultaneously for solving re meshing problems [8] although effort to combine the two approaches does not appear to have so far been made.

Due to the difficulty of obtaining meshes which hold commercial interest the majority of researchers working in this field have had to resort to the use of training sets developed within academia [4]. The primary issue associated with this is that often the training data does not exhibit the level of complexity that you would expect in many industrial sectors. Many researchers must accept this as a limitation or agree commercial terms with an organisation in order to gain access to their models [14].

Having reviewed a variety of different AI based applications to FE the use of Inductive Logic Programming (ILP) used by Bojan Dolsak et al is of greatest interest. ILP is a machine learning method first presented by Stephen Muggleton in his 1991 paper "Inductive Logic Programming" [11]. Muggleton suggests that the traditional approaches of machine learning which rely on use of extensive data sets and statistical analysis are poor in the case of many real world problems for which data is not available [13]. Muggleton cites Human learning as an example of use of ILP style techniques where understanding of new concepts is achieved not through crunching large volumes of data points but instead the use of induction on a relatively concise set of background facts and examples obtained from previous life experiances [13].

ILP uses three types of input information in order to hypothesise additional facts about the system. These three types of input information are:

- Positive examples of what constitutes an area that is well meshed
- Negative example of areas that are poorly meshed
- Background facts

Using this information ILP is capable of hypothesising rules by determining which rules can exist within the system where given the set of background facts all positive examples are satisfied while few or none of the negative examples are. Although ILP requires a body of additional metadata associated with each mesh this is easier to obtain making ILP a highly practical solution. Along with his publication Muggletons also released his implementation of an ILP algorithm as a program titled “Golem" [12], Golem was applied by Dolsak to the problem of mesh refinement with a training set of just five meshes [5]. The resulting rule set when applied to subsequent models was able to correctly classify and re mesh areas with an average accuracy of 78% for a range of geometries [5] [6].

Dolsak's choice of metadata for the ILP method to generate mesh rules is based on the classification of edges within the FE model. Dolsak recognises that edges act as an important intersections within the model and as such provide useful items of reference when designing heruristics with which to reason about the model [5] [6]. For example if it is know that an edge has a force applied close to it based on the initial model conditions then other edges that intersect it should be additionally refined [4] [6].

The format of the rules also make them attractive for experimenting with as part of a hybrid method since the method determines how to refine the mesh based on the arrangement of edges. This detail of analysis of the mesh is at a comparable detail to that of a traditional splitting method such as h-refinement which is likely to improve the ease with which the two methods can be combined simultaneously in the latter stages of the project.

3.4 Quality metrics

Finally work has also been done on establishing valid metrics for assessing the quality of a mesh automatically [14, 9] Metrics for meshes have been researched far more extensively than AI methods due to their use for comparing different stress based refinements. There are also cases of common metrics being used for industrial meshing applications [14]. Although there are metrics for assessing a mesh on a global level such as element count score the consensus is that due to the variation in meshes this is less reliable than assessing quality based on the properties of individual elements within the mesh [14].

Localised metrics associated with the quality of each element have shown to be accurate for predicting the overall quality of a mesh when taking the average for each metric across all elements [14]. The quality of an elements shape is important since the stress values which are computed for the area within the element are calculated using the stress values at each of the nodes which enclose it [18]. Elements are typically deformed near parts of the geometry where its shape simply does not allow a uniform element to be placed, an example of where this has occurred can be seen in figure 2.

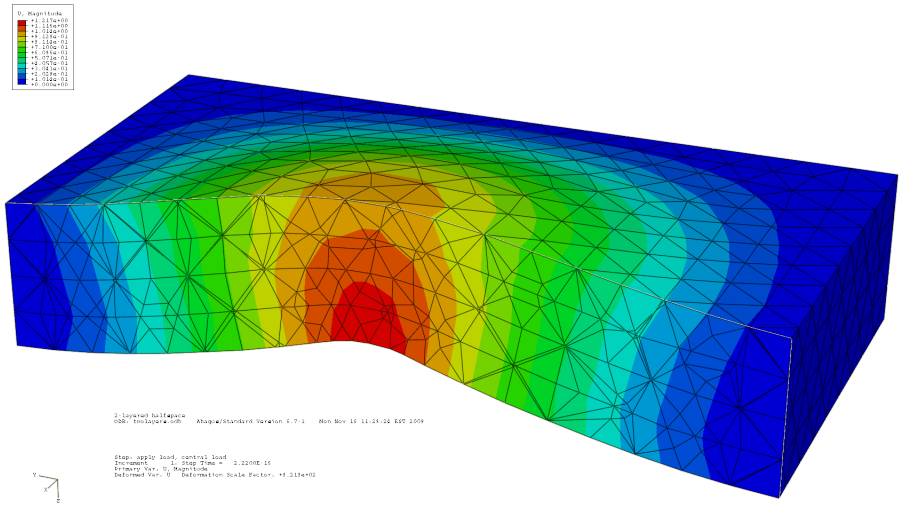


Figure 2: Example of how elements can be distorted in order to fit a geometry which will result in deterioration of gradient quality (image source: [15])

Some key shape metrics identified by Dittmer et al include (ideal values are for elements of quadrilateral type as used in the current prototypes):

A Aspect ratio – longest side / shortest side, ideal value is 1

B Maximum corner angle - widest internal angle to element, ideal is 90°

C Maximum parallel deviation - how skewed the element is, ideal is 0°

4 Description of the work

4.1 Aims and Objectives

The aim of this project was to design, build and analyse a system for refining a mesh by combining a method derived from the fields of AI or machine learning with a method relying purely on information already present in the model. The desirable end result will be a hybrid method of meshing which effectively prioritises those areas of importance whilst incurring a reduced computational cost.

The project can be broken down into thee main areas of research and implementation which have the following high level objectives:

A Research and implement both a traditional refinement procedure using data present within the model and an approach using techniques from AI developed and used by either industry or academia. These algorithms should be able to run independently on a set of example models.

B Secondly a process needs to be devised to combine the two re meshing methods to varying degrees. This will make it possible to evaluate and compare the effects of a hybrid meshing against each of the individual methods for a range of models. Through this it should be possible to establish whether or not there is potential benefit to using a hybrid approach and if so to what extent.

C The third objective will be to research and implement justifiable metrics for assessing the quality of a given mesh, this will allow objective comparisons to be made for the resulting meshes.

4.2 System specification

To demonstrate success in achieving the objectives of the project it is important to have tractability from the requirements through to the solution and lastly verification and validation. This section describes the systems initial *functional requirements* (what the system will do) and *non-functional* requirements (its constraints) based upon evaluation of the research conducted in conjunction with discussions with the project supervisor: Dr Jason Atkin. Functional requirements have primarily been listed under their respective high level subsystems that are responsible for encapsulating their functionality.

Although it has not been developed as part of the project the application responsible for solving the finite element models has been included as part of the systems requirements since it highly influences the overall scope of the project and much of the design associated with other subsystems which were developed for the project.

4.3 Functional Requirements

1. FE integration: System will be able to interface with a third party finite element application
    - 1.1. The finite element applications solver must be able to solve a mesh based on its model configuration.
    - 1.2. The finite element applications solver must be able to execute a model programmatically
    - 1.3. The finite element applications solver must be able to output stress data at different points on the mesh.
    - 1.4. The finite element application will provide a graphical representation of the model.
    - 1.5. It will be possible to manipulate the model that the finite element application uses programmatically.

1.6. It should be possible to manipulate the model that the finite element application uses from within its graphical user interface.

2. Mesh refinement: System will be able to perform different kinds of finite element mesh refinement

   2.1. The system will be able to refine a finite element mesh using a stress based refinement method.

   2.2. The system will be able to refine a finite element mesh using a non-stress based refinement method.

   2.3. A non stress based refinement method will adapt the mesh using background information about mesh design which has been previously trained.

   2.4. The system will be able to combine the two methods to produce a coherent mesh which the FE application is able to successfully solve in order to obtain results for stress and displacement.

   2.5. The system will be able to combine both methods to varying degrees that will be performed automatically by the system without direct user intervention.

   2.6. The system will re mesh using both stress and non-stress based refinement using quadrilateral elements.

   2.7. System will adapt weighting associated with each method based upon the metrics computed for the mesh in the systems previous iteration.

3. Quality assessment: System will provide the operator with results about the quality of meshes based on metrics obtained from research.

   3.1. An assessment will be conducted automatically for every mesh iteration that occurs.

   3.2. System will assess quality based on a variety of metrics to ensure overall robustness of measurement.

   3.3. The metrics will be computed for both individual element within the model and for the entire mesh.

4.4 Non-Functional Requirements

Design: The system architecture will be developed using the object oriented design principals of SOLID to allow for clear interfaces between the different functional components. Functional programming practices will be adopted through use of the .NET Language Integrated Query or LINQ framework. This will help to simplify the code and improve reliability. Where functions and classes are written their length will be kept to a minimum to reduce complexity and allow for reuse wherever possible.

Documentation: The system will be comprehensively documented at both a code level and at an architecture level. At a code level C# doc comments will be written to provide a comprehensive summary of each function. This will allow the tool Doxygen [29] to generate a full set of developer documentation upon completion of the software implementation which will be included as an appendix. Ensuring that the majority of information is present within doc comments will also help to promote a reduction of loose comments within the code and hence function size.

General applicability: In order to demonstrate that hybrid methods are a feasible means of approaching meshing problems the resulting software should be able to successfully execute on a range of models with varying geometry. The range of geometries should be representative of typical structural variation encountered by engineers.

5 System Design

5.1 System Overview

Determining the overall design of the system was initially hard since it was not clear exactly how many subsystems would be needed to mesh, evaluate and interface with LISA, what was clear was that the system would essentially be performing an optimisation procedure and as such needed to be driven iteratively towards a goal. The variable complexity and uncertainty surrounding the different parts of the project meant ensuring the architecture remained modular with well defined interfaces allowing components to easily be added or modified as the project progressed.

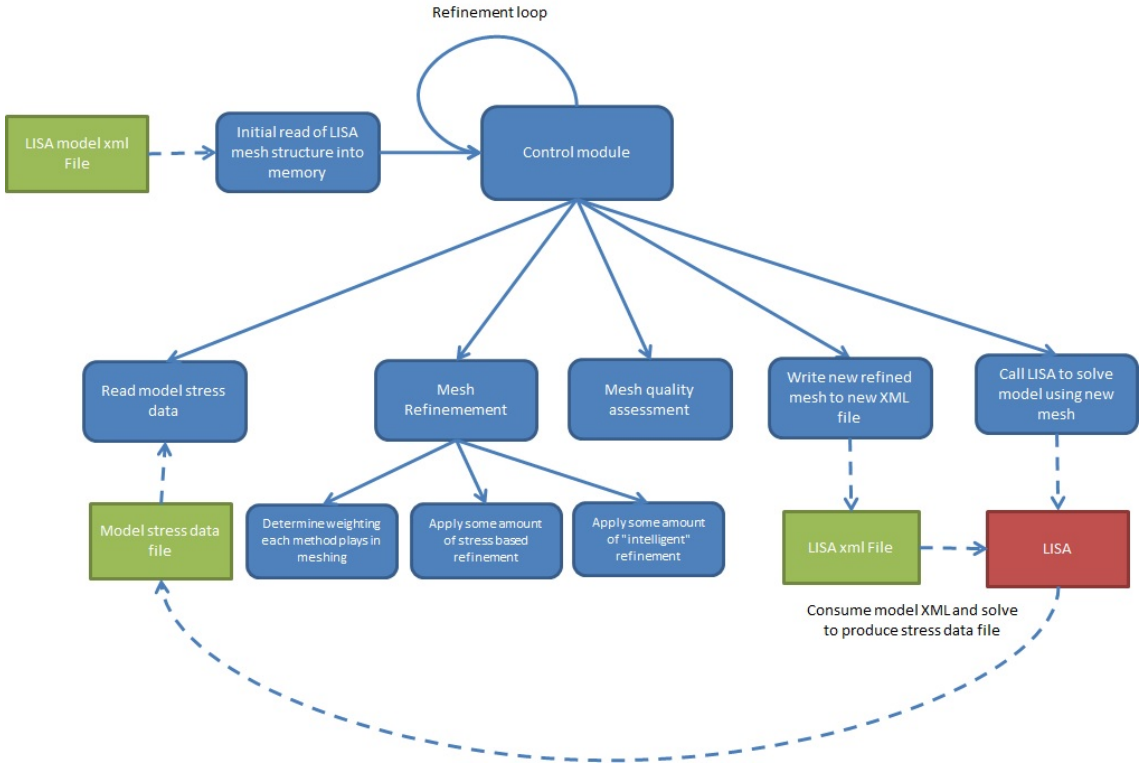


Figure 3: High level design of the system with its different modules

5.2 Modular Architecture

The modular architecture was crucial for allowing meshing algorithms and quality metrics to be replaced as necessary. At best the quality of the output could could only be predicted for each method before it was integrated into the system and executed in a range of different scenarios. To have tightly coupled these individual components would have rendered the overall system a failure in the event that any one of them failed. Instead the loose coupling of the architecture has enabled the system to be considered as more of a framework for testing the effects of combining different meshing approaches in order to generate a hybrid method.

