



# MATH100 Results

**Qwen2.5-Math-72B-Instruct+Qwen-RM**

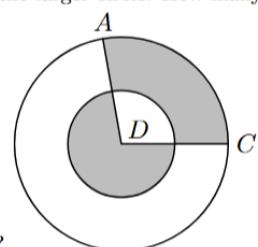
**BoN@N**

**N=256:** 共90个正确

- **id58:** false positive, 面积表示不对
  - Conceptual Error

Problem:

In the diagram, two circles, each with center  $D$ , have radii of 1 and 2. The total area of the shaded region is  $\frac{5}{12}$  of the area of the larger circle. How many



degrees are in the measure of (the smaller)  $\angle ADC$ ?

Let  $\theta$  be the measure of  $\angle ADC$  in radians. The area of the sector of the larger circle with central angle  $\theta$  is:

$$\frac{\theta}{2\pi} \times 4\pi = 2\theta.$$

The area of the sector of the smaller circle with central angle  $\theta$  is:

$$\frac{\theta}{2\pi} \times \pi = \frac{\theta}{2}.$$

The total area of the shaded region is the sum of the areas of these two sectors:

$$2\theta + \frac{\theta}{2} = \frac{4\theta}{2} + \frac{\theta}{2} = \frac{5\theta}{2}.$$

We know that this total area is equal to  $\frac{5\pi}{3}$ , so we set up the equation:

$$\frac{5\theta}{2} = \frac{5\pi}{3}.$$

To solve for  $\theta$ , we divide both sides by  $\frac{5}{2}$ :

$$\theta = \frac{5\pi}{3} \times \frac{2}{5} = \frac{2\pi}{3}.$$

To convert  $\theta$  from radians to degrees, we use the fact that  $\pi$  radians is 180 degrees:

$$\theta = \frac{2\pi}{3} \times \frac{180}{\pi} = \frac{2 \times 180}{3} = 120.$$

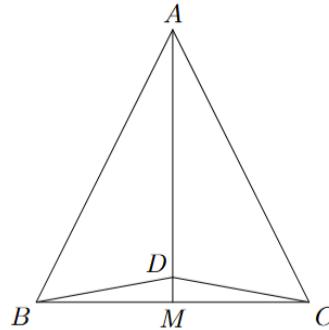
Therefore, the measure of  $\angle ADC$  is  $\boxed{120}$ .

- **id64:** false positive, 其实是  $\frac{3\theta}{2}$  和  $\frac{\theta}{2}$

- Conceptual Error

**Problem:**

Triangle  $ABC$  is isosceles, with  $AB = AC$  and altitude  $AM = 11$ . Suppose that there is a point  $D$  on  $\overline{AM}$  with  $AD = 10$  and  $\angle BDC = 3\angle BAC$ . Find the perimeter of triangle  $ABC$ .



**Solution:**

Given an isosceles triangle  $ABC$  with  $AB = AC$  and altitude  $AM = 11$ , there is a point  $D$  on  $\overline{AM}$  such that  $AD = 10$  and  $\angle BDC = 3\angle BAC$ . We need to find the perimeter of triangle  $ABC$ .

First, let  $\angle BAC = \theta$ . Then  $\angle BDC = 3\theta$ . Since  $D$  is on the altitude  $AM$ , we have  $DM = AM - AD = 11 - 10 = 1$ .

Consider the right triangle  $ABM$ . Since  $AM$  is the altitude,  $M$  is the midpoint of  $BC$ , and  $AB = AC$ , we have:

$$\angle ABM = \angle ACM = \frac{180^\circ - \theta}{2}$$

Now, consider the right triangle  $BDM$ . Using the tangent function, we have:

$$\tan \angle BDC = \tan 3\theta = \frac{BM}{DM} = \frac{BM}{1} = BM$$

Thus,  $BM = \tan 3\theta$ .

Similarly, for the right triangle  $ABM$ :

$$\tan \angle BAC = \tan \theta = \frac{BM}{AM} = \frac{BM}{11}$$

Thus,  $BM = 11 \tan \theta$ .

