Dissertation: Hybrid Methods for Finite Element Meshing

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Contents

1 Introduction

The overall aim of this project is to perform the task of refining a Finite Element Mesh (FEM) using a stress based method as is typical of FEM refinement in conjunction with an approach that uses techniques from Artificial Intelligence and Machine Learning. It should be possible to reason about the quality of a mesh produced using both methods analytically in order to evaluate the success of the approach and potential for continued use.

Finite Element Analysis (FEA) is a method widely used across different engineering domains to simulate structures under certain conditions. The method works by taking a geometry defined in continuous terms, discretize it into a mesh system before calculating the property values for each of the discrete elements. This allows engineers to observe the effect the conditions have on the entire structure, see figure 1.

Success of the project will be determined by implementation of a system that is able to combine the two approaches described above in order to refine a mesh to of a quality comparable to that of a successful stress based method.

2 Motivation and Background

Over the past forty years FEA has emerged as a prominent technology for simulating complex real world engineering problems [1, 5]. FEA works by solving a system of differential equations with each equation representing a single element in a geometric mesh. By doing this FEA is able to generate highly accurate approximations for the properties of complex physical systems [5] [18]. The method can also be highly computationally expensive with the complexity typically increasing exponentially with the model size [5]. Analysis therefore proves to be highly time consuming and costly for individuals and organisations to conduct [7]. [16]

2.1 Properties of finite element models

Although this is a computer science as opposed to a mechanical engineering dissertation it's still important to briefly outline the general principals and properties that underpin the FE method in order to have a general appreciation for how the design and evaluation of the final software system was conducted.

Finite Element Models have several key properties that need to be specified by the engineers who create them, the configuration of these properties greatly determine the results obtained from the models execution. The first of these properties that an FE model posesses is the mesh. The mesh is constructed out of nodes, points which act as intersections between the second componet- elements which are either a polygon or a polyhedron between the nodes. Nodes and elements are important concepts as they provide the theoretical framework for reasoning about the other properties of a mesh and hence the overall quality of the model [18].

In addition to the nodes and elements FE models also contain inputs and constraints. Inputs can be thought of as the phenomena from the outside world which is acting on the structure and consequently inducing some kind of physical effect on it. Inputs are used by engineers to model the conditions under which the structure will be expected to perform under when it is manufactured and enters operation.

Constraints are another fundamental concept that describe where the model is attached to the outside world. When computing stresses induced through the model there needs to be an area through which the stresses are assumed to leave. FEA is only able to calculate the path of the stresses through the model and thus the overall stress at given points using the law of conservation of energy i.e. energy cannot be created or destroyed meaning that any energy provided to the system as input through for example force needs a means by which to leave it, the constraint.

For example in figure 6 showing a suspension bridge model the simulation is to be run with the forces are induced upon the cables and the towers in the negative x direction as represented by the green vectors. The corresponding constraint area through which the force must leave is specified as the base of each bridge pillar and represented by multiple red arrows on each corresponding corner.

The final piece of information needed in order to calculate the stresses through the model is material data. Material data is associated with the models elements and is usually defined using two main parameters which are:

- Youngs Modulus - The ratio of stress over strain for a given material, i.e. for an amount of internal force endured by a material how much does it deform, a material such as rubber therefore has a low value for Young's modulus while diamond has a high value[29]
- Poissons ratio - Amount of deformation that occurs perpendicular to the force that is applied to the material.

For the sake of simplicity all structures used to evaluate the final software solution have assumed steel as their material property. Steel is a pretty common material used within manufacturing of many mechanical components and does not exhibit any abnormal properties. This is beneficial for evaluation as it removes variability in the results that could arise from selecting a more complex material.

2.2 Limitations and general considerations

An important consideration when conducting FEA is the trade-off of a models accuracy against the time in which it can be solved. A major variable determining this trade-off is the models mesh structure which discretizes the problem so that simulation can be run on it. A mesh that is finer is more computationally expensive but also produces results of greater accuracy. It is therefore desirable to generate a mesh which is fine where accuracy is most needed but coarse where it is not [17].

In every type of analysis that the FE method is used for (thermal, structural, fluid flow, electrical) there is a specific differential equation associated with each of the elements. In order to achieve overall convergence of the model the equations must be solved simultaneously to achieve a value for each of the discrete elements [18]. For this dissertation project attention will be given specifically to the problem of FEA meshing in the context of static structural analysis where the value calculated across each element is its stress. It makes sense to work on hybrid mesh refinement in the context of static structural analysis as it is likely the most common engineering application of the method and as such has the largest body of research that is relevant [5][18].

For engineers the value obtained through computing the stresses under a particular set of conditions is feedback on the quality of their design. Ideally the results from an analysis will provide a good understanding of where the design is weak and how concentrated this weakness is. This information is used to either help verify the designs' quality or alternatively inform changes to its geometry or material properties so as to reduce stress on subsequent analysis [20].

To understand the gradient of stress within a part of the model the mesh needs to be designed carefully. As each element can only display values calculated from its edge nodes a smooth gradient requires a higher concentration of elements in areas under higher stress. A high quality mesh is therefore considered to have a higher concentration of elements in areas of predicted high stress while retaining lower concentration elsewhere, thereby revealing weaknesses in the design while minimising the models runtime.

Traditionally the automated mesh refinement process consists of computing stresses for a model with an initial coarse mesh and low computation cost, once rough stresses have been computed the elements in areas of higher stress can be divided recursively into additional elements in order to achieve smoother gradients on further executions [16]. Figure 1 shows a mesh which has been refined in an area of higher stress thus providing a clearer indication of a components weakness.

Unfortunately for many large models this method for refining a mesh is still excessively costly [4]. It therefore seems worth investigating use of alternative approaches posed by the field of computer science and artificial intelligence that could support the traditional high stress meshing approach.

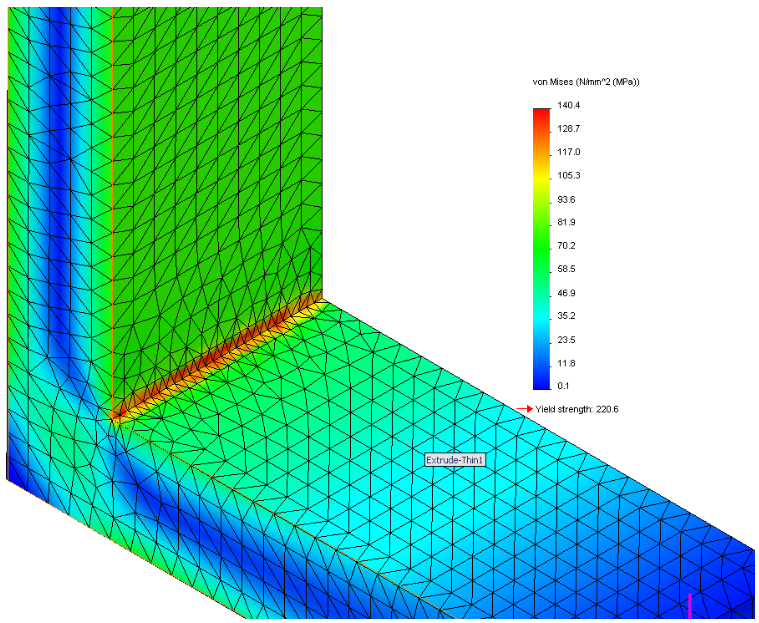


Figure 1: Mesh refinement in corner under high stress image source: ([27])

3 Related Work

Many approaches have so far been taken in an attempt to improve a computer's ability to perform the task of finite element meshing. The following subsections present an overview of the research conducted for the various aspects of the project which are more explicitly outlined under "Aims and Objectives"

3.1 Traditional Stress refinement methods

A number of methods exist to perform mesh refinement based on a stress gradient for a mesh, with the most common being higherarchical refinemententent also known as h-refinement and relocation refinement or r-refinement[2] [3]

h-refinement: H-refinement is the process of recursively refining a mesh by splitting elements into additional sub elements. This process can be performed for elements with both both triangular and quadrilateral shapes. [2].

r-refinement: R-refinement is a method which attempts to improve the quality of the stress gradient without the alteration of the mesh element count and thus the computational cost. This is achieved by the relocation of elements within the mesh which effectively increases the size of elements in areas of low stress, while reducing the size in areas where stress is high [3].

3.2 Uses of Artificial Intelligence and Machine Learning

Within the domains of AI and machine learning methods such as neural networks [9], case based reasoning [10] and inductive logic programming [5] have all been adopted to facilitate generation of meshes comparable to that of human experts. Similarly there has also been effort to combine multiple numerical methods simultaneously for solving re meshing problems [8] although effort to combine the two approaches does not appear to have so far been made.

Due to the difficulty of obtaining meshes which hold commercial interest the majority of researchers working in this field have had to result to the use of training sets developed within academia [4]. The primary issue associated with this is that often the training data does not exhibit the level of complexity that you would expect in many industrial sectors. Many researchers must accept this as a limitation or agree commercial terms with an organisation in order to gain access to their models [14].

Having reviewed a variety of different AI based applications to FE the use of Inductive Logic Programming (ILP) used by Bojan Dolsak et al is of greatest interest. ILP is a machine learning method first presented by Stephen Muggleton in his 1991 paper "Inductive Logic Programming" [11]. Muggleton suggests that the traditional approaches of machine learning which rely on use of extensive data sets and statistical analysis are poor in the case of many real world problems for which data is not available [13]. Muggleton cites Human learning as an example of use of ILP style techniques where understanding of new concepts is achieved not through crunching large volumes of data points but instead the use of induction on a relatively concise set of background facts and examples obtained from previous life experiances [13].

