WARP

Linear-Time Accumulation Schemes

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Motivation

aka
why you should care about accumulation schemes

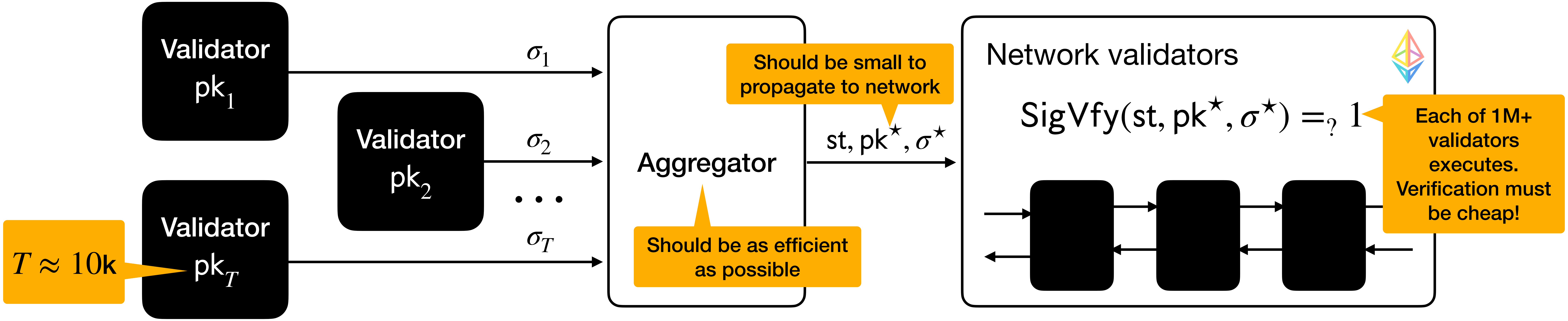
Application: PQ-signature aggregation

Ethereum's consensus

(1) Randomly chosen subcommittee of validators agrees on a state st
(2) Each validator in the committee generates a signature

(3) Aggregator batches signatures into single one
(4) & propagates to the network

(5) Each validator checks the aggregated signature



Today: BLS signatures. **Ethereum is looking for a post-quantum alternative.**

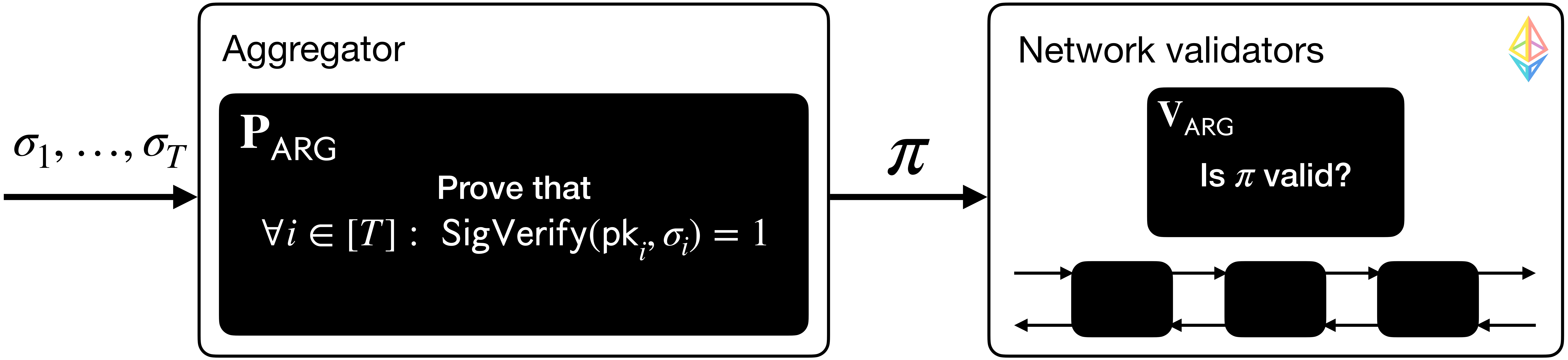
Idea: a pq-signature such as *hash-based XMSS*? **Problem:** how to efficiently aggregate? (no homomorphisms...)

Application: PQ-signature aggregation

A first idea: use a pqSNARK

Let $(\mathbf{P}_{\text{ARG}}, \mathbf{V}_{\text{ARG}})$ be a general purpose pqSNARK (e.g. Spartan+WHIR).

Wednesday at 9:00
Proof systems track



PQ secure ✅

Cheap verification ✅

Compressing $|\pi| \ll T \cdot |\sigma|$ ✅

$|\pi|$ depends on $\log T$

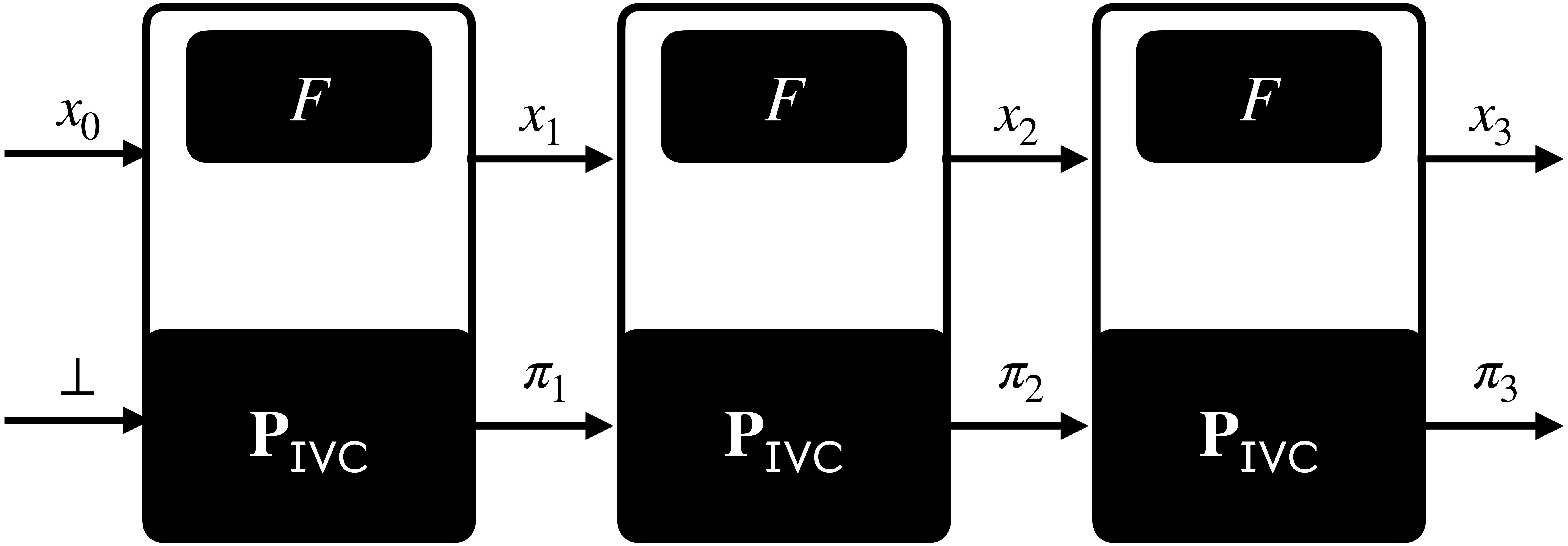
Aggregator needs memory $\Omega(T)$

Can we do better?