Since  $BM = \tan 3\theta$  and  $BM = 11 \tan \theta$ , we equate the two expressions:

$$\tan 3\theta = 11 \tan \theta$$

Using the triple angle identity for tangent,  $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$ , we substitute:

$$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = 11 \tan \theta$$

Assuming  $\tan \theta \neq 0$ , we can divide both sides by  $\tan \theta$ :

$$\frac{3 - \tan^2 \theta}{1 - 3\tan^2\theta} = 11$$

Cross-multiplying gives:

$$3 - \tan^2 \theta = 11 - 33 \tan^2 \theta$$

- **id68:** false positive, 后面的解释不对

- Logical Error

Problem:

Let  $a, b, c$  be real numbers such that

$$|ax^2 + bx + c| \leq 1$$

for all  $0 \leq x \leq 1$ . Find the largest possible value of  $|a| + |b| + |c|$ .

To confirm that 17 is indeed the maximum value, we use the fact that the Chebyshev polynomial of the first kind,  $T_2(x) = 2x^2 - 1$ , scaled and shifted to

the interval  $[0, 1]$ , gives us the polynomial  $8x^2 - 8x + 1$ . The Chebyshev polynomials are known to be extremal polynomials for the problem of minimizing the maximum deviation from zero on a given interval, which confirms that 17 is the maximum value of  $|a| + |b| + |c|$ .

Thus, the largest possible value of  $|a| + |b| + |c|$  is 17.

- **id74:** false positive, 解释的不太充分, 没有考慮无理根

- Logical Error

Since none of the rational roots satisfy  $x > 3$ , there are no solutions for  $x > 3$ .

Thus, the only solution to  $f(x) = 6$  is  $x = -3$ . Therefore,  $f^{-1}(6) = -3$ . Finally, we sum the solutions:

$$f^{-1}(0) + f^{-1}(6) = 3 + (-3) = 0.$$

The answer is:

0.

## Llama-3.1-8B-Instruct+Skywork-prm

### MCTS(BoN@N)

**N=256:** 共77个正确

- **id5:** false positive, Since  $\overline{BC}$  is parallel to the segment through  $A$ , we can infer that  $\triangle ABC$  is an isosceles triangle with  $AB = BC$ 
  - Conceptual Error
- **id9:** false positive, 因数找的不对
  - Calculation Error

Problem:

What is the result when the greatest common factor of 6432 and 132 is increased by 11?

Solution:

Step 1: Find the greatest common factor (GCF) of 6432 and 132. To find the GCF, we can first list the factors of each number. The factors of 6432 are 1, 2, 3, 4, 6, 8, 12, 16, 32, 48, 64, 96, 128, 192, 256, 384, 512, 768, 1024, 1536, 2048, 3072, 4096, 6144, 6432. The factors of 132 are 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132.

Step 2: Identify the greatest common factor from the lists. From the lists of factors, the greatest common factor of 6432 and 132 is 12.

Step 3: Increase the GCF by 11. We need to add 11 to the GCF found in Step 2.  $12 + 11 = 23$ .

Therefore, the final answer is: 23.

- **id14:** false positive, 这个有点莫名其妙.....

- Jump in Reasoning

Solution:

Step 1: Notice that  $z^5 = 1$  implies that  $z$  is a fifth root of unity. Since  $z \neq 1$ ,  $z$  must be one of the four other fifth roots of unity.

Step 2: The four fifth roots of unity can be expressed as  $\cos \frac{2\pi k}{5} + i \sin \frac{2\pi k}{5}$  for  $k = 1, 2, 3, 4$ . By De Moivre's formula,  $z^5$  must be equal to  $\cos 2\pi + i \sin 2\pi = 1$ .

Step 3: We have that  $z^5 - 1 = 0$ , which can be factored into  $(z - 1)(z^4 + z^3 + z^2 + z + 1) = 0$ . Since  $z \neq 1$ , we have that  $z^4 + z^3 + z^2 + z + 1 = 0$ .

Step 4: Multiplying this equation by  $z^{-1}$  gives  $z^3 + z^2 + z + 1 + z^{-1} = 0$ . This implies that  $z + \frac{1}{z} + z^2 + \frac{1}{z^2} = -1$ .

Step 5: Therefore, we can compute the desired expression directly. Since  $z^4 + z^3 + z^2 + z + 1 = 0$ , we can write that  $z^4 = -z^3 - z^2 - z - 1$ .

Step 6: We can plug this into the expression to obtain  $z + \frac{1}{z} + z^2 + \frac{1}{z^2} = -1$ . Substituting for  $z^4$ , we have that  $z + \frac{1}{z} + z^2 + \frac{1}{z^2} = -1$ .

Step 7: Therefore, the final answer is  $-1$ .

Therefore, the final answer is: -1.