Although the system was highly modular It was also still desirable to maintain an architecture hierarchy so that classes could be developed independently but easily integrated. Composition was therefore generally

favoured over inheritance as a means of building the architecture. Static classes and methods were also used when needing to write utility functions that were required by multiple high level subsystems and therefore did no fit especially well into any particular one. Examples of these are generic vector algebra operations such as dot product, matrix determinant and calculating surface normals.
At the highest level namespaces were used to break down the class groups appropriately, namespaces also naturally structured as folders within the Visual Studio (VS) solution explorer (see appendix D) which made navigating the project and finding components much easier as the system expanded in size.

5.3 Third Party FE Application

In order to demonstrate the potential feasibility of the hybrid approach it was first important to obtain a finite element solver which could be given a FE model containing data about forces, materials and the mesh structure and then execute the model programmatically so as to obtain stress results.

A multitude of commercial FE tools exist with there being a wide variety in both the complexity and cost associated with each tool. Finite element software is typically very expensive due to its high development cost and small customer base. Tools used within industry such as ANSYS typically require a great deal of time in order to become proficient in their usage and can cost in excess of five thousand pounds a year for a single licence [31]. It was therefore important to find a tool which was both affordable while also powerful enough to demonstrate a working prototype of the re meshing method.

5.4 LISA

After reviewing several FE applications used within industry in addition to a variety of less well known ones used within academia and by hobbyists LISA was selected as the solver application for which to implement the systems prototypes.

Strengths: LISA is a FE tool which allows the user to run models of up to 1300 element for free; This was beneficial in allowing me to experiment with the software and gauge the feasibility of my projects concept before requiring a full version. Once at a stage in the project where each problem had been solved for small models containing less than 1300 elements an academic licence for the software was purchased for the projects use.

LISA also provides a GUI which allows visual inspection of the model and its mesh; This is particularly useful for observing the output of the meshing algorithms which can often provide a human with a much better understanding of how the method has performed and whether or not there are obvious bugs. in the implementation of the meshing procedures

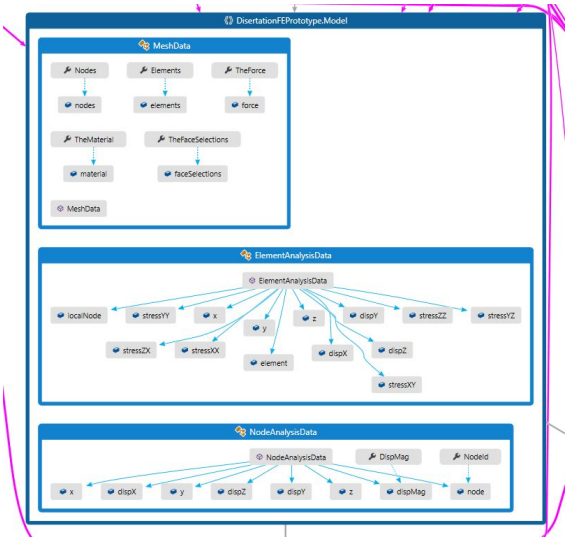
Weaknesses: Due to LISA's simplicity it does not come with an extensive API allowing for easy programmatic use of its inbuilt features, however it is still possible to interface with LISA through less direct means [19]. LISA models are stored in .liml files which use XML as a meta mark-up format. The model files contain all the information about the model including the materials used as well as loads and constraints and of course the mesh. It is therefore possible to manipulate a .liml file having parsed its contents before writing a new version of the file which LISA can be called to solve. In order to more easily alter the model it made sense to write a wrapper for the .liml files to abstract the manipulation of their content.

5.5 Simulation Data Model

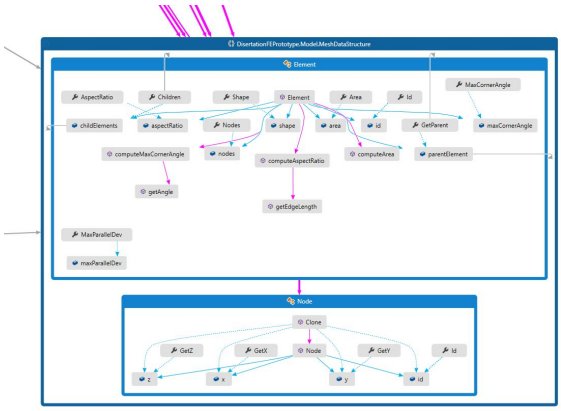
Writing an API for LISA was the first stage of development for my project for which a design had to be considered. The API was crucial in order to program the more complex aspects using basic operations and avoid having to regularly perform direct string manipulation of the input files in order to manipulate the

model.

When the first re-meshing iteration occurs the system needs to read the input .liml file into an equivalent class model which closely resembles the files schema, diagrams for which can be seen in figure 4 below. Each class in this model contains corresponding data and methods used to represent and manipulate the model. These methods are then used by each of the refinement approaches to easily alter the mesh in a controlled manner. Once the mesh has been adapted however it is required to be assessed by the modules responsible for validating its quality before finally being written back to a .liml file for LISA to solve on the subsequent iteration. Designing the data model so that it closely resembled the LISA schema not only made the higher level programming less confusing but also made serialisation of the data back to .liml format much simpler thus reducing the number of bugs arising from inconsistencies between different representations of the same data.

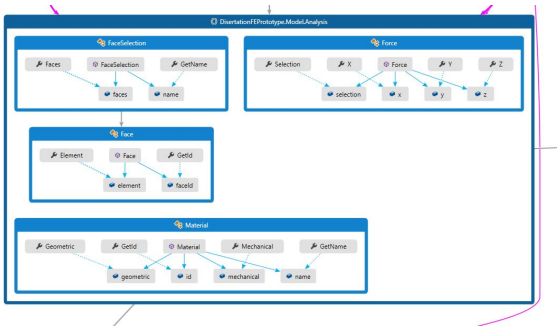


(a) Model classes

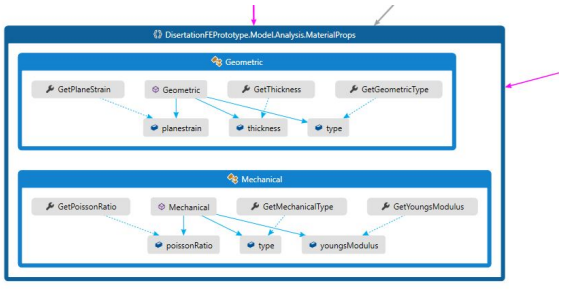


(b) Element and Node classes

One aspect of the data models design which greatly adds to the systems flexibility is the hierarchical design for representing the various Element types. At the root of this structure is the IElement interface, all new Element types must adhere to this in order for the various refinement methods to request refinement of an



(a) Model Analysis classes



(b) Material Property classes

Figure 5: Class model to represent .liml file structure used by LISA

element using its class. Implementing the interface are a range of abstract classes such as “SquareBasedElem" and “TriangleBasedElem" These classes are designed to contain methods that are generally applicable for calculating metrics and re meshing individual elements where the elements fit this abstract category but their concrete implementation specifies their dimensionality and number of nodes, see Figure 6. This is powerful since computing metrics and performing subdivision for a 3d element is simply a reduction using the code for a 2D element but over every face comprising the 3D one.

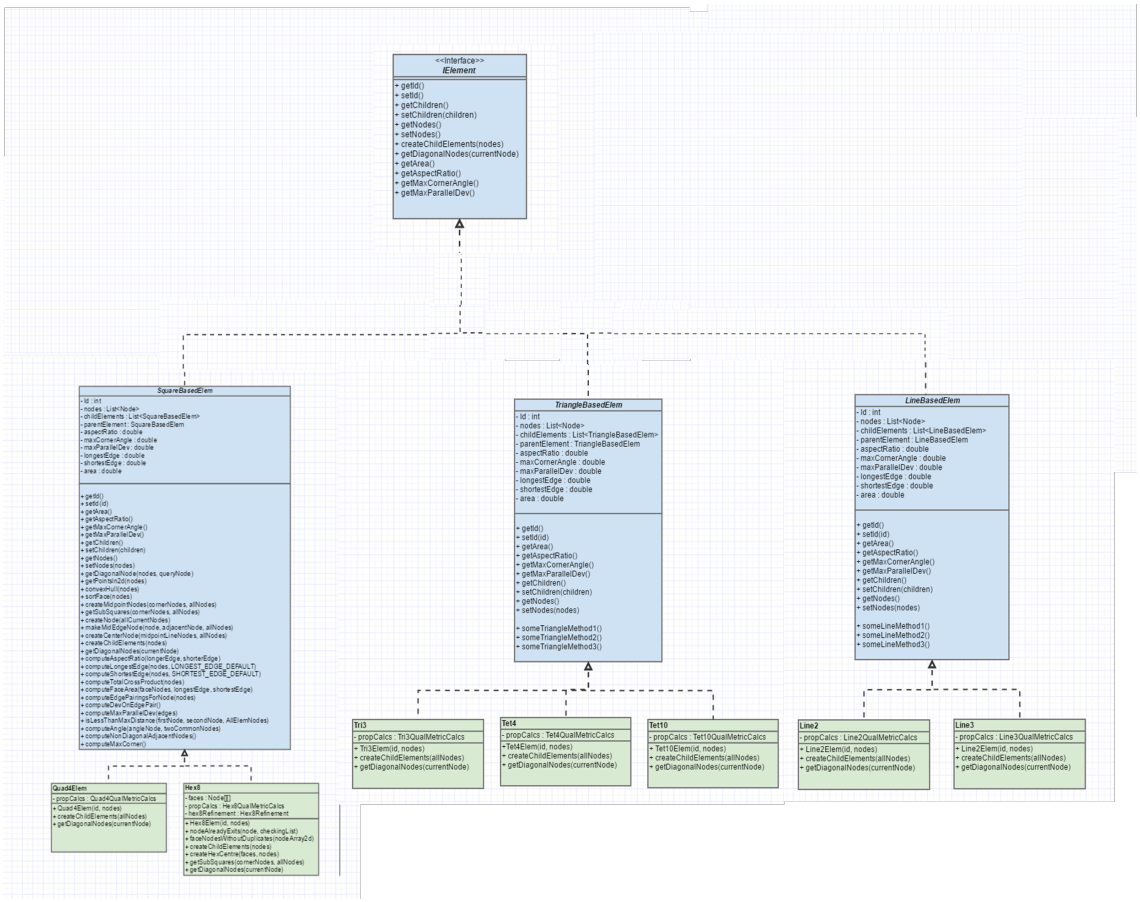


Figure 6: Class diagram showing the hierarchy of element classification within the data model, due to time limitations I was not able to implement the respective classes for triangle and line based elements, to see image representations of each element type within this class diagram refer to element type appendix

5.6 Remeshing methods approach

Element Refinement: Delegating refinement of individual elements to their respective classes made it much easier to decouple both the stress and heuristic methods allowing each of them to simply have the task of selecting elements which they considered beneficial to refine before calling the createChildElements() method on that element through the IElement interface. This allowed both of these high level refinement approaches to utilise the same low level functionality thus greatly improving code reuse and simplifying the design such that the high level meshing methods are relatively concise.

Having reviewed both h-refinement [2] and r-refinement [3] as techniques for performing element subdivision

it was concluded h-refinement was prefferable due to its simplicity and widespread use despite typically being more computationally expensive than r-refinement [2] [3]. Upon finishing subdivision and producing new child elements the refinement process also needed to flatten each tree as seen in figure 7 below to produce a new set of elements which could be stored at root level within the data model. This made the elements more difficult to manage although was another requirement imposed by LISA, which lacks an equivalent concept of element hierarchy within its representation of the mesh. As this task involved the deletion of parent elements and therefore needed to be performed by the "OptimisationManager" class responsible for combining and executing both refinement methods.

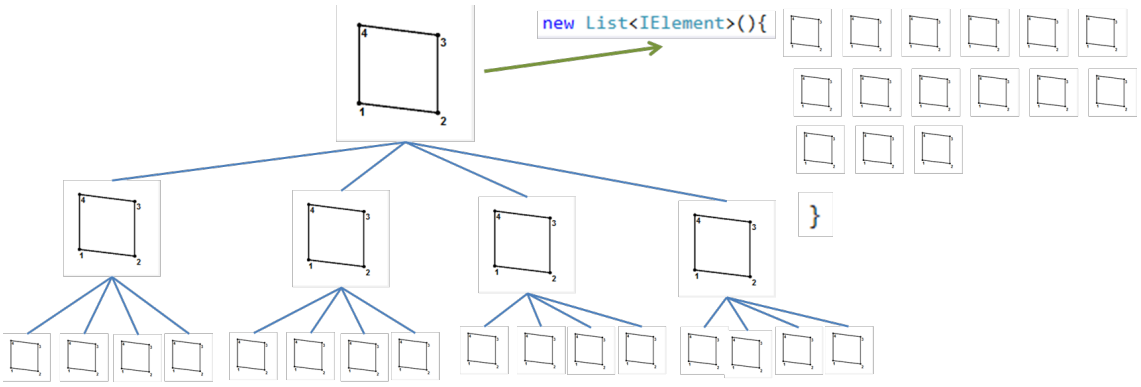


Figure 7: Process of flattening a refined element tree into a single list which can be handed back to LISA for processing

Stress and Heuristic Refinement: Having evaluated a variety of approaches from the domains of AI it was concluded that the best approach for delivering a system capable of meeting the requirements and demonstrating effectiveness of a hybrid method would be an implementation of the heuristic expert system described by Dolsak.