Incrementally Verifiable Computation (IVC)

To prove $x_T = F^T(x_0)$, prove $\exists x_1, \ldots, x_{T-1}$ such that $\forall i \in [T], x_i = F(x_{i-1})$.

In signature aggregation:
$F((\sigma_i, pk_i), b_i) := b_i \wedge \mathsf{SigVfy}(\mathsf{st}, pk_i, \sigma_i)$



$\mathbf{V}_{\mathsf{IVC}}(x_{i-1}, x_i, \pi_i)$ checks that π_i attests the whole computation!

$\mathbf{P}_{\mathsf{IVC}}$ costs independent from T ✅

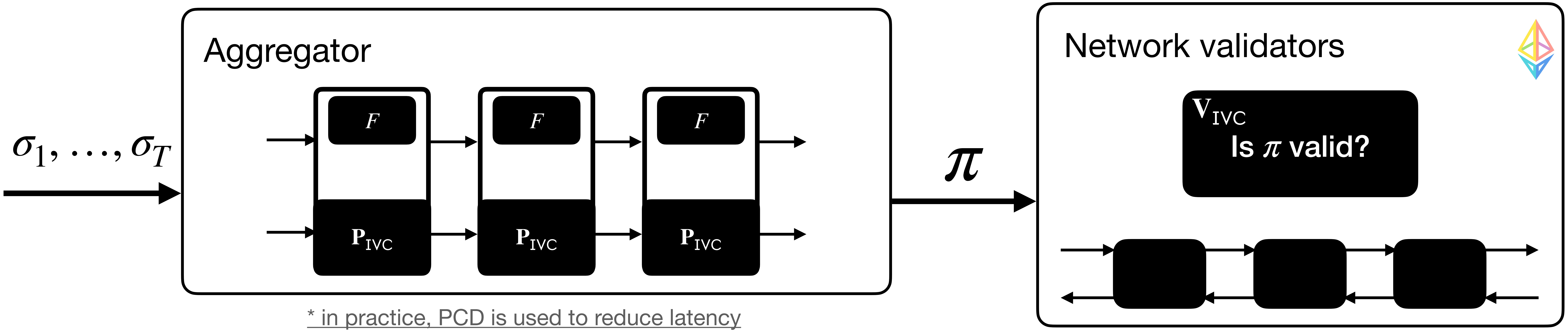
IVC can be generalized to **Proof-Carrying-Data** (PCD).
PCD considers a directed acyclic graph instead of a line.
PCD in practice is preferable to IVC, as it enables reducing the prover's latency.

Let's apply IVC to the initial idea.

Application: PQ-signature aggregation

Final blueprint:

Let $(\mathbf{P}_{\text{IVC}}, \mathbf{V}_{\text{IVC}})$ be a post-quantum secure IVC scheme.



* in practice, PCD is used to reduce latency

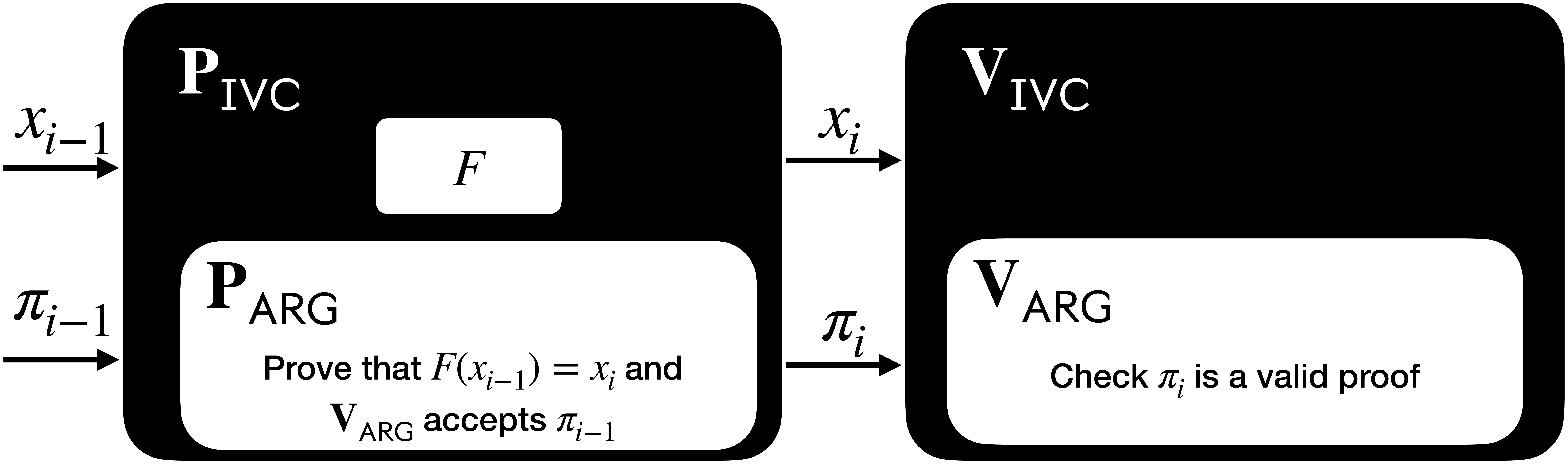
PQ secure ✅	$\lvert\pi\rvert$ independent from T ✅	Cheap aggregator ✅	Cheap verification ✅

Wonderful. Where can I get IVC?

IVC from SNARKs

Recursive proof composition

(*) more complex than this, needs preprocessing



PQ SNARK $\Longrightarrow$ PQ IVC ✅

$|\pi|$ independent from T ✅

Cheap verification ✅

Memory costs independent from T ✅

Cost of $\mathbf{P}_{\text{IVC}} \approx |F| + |\mathbf{V}_{\text{ARG}}|$

Concretely: $|\mathbf{V}_{\text{ARG}}| \approx 2^{20}$ constraints

i.e. recursive overhead is quite large

Good starting point, but can be improved!

