

Goal: To predict the value of one or more continuous target variables t given d -dim x input vectors.

Linear regression: most simple approach

SPECIFIC EXAMPLE OF Linear regression:

Type of variables: independent $x \in \mathbb{R}^d$
 dependent $y \in \mathbb{R}$

We assume a linear relationship:

$$y = x^T \beta$$

$$y = \sum_{i=1}^d x_i \beta_i$$

* NOTE THIS APPROACH IS

A NON-BAYESIAN/Frequentist.

Applying the frequentist notion of probability to the random values of the observed variables t_n .

→ FXN: polynomial curve fitting.

Generalized STATISTICAL MODEL

Generalized setting:

$$y = \beta_0 + \sum_{j=1}^d \beta_j x_j, \text{ where } \beta_0 = \text{intercept, bias term}$$

Now introduce homogeneous coordinates: $x_0 = 1$
 to rewrite generalized equation in a compact form. This way you don't worry about the offset from the origin on the y-axis

$$\Rightarrow y = x^T \beta \quad x \beta \in \mathbb{R} \rightarrow d+1 \text{ dimensions}$$

Now with noisy observations in data: $(x_i, y_i) \sim p(x, y | \beta)$
 are distributed according to some p .

You can assume the equation with noise that is iid. in which case, you are also trying to model the noise by predicting β .

$$y_i = x_i^T \beta + \epsilon_i, \text{ where } \epsilon_i \text{ is noise with iid.}$$

$\epsilon_i; 1 \leq i \leq n$ is iid.

So the question changes to; what is the probability of the ~~observed~~ observing a noise ϵ_i given a β .

$$p(\epsilon_i | \beta) = p(y_i - \underbrace{x_i^T \beta}_{\text{residuals}} | \beta)$$

If you know what ϵ_i , then you would know how $y_i - x_i^T \beta$ is distributed.

y_i = True value

$\hat{y}_i = x_i^T \beta$, value of dependent variable/(prediction) predicted by the model.

Estimation problem involves determining β from observations (training data),

$$Z = \{(x_i, y_i), 1 \leq i \leq n\}$$