ILP uses three types of input information in order to hypothesise additional facts about the system. These three types of input information are:

- Positive examples of what constitutes an area that is well meshed
- Negative example of areas that are poorly meshed
- Background facts

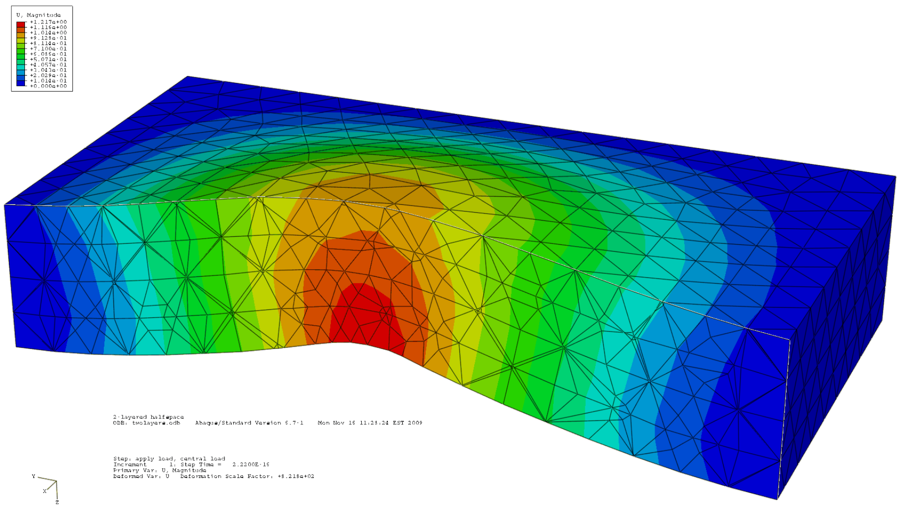


Figure 2: Example of how elements can be distorted in order to fit a geometry which will result in deterioration of gradient quality (image source: [15])

Using this information ILP is capable of hypothesising rules by determining which rules can exist within the system where given the set of background facts all positive examples are satisfied while few or none of the negative examples are. Although ILP requires a body of additional metadata associated with each mesh this is easier to obtain making ILP a highly practical solution. Along with his publication Muggletons also released his implementation of an ILP algorithm as a program titled “Golem" [12], Golem was applied by Dolsak to the problem of mesh refinement with a training set of just five meshes [5]. The resulting rule set when applied to subsequent models was able to correctly classify and re mesh areas with an average accuracy of 78% for a range of geometries [5] [6].

Dolsak’s choice of metadata for the ILP method to generate mesh rules is based on the classification of edges within the FE model which he knew to be crucial to determining their strength. For example if it is know that an edge within the model has a force applied close to it based on the initial model conditions then other edges that intersect it should be meshed to have a greater concerntration of adjacent elements prior to model execution. [4] [6].

The format of the rules make them attractive for experimenting with as part of a hybrid method since the method determines how to refine the mesh based on the arrangement of edges. This detail of analysis of the mesh is at a comparable detail to that of a traditional splitting method such as h-refinement which will likely to improve the ease at which the two methods can be combined simultaneously in the latter stages of the project.

3.3 Quality metrics

Finally work has also been done on establishing valid metrics for assessing the quality of a mesh automatically [14, 9] Metrics for meshes have been research far more extensively that AI methods due to their use for comparing different stress based refinements. There are also cases of common metrics being used for industrial meshing applications [14]. Although there are metrics for assessing a mesh on a global level such as element count score [14] the consensus is that due to the variation in meshes this is less reliable than assessing quality based on the properties of individual elements within the mesh [14]

Localised metrics associated with the quality of each element have shown to be accurate for predicting the overall quality of a mesh when taking the average for each metric across all elements [14]. The quality of an elements shape is important since the stress values which are computed for the area within the element are calculated using the stress values at each of the nodes which enclose it [18]. Elements are typically deformed near parts of the geometry where its shape simply does not allow a uniform element to be placed, an example of where this has occurred can be seen in figure 2.

Some key shape metrics identified by Dittmer et al include (ideal values are for elements of quadrilateral type as used in the current prototypes):

A Aspect ratio – longest side / shortest side, ideal value is 1

B Maximum corner angle - widest internal angle to element, ideal is 90°

C Maximum parallel deviation - how skewed the element is, ideal is 0°

4 Description of the work

4.1 Aims and Objectives

The aim of this project was to design, build and analyse a system for refining a mesh by combining a method from the domain of AI or machine learning with a stress based refinement method. The desirable end result will be a hybrid method of meshing which produces reasonably good solution accuracy whilst requiring less user intervention and reduced computational cost.

The project can be broken into thee main sections of research and implementation which can each be considered a high level objective:

A The first objective is to research and develop both a traditional stress refinement and an AI re meshing algorithm developed by industry or by academia. These algorithms should be able to run independently on a set of example models.

B Secondly a process will need to be devised for combining the two re meshing methods to varying degrees will be required. This will make it possible to test the effects of a hybrid method across the same set of models. Through this it can be established whether there is notable benefit to combining the different approaches and if so to what degree is it effective.

C The third objective will be to use justifiable metrics for assessing the quality of a given mesh. This will allow for a much greater range of hybrid variations to be tried without requiring inspection from an expert.

5 System specification

To demonstrate success in achieving the objectives of the project it is important to have tractability from the requirements through to the solution and lastly verification and validation. This section describes the current systems *functional requirements* (what the system will do) and *non-functional* requirements (its constraints) based upon evaluation of the research I have conducted in conjunction with discussions with the project supervisor: Dr Jason Atkin. Functional requirements have primarily been listed under their respective high level subsystems that are responsible for encapsulating their functionality.

Although it has not been developed as part of the project the application responsible for solving the finite element models has been included as part of the systems requirements since it highly influences the overall scope of the project and much of the design associated with other subsystems which were developed for the project.

6 Functional Requirements

1. FE integration: System will be able to interface with a third party finite element application

    1.1. The finite element applications solver must be able to solve a mesh based on its model configuration.

    1.2. The finite element applications solver must be able to execute a model programmatically

    1.3. The finite element applications solver must be able to output stress data at different points on the mesh.

    1.4. The finite element application will provide a graphical representation of the model.

1.5. It will be possible to manipulate the model that the finite element application uses programmatically.
1.6. It should be possible to manipulate the model that the finite element application uses from within its graphical user interface.

2. Mesh refinement: System will be able to perform different kinds of finite element mesh refinement
   2.1. The system will be able to refine a finite element mesh using a stress based refinement method
   2.2. The system will be able to refine a finite element mesh using a non-stress based refinement method
   2.3. A non stress based refinement method will adapt the mesh using background information about mesh design which has been previously trained.
   2.4. The system will be able to combine the two methods to produce a coherent mesh which the FE application is able to successfully solve in order to obtain results for stress and displacement.
   2.5. The system will be able to combine both methods to varying degrees will be performed automatically by the system without direct user intervention.
   2.6. The system will re mesh using both stress and non-stress based refinement using quadrilateral elements.
   2.7. System will adapt weighting associated with each method based upon the metrics computer for the mesh in the systems previous iteration.

3. Quality assessment: System will provide the operator with results about the quality of meshes based on metrics obtained from research.
   3.1. An assessment will be conducted automatically for every mesh iteration that occurs.
   3.2. System will assess quality based on a variety of metrics to ensure overall robustness of measurement.
   3.3. The metrics will be computed for both individual elements within the model and for the entire mesh.

7 Non-Functional Requirements

Design: The system architecture will be developed using the object oriented design principals of SOLID to allow for clear interfaces between the different functional components. Functional programming practices will be adopted through use of the .NET Language Integrated Query or LINQ framework. This will help to simplify the code and improve reliability. Where functions and classes are written their length will be kept to a minimum to reduce complexity and allow for reuse wherever possible.

Documentation: The system will be comprehensively documented at both a code level and at an architecture level. At a code level C# doc comments will be written to provide a comprehensive summary of each function. This will allow the tool Doxygen [24] to generate a full set of developer documentation upon completion of the software implementation which will be included as an appendix. Doc comments will also help to encourage small functions.

General applicability: In order to demonstrate that hybrid methods are a feasible means of approaching meshing problems the resulting software should be able to successfully execute on a range of models with varying geometry. The range of geometries should be representative of typical structural variation.

8 System Design

8.1 System Overview

Determining the overall design of the system was initially hard since it was not clear exactly how many subsystems would be needed to mesh, evaluate and interface with LISA, what was clear was that the system would essentially be performing an optimisation procedure and as such needed to be driven iteratively towards a goal. The complexity and uncertainty surrounding the project meant ensuring the architecture remained modular was essential with well defined interfaces allowing components could easily be added or modified as the project progressed.

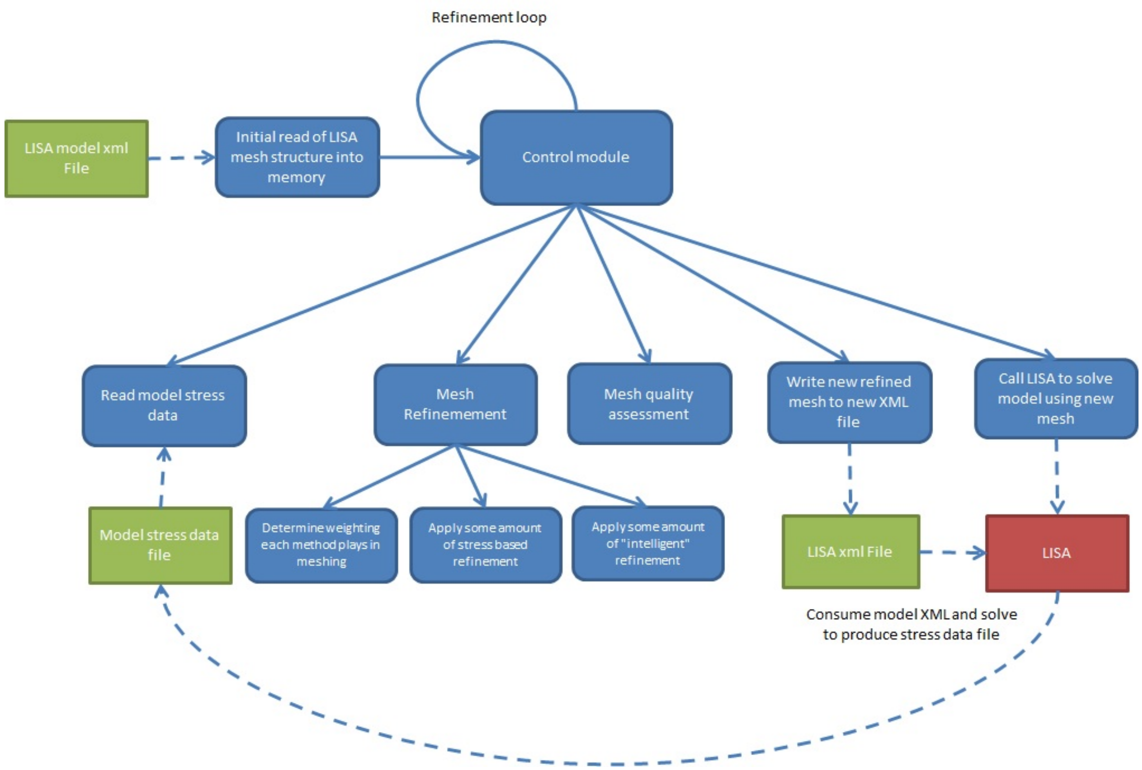


Figure 3: High level design of the system with its different modules

8.2 Modular Architecture

The modular architecture allows meshing algorithms and metrics for calculating mesh quality could be swapped interchanged was an important consideration in the design of the system. At best the quality of the output could be predicted for each method before it was integrated into the system and executed in a range of different scenarios. To have tightly coupled these individual components would have rendered the overall system a failure in the event that any one of them performed greatly worse than expected. Instead the loose coupling of the architecture has enabled the system to be considered as more of a framework for testing the effects of combining heuristic knowledge of a Finite element problem with a stress based refinement method.