Accumulation Schemes

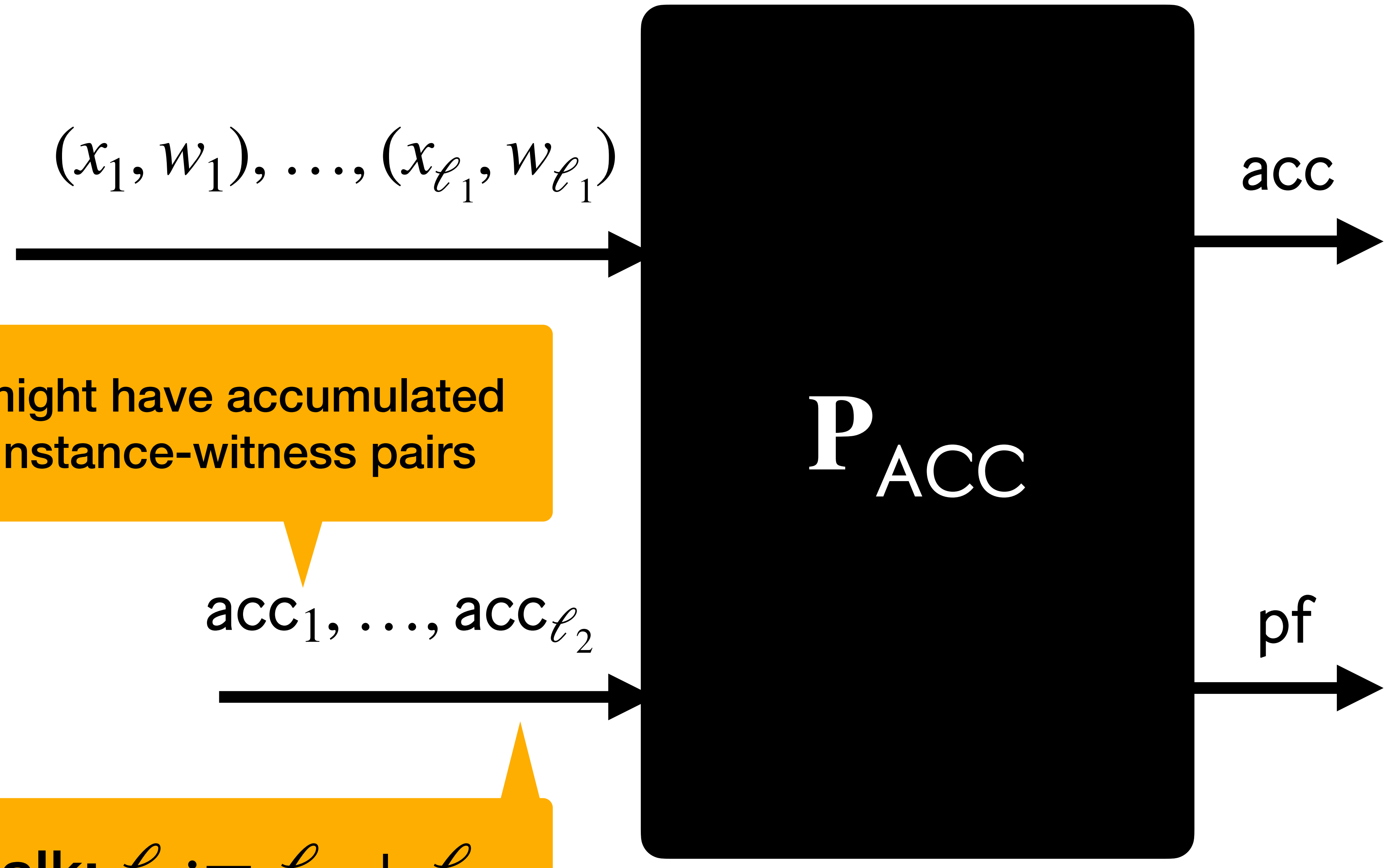
A lightweight tool for batching

Any ARG yields ACC with $|\mathbf{V}_{\mathsf{ACC}}| \approx \ell_1 \cdot |\mathbf{V}_{\mathsf{ARG}}|$.
We can do (significantly) better!

Enables batching many checks $(x_i, w_i) \in_? \mathscr{R}$ into an accumulator acc.

$\mathbf{V}_{\mathsf{ACC}}$ verifies that adding the inputs into acc was done correctly

$\mathbf{D}_{\mathsf{ACC}}$ decides whether acc is valid.



These might have accumulated many instance-witness pairs

This talk: $\ell := \ell_1 + \ell_2$

If:

a) $\mathbf{V}_{\mathsf{ACC}}((x_i)_i, (\mathsf{acc}_j)_j, \mathsf{acc}, \mathsf{pf}) = 1$

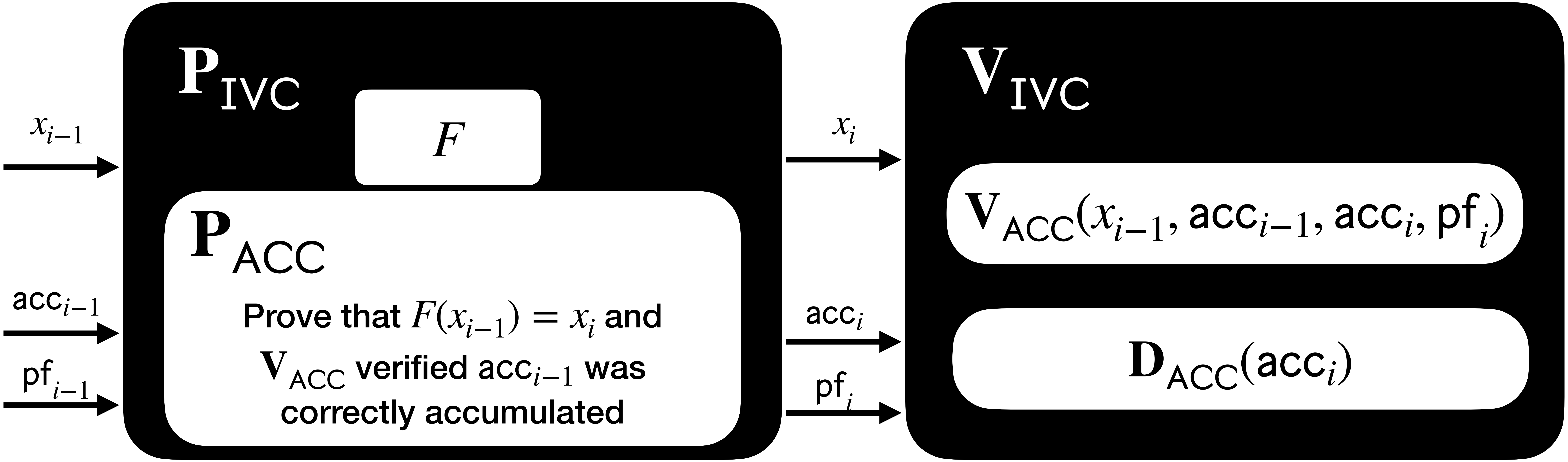
b) $\mathbf{D}_{\mathsf{ACC}}(\mathsf{acc}) = 1$