One key strength of selecting this approach as the alternative method by which to mesh was clear separation of the underlying AI method from what had to be implemented. This not only meant that focus could be given towards the design, implementation and evaluation of the general purpose system but demonstrated that the meshing procedure can for the most part be interchanged depending on the specific type of finite element analysis.

5.7 Input Files

The system requires three basic input files which should be placed within a directory that is given to the program as a parameter, these are files are:

- A structural model represented as a .liml file which LISA can solve.
- An initial stress data file generated manually so the system has a starting point.
- A JSON file containing important edges and associated meta data as identified by an engineer looking at the model.

An example of the content and format for each of these input files can be seen in Appendix B

5.7.1 Combining methods

Since each refinement method performed a discrete amount of subdivision every time it was called it made sense when developing a hybrid approach to simply enumerate the possible combinations for how much each method could be applied each iteration resulting in a set of two valued tuples up to some value :

$$\{(a, b) \mid a, b \in \mathbb{N}\ a, b < k\}$$

not sure if anyone actually cares about multithreading

Each tuple could then be considered one weighting configuration for combining the two methods and could be executed independent of the others to obtain a set of results for that weighting. Consequently it was possible to improve performance when conducting multiple evaluations through parallelism of the different weighting configurations as experiments onto independent threads. When started each thread creates its own directory which it copies the three input files to and runs for its designating weighting configuration.

6 Software Implementation

The following subsections detail the implementation of the final software solution that has been written to meet the objectives posed previously of this dissertation.

6.1 Languages and platforms

The final system has been written entirely using the C# programming language (version 5.0) with Visual Studio 2015 as the development environment on a Windows 10 system. C# is an application development language built on the .NET framework. Although any number of programming languages could have been used to implement the solution C# offered a good compromise for developing a system with both structural rigidity through static typing and object orientation in addition to functionality to allow for rapid prototyping. C# does this well through use of LINQ, a part of the standard library that provides a large number of higher order functions which allow for operations to be performed over any data structure that implements the built in IEnumerable interface. Given that much of the code within the project performs the same operation on collections of nodes and elements stored in lists, arrays and dictionaries which all implement IEnumerable the ability to write much of the project using this capability dramatically reduced the number of errors encountered and increased development speed.

6.2 Implementation Methodology

The growing size of the software meant it was important to work systematically to continuously drive the project in the right direction and avoid the introduction of unnecessary complexity. This was achieved through regularly reviewing and refactoring the code which dramatically helped to reduced the amount of bugs introduced.

For the duration of the project the spiral methodology was adhered to. This enforced multiple deliverable stages that were concluded with a supervisor meeting every one or two weeks. Adopting the spiral methodology also provided flexibility regarding the order in which tasks were able to take place outside of a spiral iteration. This was necessary when conducting a research driven project where direction of work for subsequent development iterations was largely driven by the findings of the work in the previous ones.

Tasks were chosen every week for the project, the number of tasks and their complexity was determined using a combination of factors including their complexity, the criticality of the task e.g. Did it need to be completed for other important tasks to be started and the time available to me as the individual undertaking the project (More tasks typically performed on weeks when less work was due for other modules.

6.3 Hierarchical Refinement

Elements within traditional FEA can typically be classified as either triangle or square based elements, each of these provide different strengths and weaknesses when used to mesh and solve models. Within industry triangular elements are typically preferable since it is always possible to generate an initial triangular mesh from any arbitrary CAD geometry algorithmically. This is done by simply making smaller triangles until all gaps along the edge of the geometry are filled [38]. The same cannot always be said when meshing using square elements. For proof of the solutions concept however it was concluded that square based elements were preferable to triangular ones since the steps required for a basic refinement are much simpler, just take the corners of an element that already exists, add their corresponding x, y and z components before dividing each component by two to achieve the new midpoint.

In addition to refinement it is also significantly easier to define edges which the ILP rules can be applied to when edges naturally form within a structure through a chain of nodes along the edges of square elements.

Unfortunately Triangular meshes also generally incur a higher computational cost than an equivalent square element mesh due to added complexity of performing the calculations required to remesh in addition to requiring more elements over a given area to achieve the same accuracy.

From an implementation standpoint writing a square based remeshing algorithm was also substantially easier if given a mesh of exclusively square elements since the primary task to be performed is to repeatedly divide each element into four sub elements, by contrast methods uses to re mesh triangular meshes are typically more complex and have corresponding initial meshes that are harder to generate manually by human operators [**?**].

6.4 Stress Based Refinement

To focus meshing in areas of high stress each iteration needed to parse the results file from the previous iterations execution of LISA. LISA result files are in csv format by default and contain the displacements and stresses associated with each node within the model once it has been solved.

Once the data in the output file has been parsed the nodal values for which displacement is known for can be cross referenced against those in the current model by intersecting the lists of node data on node Ids. An evaluation function is then able to determine whether or not any element handed to it meets the criteria for refinement by simply looking at the sum of the stress at its given nodes. If an element is determined to be over the threshold to justify refinement the elements "createChildElements()" method is called to subdivide it further.

6.5 Heuristic/Rule Based Refinement

Each rule is represented as a function within the implementation, this closely resembles the format presented by Dolsak [4, 5, 6] [7]. The rules resides within the "RuleManager" class and each take a number of the defined edges as parameters. When an instance of the RuleManager is created it parses the edges file provided by the user into a list of edges that the rules can then be executed on. Every rule then checks the properties of a particular edge against properties which have been identified through the ILP learning algorithm as being important when the model executes. In cases where the rules accept more than one edge as an argument the system attempts to apply the rule to each pair of different edges in the edge list giving a time complexity of $O(n^2)$ where n is the total number of defined edges.

If a rule detects a relationship in the model the edge is assigned a criticality rating as defined by the rule, the value is then used by the meshing procedure to determine how many times it should re mesh the elements along that edge.

The properties that can exist between two edges when compared are the following:

- Edges opposite one another - the edges run alongside one another closely, look at the distance between each of the corresponding nodes and check whether this distance is less than some threshold amount
- Edges posses the same form - Still need to write about this
- Edges are considered the same - to meet this requirement both edges must be almost the same length, opposite one another and posses the same form.

Since this system relied on a persistent definition of edges across multiple refinement iterations another challenge was to correctly redefine edges in terms of the newly created nodes so that after meshing had occurred the rules could be re applied to a refined edge to potentially refine it further.

```
1 reference | 0 changes | 0 authors, 0 changes
private void rule7(Edge edgeA, Edge edgeB)
{

    const int INVOLVED_EDGES = 3;

    bool b1 = edgeA.GetEdgeType() == Edge.EdgeType.important;
    bool b2 = edgeA.GetLoadType() == Edge.LoadingType.notLoaded;
    bool b3 = edgeA.isNeighbour(edgeB);
    bool b4 = edgeB.GetEdgeType() == Edge.EdgeType.importantShort;

    if(b1 && b2 && b3 && b4)
    {
        edgeA.ElementCount = INVOLVED_EDGES;
    }
}
```

(a) Code implementation of rule 7 provided by Dolsak within the RuleManger class

$$
\begin{aligned}
&\text{mesh(A,3) :-}\\
&\quad \text{important(A),}\\
&\quad \text{not\_loaded(A),}\\
&\quad \text{neighbour(B,A),}\\
&\quad \text{important\_short(B).}
\end{aligned}
$$

(b) Rule 7 as stated by dolsak in his papers [6]

6.6 Mesh Quality Assessment

Dittmers rules for computing the quality of both individual elements and the entire mesh are built into their own “MeshQualtyAssessments" and “ElementQualityMetrics" classes, the latter of which is encapsulated within an element object, like with refinement this allows each element to assess its own quality removing the need for additional utility classes and static methods.

Since each element is initialised with the nodes that comprise it, it is also possible to derive all the geometric characteristics and thus its quality metrics upon its initialisation. This allows the metrics for each element to also be calculated upon its initialisation and thus removing the risk of null values being returned when other parts of the system request this information.

6.7 Implementation Challenges

Implementation of the system was not without its share of challenges, some of which required fundamentally re addressing the approach used to tackle the problem. This section outlines the main instances of such cases during the projects development where as a consequence a notable change to the implementation was made often requiring additional research.

6.7.1 Fast Node Lookup and Update

A key requirement for the design of the data model generated by the hierarchical re meshing process was the need to perform fast lookup of nodes already present in the mesh. Lookup is important within the meshing methods as a means of checking whether a node that is about to be created already exists within the model, in the event that no such node already exists a new one can be created however if it does then instead of creating a new node the node that already exists needs to be connected to a node in an adjacent element that is currently being refined. If nodes are not linked correctly form correct elements the physics solver is unable to assume the stress moves through one element to another despite both having nodes at the same coordinates, this results in inaccurate output or potentially an error being thrown by LISA.

This issue arose partly as a result of the systems design, as previously mentioned subdivision for every individual element is the responsibility of that element which from a software engineering perspective is very good since it means the low level meshing process for each different type of element could be written within that elements class. This avoids the need for much heavier generalised refinement classes that would have needed to know how to perform the meshing for all elements in the model at once and for each of the different potential element types. A consequence of this was despite every Element being capable of meshing itself perfectly adjacent elements that also requiring refinement needed the ability to reconnect the new nodes along their edges to those that have been created by the adjacent element, this can be seen below in figure

9.

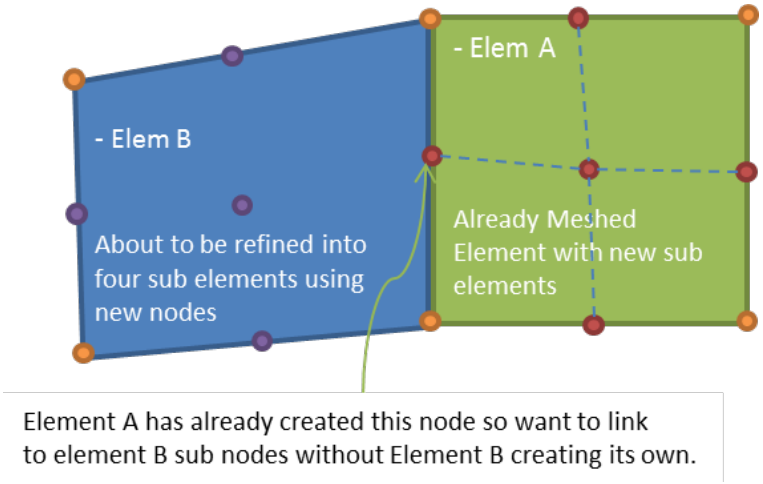


Figure 9: The need for an element to check for existing adjacent nodes when subdividing itself during refinement,

Orange Nodes - An original node for one or more elements
Red Nodes - new nodes made by Elem A
Purple Nodes - new nodes made by Elem B

The solution to this problem was to store all the nodes in the mesh model within a C# dictionary structure a reference to which is passed to each element within the model. The dictionary can be indexed using a Tuple of the x, y and z coordinates for the new potential element which will either return a node already at that location or indicate that no such node exists, in which case that element is then responsible for creating the node as its first instance. Dictionaries in C# represent a generalised instance of a hash table ensuring that lookup and insert are both constant time on average.

6.7.2 Sorting Element Nodes

One issue faced when working with LISA was an interface requirement specified requiring nodes for each type of element to be sorted in a specific geometric order. The general rule for node ordering within LISA is to have them form a perimeter around the edge of an element in 3d space without edges crossing one another internal to the element.

When addressing this problem for simple models constructed from Quad4 elements the most straightforward approach was to simply traverse each of the nodes in the order specified by LISA and with the resulting traversal list being ordered for LISA. The resulting traversal process resembles the following:

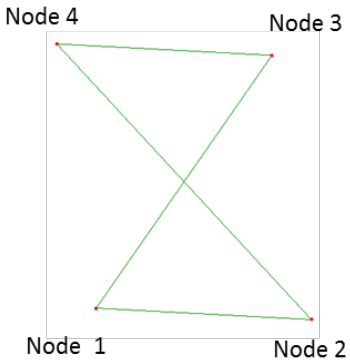


Figure 10: Element with 3d skew resulting in edges between diagonals being shortest by a small amount.