Although the system was highly modular It was also still desirable to maintain a hierarchy within the classes so that smaller components could be developed independently and contained within the larger ones.

Composition was therefore generally favoured over inheritance as a means of building the architecture. Static classes and methods were also used when needing to write utility functions that were required by multiple high level subsystems and therefore did no fit especially well into any particular one. Examples of these are generic vector algebra operations such as dot product, matrix determinants and normal vectors.

8.3 Input Files

The system requires three basic input files which should be placed within a directory that is given to the program as as parameter, these are files are:

- A structural model represented as a .liml file which LISA can solve
- An initial stress data file generated manually so the system has some idea about where to start working
- A JSON file containing important edges as identified by an engineer along with associated meta data.

An example of the content and format for each of these input files can be seen in Appendix B

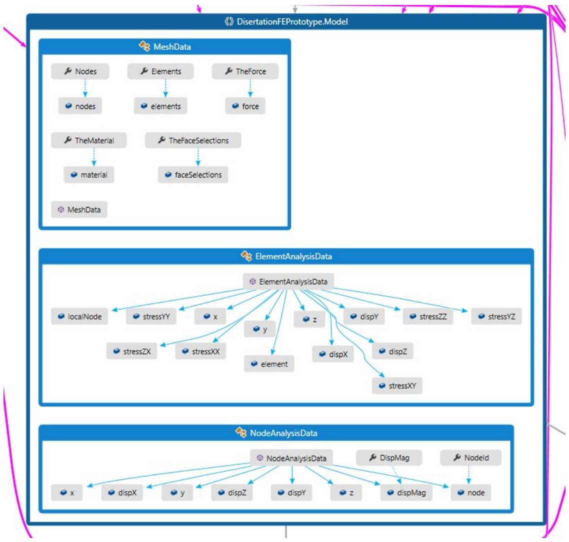
8.4 Simulation Data Model

Writing an API for LISA was the first clear stage of development for my project for which a design had to be considered. The API was crucial in order to program the more complex aspects using basic operations and avoid having to regularly perform direct string manipulation of the input files in order to manipulate the model.

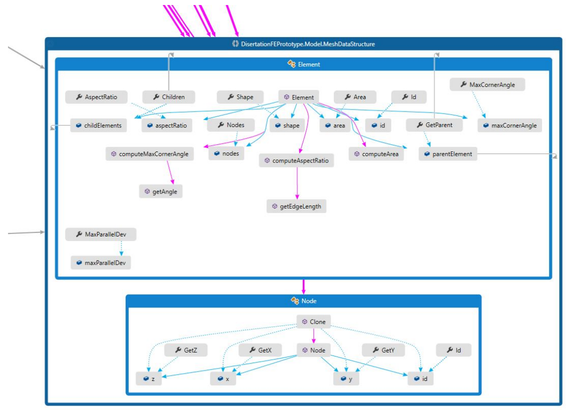
When the first re-meshing iteration occurs the system needs to read the input .liml file into an equivalent class model which closely resembles the schema specified within the .liml file, diagrams for which can be seen in figure 4 below. Each class in this model contains corresponding data and methods used to represent and manipulate the model. These methods are then used by the different refinement methods that exist in the different optimisation classes to alter the mesh, this adapted version of the mesh represented as a data model can then be assessed by the module responsible for validating mesh quality before finally being written back to a .liml file for LISA to solve on the subsequent iteration. When designing the data model to represent the state within LISA it was desirable to maintain coherence with the structure specified by LISA as closely as possible, this allowed for the data model to more easily be serialised when communicating with LISA without errors resulting from inconsistencies between the two representations.
One aspect of the data models design which greatly adds to the systems flexibility is the hierarchical design for representing the various Element types. At the root of this structure is the IElement interface, all new Element types must adhere to this in order for the various refinement methods to run. Implementing the interface are a range of abstract classes such as “SquareBasedElem" and “TriangleBasedElem" These classes are designed to contain methods that are generally applicable for calculating metrics and re meshing individual elements for elements of this type that are both in two and three dimensions. This is powerful since computing this for a 3d element is simply a reduction of the method for a 2d element over every face which comprises the 3d one. Finally underneath this level are the concrete classes which represent each specific type of element with its own bespoke characteristics.

8.5 Remeshing methods approach

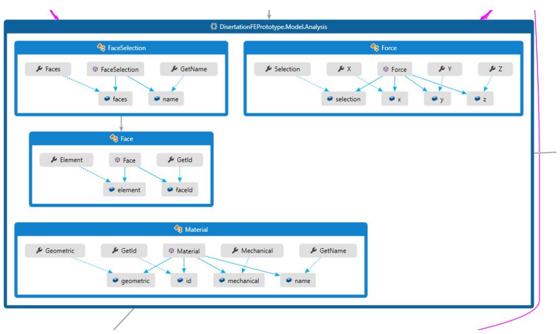
Due to the modular design of the system the low level methods for performing refinement for each element type were contained within the elements abstract class or if more bespoke its subclass, see Figure 6. This made it much easier to decouple both the stress and heuristic methods allowing each of them to simply have the task of selecting elements which they considered beneficial to refine before delegating the refinement to each element itself. This allowed both high level refinement approaches to utilise the same low level functionality which greatly improved the systems reliability and simplified the design.



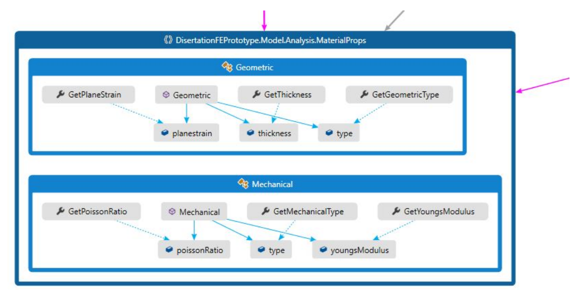
(a) Model classes



(b) Element and Node classes



(a) Model Analysis classes



(b) Material Property classes

Figure 5: Class model to represent .liml file structure used by LISA

8.5.1 Combining methods

Since each refinement method could be executed independently of the other it made sense when testing combinations both to simply enumerate the combinations that each could be applied for each iteration before running each specific combination iteratively on a model. This results in a set of two valued tuples up to some value :

$$\{(a,b) \mid a,b \in \mathbb{N}a, b < k\}$$

8.5.2 Multi Threading

To allow allow for better performance when evaluating a range of different weightings of the stress and heuristic methods multi threading was used to allow each configuration to be run independently of the others. When started each thread creates its own directory which it copies the three input files to and runs for its designating weighting configuration. Use of multi threading made assessment of the system much more rapid in the later stages of the p

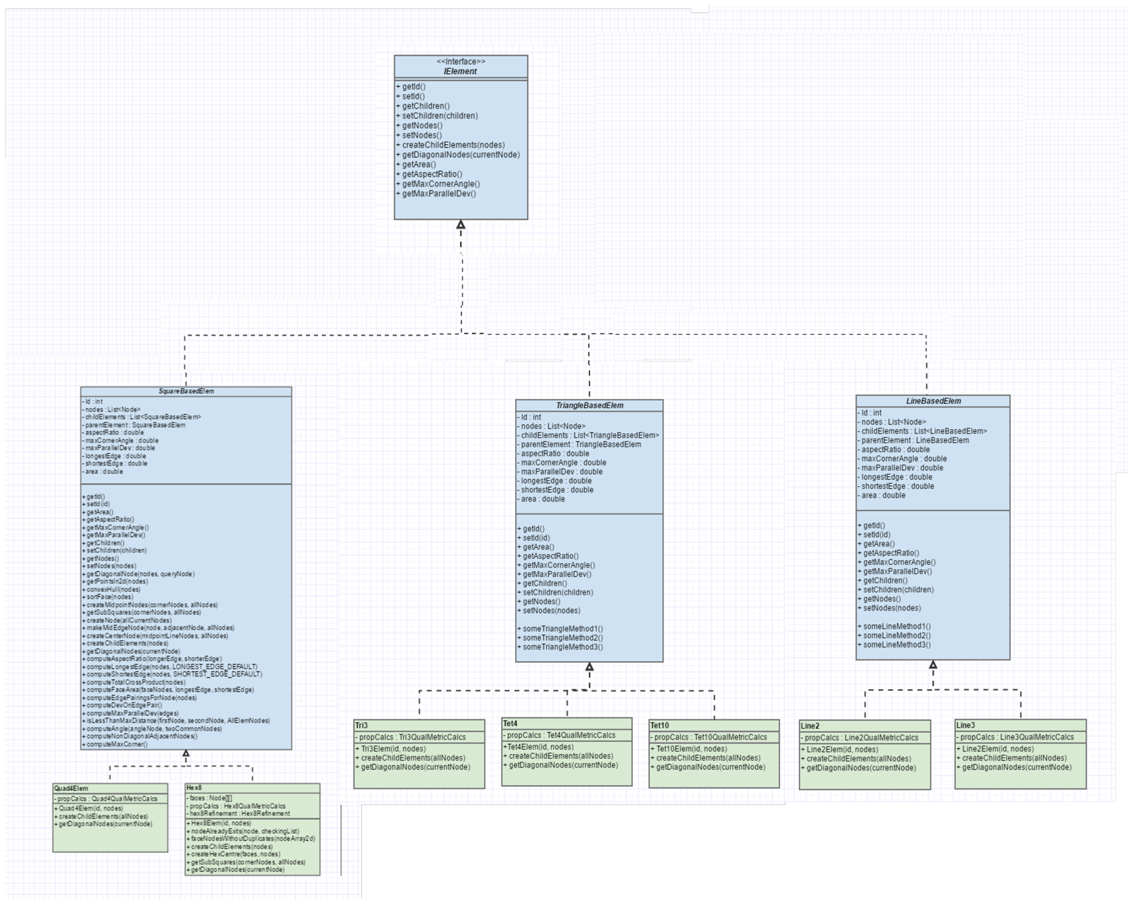


Figure 6: Class diagram showing the hierarchy of element classification within the data model, due to time limitations I was not able to implement the respective classes for triangle and line based elements, to see image representations of each element type within this class diagram refer to element type appendix

9 Software Implementation

The following subsections detail the implementation of the final software solution that has been written to meet the objectives posed at the beginning of this dissertation.