Then w.h.p:

$$\forall i \in [\ell_1] : \ (x_i, w_i) \in \mathscr{R}$$
$$\forall j \in [\ell_2] : \ \mathbf{D}_{\mathsf{ACC}}(\mathsf{acc}_j) = 1$$

IVC from accumulation

(*) actually we need a more refined notion: "split" accumulation schemes



PQ Accumulation $\implies$ PQ IVC ✅

Memory costs independent from T ✅

$|\pi|$ independent from T ✅

Cost of $\mathbf{P}_{\mathsf{IVC}} \approx |F| + |\mathbf{V}_{\mathsf{ACC}}|$ ✅

$\ll |\mathbf{V}_{\mathsf{ARG}}|$

Not succinct

Cost of $\mathbf{V}_{\mathsf{IVC}} \approx |\mathbf{V}_{\mathsf{ACC}}| + |\mathbf{D}_{\mathsf{ACC}}|$

Wrap with a final SNARK $\implies$ succinct verification ✅

One more thing…

ACC is not limited to signature aggregation

Accumulation schemes are broadly useful for integrity in distributed systems with repeated computations.

Verifiable Virtual Machines (VVMs)

SP1

RISC ZERO

NEXUS

OpenVM

+ At least 20 more…

Digital provenance

VIMz: Private Proofs of Image Manipulation using Folding-based zkSNARKs*
Stefan Dziembowski Shahriar Ebrahimi Parisa Hassanizadeh

Eva: Efficient Privacy-Preserving Proof of Authenticity for Lossily Encoded Videos
Chengru Zhang[1], Xiao Yang[2], David Oswald[2], Mark Ryan[2], and Philipp Jovanovic[3]

Consensus

Breaking the $O(\sqrt{n})$-Bit Barrier: Byzantine Agreement with Polylog Bits Per Party
Elette Boyle* Ran Cohen† Aarushi Goel‡

And more…

Reef: Fast Succinct Non-Interactive Zero-Knowledge Regex Proofs
Sebastian Angel* Eleftherios Ioannidis* Elizabeth Margolin* Srinath Setty† Jess Woods*
*University of Pennsylvania †Microsoft Research

ALPACA: Anonymous Blocklisting with Constant-Sized Updatable Proofs
Jiwon Kim, University of Michigan Abhiram Kothapalli, University of California, Berkeley Orestis Chardouvelis, Carnegie Mellon University
Riad S. Wahby, Carnegie Mellon University Paul Grubbs, University of Michigan

Mangrove: A Scalable Framework for Folding-based SNARKs
Wilson Nguyen Trisha Datta Binyi Chen Nirvan Tyagi Dan Boneh

Accumulation schemes:

Group-based

Nova, Supernova, Hypernova, Protostar, Protogalaxy, NeutronNova, KZHFold, …

Must use 256-bit fields, accumulation time super-linear, cycles of curves required for recursion, not pq

Lattice-based

Latticefold, Lova, Latticefold+, Neo

Very promising, accumulation costs super-linear, plausibly pq some field flexibility

Hash-based

Awh, ARC, [TODAY]

Accumulation costs can be linear, plausibly pq, full field flexibility

Our results

Polynomial Equation Satisfiability

$$\mathcal{R}_{\mathsf{PESAT}}(\mathbb{F}) = \left\{ (i, x, w) : \begin{array}{c} i = (\hat{\mathbf{p}}, M, N, k) \\ x \in \mathbb{F}^{N-k} \\ w \in \mathbb{F}^{k} \\ \forall i \in [M] : \ \hat{\mathbf{p}}_i(x, w) = 0 \end{array} \right\}$$

Polynomial over $\mathbb{F}$ in N variables.

PESAT generalizes:
R1CS, CCS, GR1CS…

e.g. R1CS: for $\mathbf{A}, \mathbf{B}, \mathbf{C} \in \mathbb{F}^{M \times N}$ and $x \in \mathbb{F}^{N-k}$: $\exists w \in \mathbb{F}^{N-k}$ such that $\mathbf{A}\begin{bmatrix} x \\ w \end{bmatrix} \circ \mathbf{B}\begin{bmatrix} x \\ w \end{bmatrix} = \mathbf{C}\begin{bmatrix} x \\ w \end{bmatrix}$

Define $\hat{\mathbf{p}}_i(\mathbf{Z}) = \langle \mathbf{a}_i, \mathbf{Z} \rangle \cdot \langle \mathbf{b}_i, \mathbf{z} \rangle - \langle \mathbf{c}_i, \mathbf{z} \rangle$. The equivalent PESAT condition becomes:

"$\exists w \in \mathbb{F}^{N-k}$ such that $\forall i \in [M] : \ \hat{\mathbf{p}}_i(x, w) = 0$"

WARP 🌀

An essentially optimal hash-based accumulation scheme

To accumulate ℓ instances of $\mathscr{R}_{\mathsf{PESAT}}(\mathbb{F})$ and accumulators

Same complexity as deciding the instances and accumulators!

Prover cost: $O(\ell \cdot |\hat{\mathbf{p}}|)$ $\mathbb{F}$-ops and $O(k)$ random oracle queries

Verifier cost: $O(\ell \cdot (\log N + \log M + \lambda))$ $\mathbb{F}$-ops and
$O(\ell \cdot \lambda \cdot \log k)$ random oracle queries

Optimal for hash-based

Decider cost: $O(\hat{\mathbf{p}})$ $\mathbb{F}$-ops and $O(k)$ random oracle queries

Secure in the pure random oracle model (no other cryptography needed).

Can be instantiated over every $\mathbb{F}$ that is sufficiently large for soundness.

In fact, can be instantiated over every $\mathbb{F}$ using field extensions. Asymptotics vary.