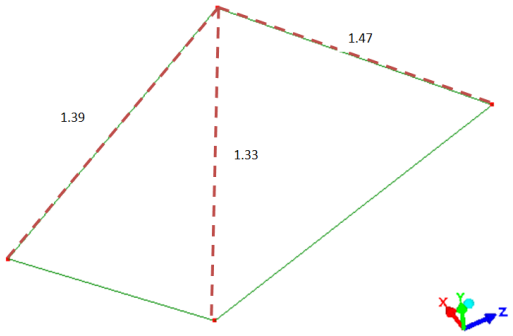
```
while ∃node ∈ UnstortedNodes do
    Get distance between current node and the next two nodes in UnstortedNodes;
    if Nodes left == 1 then
    else
    end
    if distanceToNode1 < distanceToNode2 then
        currentNode Node1;
        sortedNodes.Add(Node1);
        UnstortedNodes.Remove(Node1);
    else
        currentNode Node2;
        sortedNodes.Add(Node2);
        UnstortedNodes.Remove(Node2);
    end
end
```

Algorithm 1: How to write algorithms

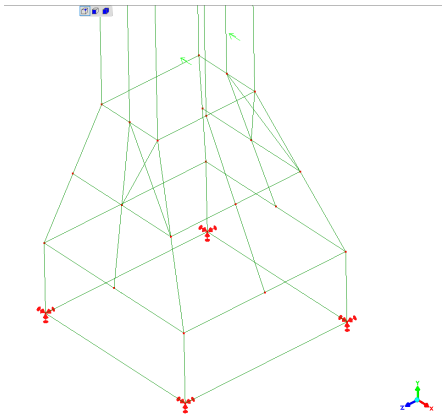
For the most part this approach was both fast and correct for Quad4 elements although in cases where elements were particularly skewed in 3d space it was sometimes possible for the internal diagonals to be shorter than the actual sides as seen in Figure 9 below. This proved to be a significant flaw in the method and brought about the realisation that a reliable approach would not be able to depend simply upon highly variable properties such as node distances.

Attempting to arrive at a more general solution focus was directed towards sorting the more complex Hex8 element type as this represented a more complete instance of the problem. Analysis of this led to the realisation that the most important task in sorting nodes for an arbitrary type is to simply establish the corner nodes relative to that type. Having established corners correctly sorting then simply required adding them to a list in the order specified by LISA.

The subsequent method which was used to successfully establish corners for both Quad4 and Hex8 elements was to split nodes for each element using planes running along the x, y and z axis as can be seen in Figure 9 below.
Although this approach resolved the initial problems resulting from simply trying to traverse the nodes it did not offer a strong general case solution to the problem with the code for a Hex8 element needing to be significantly different and more complex than that of a Quad4 and with the potential for the most complex

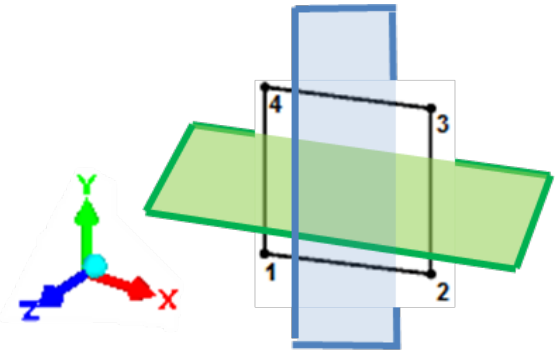


(a) Dividing a Quad4 along planes to establish each node as a corner point

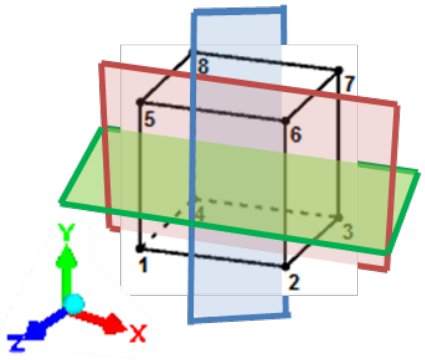


(b) Skew in bridge model elements resulting in rejection of the model by LISA

Figure 11: Incorrectly sorted elements arising from failure of traversal routine for skewed elements



(a) Dividing a Quad4 along planes to establish each node as a corner point



(b) Dividing a Hex8 along planes to establish each node as a corner point

Figure 12: Splitting Element points using x, y and z planes in order to perform ordering for LISA

FE element types such as wedge15, hex20, and pyr13 requiring implementations with even greater number of plane divisions and groupings in order to successfully identify every node.

Having already devised two solutions it seemed likely that there would be some body of research surrounding the problem worth investigating. with research leading to a set of possible alternaives known as convex hull algorithms. As the name suggests the goal of a convex hull algorithm is to generate a convex hull, convex hulls have several definitions but the simplest of these as described by [22] is for a set of points in some space a subset S of those points is convex if for any two points P and Q inside S the line between the two should also be inside S. This is directly applicable in the case of quad4 elements where the LISA sort order is the convex hull of the points, in the case of more complex elements the algorithm can be applied repeatedly to different faced divided though plane splitting before sorting the nodes at the end with knowledge of node ordering within each individual face.

Having considered several convex hull algorithms including Graham scan [21] $O(n\ log\ n)$ and brute force scan $O(n^4)$ [22] [23] before deciding to trial the following C# implementation of the Monotone Chain algorithm,

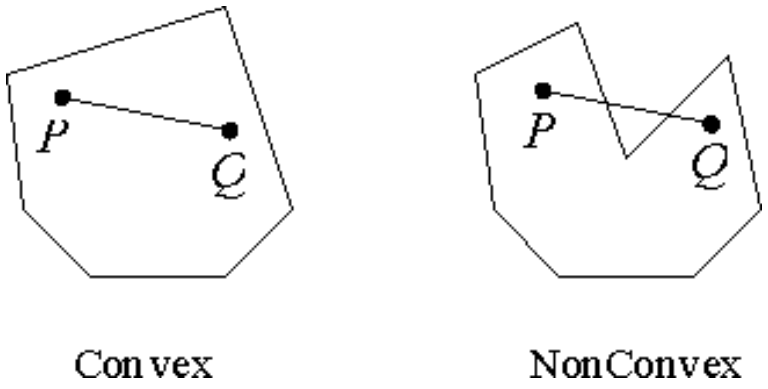


Figure 13: Illustration of convex hull definition, imace source: [22]

also $O(n\ log\ n)$ [35].

The Monotone Chain algorithm algorithm was developed shortly after Graham scan and builds upon the concepts introduced in the former. Graham's scan works by initially finding the point in the data set with the lowest y coordinate which can be called P. Having found this point the other points in the set are sorted based on the angle created between them and P. Combining both these steps gives a complexity of $O(n\ log\ n)$, with $O(n)$ to find P and $O(n\ log\ n)$ to perform a general sort of the angles. Moving through each point in the sorted array Graham scan determines whether moving to this point results in making a right or left hand turn based on the two previous points. If a right turn is made then the second do last point has caused a concave shape which has violated the requirement of the convex hull path. In this scenario is repeated the algorithm excludes the point from the convex set and resumes with the previous two points being those on the path before the rejected point. A stack structure is therefore typically used to keep track of the point ordering as is the case with the Monotone Chain implementation within the system [33].

The Monotone Chain algorithm performs essentially the same procedure however instead of sorting using simply y values Monoton Chain sorts using both x and y values. This allows the algorithm to sort the points in two separate groups which form the top and the bottom of the hull and a reduction in the complexity of the sort comparison function.

```
Sort the points of P by x-coordinate (in case of a tie, sort by y-coordinate);
Make two empty lists I and L Lists hold vertices of upper and lower hull;
while i = 1; i < n; i++ do
    while L contains at least two points and the sequence of last two points of L and the point P[i]
     does not make a counter clockwise turn do
        Remove the last Point from L;
    end
end
while i = n; i > 1; i- - do
    while L contains at least two points and the sequence of last two points of L and the point P[i]
     does not make a counter clockwise turn do
        Remove the last Point from U;
    end
end
Remove the last point of each list (it's the same as the first point of the other list).;
Concatenate L and U to obtain the convex hull of P.;
Points in the result will be listed in counter-clockwise order.;
```

Algorithm 2: Monotone Chain algorithm for generating convex hull, pseudocode description credit: [34]

The additional complexity of implementing a 3D convex hull algorithm meant it was much easier to experiment the with the approach as a potential solution to the problem using a 2D implementation by simply reducing the problem to a 2D equivalent. This was for quad4 elements by calculating the maximum delta between the max and min value on each axis and eliminating the axis with the smallest delta. These new 2D points could be given to the algorithm which when used in conjunction with the approach already taken was able to solve all instances of node ordering within the models. The only instances in which this approach failed were where highly elements would lie on a perfectly diagonal plane resulting in two axis of elimination using this method. This problem was avoid however by using the basic traversal to sort these elements.

6.7.3 Attempts to Automatically Define Edges Within Models

As discussed under evaluation another central faced was identifying where the system behaved poorly as a result of weakness in the underlying methods or poor design and implementation as opposed to poor edge definitions as input for the heuristic method. One way to avoid this issue of evaluation would be to remove all user intervention from the process besides the configuration of the initial model.
The obvious way by which to do this was automatic identification of interesting edges which would normally be the responsibility of the trained engineer. Such an approach initially seems promising since the quality of edges as with the mesh can be judged using a small number of key properties. For example it is known that edge importance directly correlates with the size of the edge and how much force is applied near it, since this information exists within the data model it should be possible to identify edges from it and generate them automatically for Dolsaks rules to process.

In practice there were multiple complications surrounding this, several of these arise from ambiguity in Dolsaks paper regarding what constitutes a an edge which is for example "long" or "important". As a result edges are only able to be defined to the extent that a user has confidence in their understanding of these concepts.

Assuming it is possible to generate a system capable of producing edges of interest for a model the problem of evaluating system correctness persists. Bad results then indicate that one of the two systems is broken but provide no clear indication as to which one is.

Having considered a method using these properties of the mesh several days were then spent attempting to implement it with poor success achieved.

7 Evaluation of Project

Evaluation of the project consisted of multiple stages ranging from verification of the functional requirements through simple black box testing to evaluation of refined mesh quality using methods provided by Dittmer [14].

7.1 Validation Against Functional Requirements

In order to validate the system against many of the functional requirements the system only needs to be run on several basic models with different input configurations.

The output clearly demonstrates the systems ability to evaluate the quality of meshes using a range of metrics and refinement occurring as a result of both the stresses induced by the user and on their categorisation of edges.

7.2 Validation Against Non Functional Requirements

Validation of non functional requirements was made simple due to the limited number of them, this was partly a consequence of the system not being designed for a specific user base resulting in expectations regarding the systems design to improve usability and guide interaction. It was also not possible to define the general accuracy and performance of the system during the requirement elicitation phase since this could only be determined once the required research and trial on the finished system gave indication to both of these.

In the case of quality for the systems design and documentation evidence is present to indicate that this adheres to the requirements specified with the project submission containing detailed documentation in the form of a Deoxygen guide and sophisticated use of object oriented and functional software design as seen within the codebase. General applicability of functionality has also been demonstrated through evaluation using a variety of both models and conditions when performing simulations.

7.3 Unit Testing

Holistic evaluation provided evidence of the overall systems effectiveness however without verification of individual components it would not have been possible to assert the accuracy of the results produced. Unit testing was also conducted from within Visual Studio using the NUnit framework and structured as a separate VS project. This Guaranteed that the system was not able to interact with the tests and that testing was conducted through the class and function interfaces provided by the implemented solution. Tests were also grouped into classes with each test class corresponding approximately to one class within the system. Each test class then contains a number of test functions each of which performing the asserts necessary to deem its associated function as correct. This layout provided clear traceability from each item of function to its associated test making assessment of the test coverage much easier.

It was important when unit testing not only to write tests for the different aspects of system functionality.
The design and execution of the tests did manage to reveal a bug within the system .
Much of the software
this allowed test names to replicate their partner functions making the

7.4 Software Quality and Management

The quality of the design and implementation of the system reflects my experience not only as a computer science undergraduate but as a developer with one year industrial experience, although not directly effecting the execution of the program properties such as appropriate variable naming, loose coupling of classes, use of abstractions and descriptive error messages make the software easier to read and debug for any potential

future developers.

Visual Studio also enabled calculation of various software quality metrics for the code base automatically. This made selecting parts of the codebase for refactoring much easier when time was allocated for this. Upon completion of the project the average maintainability index across all modules was 75 with the lowest score for any high level module being 60 and the highest 92. According to the Microsoft Developer Network (MSDN) website code with an index of between 0 and 9 indicates low maintainability, 10 to 19 indicates moderately maintainable and 20 to 100 high maintainabilty [42].

In order to ensure progress was responsibly backed up and new features easily managed a private Github repository was set up and all progress made to the project pushed every couple of days. This proved invaluable on at least one occasion where a bug was accidentally introduced and despite efforts could not be removed manually.

7.5 Documentation

The process of continuously writing descriptive documentation was important to the success of the project and was treated as an integral part to meeting the goals of the project development methodology which aimed to reduce the systems complexity and improve readability. Through the writing Doc comments corresponding to every function within the codebase it was possible to generate documentation files automatically through use of the tool Doxygen. This allows anyone with the solution to view descriptions of each of its functions either in the codebase or alternatively through the manual produced automatically by Doxygen.