9.1 Third Party FE Application

In order to demonstrate the potential feasibility of the hybrid approach it was first important to obtain a finite element solver which could be given a FE model containing data about forces, materials and the mesh structure and then execute the model pro grammatically so as to obtain stress results.

A multitude of commercial FE tools exist with there being a wide variety in both the complexity and cost associated with each tool. Finite element software is typically very expensive due to its high development cost and small customer base. Tools used within industry such as ANSYS typically require a great deal of time in order to become proficient in their usage and can cost in excess of five thousand pounds a year for a single licence [26]. It was therefore important to find a tool which was both affordable while also powerful enough to demonstrate a working prototype of the re meshing method.

9.2 LISA

After reviewing several FE applications used within industry in addition to a variety of less well known ones used within academia and by hobbyists LISA was selected as the solver application for which to implement the systems prototypes.

Strengths: LISA as a FE tool which allows the user to run models of up to 1300 element for free; This was beneficial in allowing me to experiment with the software and gauge the feasibility of my projects concept before requiring a full version. Once at a stage in the project where each problem had been solved for small models containing less than 1300 elements an academic licence for the software was purchased for the projects use.

LISA also provides a GUI which allows visual inspection of the model and its mesh; This is particularly useful for observing the output of the meshing algorithms which can often provide a human with a much better understanding of how the method has performed and whether or not there are obvious bugs. in the implementation of the meshing procedures

Weaknesses: Due to LISA's simplicity it does not come with an extensive API allowing for easy programmatic use of its inbuilt features, however it is still possible to interface with LISA through less direct means [19]. LISA models are stored in .liml files which use XML as a meta mark-up format. The model files contain all the information about the model including the materials used as well as loads and constraints and of course the mesh. It is therefore possible to manipulate a .liml file having parsed its contents before writing a new version of the file which LISA can be called to solve. In order to more easily alter the model it made sense to write a wrapper for the .liml files to abstract the manipulation of their content.

9.3 Languages and platforms

The final system has been written entirely using the C# programming language (version 5.0) with Visual Studio 2015 as the development environment on a Windows 10 system. C# is an application development language built on the .NET framework. Although any number of programming languages could have been used to implement the solution C# offered a good compromise for developing a system with both structural rigidity through static typing and object orientation in addition to functionality to allow for rapid prototyping. C# does this well through use of LINQ, a part of the standard library that provides a large number of higher order functions which allow for operations to be performed over any data structure that implements the built in IEnumerable interface. Given that much of the code within the project performs the same operation on collections of nodes and elements stored in Lists arrays and dictionaries which all implement IEnumerable the ability to write much of the project using this capability dramatically reduced the number of errors encountered and increased development speed.

9.4 Implementation Methodology

Growing size of the software meant it was important to work systematically to continuously drive the project in the right direction and avoid the introduction of unnecessary complexity. This was achieved through regularly reviewing and refactoring the code which dramatically helped to reduced the amount of bugs introduced.

For the duration of the project the spiral methodology was adhered to. This enforced multiple deliverable stages that were concluded with a supervisor meeting every one or two weeks. Adopting the spiral methodology also provided flexibility regarding the order in which tasks were able to take place outside of a spiral iteration. This was necessary when conducing a research driven project where direction of work for subsequent development iterations was largely driven by the findings of the work in the previous ones.

Tasks were chosen every week for the project, the number of tasks and their complexity was determined using a combination of factors including their complexity, the criticality of the task e.g. Did it need to be completed for other important tasks to be started and the time available to me as the individual undertaking the project (More tasks typically performed on weeks when less work was due for other modules.

9.5 Implementation of Subsystems

This section describes the implementation details of the various different subsystems which combine to form the overall solution.

9.5.1 Re-meshing using hierarchical refinement

After reviewing both h-refinement [2] and r-refinement [3] it was concluded h-refinement would be best the best approach to adopt due to its simplicity and more widespread use [2] despite the fact that the mesh is usually more computationally expensive than it would have been if it was created using r-refinement [3]

Elements within traditional FEA can typically be classified as either triangle or square based elements, each of these provide different strengths and weaknesses when required to mesh and solve models. Within industry triangular elements are typically preferable since it is always possible to generate an initial triangular mesh from any arbitrary CAD geometry algorithmically. This is done by simply making smaller triangles until all gaps along the edge of the geometry are filled [31]. The same cannot always be said when meshing using square elements. For proof of the solutions concept however it was concluded that square based elements were preferable to triangular ones with since the steps required for a basic refinement are much simpler, just take the corners of an element that already exists, add their x, y and z components before dividing each component by two to achieve the coordinate for a node which is halfway between the two.

In addition to refinement it is also significantly easier to define edges which the ILP rules can be applied to when edges naturally form within a structure through a chain of nodes along the edges of square elements. Unfortunately Triangular meshes also generally incur a higher computational cost than an equivalent square element mesh due to added complexity of performing the calculations required to remesh in addition to requiring more elements over a given area to achieve the same accuracy.

From an implementation standpoint writing a square based remeshing algorithm is also substantially easier as given an initial mesh made just of square elements the only task is to repeatedly divide each element into four sub elements, by contrast methods uses to re mesh triangular meshes vary greatly are typically more complex and have corresponding initial meshes that are harder to generate manually by human operators [?].

9.6 Fast node lookup and update of nodes

A key requirement for the design of the data model generated by the hierarchical re meshing process was the need to perform fast lookup of nodes already present in the mesh. Lookup is important within the meshing methods as a means of checking whether a node that is about to be created already exists within the model, in the event that no such node already exists a new one can be created however if it does then instead of creating a new node the node that already exists needs to be connected to a node in an adjacent element that is currently being refined. If nodes are not linked correctly form correct elements the physics solver is unable to assume the stress moves through one element to another despite both having nodes at the same coordinates, this results in inaccurate output or potentially an error being thrown by LISA.

This issue arose as a result of partly as a result of the systems design, as previously mentioned subdivision for every individual element is the responsibility of that element which from a software engineering perspective is very good since it means the low level meshing process for each different type of element could be written within that elements class. This avoids the need for much heavier generalised refinement classes that would have needed to know how to perform the meshing for all elements in the model at once and for each of the different potential element types. A consequence of this was despite every Element being capable of meshing itself perfectly adjacent elements that also requiring refinement needed the ability to reconnect the new nodes along their edges to those that had created by the adjacent element. This can be seen below in figure x

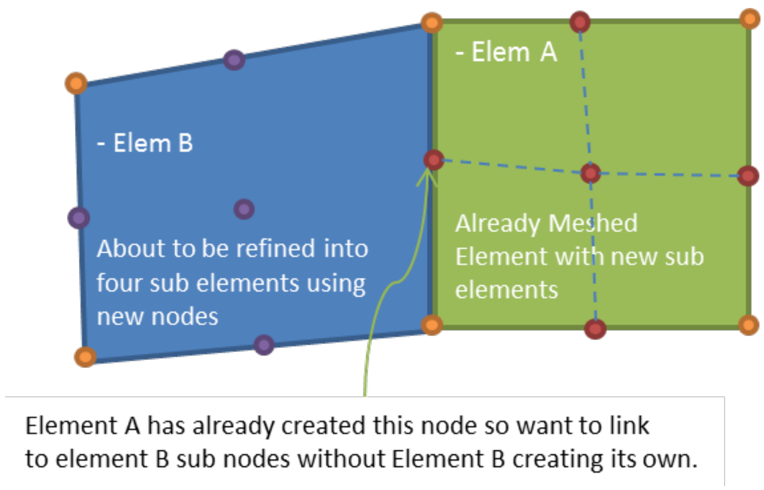


Figure 7: The need for an element to check for existing adjacent nodes when subdividing itself during refinement,

Orange Nodes - An original node for one or more elements
Red Nodes - new nodes made by Elem A
Purple Nodes - new nodes made by Elem B

The solution to this problem was to store all the nodes in the mesh model within a C# dictionary structure a reference to which is passed to each element within the model. The dictionary can be indexed using a Tuple of the x, y and z coordinates for the new potential element which will either return a node already at that location or indicate that no such node exists, in which case that element is then responsible for creating the node as its first instance. Dictionaries in C# represent a generalised instance of a hash table ensuring that lookup and insert are both constant time on average.

9.7 Sorting Element Nodes in 3d space using convex hull Algorithm

A significant issue encountered when working with LISA on the project was an interface requirement specified by LISA requiring nodes for each type of element to be sorted in a specific geometric order. The general rule for node ordering within LISA is to order the nodes as a perimeter around an element in 3d space without paths between nodes crossing one another internal to the element. When first addressing this problem for simple models using elements of Quad4 type the obvious approach was to think of a simple quadrilateral resembling a square, write an algorithm to handle that as a base case before considering a more complex case. The resulting approach was the following approach:
This approach was sufficient for the vast majority of elements within the different models, however as model complexity increased multiple iterations of the heuristics resulted in increasingly distorted element shapes, resulting in rejection of elements with incorrectly sorted nodes by LISA.