Comparison

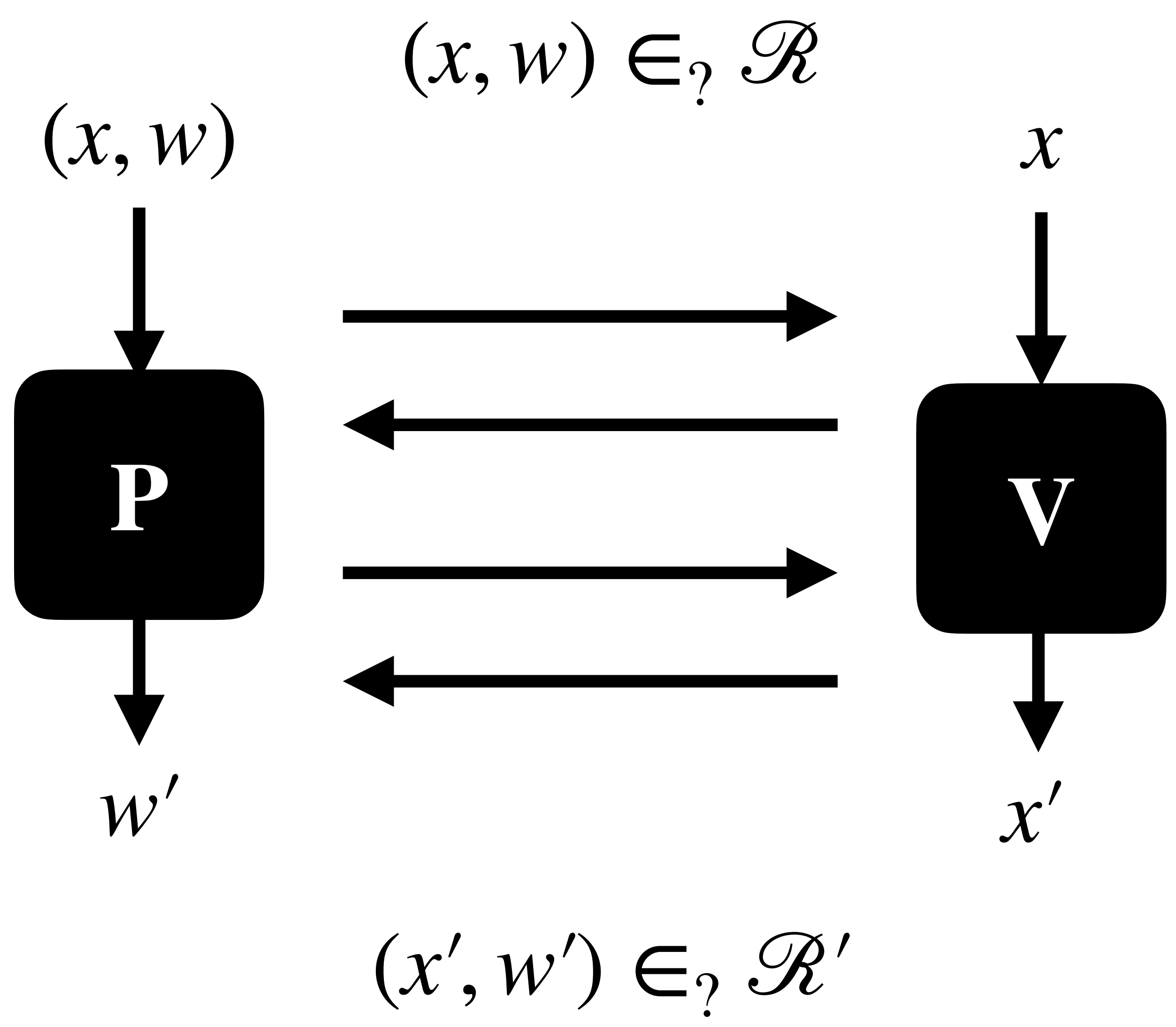
In this slide
$\ell = O(1)$

	hash-based?	linear prover?	verifier size (RO queries)
Brakedown	✅	✅	$O(\lambda \cdot \sqrt{k})$
Blaze	✅	✅	$O(\lambda \cdot \log^2 k)$
Group or lattice-based accumulation (Nova, etc.)	❌	❌	$O(1)$
Arc	✅	❌	$O(\lambda \cdot \log k)$
This work	✅	✅	$O(\lambda \cdot \log k)$
FACS (concurrent)	✅	✅	$O(\lambda \cdot \log k)$

On Hash-Based Accumulation

Hash-Based Reductions

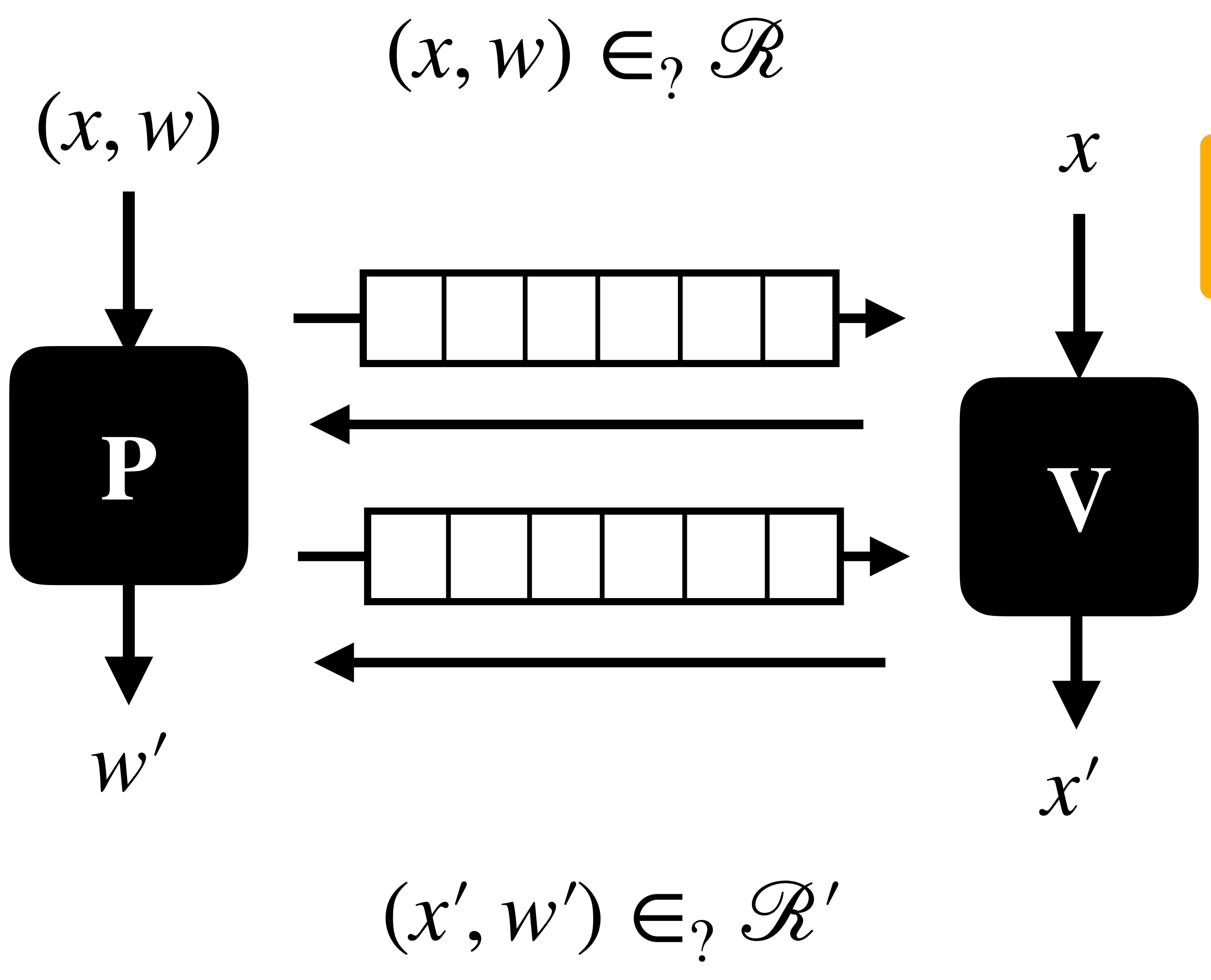
Interactive reduction
$\mathscr{R} \to \mathscr{R}'$



