The model built is a balanced network flow problem exploiting demands, supplies and distances. The dataset includes walking distances in seconds across 47 different locations. Six locations are the centres representing the locations where the home care staff are based at, while the remainder locations are the clients indicating the locations of the houses of the citizens needing the home care services. There are some zero entries in the distance matrix meaning that there is no information for these distances and hence, we assume that there is no connection between the two locations. The aim of the model is to determine how many clients each centre will serve in order to keep the minimum walking distance.

The decision variable is the flow variable which indicates the flow between centre and client. The objective function is to minimize the total distance walked across valid flows, namely the flows that do not have zero distance. Each of the 41 clients has a demand of one task to be carried and two out of six centres have limited capacities of maximum of six and seven tasks respectively. To initialize the model, we specify that the supply is equal to zero and the demand equal to 1 for all clients. Similarly, we set the demand as zero for all centres and select a huge number of supplies for the centres with no capacity constraint and the maximum value for the two centres with the capacity limitations.

After reading the entire distance matrix in FICO Xpress, a dummy variable (connected) is created to track if a client location is connected with at least one of the centres otherwise if there is no connection between the client and the centres, the client can not be served by the specific centres. This variable helps to balance the network in order the total supply can be equal to the total demand. For the clients that can be served by the specific centres, we add a constraint which does not allow connections when the distance is zero. Thus, the objective function cannot select the zero distance when minimizing. Subsequently, we ensure that for each demand node (clients) the incoming flow is equal to demand since there is no outgoing flow and we multiplied the demand with the dummy variable (“connected”) in order to exclude the demand of the clients that cannot be served by the specific centres due to zero connection. It is worth mentioning that there are eight out of 41 clients that have no connection with any of the six centres, hence, the net supplies should be 33 instead of 41 in order to ensure the balance of the network. Ultimately, we set the decision variable to be equal or less than the maximum supply for each centre in order to ensure that the outgoing flow is equal or less than the maximum supply since the incoming flow is zero.

The results of the network problem are illustrated in the below table:

|  |  |
| --- | --- |
| **Centres** | **Number of Clients** |
| Centre 1 | 7 |
| Centre 2 | 5 |
| Centre 3 | 7 |
| Centre 4 | 10 |
| Centre 5 | 4 |

As demonstrated in the results, covering the demand based on minimizing the total distance walked, we utilise five out of six centres. This can be justified because the missing centre has longer walking distances compared to other five and since there are no capacity constraints on the number of tasks for 4 centres, the specific centre can not be selected. Another point worth noting is that the flow variable has integer values because both net supplies and the bounds are integers, therefore, the optimal solution obtained is integer as well.