In order to resolve this issue I needed to research and apply a convex hull algorithm to this problem. The subsequent solution was the following:

Despite the existence of algorithms for generating a convex hull in three dimensions due to the small number of points it made sense to assess the effectiveness of a 2d method first before committing and assess the effectiveness of that for resolving my problem. In order to do this I simply took my Quad4 elements and flattened them to a 2d representation by calculating the maximum delta between the greatest and smallest value on each axis and eliminating the axis with the smallest delta. This proved successful, when applied to the model this successfully removed incorrect ordering from elements that had been particularly skewed.

This method has O(n log n) time complexity however due to the size of n being 4 in all cases the complexity of sorting an individual element is constant, with the overall complexity of sorting all elements in the model being O(n) where n is the number of elements.

9.8 Stress Based Refinement

To focus meshing in areas of high stress each iteration needed to parse the results file from the previous iterations execution of LISA. LISA result files are in csv format by default and contain the displacements and stresses associated with each node within the model once it has been solved.

Once the data in the output file has been parsed meshing can be conducted for high stress areas by averaging the stresses across the whole model and then cross referencing the node Ids in the results which have stresses above the average against the main data model allows a list of elements that can be instructed to refine themselves.

9.9 Rule Based Refinement

Each rule is represented as a function within the implementation, this closely resembles the format presented by Dolsak [4, 5, 6] [7]. Each of the rules resides within the "RuleManager" class and takes a number of the defined edges as parameters. Each rule then checks the properties of a particular edge against properties which have been identified through the ILP learning mechanism as being important when the model executes. In cases where the rules accept more than one edge as an argument the rule is applied to all combinations of possible input edges. If a rule detects a relationship in the model the edge is assigned a criticality rating as defined by the rule, the value is then used by the meshing procedure to determine how many times it should repeatedly re mesh the elements along that edge.

The properties that can exist between two edges when compared are the following:

- Edges opposite one another - the edges run alongside one another closely
- Edges posses the same form -
- Edges are considered the same - to meet this requirement both edges must be almost the same length, opposite one another and posses the same form.

```
1 reference | 0 changes | 0 authors, 0 changes
private void rule7(Edge edgeA, Edge edgeB)
{

    const int INVOLVED_EDGES = 3;

    bool b1 = edgeA.GetEdgeType() == Edge.EdgeType.important;
    bool b2 = edgeA.GetLoadType() == Edge.LoadingType.notLoaded;
    bool b3 = edgeA.isNeighbour(edgeB);
    bool b4 = edgeB.GetEdgeType() == Edge.EdgeType.importantShort;

    if(b1 && b2 && b3 && b4)
    {
        edgeA.ElementCount = INVOLVED_EDGES;
    }
}
```

(a) Code implementation of rule 7 provided by dolsak within the RuleManger class

```
mesh(A,3) :-
    important(A),
    not_loaded(A),
    neighbour(B,A),
    important_short(B).
```

(b) Rule 7 as stated by dolsak in his papers [6]

9.10 Mesh Quality Assessment

Dittmers rules for computing the quality of both individual elements and the entire mesh are built into their own "MeshQualtyAssessments" and "ElementQualityMetrics" classes, the latter of which is encapsulated within an element object, like with refinement this allows each element to assess its own quality removing the need for additional utility classes containing static methods.

Since each element is initialised with the nodes that comprise it, it is also possible to derive all the geometric characteristics and thus its quality metrics upon its initialisation. This allows the metrics for each node to also be calculated upon its initialisation removing the risk of elements returning null when asked for them.

9.11 Implementation Issues

Implementation of the design was not without its difficulties, many of which arising as a consequence of unforeseeable complications when implementing the well understood theoretical aspects.

9.11.1 Attempts to Automatically Define edges within models

As discussed under evaluation one of the main issues faced was identifying where the system behaved poorly through weakness in the methods or poor design and implementation as opposed to poor output generated as a consequence of poor input by the operator. In order to avoid this issue it was desirable to try and remove user intervention besides the configuration of the initial model. The obvious way by which to do this was automatic identification of interesting edges within the model. A crucial property which made this approach appear promising is the fact that it is known that edge importance directly correlates with the size of the edge and how much force is applied near it, since both this information exists within the data model it should be possible to identify edges from it and generate them automatically for Dolsaks rules to process.

In practice there are multiple complications surrounding this, several of these arise from ambiguity in Dolsaks paper regarding what constitutes a an edge which is for example "long" or "important". As a result edges are only able to be defined to the extent that a user has confidence in their understanding of these concepts.

Having considered a method using these properties of the mesh several days were then spent attempting to implement it with poor success achieved.

10 Evaluation of Project

Evaluation of the project consisted of multiple stages ranging from verification of the functional requirements through simple black box testing to evaluation of refined mesh quality using methods provided by Dittmer [14].

10.1 Validation against Functional Requirements

In order to validate the system against many of the functional requirements the system only needs to be run on several basic models with different input configurations.

output clearly demonstrates the systems ability to evaluate the quality of meshes using a range of metrics and refinement occurring as a result of both the stresses induced by the user and on their categorisation of edges.

10.2 Unit Testing

Holistic evaluation provided evidence of the overall systems effectiveness however without verification of individual components used to

10.3 Validation against Non Functional

Validation of non functional requirements was made simple due to the limited number of them, this was partly a consequence of the system not being designed for a specific user base resulting in expectations regarding the systems design to improve usability and guide interaction. It was also not possible to define the general accuracy and performance of the system during the requirement elicitation phase since this could only be determined once the required research and trial on the finished system gave indication to both of these.

In the case of quality for the systems design and documentation evidence is present to indicate that this adheres to the requirements specified with the project submission containing detailed documentation in the form of a Deoxygen guide and sophisticated use of object oriented and functional software design as seen within the codebase. General applicability of functionality has also been demonstrated through evaluation using a variety of both models and conditions when performing simulations.

10.4 Evaluation Of The system For Various Models

several models have been created resembling basic equivalents of that used for real engineering applications.

do this reliably and validate the heuristic component against the results obtained in academic papers two of the three models were based on those previously used by Dolsak (Paper mill and Cylinder structures). In addition to these two a suspension bridge model was developed to demonstrate the systems capacity to work beyond Dolsaks basic validation examples.

These initial models were constructed manually using LISA's graphical user interface using measurements by Dolsak and in the case of the suspension bridge from documents available on the web [4] [**?**].

10.4.1 Suspension Bridge structure

The basic bridge structure consists of 196 elements and 212 nodes, which is very coarse for a FE mesh structure.

The primary model for which work on the latter half of the project was based upon was the following suspension bridge model:

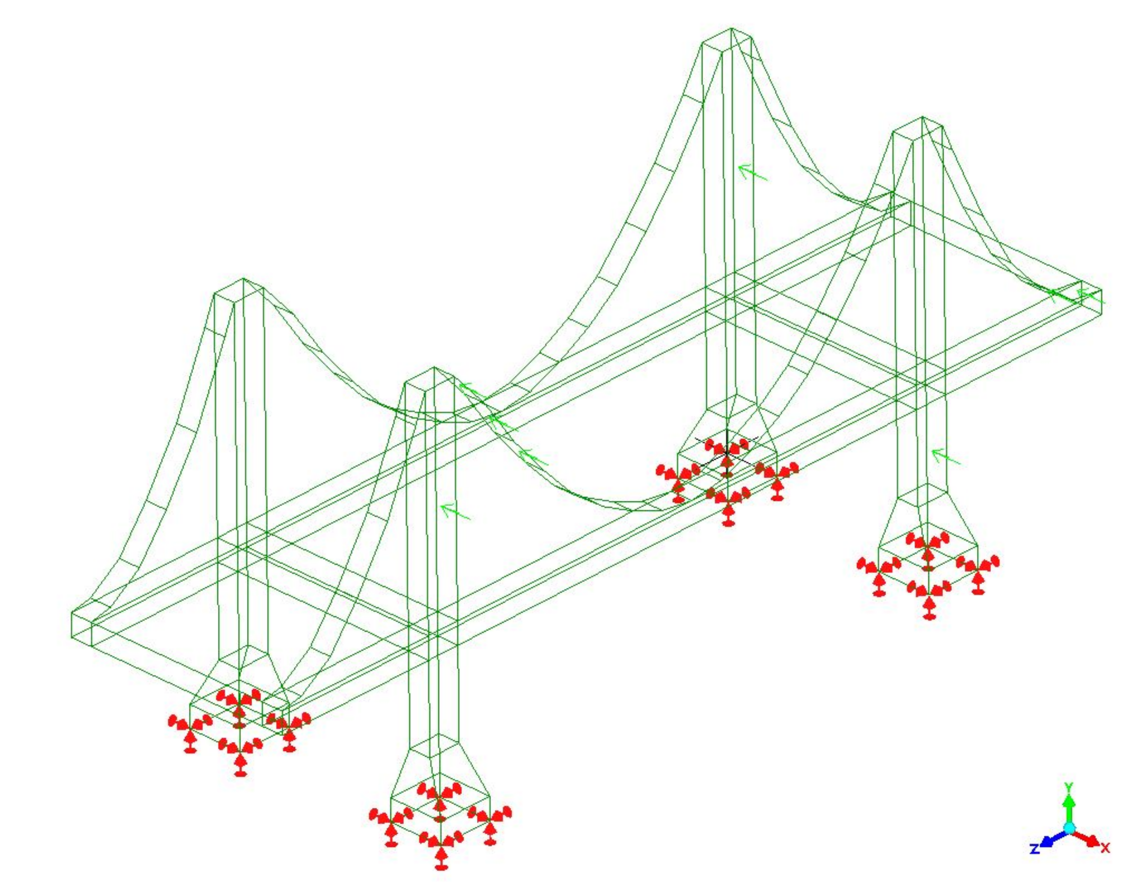


Figure 9: Diagram representing the general structure of the system with interactions between non internal entities such as files and LIA shown using dashed lines

A strength of the bridge model is its high versatility with regards to different simulation configurations. This meant during evaluation the bridge could be run with forces placed on a range of different faces with corresponding edge rules and a range of varying results obtained. For example a force on the desk of the bridge could be used to simulate the weight of a vehicle moving over it while forces placed sideways on the towers and cables could be used to model stress induced from strong crosswinds.