e.g. sumcheck protocol

Typically, want to reduce
$\mathscr{R}^\ell \to \mathscr{R}$

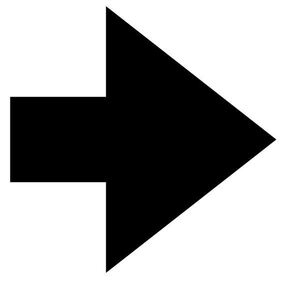
Interactive oracle reduction



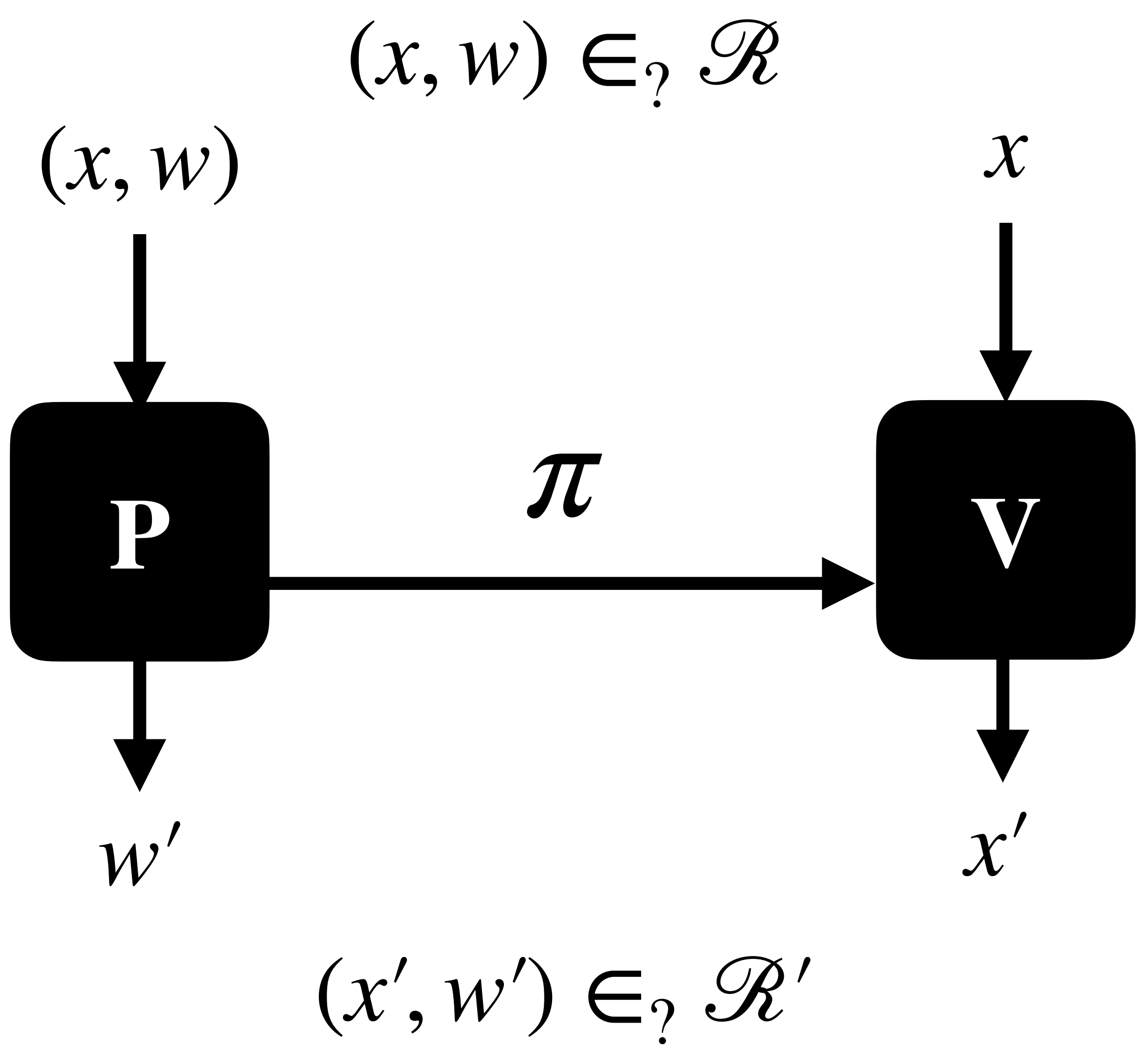
Oracles allow for
succinct verification

Our focus!

