Integral Tentu

Contoh :

① $\int_1^2 \frac{3}{1} x^5 \, dx = \frac{3}{4} \cdot \frac{1}{5+1} x^{5+1} \Big|_0^2$

$\Longleftrightarrow \frac{1}{8} x^6 \Big|_0^2$

$= \left\{ \frac{1}{8} \cdot 2^6 \right\} - \left\{ \frac{1}{8} \cdot 0^6 \right\}$

$= \frac{1}{8} \cdot 64 - 0$

$= 8$

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② $\int_{-1}^{3} \left(\frac{2}{5} x^7 + 6x^8 \right) dx$

$\Longleftrightarrow \int_{-1}^{3} \frac{2}{5} x^7 \, dx + \int 6x^8 \, dx$

$= \frac{2}{5} \cdot \frac{1}{7+1} x^{7+1} + \frac{62}{8+1} x^{8+1} \Big|_{-1}^{3}$

$\frac{8}{4} \quad \frac{2}{1} \quad \frac{9}{3} \quad \frac{3}{1}$

$$= \frac{1}{20} x^8 + \frac{2}{3} x^9 \Big|_{-1}^{3}$$

$$= \left\{ \frac{1}{20} \cdot 3^8 + \frac{2}{3} \cdot 3^9 \right\} - \left\{ \frac{1}{20} (-1)^8 + \frac{2}{3} (-1)^9 \right\}$$

$$= \left\{ \frac{1}{20} \cdot 6561 + \frac{2}{3} \cdot 19.683 \right\} - \left\{ \frac{1}{20} \cdot 1 + \frac{2}{3} (-1) \right\}$$

$$= \left\{ \frac{6561}{20} + 13122 \right\} - \left\{ \frac{3 - 40}{60} \right\}$$

$$= \left\{ \frac{6561 + 262440}{20} \right\} - \left\{ -\frac{37}{60} \right\}$$

$$= \frac{269001}{20} + \frac{37}{60}$$

$$= \frac{807003 + 37}{60}$$

$$= \frac{807\,040}{60} = \frac{40352}{3}$$

③ $\int_1^3 2x\,(3x^2+4)^3\,dx$

$u = x^2 + 4$

$u' = \frac{du}{dx} = 2 \cdot 3 \cdot x^{2-1}$

$= 6x$

$dx = \frac{1}{6x}\,du$

shg: $\int_1^3 2x\,(3x^{(2)}+4)^3\,dx$

$\Longleftrightarrow \int_1^3 \cancel{2x} \cdot u^3 \cdot \frac{1}{\cancel{6x}_3}\,du$

$= \int_1^3 \frac{1}{3} \cdot u^3\,du$

$= \frac{1}{3} \cdot \frac{1}{3+1}\,u^{3+1}\Big|_1^3$

$$= \frac{1}{12} u^4 \Big|_1^3$$

$$= \frac{1}{12} (3x^2 + 4)^4 \Big|_1^3$$

$$= \left\{\frac{1}{12}(3 \cdot 3^2 + 4)^4\right\} - \left\{\frac{1}{12}(3 \cdot 1^2 + 4)^4\right\}$$

$$= \left\{\frac{1}{12} \cdot 31^4 - \frac{1}{12} \cdot 7^4\right\}$$

$$= \frac{923521}{12} - \frac{2401}{12}$$

$$= \frac{921120}{12} = 76.760$$

(4) $\int_0^3 3(4x - 5)^2 \, dx$

misal : $u = 4x - 5$

$$u' = \frac{du}{dx} = 4$$

$$dx = \frac{1}{4}\,du$$

$$\text{sbg} : \int_0^3 3(4x-5)^2\,dx$$

$$\Leftrightarrow \int_0^3 3 \cdot u^2 \cdot \frac{1}{4}\,du$$

$$= \frac{3}{4} \cdot \frac{1}{2+1} u^{2+1} \Big|_0^3$$

$$= \frac{1}{4}(4x-5)^3 \Big|_0^3$$

$$= \left\{\frac{1}{4}(4\cdot 3-5)^3\right\} - \left\{\frac{1}{4}(4\cdot 0-5)^3\right\}$$

$$= \frac{1}{4}\cdot 7^3 - \frac{1}{4}\cdot(-5)^3$$

$$= \frac{343}{4} - \frac{(-125)}{4}$$

$$= \frac{343+125}{4} = \frac{468}{4} = 117$$

(5) $\int_{-1}^{2} (2x+5)(x^2+5x)^3 \, dx$

Misal : $u = x^2 + 5x$

$u' = \frac{du}{dx} = 2x + 5$

$dx = \frac{1}{2x+5} \, du$

Shg : $\int_{-1}^{2} (2x+5)(x^2+5x)^3 \, dx$

$= \int_{-1}^{2} \cancel{(2x+5)} \, u^3 \, \frac{1}{\cancel{2x+5}} \, du$

$= \int_{-1}^{2} u^3 \, du = \frac{1}{3+1} u^{3+1} \Big|_{-1}^{2}$

$= \frac{1}{4} (x^2+5x)^4 \Big|_{-1}^{2}$

$= \left\{ \frac{1}{4} (2^2 + 5 \cdot 2)^4 \right\} - \left\{ \frac{1}{4} \cdot ((-1)^2 + 5 \cdot (-1))^4 \right\}$

$$= \left\{ \frac{1}{4} \cdot 14^4 - \frac{1}{4}(-4)^4 \right\}$$

$$= \frac{38416}{4} - \frac{256}{4} = \frac{38160}{4}$$

$$= 9540$$