10.4.2 Evaluation of bridge for sideways loading

Simulating effects of forces across the bridge provided an opportunity to test the system when simulating a scenario that could feasibly effect the structure in the real world such as crosswinds.

[?]

In order to mesh successfully in order to focus refinement on these areas the following rules can be suggested for the bridge structure when crosswinds are applied from a negative x direction.

A total of x experiments were run for this configuration with four refinement iterations for each experiment

10.4.3 Evaluation of bridge for base loading

Loading the base of a bridge is yet another typical analysis conducted within civil engineering. The loading can be used to represent anything which moves over the bridge that needs to be supported by it such as vehicles or pedestrians.

10.4.4 Validating system against previously obtained research results

Due to a lack of specific implementation details provided by Dolsak and Muggleton in their papers for generating meshes it was important to demonstrate that the underlying implementation for generating meshes based upon the rules was essentially equivalent.

Dolsak did however provide information about the various models

this the models used by Dolsak in order to train his initial ILP system responsible for generating the rules were re crated. This allowed for verification of the heuristic component of the project through comparison of both implentations outputs. [4].

10.4.5 Paper Mill

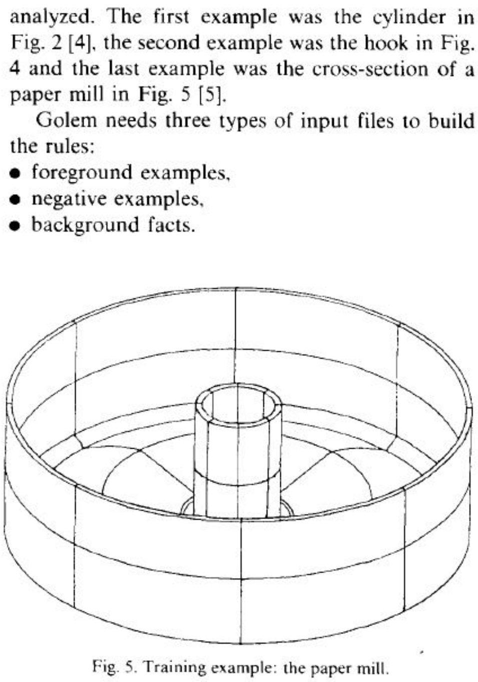
Disks are used in multiple places within the paper manufacturing process in order to press the paper as it undergoes pressing and drying. Unlike the bridge example where stresses are for the most part induced by external loading disks within paper mills are primarily stressed as a result of centripetal loading [] Consequently stresses are much more even assuming symmetry in the geometry [32].

Assuming a disk is rotating at a particular speed it is possible to calculate the centrifugal force exerted outwards on it. This has been done in the appendix section for various points on the mill to calculate feasible input for the system [32].

10.4.6 Cylinder

Obtaining results based on the edge relationships described within the research literature was the first step required when performing evaluation of the cylinder. The rules provided in Dolasks paper used to generate the initial JSON file were the following:

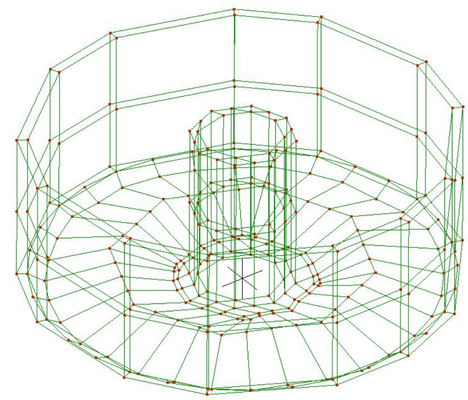
The resulting mesh generated by the system after increasing numbers of iterations based primarily


analyzed. The first example was the cylinder in Fig. 2 [4], the second example was the hook in Fig. 4 and the last example was the cross-section of a paper mill in Fig. 5 [5].

Golem needs three types of input files to build the rules:
- foreground examples,
- negative examples,
- background facts.

Fig. 5. Training example: the paper mill.

(a) Paper Mill presented by Dolsak in his paper as input for training Golem



(b) Representation of paper mill within LISA before applying meshing rules to validate

10.5 Evaluation of Subsystems based on model evaluation

Running the system for the above models gave clear indications as to the overall effectiveness of both methods

Having run the system on the range of models described above it was possible to begin assessing it's ability to mesh and the use of Dittmers metrics for assessing the quality of each mesh.

10.5.1 Analysis of Methods Using Metrics

When evaluating Dittmers metrics for distinguishing between meshes of different qualities there were several key observations about the metrics in particular which resulted in re assessment of how to evaluate the mesh qualities in general.

Central to these observations was that although metrics provided by Dittmer give a clear indication of the general accuracy of stress across a mesh or a particular element in the majority of cases they give no further insight into the strengths of the particular element arrangement within a given mesh. Consequently it was possible to demonstrate that the meshes produced as a result of the methods retailed general quality that is desired within general meshes but not that the mesh was especially good given the particular model it was based on.

A more conclusive evaluation therefore required a metric that would instead indicate the effectiveness of each approach when comparing them to one another. Ideally each method should provide as much information as possible to an engineer while retaining the lowest computational overhead possible, essentially how much additional information is revealed through each additional iteration of a given approach. We can also assume in this case that the concentration of stress is what we are interested in finding. Having reasoned about this I concluded that the best means of evaluating each method was was to cross reference the resultant

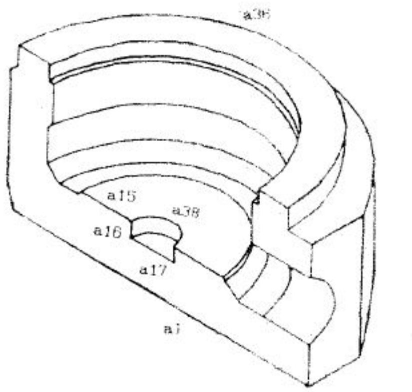


Fig. 3. Labelled edges of the structure in Fig. 1.

(a) Cylinder model used by Dolsak with edges labeled, each labelling corresponds to a rule generated by the Golem algorithm [4]

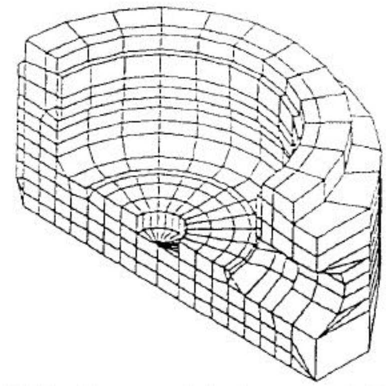
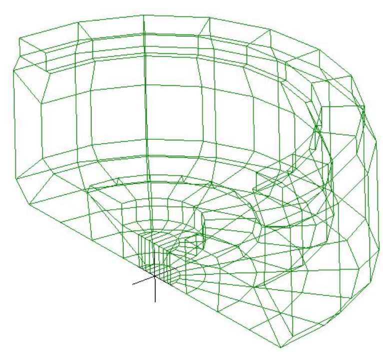
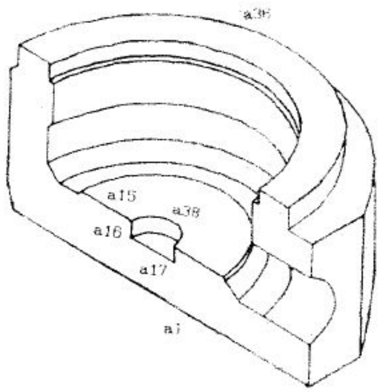


Fig. 2. Finite element mesh for the structure in Fig. 1.

(b) Diagram representing the general structure of the system with interactions between non internal entities such as files and LIA shown using dashed lines



(a) Cylinder cross section initial construction within Lisa [4]



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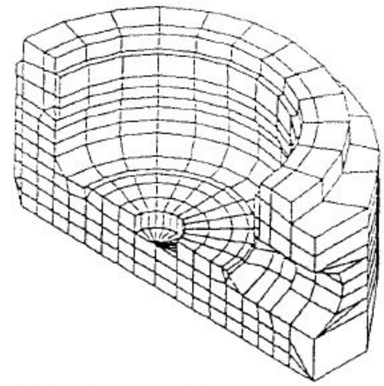


Fig. 2. Finite element mesh for the structure in Fig. 1.

(b) Diagram representing the general structure of the system with interactions between non internal entities such as files and LIA shown using dashed lines

stress data for each iteration against the newly refined regions generated by each approach, the overall stress revealed within the output as a result of that part of the mesh being refined could then be summed and divided by the number of nodes created in order to reveal it through evaluation.

Having coded this into the MeshQualityAssessment class the results for this metric could be plotted to evaluate each approach over a number of iterations for the different simulations. I also varied the selection of edges provided to heuristic refinement method which highlighted just how dependent the heuristic approach was on the quality of user expertise.

10.5.2 Evaluation of stress refinement effectiveness

Comparisons across multiple models suggests that for general purpose scenarios where results of the simulation are highly unpredictable and the geometry irregular stress based refinement will continue to provide engineers with reliable results within a predictable although potentially costly amount of time. This scenario likely encapsulates the majority of use cases for FE analysis in the real world and therefore is likely to remain the prominent approach by practitioners.

10.5.3 Evaluation of heuristic refinement effectiveness

Heuristic refinement can be seen as producing output of highly variable quality

10.6 Increase in performance through parallel execution

The following

The average speed up (Time of serial execution/ Time of parallel execution) was

The Efficiency of the parallel execution (speedup / number of processors) with an Intel i5 processor with 4 cores the average efficiency was calculated as

10.7 Software Quality and Maintainability

The quality of the design and implementation of the system reflects my experiance not only as a computer science undergraduate but as a developer with one year industrial experience, although not directly effecting the execution of the program properties such as appropriate variable naming, loose coupling of classes, use of abstractions and descriptive error messages make the software easier to read and debug for any potential future developers.