Standard techniques:
Merkle Trees + FS



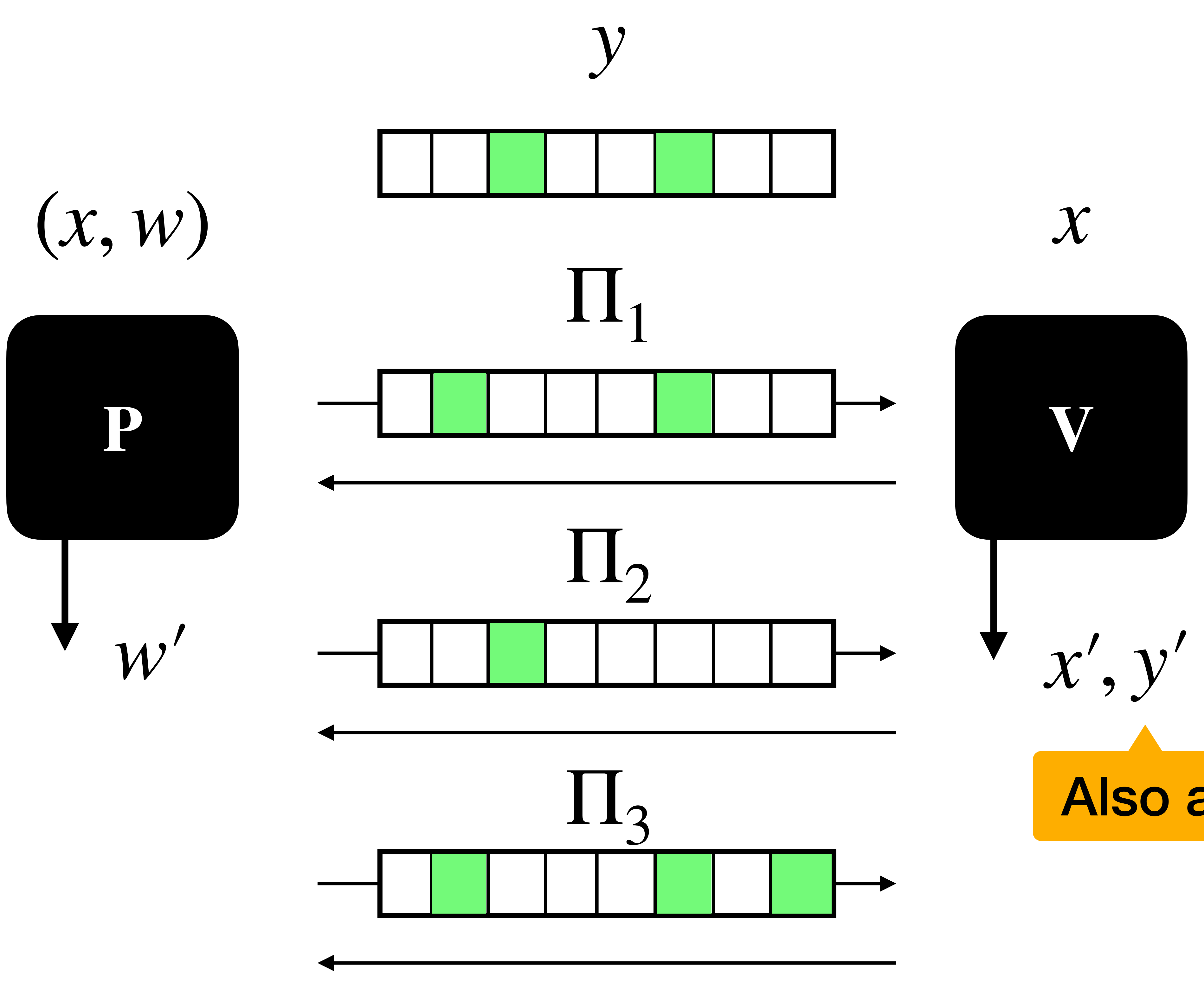
Hash-Based
(Non-Interactive) Reduction



Core of hash-based
accumulation schemes

IORs of Proximity

IOPP : ARG = IORP : ACC



Also an oracle

Completeness

y' can depend on $(y, \Pi_1, \Pi_2, \ldots,)$

If $(x, y, w) \in R$ then $(x', y', w') \in R$

Soundness

If $\Delta(y, R[x]) > \delta$ then w.h.p. $\Delta(y', R[x']) > \delta'$

Not enough, must be state-restoration sound for FS security

Not enough must be knowledge-sound too

Large, think 2^{20}

Proof length l $\approx O(k)$

Queries q $\approx O(\lambda)$

Small, think ~100

+ RO

Prover RO queries $O(\mathsf{l})$

Verifier RO queries $O(\mathsf{q} \cdot \log \mathsf{l})$

Accumulation from IORs

PESAT IOR_1

Reduce PESAT to proximity of an (encoded) witness to a relation

$$\mathscr{R}_{\mathsf{PESAT}}(\mathbb{F}) \to \mathscr{R}_{\mathsf{acc}}$$

Batching IORP_2

Batches many instances of accumulation relation into a single one

$$\mathscr{R}_{\mathsf{acc}}^{\ell} \to \mathscr{R}_{\mathsf{acc}}$$

Hash-based accumulation constructed by compiling with Merkle Trees and Fiat-Shamir

Final IOR $\mathscr{R}_{\mathsf{PESAT}}(\mathbb{F})^{\ell_1} \times \mathscr{R}_{\mathsf{acc}}^{\ell_2} \to \mathscr{R}_{\mathsf{acc}}$

ℓ_1 instances of the relation

$\mathscr{R}_{\mathsf{PESAT}}(\mathbb{F}) \quad \cdots \quad \mathscr{R}_{\mathsf{PESAT}}(\mathbb{F})$

ℓ_2 accumulators

$\mathscr{R}_{\mathsf{acc}} \quad \cdots \quad \mathscr{R}_{\mathsf{acc}}$

$\mathrm{IOR}_1 \quad \cdots \quad \mathrm{IOR}_1$

ℓ accumulators

$\mathscr{R}_{\mathsf{acc}} \quad \cdots \quad \mathscr{R}_{\mathsf{acc}} \quad \mathscr{R}_{\mathsf{acc}} \quad \cdots \quad \mathscr{R}_{\mathsf{acc}}$