- **id29:** false positive

- Calculation Error

- **id55:** false positive

- Calculation Error

- **id59:** false positive

- Calculation Error

- **id65:** false positive, 前面一顿胡说八道, 后面也有点问题

- Calculation Error

Step 13: Combine the inequalities We only need to consider the interval  $-\sqrt{3} < x < \sqrt{3}$ .

Step 14: Account for the original inequality However, we must also consider the inequality  $-4 < x^4 + 4x^2$ , which is equivalent to  $x^4 + 4x^2 + 4 > 4$ , or  $(x^2 + 2)^2 > 4$ . This means that  $x^2 + 2 > 2$ , or  $x^2 > 0$ . Since the square of any real number is non-negative, we do not need to consider this inequality.

Step 15: Combine the inequalities We only need to consider the interval  $-\sqrt{3} < x < \sqrt{3}$ .

Step 16: Account for the original inequality We must also consider the inequality  $x^4 + 4x^2 < 21$ . We already factored this inequality in Step 8, and found that it is equivalent to  $(x^2 + 7)(x^2 - 3) < 0$ . We found the roots of  $x^2 - 3 = 0$  in Step 9, and we know that  $x^2 + 7 > 0$  for any real  $x$ . Therefore, the inequality  $(x^2 + 7)(x^2 - 3) < 0$  is true for  $-\sqrt{3} < x < \sqrt{3}$ .

Step 17: Combine the inequalities We only need to consider the interval  $-\sqrt{3} < x < \sqrt{3}$ .

Step 18: Exclude values of  $x$  that do not satisfy  $x^4 < 37$  We must exclude the values of  $x$  that do not satisfy  $x^4 < 37$ . We already found the values of  $x$  that satisfy this inequality in Step 7, and we found that they are  $|x| < \sqrt[4]{37}$ . Since  $\sqrt[4]{37} < \sqrt{3}$ , we do not need to exclude any values of  $x$ .

Step 19: Combine the inequalities We only need to consider the interval  $-\sqrt{3} < x < \sqrt{3}$ .

Step 20: Conclude the solution We have found that the values of  $x$  that satisfy the inequality  $-4 < x^4 + 4x^2 < 21$  are  $-\sqrt{3} < x < \sqrt{3}$ .

Therefore, the final answer is:  $(-\sqrt{3}, \sqrt{3})$ .

- **id70:** false positive, 应该看 $4^{24}$

- Logical Error

Step 1: Understand the definition of superfactorial The superfactorial of a number  $n$  is defined as  $n\$ = \underbrace{n!^{n!^{n!^{...}}}}_{n!}$ , which involves raising  $n!$  to a power of itself, repeated  $n!$  times.

Step 2: Determine the units digit of  $4!$  First, we find the units digit of  $4!$ .  $4! = 4 \times 3 \times 2 \times 1 = 24$ . The units digit of  $4!$  is 4.

Step 3: Raise the units digit of  $4!$  to the power of itself We need to raise the units digit of  $4!$  (which is 4) to the power of itself.  $4^4 = 256$ , so the units digit of  $4^4$  is 6.

Step 4: Raise the result to the power of the units digit of  $4!$  again We raise the units digit from the previous step (which is 6) to the power of the units digit of  $4!$  (which is 4).  $6^4$  is not directly necessary since we're only interested in the pattern of the units digit when raising powers of 6. Since the units digit of 6 raised to any power is always 6, the units digit of the superfactorial  $4\$$  will be 6.

Step 5: Determine the units digit of  $4\$$  The units digit of  $4\$$  is equal to the units digit found in the previous step, which we determined to be 6.

Therefore, the final answer is: 6.

**N=64:** 共78个正确

- **id5:** false positive, Since  $\overline{BC}$  is parallel to the segment through  $A$ , we can infer that  $\triangle ABC$  is an isosceles triangle with  $AB = BC$ .
- **id9:** false positive, 因数找的不对

Solution:

Step 1: Find the greatest common factor (GCF) of 6432 and 132 To find the GCF, we can first list the factors of each number. The factors of 6432 are 1, 2, 3, 4, 6, 8, 12, 16, 32, 48, 64, 96, 128, 192, 256, 384, 512, 768, 1024, 1536, 2048, 3072, 4096, 6144, 6432. The factors of 132 are 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132.

Step 2: Identify the greatest common factor from the lists From the lists of factors, the greatest common factor of 6432 and 132 is 12.

Step 3: Increase the GCF by 11 Adding 11 to the GCF, we get  $12 + 11 = 23$ .

Therefore, the final answer is: 23.