7.6 Increase in performance through parallel execution

The following

The average speed up (Time of serial execution/ Time of parallel execution) was

The Efficiency of the parallel execution (speedup / number of processors) was

7.7 Evaluation Of System For Model Simulations

Several models have been created resembling basic equivalents of that used for real engineering applications.

do this reliably and validate the heuristic component against the results obtained in academic papers two of the three models were based on those previously used by Dolsak (Paper mill and Cylinder structures). In addition to these two a suspension bridge model was developed to demonstrate the systems capacity to work beyond Dolsaks basic validation examples.

These initial models were constructed manually using LISA's graphical user interface using measurements by Dolsak and in the case of the suspension bridge from documents available on the web [4] [?].

7.7.1 Suspension Bridge structure

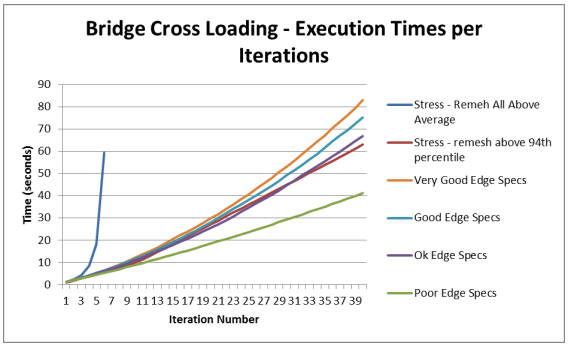
The main model used for testing the methods was a suspension bridge structure. The bridge seemed like a good model to perform initial experimentation on since its structure is

Constructed manually out of Quad4 LISA elements it has four key constraint points at the base of each supporting column. The bridge itself initially consists of 196 elements and 212 nodes before remeshing, which is considered coarse for such an FE structure.

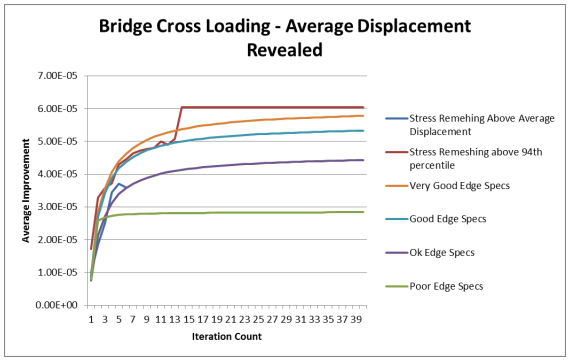
When testing the system with the suspension bridge model the first step was to test each method independently of the other in order to evaluate both approaches before attempting to combine the two. Testing each method individually with multiple configurations provided a better understanding of the variation within

each allowing more accurate high level reasoning about the results of a hybrid.

Variation of inputs was particularly important when testing Heuristic method as experimentation revealed a high degree of variation in the results both in terms of runtime and the methods ability to reveal high stress or displacement areas, this can be seen looking at the graph in Figure 14 below and at the meshing results for the structure in appendix F. Four main tests were run for the heuristic approach with each test having a different edge specification file of some level of a designated level of quality. With the very good and good edge specifications I made sure to run the model first using stress refinement and then observe areas of high displacement, this allowed specification of edges around areas I knew in advance would benefit from being specified. The idea being a more experienced FE engineer would be able to do this without having to run the stress refinement first. In addition to the two good quality edge specification sets an OK one was also produced where some edges were specified as existing over the highest stress area while others focused mainly on the lower stress areas, and finally a set of poor edges none of which overlapped the high stress area.



(a) Execution time in seconds for each method to reach 40 refinement iterations



(b) Average amount of displacement and thus stress revealed by each approach as number of iterations increases

Figure 14: Execution time increase compared to the amount of information revealed for the different approaches

For stress refinement the varied parameter was the threshold used to determine whether elements were considered under high stress and should be further refined. The main consequence of varying this parameter was change in both the node count and the programs runtime as can be seen in Figure 14a. Despite time complexity for subdivision being $O(n^2)$ for all methods using a low threshold results in large values of n and consequently a rapid increase in both runtime and element count.

In order to perform a reasonable comparison between both the stress and heuristic refinement methods it was important to specify the threshold such that approximately equal amounts were performed by each approach per unit of weighting for each iteration (see combining methods under System Design), thus allowing each method to be evaluated on its merit to select elements appropriately and a hybrid method through its designated weightings. To do this the increase in element count was tracked for the different heuristic methods, as can be seen in Figure 10. The average increase in elements per iteration could then be calculated as 6% of the total number of elements within the model. This meant stress refinement could be reasonably compared to heuristic refinement if for each use of the refinement it only re meshed those elements considered to be above the 94th percentile in terms of stress.

Executing the bridge model for a variety of hybrid weightings revealed the following general trends in the amount of stress discovered per iteration:

Looking at the data and considering the distribution of stress present within the bridge model it is clear

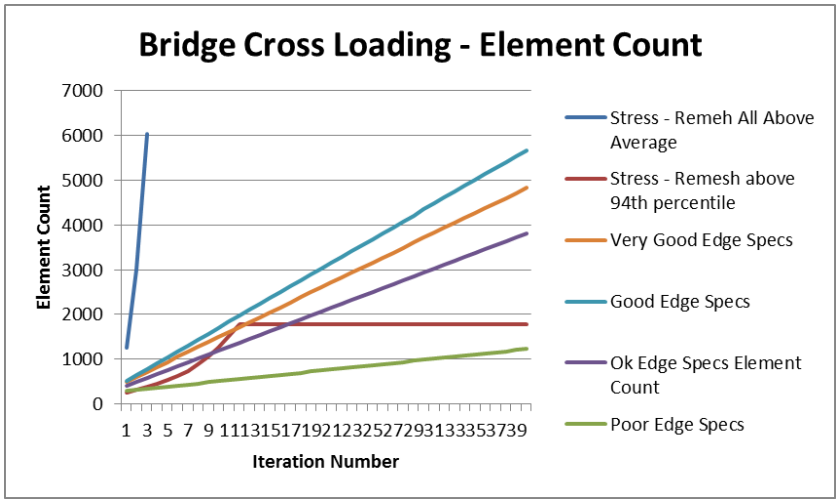


Figure 15: Increase in element count for the various refinement methods over forty iterations

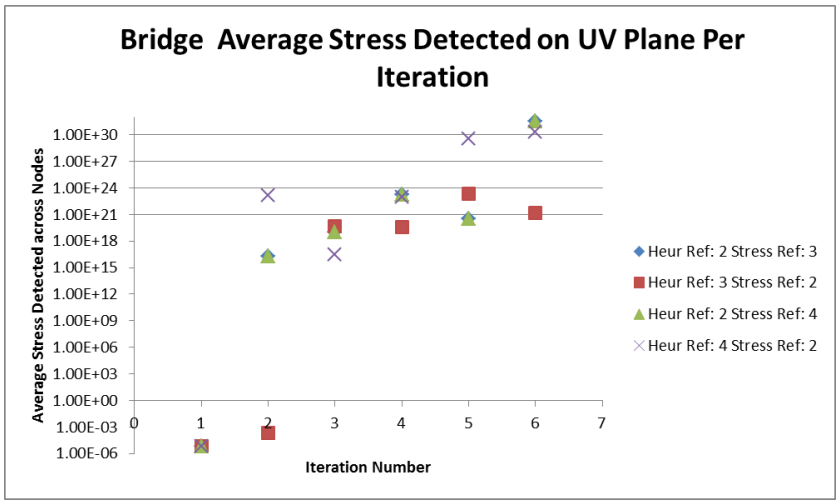
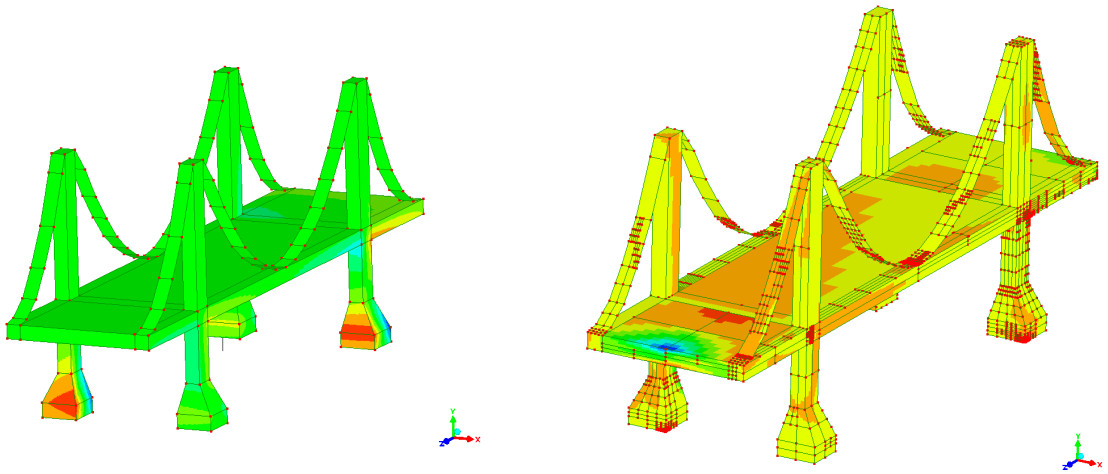


Figure 16: Increase in element count for the various refinement methods over forty iterations

that the system very quickly identifies areas under very high stress and continues to focus meshing on those areas. This results in hugely exponential increase in the average stress covered by the elements in the model after only two iterations. This rapid increase in the average across the elements shows that both methods if effectively used are able to focus quickly on the highly specific areas of the structure that are weakest.

Comparing the amount of improvement performed by both the stress and heuristic refinement processes with varying weightings that in general heuristics performed better than stress refinement although interestingly more weighting did not necessarily correlate to better results as can be be seen with heuristic weighting of two doing a better job than weighting four in the final iteration.

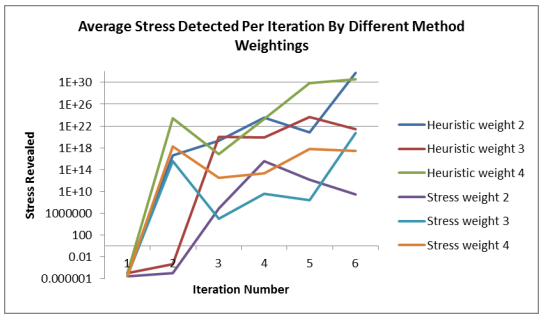
Effective comparison of the two methods indicates that the system works as a tool by which comparisons of different refinement methods can effectively be made.



(a) Stress Revealed through the initial highly coarse bridge mesh without running any iterations for either method

(b) Stress revealed within model after just 4 iterations with a heuristic stress method weighting of 3-2, edge heuristics also considered good in this case

Figure 17: Execution time increase compared to the amount of information revealed for the different approaches



Per Iteration By Different Method Weightings.png

Figure 18: Increase in element count for the various refinement methods over forty iterations

7.7.2 Evaluation Issues

A significant issue faced in attempting to demonstrate the effectiveness of the system was to provide an indication of how well the system worked without taking into account the ability of the user who may be providing the edge rules for a particular model. Not taking this into account would result in an inaccurate representation of its ability.

7.8 Strengths and Weaknesses

The resulting system successfully satisfied both the functional and non functional requirements in addition to providing insights into the possibilities of a hybrid technique for effective finite element meshing, something that was optimistic at the start of the project but highly desirable. The project was well managed with all of the objectives being delivered as per the initial time plan. Quality was also maintained throughout the project by the application of good software engineering practice.

One of the great strengths of the finished system was its modular architecture which allows for a great amount of potential extendibility in the future. Although little focus has so far been given to the systems

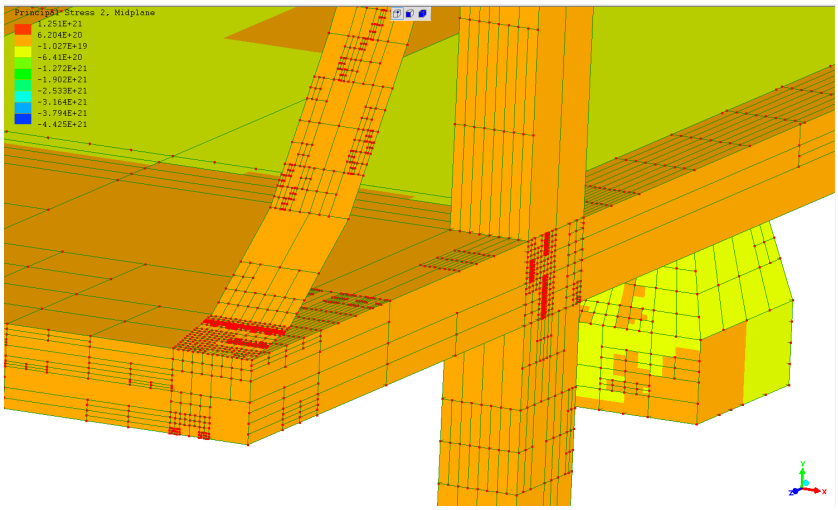


Figure 19: Increase in element count for the various refinement methods over forty iterations

usability it could be developed and distributed as a public tool for experimentation with hybrid meshing with limited additional effort.