IORP_2

1 accumulator

$\mathscr{R}_{\mathsf{acc}}$

Conclusion

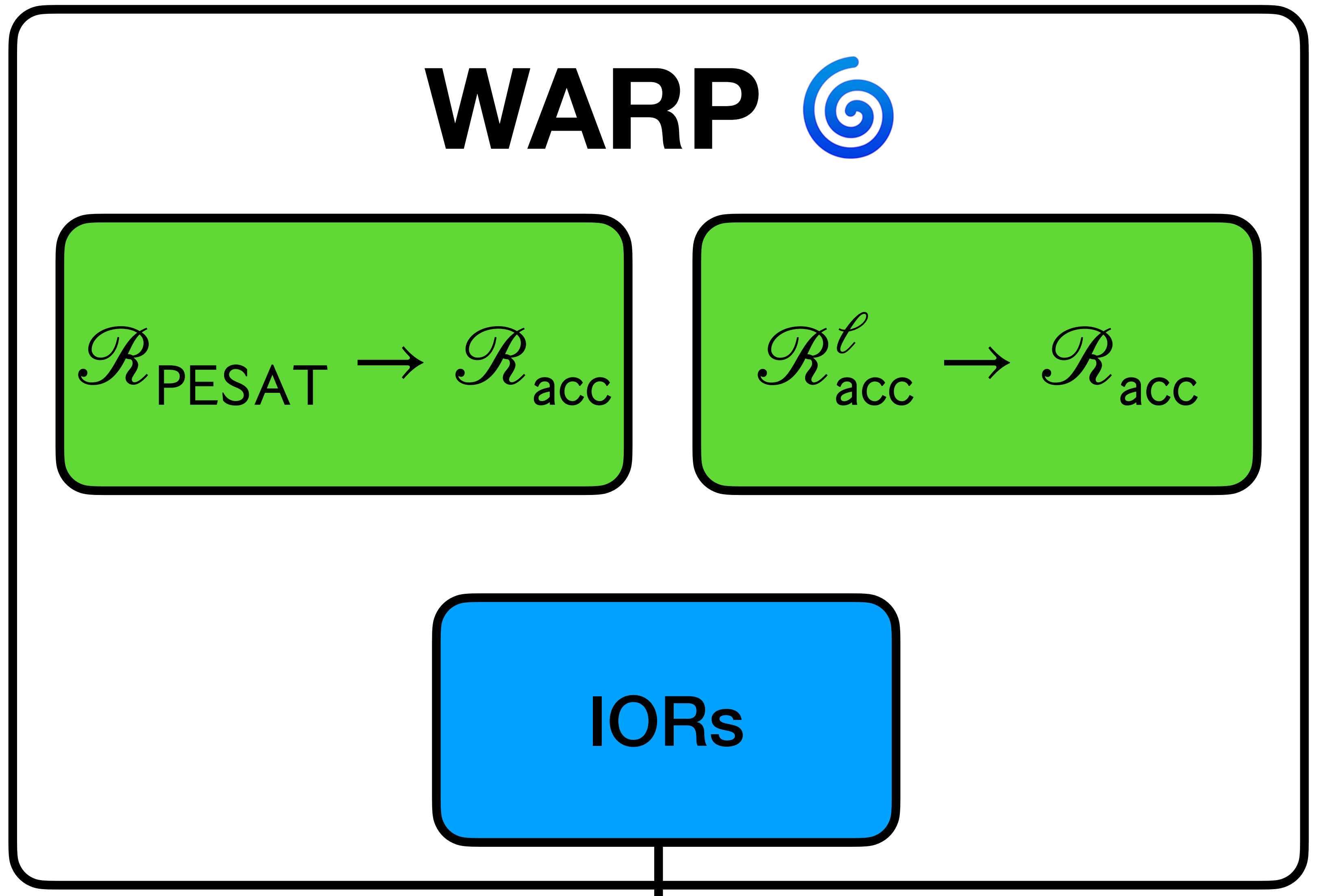
Recap

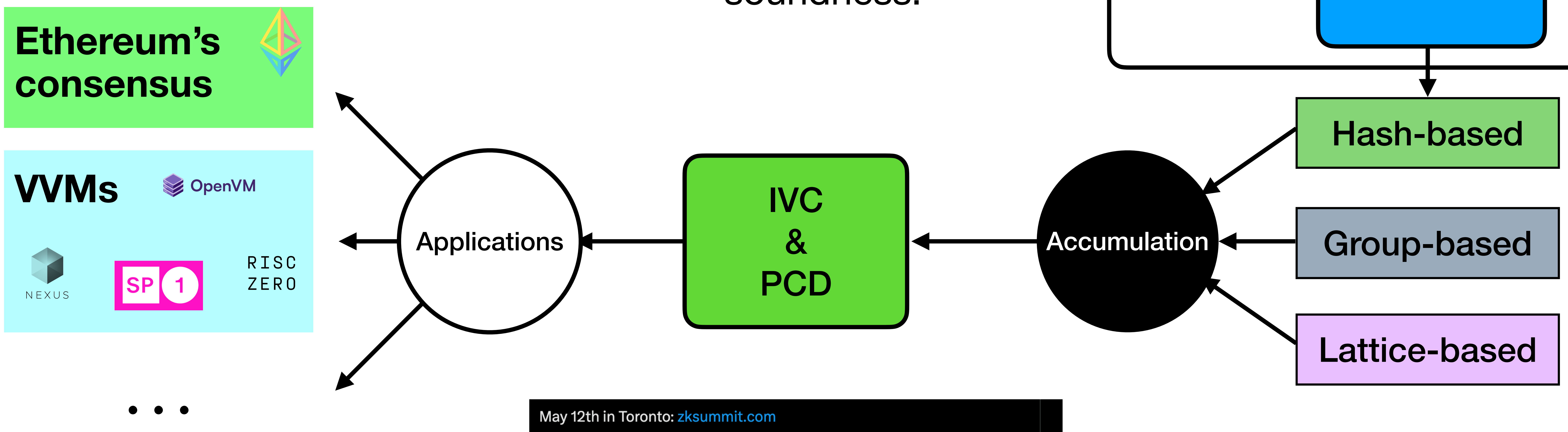
Lots I could not cover today!

Out of domain samples for general linear codes

Twin-constraint pseudobatching

New notions of round-by-round knowledge soundness!





Want to hear more?

William will present WARP 🌀!
May 12th in Toronto.
More details @ **zksummit.com**

Extra slides

Application: PQ-signature aggregation

Ethereum's consensus

- Ethereum's consensus requires validator to sign a message, which is aggregated to a single signature and distributed to the network. Currently using BLS signatures (**vulnerable** to quantum attacks).

- Replace the signature with hash-based XMSS. **Problem:** how to efficiently aggregate? No homomorphic structure to exploit.

Approach a): use pqSNARK to show:
$\forall i \in [T]: \ \mathsf{SigVfy}(\mathsf{pk}_i, m, \sigma_i)$

Pros:

- $|\pi| \ll T \cdot |\sigma_i|$
- PQ security

Cons:

- $|\pi| = O(T)$
- Memory usage is also $O(T)$

Approach b): use IVC with:
$F(i, \sigma_i) = \mathsf{SigVfy}(\mathsf{pk}_i, m, \sigma_i)$

- $|\pi|$ **independent** of T
- Memory usage also **independent of** T