- **id14:** false positive, 这个有点莫名其妙.....

Step 1: Notice that  $z^5 = 1$  implies that  $z$  is a fifth root of unity. Since  $z \neq 1$ ,  $z$  must be one of the four other fifth roots of unity.

Step 2: The four fifth roots of unity can be expressed as  $\cos \frac{2\pi k}{5} + i \sin \frac{2\pi k}{5}$  for  $k = 1, 2, 3, 4$ . By De Moivre's formula,  $z^5$  must be equal to  $\cos 2\pi + i \sin 2\pi = 1$ .

Step 3: We have that  $z^5 - 1 = 0$ , which can be factored into  $(z - 1)(z^4 + z^3 + z^2 + z + 1) = 0$ . Since  $z \neq 1$ , we have that  $z^4 + z^3 + z^2 + z + 1 = 0$ .

Step 4: Multiplying this equation by  $z^{-1}$  gives  $z^3 + z^2 + z + 1 + z^{-1} = 0$ . This implies that  $z + \frac{1}{z} + z^2 + \frac{1}{z^2} = -1$ .

Therefore, the final answer is: -1.

- **id29:** false positive
- **id51:** false positive

Step 4: Multiply both sides by -1 to simplify the equation further Multiplying by -1 gives us  $2r^2 - 10r + 28 = 0$ .

Step 5: Factor the quadratic equation Factoring gives us  $(2r - 7)(r - 4) = 0$ .

Step 6: Solve for  $r$  by setting each factor equal to zero Setting each factor equal to zero and solving for  $r$  gives us two possible solutions:  $r = \frac{7}{2}$  and  $r = 4$ .

Step 7: Check if both solutions are valid by plugging them back into the original equation Plugging  $r = \frac{7}{2}$  into the original equation gives us  $\frac{6(\frac{7}{2})^2 - 19(\frac{7}{2}) - 7}{2(\frac{7}{2}) - 7} = \frac{6(\frac{49}{4}) - 19(\frac{7}{2}) - 7}{\frac{14}{2} - 7} = \frac{\frac{147}{4} - \frac{133}{2} - 7}{0}$ , which is undefined. Therefore,  $r = \frac{7}{2}$  is not a valid solution. Plugging  $r = 4$  into the original equation gives us  $\frac{6(4)^2 - 19(4) - 7}{2(4) - 7} = \frac{6(16) - 19(4) - 7}{8 - 7} = \frac{96 - 76 - 7}{1} = \frac{13}{1} = 13$ , which equals  $4(4) - 3 = 13$ . Therefore,  $r = 4$  is a valid solution.

Therefore, the final answer is: 4.

- **id55:** false positive
- **id59:** false positive
- **id65:** false positive, 前面一顿胡说八道, 后面也有点问题

Step 13: Combine the inequalities We only need to consider the interval  $-\sqrt{3} < x < \sqrt{3}$ .

Step 14: Account for the original inequality However, we must also consider the inequality  $-4 < x^4 + 4x^2$ , which is equivalent to  $x^4 + 4x^2 + 4 > 4$ , or  $(x^2 + 2)^2 > 4$ . This means that  $x^2 + 2 > 2$ , or  $x^2 > 0$ . Since the square of any real number is non-negative, we do not need to consider this inequality.

Step 15: Combine the inequalities We only need to consider the interval  $-\sqrt{3} < x < \sqrt{3}$ .

Step 16: Account for the original inequality We must also consider the inequality  $x^4 + 4x^2 < 21$ . We already factored this inequality in Step 8, and found that it is equivalent to  $(x^2 + 7)(x^2 - 3) < 0$ . We found the roots of  $x^2 - 3 = 0$  in Step 9, and we know that  $x^2 + 7 > 0$  for any real  $x$ . Therefore, the inequality  $(x^2 + 7)(x^2 - 3) < 0$  is true for  $-\sqrt{3} < x < \sqrt{3}$ .

Step 17: Combine the inequalities We only need to consider the interval  $-\sqrt{3} < x < \sqrt{3}$ .

Step 18: Exclude values of  $x$  that do not satisfy  $x^4 < 37$  We must exclude the values of  $x$  that do not satisfy  $x^4 < 37$ . We already found the values of  $x$  that satisfy this inequality in Step 7, and we found that they are  $|x| < \sqrt[4]{37}$ . Since  $\sqrt[4]{37} < \sqrt{3}$ , we do not need to exclude any values of  $x$ .