Another strength of the system is its ability to accept any heuristic definition in terms of edges within a mesh structure. Theoretically this means the final system is also capable of using the same types of edge specifications for any type of FE analysis such as fluid flow or heat transfer given a corresponding rule set by which to mesh with.

A downside of the current design is the need for the user to manually specify the edges by the user directly into the JSON input file which is both time consuming and prone to error despite the relatively small size of the models analysed in this dissertation. Comparing the size of these with those used in industry it is clear that this process is simply not practical for engineers conducting FE analysis. To change this better tools are required that will allow engineers to automatically generate edge specifications quickly, most likely through some GUI or a bespoke high level language capable of combining knowledge about the mesh structure and different types of edges to generate specific rules. Again this is beyond the scope of the project and would likely be a dissertation in its own right.

Although the system had a strong subsystem and class level architecture many of its weaknesses could be attributed to needing to prioritise the ability to perform rapid prototyping over efficient implementation of the various algorithms and methods described in this dissertation. Much of this a consequence of overusing the functional programming capabilities within the C# LINQ library. Widespread adoption of functional programming practices was stated as a desirable aspect of the final system implementation within the non-functional requirements. This has largely been adhered to with higher order and lambda functions widespread throughout the codebase. In the later stages of the project it became apparent however that in many cases reliance on these features resulted in reduced readability and performance for many of underlying algorithms described in this dissertation.

7.9 Evaluation summary

8 Further Work

This section details some areas which given additional time to work on the project would at the very least be investigated, if not implemented. Each of these areas would hopefully provide some benefit in assisting to demonstrate the possibilities of hybrid methods.

8.1 Gathering feedback from experienced engineers

Approaching the end of the project it became clear that in order to better identify the systems strengths and weaknesses would require additional user testing by engineers who have experience conducting this type of analysis. Despite a lack of available time obtaining feedback from engineers with applied industrial experience along with that of academics this would have allowed for a more conclusive analysis of the systems and its ability to work across a greater number of general case scenarios. User feedback was largely not obtained within the duration of the project as a result of time constraints and the complexity inherent in simply implementing and validating the project for a selection of basic models. As such even if time had been available the ethical clearance required to collect user feedback at the start was not obtained.

8.2 Improving Usability Through A Web Interface

Although it would have been possible to visit various engineers in order to conduct feedback the process would have been both time consuming on my part and inconvenient for the participant as a time at which to meet must be scheduled, also a laptop containing the working software would need to be brought to them on which they must design or transfer their model to before running it multiple times to obtain results. This scenario is at best inconvenient for the participants and pressures them into arriving at a conclusion within a relatively short period of time after starting to experiment with it.

An alternative approach would be to develop a Web Interface so as to allow users to interact with the system in a more efficient manner.The system and allow engineers to submit feedback digitally and allow feedback to be obtained and aggregated from a much wider range of different sources separated by significant geographic distance.

To use the web interface an engineer would simply need to submit a model they have already created along with a JSON file containing edges they have designated as important for their model. LISA supports imports from multiple CAD formats including the Standard for the Exchange of Product model data (STEP) and Initial Graphics Exchange Specification (IGES)) [19]. Upon receiving the request the web server would run the system using their input data and having finished allow them to download the re meshed model along with the calculated stress data for analysis.

9 Personal Reflections and Summary

Overall I felt the project was a success. The final software solution was evidently capable of facilitating execution of the specified functions and therefore allowed comparison to be made of both heuristic and stress based mesh refinement techniques. Not only did the final system allow this comparison for the two specific approaches selected, it provided a flexible framework allowing experimentation for hybrid meshing using any two meshing approaches.

From my perspective I wanted to use this project as an opportunity to improve my understanding of a technology that I previously had limited knowledge of through its use on my industrial placement year. My prior experience with FE analysis was very much confined to that of a typical engineer making use of the method through a licensed desktop application with many of the technicalities that are of most interest to a computer scientist hidden. I therefore found the project highly enjoyable as an opportunity to learn

more about the underlying processes through both research and practical experimentation. As a means of facilitating my personal learning as an individual I therefore also consider the project a success.
Despite working on larger software projects during my year within industry this was certainly the most complex I have undertaken as an individual. As the lead software developer on my own project I encountered many challenges which as a junior developer within industry were not my responsibility but which I observed team leaders and senior developers encountering regularly. Such tasks were those that required high level analysis of the systems purpose and design in order to continuously steer the project in the right direction. Only through doing this was it possible to ensure successful delivery within the specified time scales. In many ways the project therefore provided me with a greater appreciation of the difficulties associated with delivering a software project in its entirety.
Research and evaluation phases were also particularly difficult. Upon completion of the project I came to realise this was largely due to my lack of formal education in mechanical engineering which meant I had to work a lot harder both to understand the initial problems associated with the methods and subsequently to correctly evaluate the results I obtained. Had I chosen a more traditional computer science topic I believe both of these aspects would have been much easier. The increasing scope of the project as each aspect revealed ever more complex challenges meant it became an ever growing difficulty to understanding each of the sub topics required in order to implement the different aspects of the project. To add to this issue much of the academic literature surrounding finite element approaches transpired to be highly bespoke for particular scenarios and appeared too complicated for anyone with less than a postdoctoral level of understanding of the topic.

Having completed the research too much time was then spent concerned with the specifics of the implementation, much aof which was associated with integrating the functionality of LISA into my system. Although LISAs simplicity was its great strength and helped in simplifying many of the initial design and testing aspects of the project its lack of an extensive API resulted in a large amount of the projects time being focused towards system integration issues which were not apparent in the system design and research stages. Although these problems such as element sorting and data modelling proved interesting challenges. Solving them was considerably more time consuming than would have initially been predicted and thus reducing the amount of time that could be directed towards the other components. Given the change to repeat the project and having learnt a lot about the design of finite element programs I feel I would be better placed to evaluate a greater range of potential choices so as to select an alternative which would have come with a comprehensive API out of the box.

Throughout the majority of the project I felt organisation of time and planning of activities was done well. Work on the project began early with the goal of easing pressure in the later stages and work continued despite deadlines for coursework associated with other modules. A crucial mistake made was to reduce effort two months before the deadline having completed the software implementation and written much of the initial sections of the dissertation despite not completing evaluation of the results. Evaluation of the software turned out to be substantially more time consuming than expected which created additional stresses towards the deadline.

By contrast I found the system design and programming components
planning and research components
managerial architectural aspects which I had not previously had that great an involvement with

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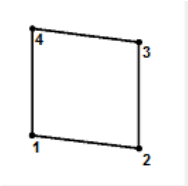
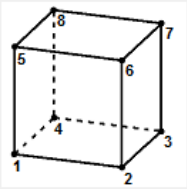
A Element Types within LISA

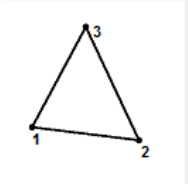
Here are shown the the visual specifications LISA provides for the ordering and layout of nodes for defining each type of element supported. Each of these types can be classified using the



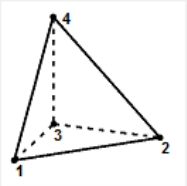
(a) quad4 element



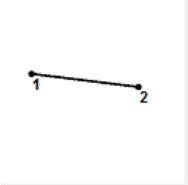
(b) hex8 element



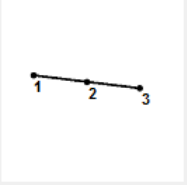
(a) Specification for node ordering of tri3 element within LISA



(b) tet4 element



(a) line2 element



(b) line3 element

B Unit Testing

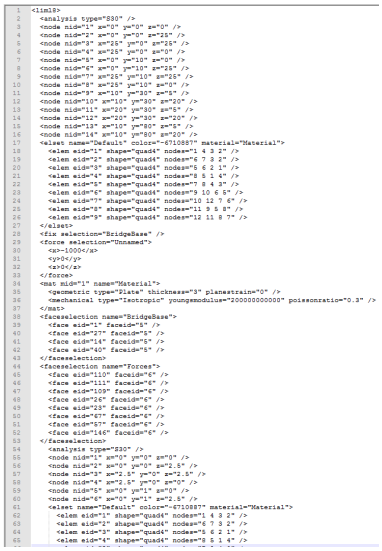
[41]

C Input and output files

Below can be seen the format of the input files for the system, a LISA .liml and a .json edge definition file

D Project Layout in Solution Explorer

Below show the Visual Studio Solution Explorer which provides a general idea of the layout of the project with namespace hierarchies from within an IDE.



(a) Cut down .liml file to show general content which largely defined the schema for the systems data model

(b) A json file containing the edges of interest specified by an engineer, this is parsed and the rules are applied to determine the models meshing based on the input

E Software Quality Metrics

F Bridge Cross Loading Simulation Results

Below can be seen the results of executing both the heuristics and the stress refinement method on the suspension bridge model when loads are placed against the base of the bridge in the negative x direction.

F.1 Heuristic Refinement

G Stress Refinement

H Bridge Base Loading Simulation Results

I Paper Mill Simulation Results

J Gantt Chart for Project Time Management

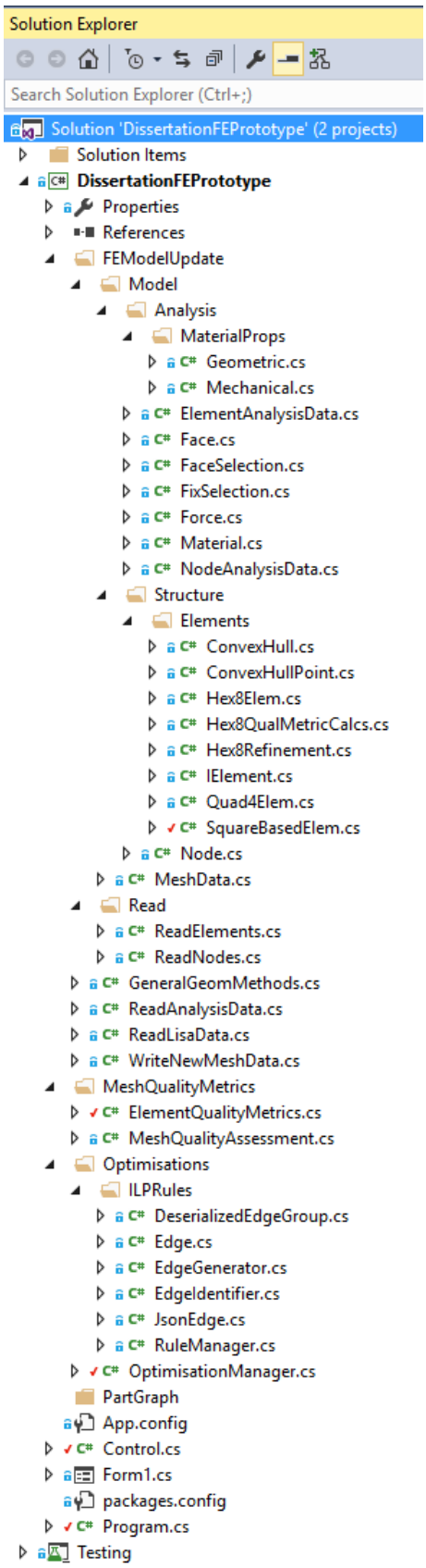
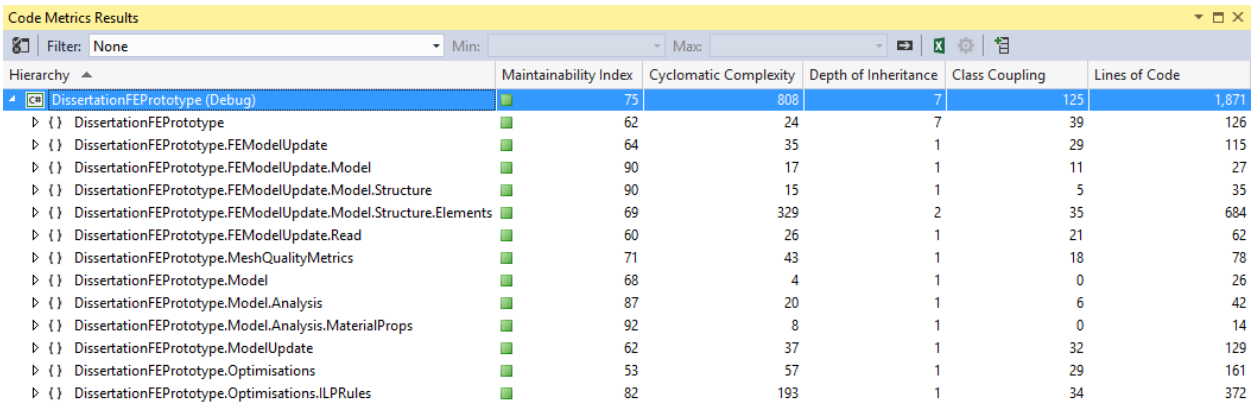
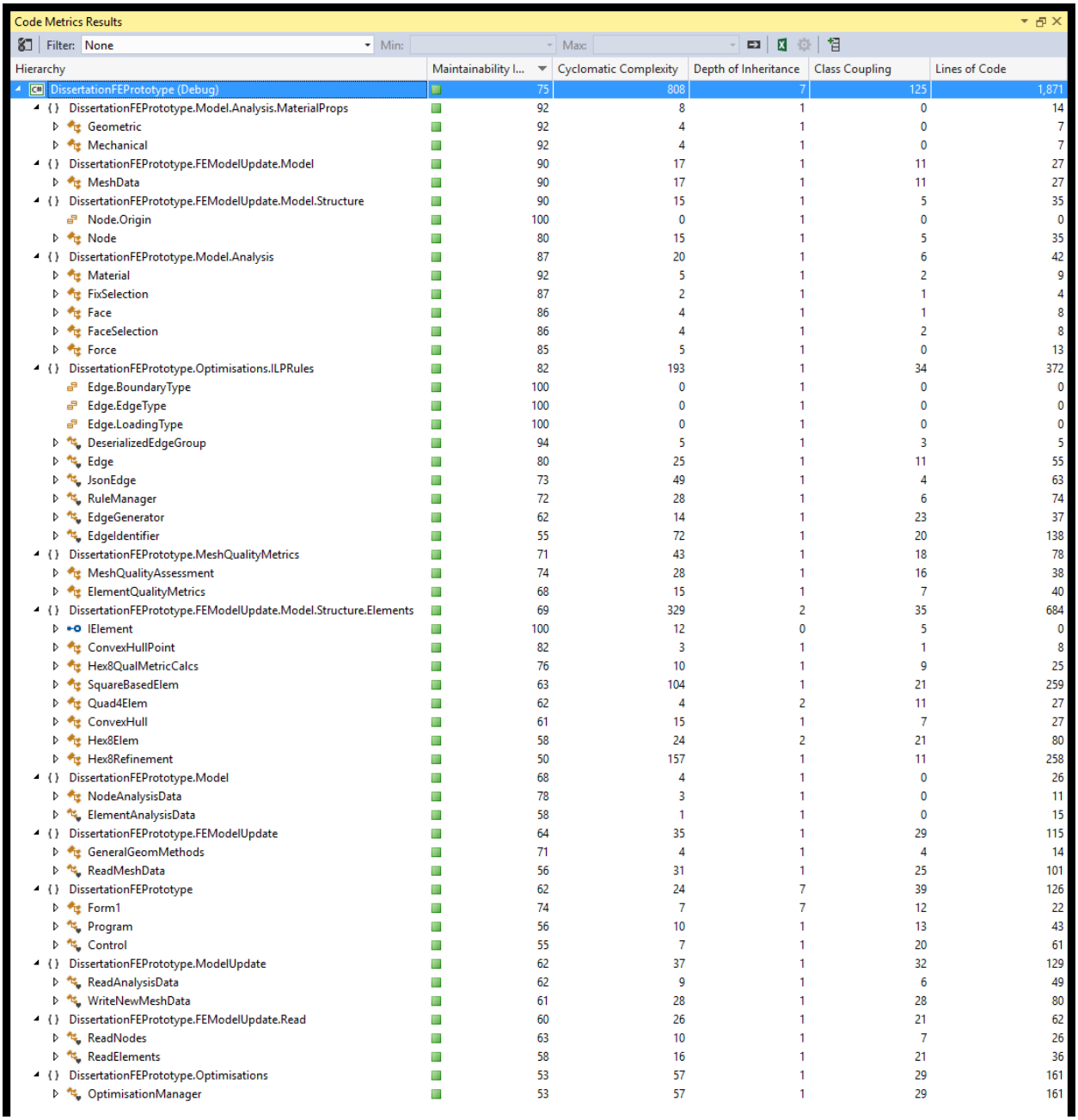