Visual Studio also enabled calculation of various software quality metrics for the code base automatically. This made selecting parts of the codebase for refactoring much easier when time was allocated for this. Upon completion of the project the average maintainability index across all modules was 75 with the lowest score for any high level module being 60 and the highest 92. According to the Microsoft Developer Network (MSDN) website code with an index of between 0 and 9 indicates low maintainability, 10 to 19 indicates moderately maintainable and 20 to 100 high maintainabilty [35].

10.8 Documentation

The process of continuously writing descriptive documentation was important to the success of the project and was treated as an integral part to meeting to the goals of the project development methodology which aimed to reduce the systems complexity and improve readability. Through the writing Doc comments corresponding to every function within the codebase it was possible to generate documentation files automatically through use of the tool Doxygen. This allows anyone with the solution to view descriptions of each of its functions either in the codebase or alternatively through the manual produced automatically by Doxygen.

10.8.1 Evaluation Issues

A significant issue faced in attempting to demonstrate the effectiveness of the system was to provide an indication of how well the system worked without taking into account the ability of the user who may be providing the edge rules for a particular model. Not taking this into account would result in an inaccurate representation of its ability.

10.9 Evaluation of overall project

Overall I

10.10 Evaluation summary

10.11 Successes and Limitations of project

11 Further Work

This section details some areas which given additional time to work on the project would at the very least be investigated, if not implemented. Each of these areas would hopefully provide some benefit in assisting to demonstrate the possibilities of hybrid methods.

11.1 Gathering feedback from experienced engineers

Approaching the end of the project it became clear that in order to better identify the systems strengths and weaknesses would require additional user testing by engineers who have experience conducting this type of analysis. Despite a lack of available time obtaining feedback from engineers with extensive applied industrial experience along with that of academics would have hopefully allowed for a more conclusive analysis of the systems and its ability to work across a greater number of general case scenarios.

Part of the reason feedback for not obtaining user feedback was the difficulty of doing so given the available time available simply to implement the project and validate it for a selection of basic models. As such even if time had been available the ethical clearance required to collect user feedback at the start was not obtained.

11.2 Improving usability through a web interface

Although possible to visit various engineers in order to conduct feedback the process is both time consuming on my part and inconvenient for the participant as a rigid time for which to meet must be scheduled and a laptop containing the working software brought to them which they must design or transfer their model to before running it multiple times to obtain results. This scenario is at best inconvenient for the participants and pressures them into arriving at a conclusion within a relatively small time of experimenting with it.

Instead by facilitating interaction with the system by means of a web interface the engineers would be able spend as little or as much time as they like experimenting with the system and allow them to submit feedback digitally allowing feedback to be obtained and aggregated from a much wider range of different sources separated by significant geographic distance.

To use the interface an engineer would simply need to submit a model they have already created along with a JSON file containing edges they have designated as important for their model. LISA supports imports from multiple CAD formats including Standard for the Exchange of Product model data (STEP) and Initial Graphics Exchange Specification (IGES)) [19]. Upon receiving the request the web server would the current project with their input data and having finished allow them to download the re meshed model along with the calculated stress data for analysis.

12 Personal Reflections and Summary

Overall I felt the project was a success. The final software solution was evidently capable of facilitating execution and therefore allowing comparison to be made of both heuristic and stress based mesh refinement techniques. Not only did the final system allow this comparison for the two specific approaches that were coded by me in order to validate the system it provided a flexible framework allowing for a potentially unlimited set of configurations when performing future experimentation.

From my perspective I wanted to use this project as an opportunity to improve my understanding of a technology that I previously had limited knowledge of through its use on my industrial placement year. My prior experience with with FE analysis was very much confined to that of a typical engineer making use of the method through a licensed desktop application with many of the technicalities that are of most interest to a computer scientist hidden. I therefore found the project highly enjoyable as an opportunity to learn

more about the underlying processes through both research and practical experimentation. As a means of facilitating my own learning as an individual I therefore also consider the project a success.
Despite working on larger software projects during my year within industry this was certainly the most complex I have undertaken as an individual. As the lead software developer on my own project I encountered many challenges which as a junior developer within industry were not my responsibility but which I observed team leaders and senior developers encountering regularly. Such tasks were those that required high level analysis of the overall solution in order to continuously steer the project in the right direction and ensure successful delivery within the specified time scales. In many ways the project therefore gave me a good appreciation of the overall difficulties associated with delivering a software project in its entirety and one that has the potential change fluidly throughout.

In particular I found the research and evaluation phases particularly difficult. Upon completion I came to realise this was largely due to my lack of formal education in mechanical engineering which meant I had to work a lot harder both to understand the initial problems associated with the methods and subsequently to correctly evaluate the results I obtained. Had I chosen a more traditional computer science topic I believe both of these aspects would have been much easier.

Having completed the research too much time was then spent concerned with specifics of the implementation.

A negative consequence of the bespoke nature of the system was that obtaining details of designs results for comparable systems published by the wider academic community was both challenging and highly time consuming.

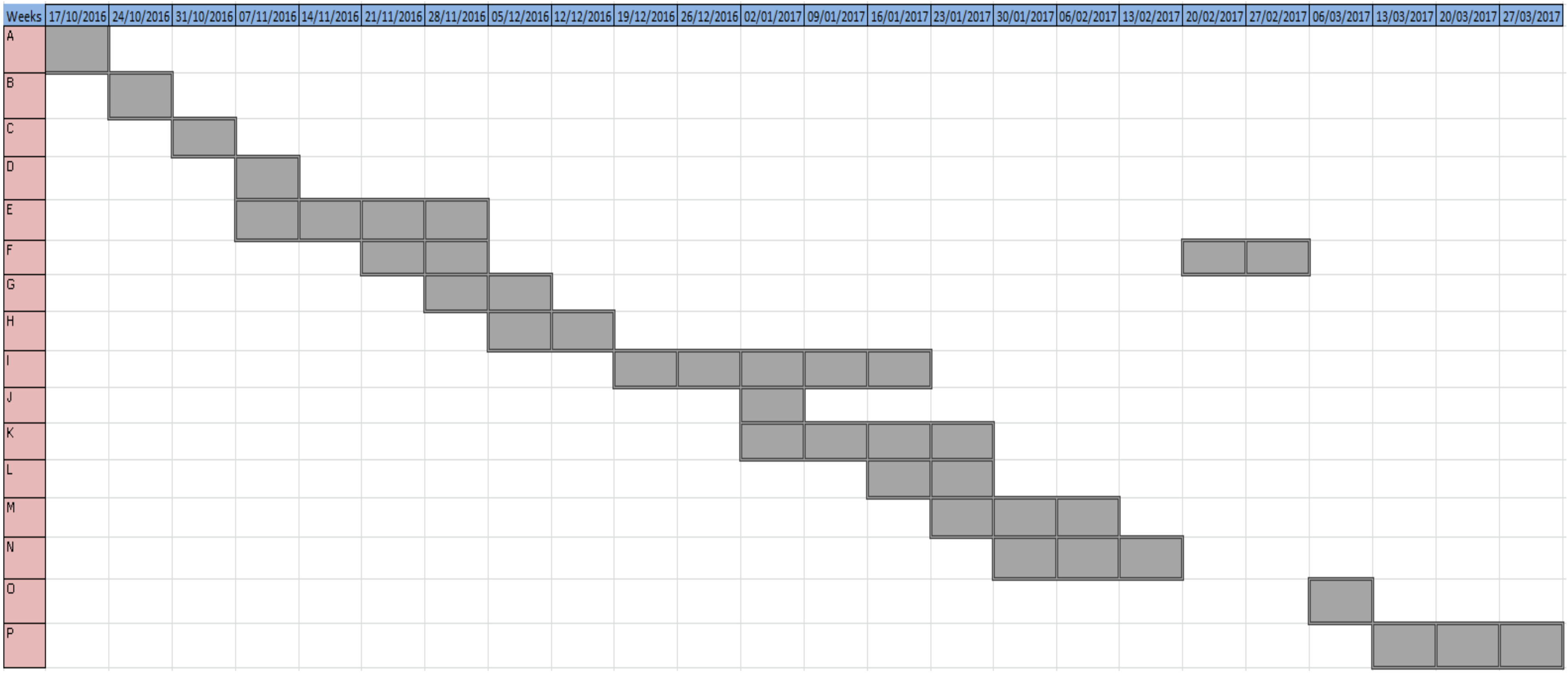
Throughout the majority of the project I felt organisation of time and planning of activities was done well. Work on the project began early with the goal of easing pressure in the later stages and work continued despite deadlines for coursework associated with other modules. A crucial mistake made was to reduce effort two months before the deadline having completed the software implementation and written much of the initial sections of the dissertation despite not completing evaluation of the results. Evaluation of the software turned out to be substantially more time consuming than expected in additional stresses towards the deadline.

Given the chance to redo the project I would have liked to conducted more research to better assess an even wider scope of research surrounding meshing methods. In many cases the academic literature surrounding bespoke meshing techniques became complicated very quickly and was clearly aimed at individuals with a high degree of prior experience.

By contrast I found the system design and programming components

planning and research components

managerial architectural aspects which I had not previously had that great an involvement with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24/10/2016
31/10/2016
07/11/2016
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19/12/2016
26/12/2016
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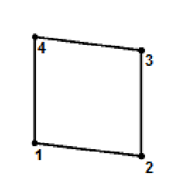
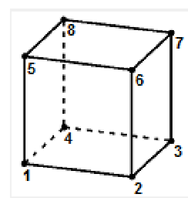
A Element Types within LISA

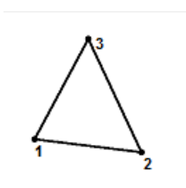
Here are shown the the visual specifications LISA provides for the ordering and layout of nodes for defining each type of element supported. Each of these types can be classified using the



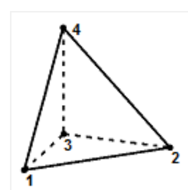
(a) quad4 element



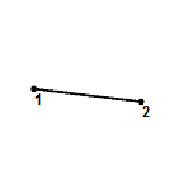
(b) hex8 element



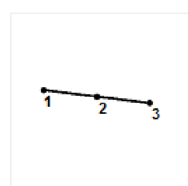
(a) Specification for node ordering of tri3 element within LISA



(b) tet4 element



(a) line2 element



(b) line3 element

B Calculating Centripetal Force For Paper Mill

Assuming a constant speed of the paper mill disk at the following standard calculation was done to compute a forces that could be specified for different elements in order to simulate the effects on the model.