Step 19: Combine the inequalities We only need to consider the interval  $-\sqrt{3} < x < \sqrt{3}$ .

Step 20: Conclude the solution We have found that the values of  $x$  that satisfy the inequality  $-4 < x^4 + 4x^2 < 21$  are  $-\sqrt{3} < x < \sqrt{3}$ .

Therefore, the final answer is:  $(-\sqrt{3}, \sqrt{3})$ .

- **id70: false positive, 应该看 $4^{24}$**

Step 1: Understand the definition of superfactorial The superfactorial of a number  $n$  is defined as  $n\$ = \underbrace{n! \cdot \dots \cdot n!}_{n!}$ , which involves raising  $n!$  to a power of itself, repeated  $n!$  times.

Step 2: Determine the units digit of  $4!$  First, we find the units digit of  $4!$ .  $4! = 4 \times 3 \times 2 \times 1 = 24$ . The units digit of  $4!$  is 4.

Step 3: Raise the units digit of  $4!$  to the power of itself We need to raise the units digit of  $4!$  (which is 4) to the power of itself.  $4^4 = 256$ , so the units digit of  $4^4$  is 6.

Step 4: Raise the result to the power of the units digit of  $4!$  again We raise the units digit from the previous step (which is 6) to the power of the units digit of  $4!$  (which is 4).  $6^4$  is not directly necessary since we're only interested in the pattern of the units digit when raising powers of 6. Since the units digit of 6 raised to any power is always 6, the units digit of the superfactorial  $4\$$  will be 6.

Step 5: Determine the units digit of  $4\$$  The units digit of  $4\$$  is equal to the units digit found in the previous step, which we determined to be 6.

Therefore, the final answer is:  $[6]$ .

- **id95: false positive**

Solution:

Step 1: The polynomial  $f(x)$  is given as  $f(x) = 2x^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + 66$ . We know that the product of the roots of the polynomial is equal to  $(-1)^n \times \frac{a_0}{a_n} = (-1)^n \times \frac{66}{2} = (-1)^n \times 33$ .

Step 2: Since the roots are distinct integers and their product is  $(-1)^n \times 33$ , we need to find two distinct integers whose product is 33. The possible pairs of distinct integers whose product is 33 are: (1, 33) or (-1, -33) or (3, 11) or (-3, -11).

Step 3: Since we want to minimize the value of  $|a_{n-1}|$ , we need to minimize the sum of the absolute values of the roots. When  $n$  is even, the sum of the roots will be  $1 + 33 = 34$  or  $-1 - 33 = -34$  for the pairs (1, 33) and (-1, -33) respectively. When  $n$  is odd, the sum of the roots will be  $3 + 11 = 14$  or  $-3 - 11 = -14$  for the pairs (3, 11) and (-3, -11) respectively.

Step 4: The coefficient  $a_{n-1}$  is equal to the negative sum of the roots (or the positive sum of the roots if  $n$  is even). When  $n$  is even,  $a_{n-1} = 34$  or  $-34$ . When  $n$  is odd,  $a_{n-1} = 14$  or  $-14$ . The absolute value of these values are  $|34| = 34$  or  $|-34| = 34$  or  $|14| = 14$  or  $|-14| = 14$ .

Step 5: Among these values, the smallest absolute value of  $a_{n-1}$  is 14. Therefore, the least possible value of  $|a_{n-1}|$  is  $\boxed{14}$ .

Therefore, the final answer is:  $\boxed{14}$ .

**N=16:** 共77个正确

- **id5:** false positive, Since  $\overline{BC}$  is parallel to the segment through  $A$ , we can infer that  $\triangle ABC$  is an isosceles triangle with  $AB = BC$
- **id9:** false positive, 因数找的不对

Problem:

What is the result when the greatest common factor of 6432 and 132 is increased by 11?

Solution:

Step 1: Find the greatest common factor (GCF) of 6432 and 132. To find the GCF, we can first list the factors of each number. The factors of 6432 are 1, 2, 3, 4, 6, 8, 12, 16, 32, 48, 64, 96, 128, 192, 256, 384, 512, 768, 1024, 1536, 2048, 3072, 4096, 6144, 6432. The factors of 132 are 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132.

Step 2: Identify the greatest common factor from the lists. From the lists of factors, the greatest common factor of 6432 and 132 is 12.

Step 3: Increase the GCF by 11. We need to add 11 to the GCF found in Step 2.  $12 + 11 = 23$ .

Therefore, the final answer is:  $\boxed{23}$ .