Figure 24: The metrics calculated by visual studio for all high level modules in the system



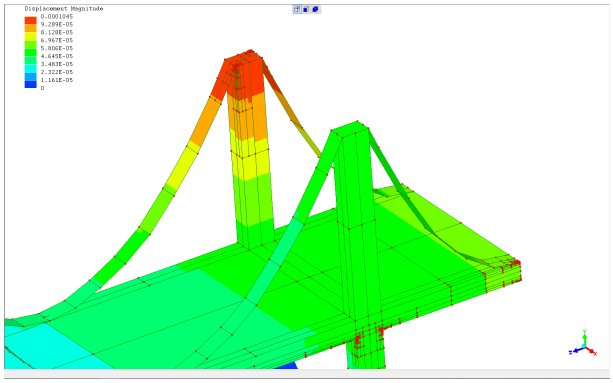
Hierarchy	Maintainability Index	Cyclomatic Complexity	Depth of Inheritance	Class Coupling	Lines of Code
DissertationFEPrototype (Debug)	75	808	7	125	1,871
{ } DissertationFEPrototype	62	24	7	39	126
{ } DissertationFEPrototype.FEModelUpdate	64	35	1	29	115
{ } DissertationFEPrototype.FEModelUpdate.Model	90	17	1	11	27
{ } DissertationFEPrototype.FEModelUpdate.Model.Structure	90	15	1	5	35
{ } DissertationFEPrototype.FEModelUpdate.Model.Structure.Elements	69	329	2	35	684
{ } DissertationFEPrototype.FEModelUpdate.Read	60	26	1	21	62
{ } DissertationFEPrototype.MeshQualityMetrics	71	43	1	18	78
{ } DissertationFEPrototype.Model	68	4	1	0	26
{ } DissertationFEPrototype.Model.Analysis	87	20	1	6	42
{ } DissertationFEPrototype.Model.Analysis.MaterialProps	92	8	1	0	14
{ } DissertationFEPrototype.ModelUpdate	62	37	1	32	129
{ } DissertationFEPrototype.Optimisations	53	57	1	29	161
{ } DissertationFEPrototype.Optimisations.ILPRules	82	193	1	34	372

Figure 25: The metrics calculated by visual studio for all high level modules in the system

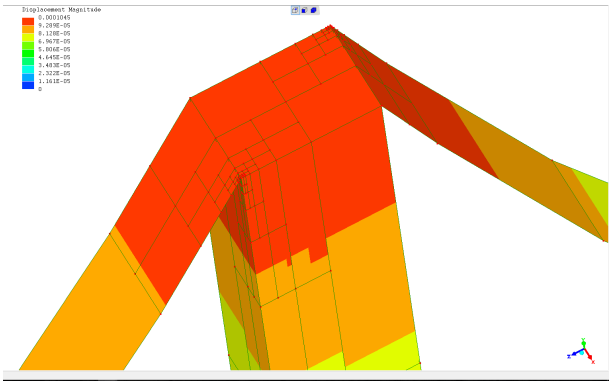


Hierarchy	Maintainability I...	Cyclomatic Complexity	Depth of Inheritance	Class Coupling	Lines of Code
DissertationFEPrototype (Debug)	75	808	7	125	1,871
DissertationFEPrototype.Model.Analysis.MaterialProps	92	8	1	0	14
Geometric	92	4	1	0	7
Mechanical	92	4	1	0	7
DissertationFEPrototype.FEModelUpdate.Model	90	17	1	11	27
MeshData	90	17	1	11	27
DissertationFEPrototype.FEModelUpdate.Model.Structure	90	15	1	5	35
Node.Origin	100	0	1	0	0
Node	80	15	1	5	35
DissertationFEPrototype.Model.Analysis	87	20	1	6	42
Material	92	5	1	2	9
FixSelection	87	2	1	1	4
Face	86	4	1	1	8
FaceSelection	86	4	1	2	8
Force	85	5	1	0	13
DissertationFEPrototype.Optimisations.ILPRules	82	193	1	34	372
Edge.BoundaryType	100	0	1	0	0
Edge.EdgeType	100	0	1	0	0
Edge.LoadingType	100	0	1	0	0
DeserializedEdgeGroup	94	5	1	3	5
Edge	80	25	1	11	55
JsonEdge	73	49	1	4	63
RuleManager	72	28	1	6	74
EdgeGenerator	62	14	1	23	37
EdgeIdentifier	55	72	1	20	138
DissertationFEPrototype.MeshQualityMetrics	71	43	1	18	78
MeshQualityAssessment	74	28	1	16	38
ElementQualityMetrics	68	15	1	7	40
DissertationFEPrototype.FEModelUpdate.Model.Structure.Elements	69	329	2	35	684
IElement	100	12	0	5	0
ConvexHullPoint	82	3	1	1	8
Hex8QualMetricCalcs	76	10	1	9	25
SquareBasedElem	63	104	1	21	259
Quad4Elem	62	4	2	11	27
ConvexHull	61	15	1	7	27
Hex8Elem	58	24	2	21	80
Hex8Refinement	50	157	1	11	258
DissertationFEPrototype.Model	68	4	1	0	26
NodeAnalysisData	78	3	1	0	11
ElementAnalysisData	58	1	1	0	15
DissertationFEPrototype.FEModelUpdate	64	35	1	29	115
GeneralGeomMethods	71	4	1	4	14
ReadMeshData	56	31	1	25	101
DissertationFEPrototype	62	24	7	39	126
Form1	74	7	7	12	22
Program	56	10	1	13	43
Control	55	7	1	20	61
DissertationFEPrototype.ModelUpdate	62	37	1	32	129
ReadAnalysisData	62	9	1	6	49
WriteNewMeshData	61	28	1	28	80
DissertationFEPrototype.FEModelUpdate.Read	60	26	1	21	62
ReadNodes	63	10	1	7	26
ReadElements	58	16	1	21	36
DissertationFEPrototype.Optimisations	53	57	1	29	161
OptimisationManager	53	57	1	29	161

Figure 26: The metrics calculated by visual studio for the all classes in the final system



(a) Important edges specified effectively to facilitate pre-emptive meshing of area which undergoes high displacement and thus stress



(b) Close up view of refinement for high displacement areas

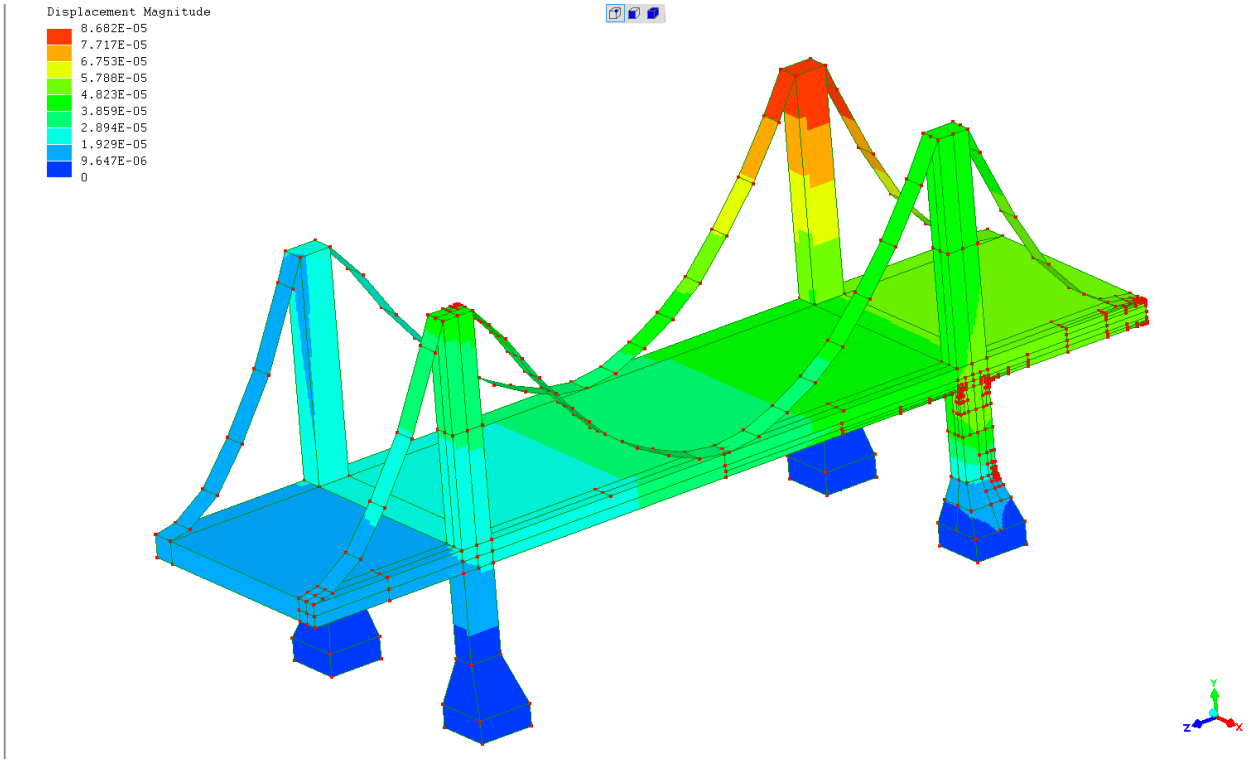


Figure 28: Important edges more poorly specified missing high displacement region on top of furthest suspension bridge tower