F = m ω^2 r

where:
m - mass of object
r - radius from centre
ω - angular velocity (radians per second)

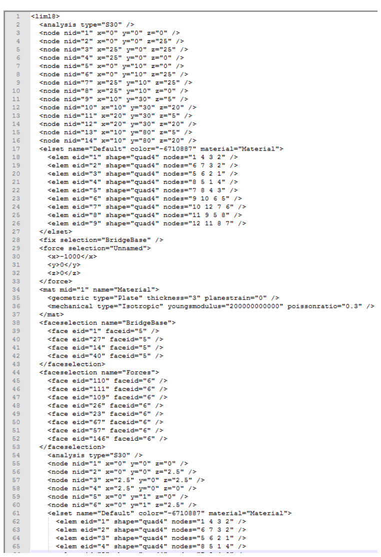
Using the following values for each variable for the plates forming the outside of the paper mill disk the force could be calculated as:

mass- paper mill is made of steel with each plate having a volume of approximately $24cm^3$ which gives a mass of 188 grams

[34]

C Input and output files

Below can be seen the format of the input files for the system (A LISA .liml and a .json edge definition file




```
<liml8>
  <analysis type="S30" />
  <node nid="1" x="0" y="0" z="0" />
  <node nid="2" x="0" y="0" z="25" />
  <node nid="3" x="25" y="0" z="25" />
  <node nid="4" x="25" y="0" z="0" />
  <node nid="5" x="0" y="10" z="0" />
  <node nid="6" x="0" y="10" z="25" />
  <node nid="7" x="25" y="10" z="25" />
  <node nid="8" x="25" y="10" z="0" />
  <node nid="9" x="10" y="30" z="5" />
  <node nid="10" x="10" y="30" z="20" />
  <node nid="11" x="20" y="30" z="5" />
  <node nid="12" x="20" y="30" z="20" />
  <node nid="13" x="10" y="80" z="5" />
  <node nid="14" x="10" y="80" z="20" />
  <elset name="Default" color="-6710887" material="Material">
    <elem eid="1" shape="quad4" nodes="1 4 3 2" />
    <elem eid="2" shape="quad4" nodes="6 7 3 2" />
    <elem eid="3" shape="quad4" nodes="5 6 2 1" />
    <elem eid="4" shape="quad4" nodes="8 5 1 4" />
    <elem eid="5" shape="quad4" nodes="7 8 4 3" />
    <elem eid="6" shape="quad4" nodes="9 10 6 5" />
    <elem eid="7" shape="quad4" nodes="10 12 7 6" />
    <elem eid="8" shape="quad4" nodes="11 9 5 8" />
    <elem eid="9" shape="quad4" nodes="12 11 8 7" />
  </elset>
  <fix selection="BridgeBase" />
  <force selection="Unnamed">
    <x>-1000</x>
    <y>0</y>
    <z>0</z>
  </force>
  <mat mid="1" name="Material">
    <geometric type="Plate" thickness="3" planestrain="0" />
    <mechanical type="Isotropic" youngsmodulus="200000000000" poissonratio="0.3" />
  </mat>
  <faceselection name="BridgeBase">
    <face eid="1" faceid="5" />
    <face eid="27" faceid="5" />
    <face eid="14" faceid="5" />
    <face eid="40" faceid="5" />
  </faceselection>
  <faceselection name="Forces">
    <face eid="110" faceid="6" />
    <face eid="111" faceid="6" />
    <face eid="109" faceid="6" />
    <face eid="26" faceid="6" />
    <face eid="23" faceid="6" />
    <face eid="67" faceid="6" />
    <face eid="57" faceid="6" />
    <face eid="146" faceid="6" />
  </faceselection>
    <analysis type="S30" />
    <node nid="1" x="0" y="0" z="0" />
    <node nid="2" x="0" y="0" z="2.5" />
    <node nid="3" x="2.5" y="0" z="2.5" />
    <node nid="4" x="2.5" y="0" z="0" />
    <node nid="5" x="0" y="1" z="0" />
    <node nid="6" x="0" y="1" z="2.5" />
    <elset name="Default" color="-6710887" material="Material">
      <elem eid="1" shape="quad4" nodes="1 4 3 2" />
      <elem eid="2" shape="quad4" nodes="6 7 3 2" />
      <elem eid="3" shape="quad4" nodes="5 6 2 1" />
      <elem eid="4" shape="quad4" nodes="8 5 1 4" />
```


(a) Cut down .liml file to show general content which largely defined the schema for the systems data model


```
{
    "Edges": [{
        "Id": 1,
        "edgeType": "circuit",
        "loadType": "oneSideLoaded",
        "boundaryType": "free",
        "nodePath": [42, 198, 197, 41]
    },
    {
        "Id": 2,
        "edgeType": "importantLong",
        "loadType": "oneSideLoaded",
        "boundaryType": "fixedTwoSides",
        "nodePath": [70, 62, 61, 46, 45, 67]
    },
    {
        "Id": 3,
        "edgeType": "importantShort",
        "loadType": "oneSideLoaded",
        "boundaryType": "fixedTwoSides",
        "nodePath": [41, 197]
    },
    {
        "Id": 4,
        "edgeType": "notImportant",
        "loadType": "notLoaded",
        "boundaryType": "fixedCompletely",
        "nodePath": [197, 207]
    },
    {
        "Id": 5,
        "edgeType": "circuit",
        "loadType": "continuiousLoading",
        "boundaryType": "fixedOneSide",
        "nodePath": [88, 142, 148, 147, 140, 141, 135, 85]
    }
    ]
}
```


(b) A json file containing the edges of interest specified by an engineer, this is parsed and the rules are applied to determine the models meshing based on the input

D Software Quality Metrics

Code Metrics Results

Filter: None Min: Max:

Hierarchy	Maintainability Index	Cyclomatic Complexity	Depth of Inheritance	Class Coupling	Lines of Code
DissertationFEPrototype (Debug)	75	808	7	125	1,871
DissertationFEPrototype	62	24	7	39	126
DissertationFEPrototype.FEModelUpdate	64	35	1	29	115
DissertationFEPrototype.FEModelUpdate.Model	90	17	1	11	27
DissertationFEPrototype.FEModelUpdate.Model.Structure	90	15	1	5	35
DissertationFEPrototype.FEModelUpdate.Model.Structure.Elements	69	329	2	35	684
DissertationFEPrototype.FEModelUpdate.Read	60	26	1	21	62
DissertationFEPrototype.MeshQualityMetrics	71	43	1	18	78
DissertationFEPrototype.Model	68	4	1	0	26
DissertationFEPrototype.Model.Analysis	87	20	1	6	42
DissertationFEPrototype.Model.Analysis.MaterialProps	92	8	1	0	14
DissertationFEPrototype.ModelUpdate	62	37	1	32	129
DissertationFEPrototype.Optimisations	53	57	1	29	161
DissertationFEPrototype.Optimisations.ILPRules	82	193	1	34	372

Figure 18: The metrics calculated by visual studio for all high level modules in the system

Code Metrics Results

Filter: None Min: Max:

Hierarchy	Maintainability I...	Cyclomatic Complexity	Depth of Inheritance	Class Coupling	Lines of Code
DissertationFEPrototype (Debug)	75	808	7	125	1,871
DissertationFEPrototype.Model.Analysis.MaterialProps	92	8	1	0	14
Geometric	92	4	1	0	7
Mechanical	92	4	1	0	7
DissertationFEPrototype.FEModelUpdate.Model	90	17	1	11	27
MeshData	90	17	1	11	27
DissertationFEPrototype.FEModelUpdate.Model.Structure	90	15	1	5	35
Node.Origin	100	0	1	0	0
Node	80	15	1	5	35
DissertationFEPrototype.Model.Analysis	87	20	1	6	42
Material	92	5	1	2	9
FixSelection	87	2	1	1	4
Face	86	4	1	1	8
FaceSelection	86	4	1	2	8
Force	85	5	1	0	13
DissertationFEPrototype.Optimisations.ILPRules	82	193	1	34	372
Edge.BoundaryType	100	0	1	0	0
Edge.EdgeType	100	0	1	0	0
Edge.LoadingType	100	0	1	0	0
DeserializedEdgeGroup	94	5	1	3	5
Edge	80	25	1	11	55
JsonEdge	73	49	1	4	63
RuleManager	72	28	1	6	74
EdgeGenerator	62	14	1	23	37
EdgeIdentifier	55	72	1	20	138
DissertationFEPrototype.MeshQualityMetrics	71	43	1	18	78
MeshQualityAssessment	74	28	1	16	38
ElementQualityMetrics	68	15	1	7	40
DissertationFEPrototype.FEModelUpdate.Model.Structure.Elements	69	329	2	35	684
IElement	100	12	0	5	0
ConvexHullPoint	82	3	1	1	8
Hex8QualMetricCalcs	76	10	1	9	25
SquareBasedElem	63	104	1	21	259
Quad4Elem	62	4	2	11	27
ConvexHull	61	15	1	7	27
Hex8Elem	58	24	2	21	80
Hex8Refinement	50	157	1	11	258
DissertationFEPrototype.Model	68	4	1	0	26
NodeAnalysisData	78	3	1	0	11
ElementAnalysisData	58	1	1	0	15
DissertationFEPrototype.FEModelUpdate	64	35	1	29	115
GeneralGeomMethods	71	4	1	4	14
ReadMeshData	56	31	1	25	101
DissertationFEPrototype	62	24	7	39	126
Form1	74	7	7	12	22
Program	56	10	1	13	43
Control	55	7	1	20	61
DissertationFEPrototype.ModelUpdate	62	37	1	32	129
ReadAnalysisData	62	9	1	6	49
WriteNewMeshData	61	28	1	28	80
DissertationFEPrototype.FEModelUpdate.Read	60	26	1	21	62
ReadNodes	63	10	1	7	26
ReadElements	58	16	1	21	36
DissertationFEPrototype.Optimisations	53	57	1	29	161
OptimisationManager	53	57	1	29	161

Figure 19: The metrics calculated by visual studio for the all classes in the final system