- **id14:** false positive, 这个有点莫名其妙.....

Step 1: Notice that  $z^5 = 1$  implies that  $z$  is a fifth root of unity. Since  $z \neq 1$ ,  $z$  must be one of the four other fifth roots of unity.

Step 2: The four fifth roots of unity can be expressed as  $\cos \frac{2\pi k}{5} + i \sin \frac{2\pi k}{5}$  for  $k = 1, 2, 3, 4$ . By De Moivre's formula,  $z^5$  must be equal to  $\cos 2\pi + i \sin 2\pi = 1$ .

Step 3: We have that  $z^5 - 1 = 0$ , which can be factored into  $(z - 1)(z^4 + z^3 + z^2 + z + 1) = 0$ . Since  $z \neq 1$ , we have that  $z^4 + z^3 + z^2 + z + 1 = 0$ .

Step 4: Multiplying this equation by  $z^{-1}$  gives  $z^3 + z^2 + z + 1 + z^{-1} = 0$ . This implies that  $z + \frac{1}{z} + z^2 + \frac{1}{z^2} = -1$ .

Therefore, the final answer is:  $\boxed{-1}$ .

- **id29:** false positive
- **id55:** false positive
- **id59:** false positive

- **id70:** false positive, 应该看 $4^{24}$

Solution:

Step 1: Understand the definition of superfactorial The superfactorial of a number  $n$  is defined as  $n\$ = \underbrace{n!^{n!^{\dots^{n!}}}}_{n!}$ , which involves raising  $n!$  to a power of itself, repeated  $n!$  times.

Step 2: Determine the units digit of  $4!$  First, we find the units digit of  $4!$ , which is  $4! = 4 \times 3 \times 2 \times 1 = 24$ . The units digit of  $4!$  is 4.

Step 3: Raise  $4!$  to the power of itself Since the units digit of  $4!$  is 4, we need to find the units digit of  $4^4$  because of the way superfactorials are defined.  $4^4 = 256$ , and the units digit of  $4^4$  is 6.

Step 4: Determine the units digit of  $4\$$  Since  $4\$$  involves raising  $4!$  to a power of itself, repeated  $4!$  times, and we know the units digit of  $4!^4$  is 6, we only need to consider the units digit of 6 raised to various powers to see if there is any pattern or repetition.  $6^1 = 6$ ,  $6^2 = 36$  (units digit is 6),  $6^3 = 216$  (units digit is 6), and  $6^4 = 1296$  (units digit is 6). It is clear that the units digit of 6 raised to any power is always 6.

Step 5: Conclusion Therefore, the units digit of  $4\$$  is the units digit of 6, which is 6.

The final answer is: 6.

- **id95:** false positive

Step 6: Analyze possible values of  $N$  that satisfy the conditions The possible values of  $N$  that make the units digit of the number even are 0, 2, 4, 6, and 8. We need to find the value of  $N$  for which  $9 + N$  is divisible by 3.

Step 7: Check the divisibility of  $9 + N$  by 3 for the possible values of  $N$  Checking each possible value of  $N$ :  $9 + 0 = 9$ ,  $9 + 2 = 11$ ,  $9 + 4 = 13$ ,  $9 + 6 = 15$ ,  $9 + 8 = 17$ . Only  $9 + 6 = 15$  is divisible by 3.

Step 8: Choose the correct value of  $N$  Therefore, the correct value of  $N$  that makes the number  $21420N$  divisible by 6 is 6.

The final answer is: 6.

N=4: 共71个正确

- **id4:** false positive, step2其实漏了一项

Solution:

Step 1: To rotate a complex number  $z$  around another complex number  $c$  by an angle  $\theta$ , we can use the formula  $w = c + (z - c)e^{i\theta}$ .

Step 2: First, we need to find  $z - c$ . We have  $z = 2 + \sqrt{2} - (3 + 3\sqrt{2})i$  and  $c = 2 - 3i$ . So,  $z - c = (2 + \sqrt{2} - 2) + (-3\sqrt{2} + 3)i = \sqrt{2} - 3\sqrt{2}i$ .

Step 3: Next, we need to find  $e^{i\theta}$  where  $\theta = \frac{\pi}{4}$ . We have  $e^{i\pi/4} = \cos(\frac{\pi}{4}) + i \sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ .

Step 4: Now we substitute  $z - c$  and  $e^{i\theta}$  into the formula for  $w$ . We get  $w = c + (\sqrt{2} - 3\sqrt{2}i)(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)$ .