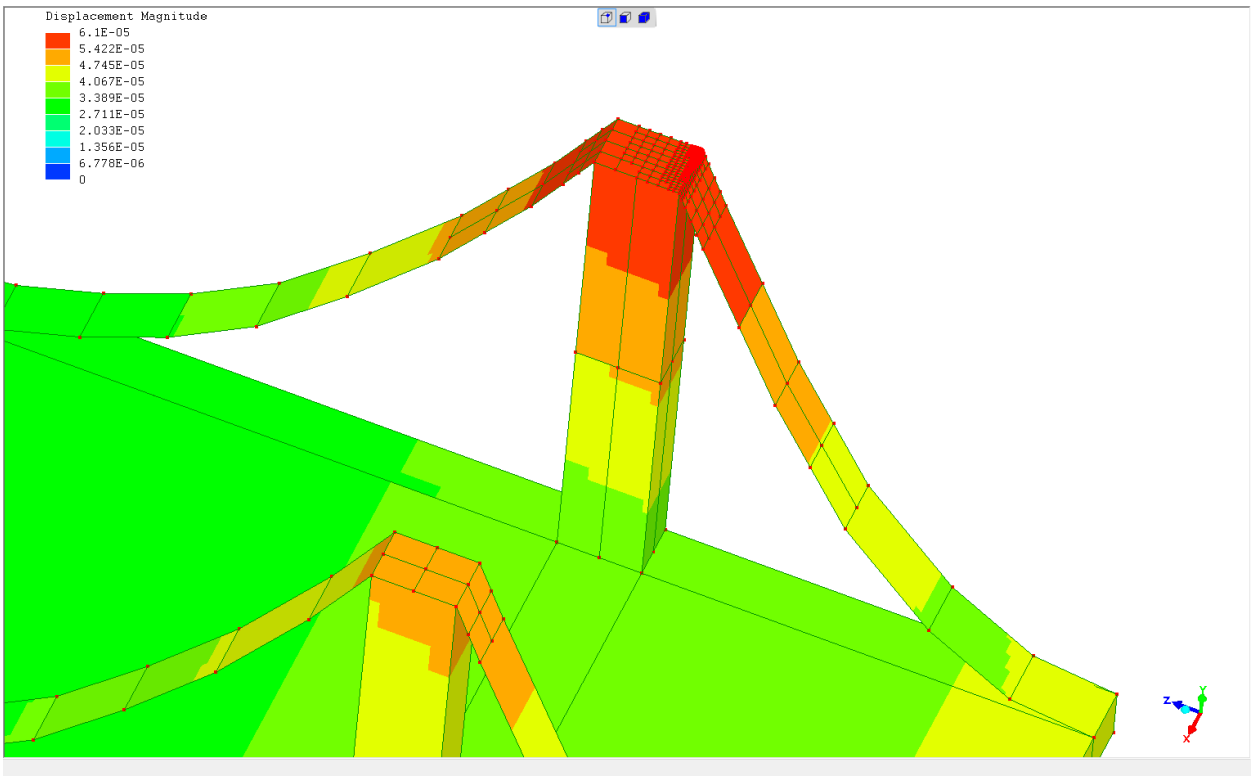


Figure 29: Iterative stress/ displacement refinement method used to focus meshing on the top 6% most displaced region of model

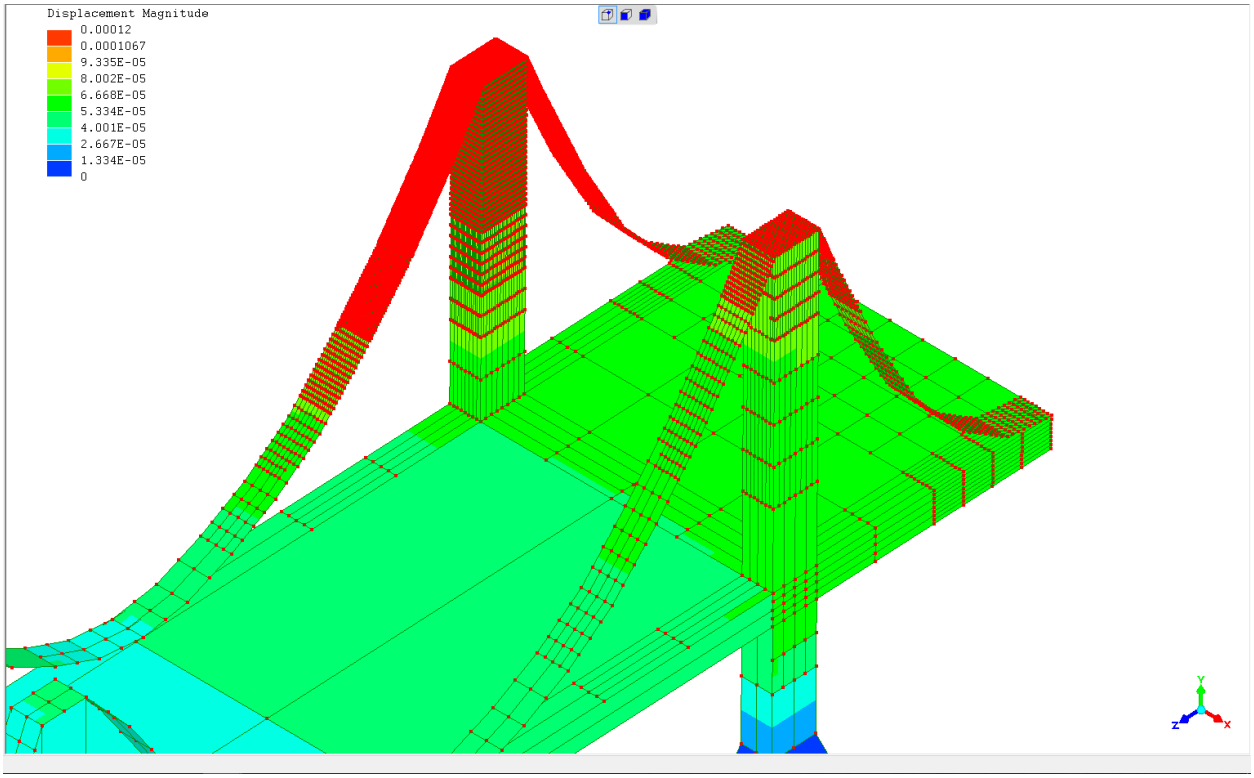


Figure 30: Iterative refinement of high displacement but with the remesh threshold specified as the average displacement across the whole model. A consequence of this is the gradient of refinement fidelity that can be seen corresponding to the importance of that part of the structure

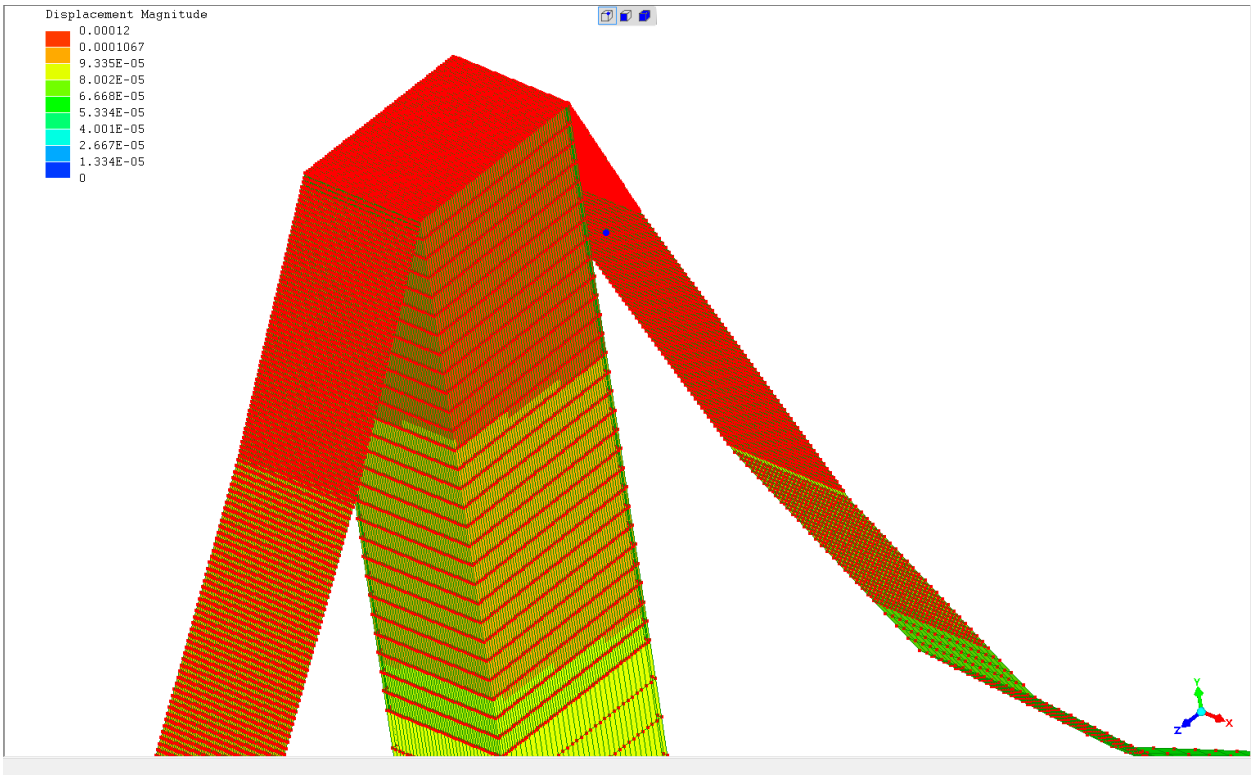
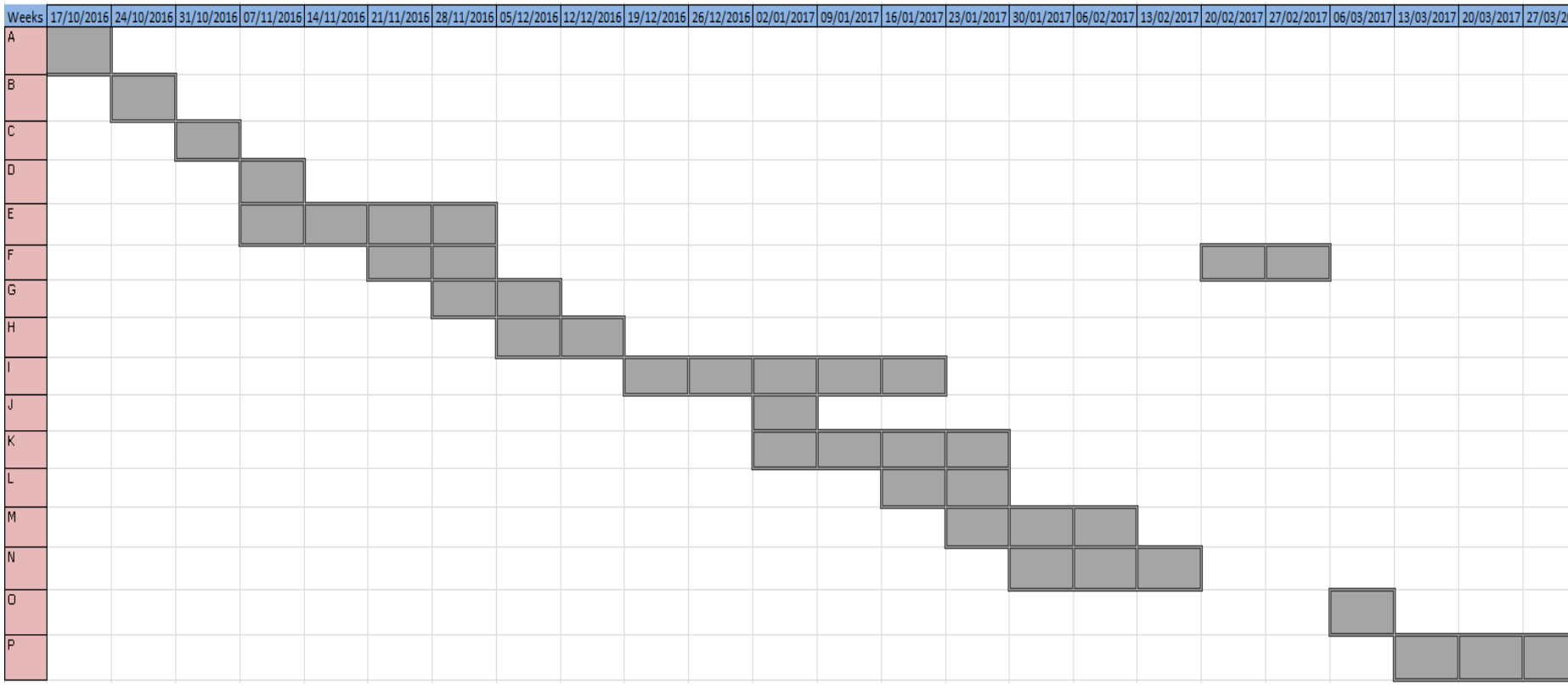


Figure 31: The metrics calculated by visual studio for the all classes in the final system


Weeks
17/10/2016
24/10/2016
31/10/2016
07/11/2016
14/11/2016
21/11/2016
28/11/2016
05/12/2016
12/12/2016
19/12/2016
26/12/2016
02/01/2017
09/01/2017
16/01/2017
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30/01/2017
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20/02/2017
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