Step 5: To simplify this, we need to multiply out the complex numbers:  $w = c + (\sqrt{2}(\frac{1}{\sqrt{2}}) - 3\sqrt{2}(\frac{1}{\sqrt{2}})i + \sqrt{2}(\frac{1}{\sqrt{2}})i - 3\sqrt{2}(\frac{1}{\sqrt{2}})i^2)$ . Since  $i^2 = -1$ , this simplifies to  $w = c + (1 - 3i + i + 3)$ .

Step 6: Combine like terms to get  $w = c + (4 - 2i)$ .

Step 7: Substitute in the value of  $c$  to get  $w = (2 - 3i) + (4 - 2i)$ .

Step 8: Combine like terms again to get  $w = 6 - 5i$ .

The final answer is: 6 - 5i

- **id5:** false positive, Since  $\overline{BC}$  is parallel to the segment through  $A$ , we can infer that  $\triangle ABC$  is an isosceles triangle with  $AB = BC$
- **id9:** false positive, 因数找的不对

Problem:

What is the result when the greatest common factor of 6432 and 132 is increased by 11?

Solution:

Step 1: Find the greatest common factor (GCF) of 6432 and 132. To find the GCF, we can first list the factors of each number. The factors of 6432 are 1, 2, 3, 4, 6, 8, 12, 16, 32, 48, 64, 96, 128, 192, 256, 384, 512, 768, 1024, 1536, 2048, 3072, 4096, 6144, 6432. The factors of 132 are 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132.

Step 2: Identify the greatest common factor from the lists. From the lists of factors, the greatest common factor of 6432 and 132 is 12.

Step 3: Increase the GCF by 11. We need to add 11 to the GCF found in Step 2.  $12 + 11 = 23$ .

Therefore, the final answer is: 23.

- **id14:** false positive, step3多了一项出来

Solution:

Step 1: We are given that  $z^5 = 1$  and  $z \neq 1$ . To find  $z + \frac{1}{z} + z^2 + \frac{1}{z^2}$ , we can start by expressing  $z^5$  as 1 and factoring it as  $(z - 1)(z^4 + z^3 + z^2 + z + 1) = 0$ . Since  $z \neq 1$ , it follows that  $z^4 + z^3 + z^2 + z + 1 = 0$ .

Step 2: Now we can use this expression to find  $z + \frac{1}{z} + z^2 + \frac{1}{z^2}$ . We have  $z^4 + z^3 + z^2 + z + 1 = 0$ , so we can multiply the equation by  $z$  to get  $z^5 + z^4 + z^3 + z^2 + z = 0$ . We can also divide the original equation by  $z^2$  to get  $z^2 + z + 1 + z^{-1} + z^{-2} = 0$ .

Step 3: Since  $z^5 = 1$ , we can replace  $z^5$  with 1 in the equation from the previous step. We have  $1 + z^4 + z^3 + z^2 + z = 0$ . We can rearrange the terms to obtain  $z^2 + z + \frac{1}{z} + z + \frac{1}{z^2} = -1$ .

Step 4: Rearrange the equation to isolate  $z + \frac{1}{z} + z^2 + \frac{1}{z^2}$ .  $z + \frac{1}{z} + z^2 + \frac{1}{z^2} = -1$ .

Therefore, the final answer is: -1.

- **id29:** false positive
- **id55:** false positive
- **id58:** false positive
- **id70:** false positive, 应该看 $4^{24}$

Solution:

Step 1: Understand the definition of superfactorial. The superfactorial  $n\$$  is defined as  $n\$ = \underbrace{n!n! \dots n!}_{n!}$ , which means we need to calculate the factorial of  $n$  and raise it to the power of the factorial of  $n$ , repeated  $n!$  times.

Step 2: Determine the units digit of  $4!$ . To calculate the units digit of  $4\$$ , we first need to calculate the units digit of  $4!$ , which is  $4 \times 3 \times 2 \times 1 = 24$ . The units digit of 24 is 4.

Step 3: Raise the units digit of  $4!$  to the power of the units digit of  $4!$ . The units digit of  $4\$$  is the units digit of  $4!^4$ , which is  $4^4 = 256$ . The units digit of 256 is 6.

Step 4: Determine the units digit of the result. Since the units digit of  $4^4$  is 6, the units digit of  $4\$$  is 6.

Therefore, the final answer is: 6.

- **id94:** false positive, 可能的情况没有讨论全

Solution:

Step 1: We need to consider the different possible routes from each starting point to visit the other three points once. We have to calculate the total distance traveled for each possible route.

Step 2: Let's start by considering the route A-B-C-D. The distances for this route are  $AB = 6$ ,  $BC = 5$ , and  $CD = 4$ , so the total distance for this route is  $6 + 5 + 4 = 15$ .

Step 3: Next, let's consider the route A-C-B-D. The distances for this route are  $AC = 6$ ,  $CB = 5$ , and  $BD = 3$ , so the total distance for this route is  $6 + 5 + 3 = 14$ .

Step 4: Now, let's consider the route A-D-C-B. The distances for this route are  $AD = 5$ ,  $DC = 4$ , and  $CB = 5$ , so the total distance for this route is  $5 + 4 + 5 = 14$ .

Step 5: Finally, let's consider the route A-D-B-C. The distances for this route are  $AD = 5$ ,  $DB = 3$ , and  $BC = 5$ , so the total distance for this route is  $5 + 3 + 5 = 13$ .

Step 6: We also need to consider the route B-A-C-D. The distances for this route are  $BA = 6$ ,  $AC = 6$ , and  $CD = 4$ , so the total distance for this route is  $6 + 6 + 4 = 16$ .

Step 7: Next, let's consider the route B-C-A-D. The distances for this route are  $BC = 5$ ,  $CA = 6$ , and  $AD = 5$ , so the total distance for this route is  $5 + 6 + 5 = 16$ .

Step 8: Now, let's consider the route B-D-A-C. The distances for this route are  $BD = 3$ ,  $DA = 5$ , and  $AC = 6$ , so the total distance for this route is  $3 + 5 + 6 = 14$ .

Step 9: Finally, let's consider the route B-D-C-A. The distances for this route are  $BD = 3$ ,  $DC = 4$ , and  $CA = 6$ , so the total distance for this route is  $3 + 4 + 6 = 13$ .

Step 10: Among these routes, the shortest distance is 13, which is obtained for two different routes: A-D-B-C and B-D-C-A.

Therefore, the final answer is: 13.

## DVTS(BoN@N)

N=256: 共80个正确

- **id5:** false positive, Since  $\overline{BC}$  is parallel to the segment through  $A$ , this implies that  $\angle ABC$  and  $\angle CAB$  are corresponding angles, making them congruent.
  - Conceptual Error
- **id9:** false positive, 因数找的不对
  - Calculation Error

Step 1: Find the greatest common factor (GCF) of 6432 and 132. To find the GCF, we can list the factors of each number or use the Euclidean algorithm. The factors of 6432 are 1, 2, 3, 4, 6, 7, 8, 9, 12, 14, 16, 18, 21, 24, 28, 32, 36, 42, 48, 56, 64, 72, 84, 96, 112, 128, 144, 168, 192, 224, 256, 288, 336, 384, 448, 512, 576, 672, 768, 896, 1024, 1152, 1368, 1536, 1792, 2304, 3072, 4608, 6144, and 6432. The factors of 132 are 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, and 132. The greatest common factor is 12.

Step 2: Increase the GCF by 11. To increase the GCF by 11, we add 11 to 12.

- **id28:** false positive, 复数方程解的有点问题
  - Calculation Error

Step 1: Use the given equation to express each term in the form of  $r_i\bar{r}_i$ . Let's express each term  $\frac{1}{r_i\bar{r}_i}$  in the given equation. We can use the equation  $x^{10} + (13x - 1)^{10} = 0$  to find this. We have  $(13x - 1)^{10} = -x^{10}$ . Taking the 10th root of both sides, we get  $13x - 1 = \pm ix$  where  $i$  is the imaginary unit.

Step 2: Solve for  $x$  in the equation to find the values of  $r_i\bar{r}_i$ . Solving  $13x - 1 = ix$  and  $13x - 1 = -ix$ , we get  $x = \frac{1}{13+i}$ . Multiplying the numerator and denominator by the conjugate of the denominator, we get  $x = \frac{13+i}{(13)^2 - (i)^2}$ . Simplifying this expression, we get  $x = \frac{13+i}{169+i^2}$ . Since  $i^2 = -1$ , we have  $x = \frac{13+i}{169+1}$ . This simplifies to  $x = \frac{13+i}{170}$ .

Step 3: Find the product  $r_i\bar{r}_i$ . The product  $r_i\bar{r}_i$  is the square of the magnitude of  $x_i$ , which is given by  $|x_i|^2 = (\frac{13}{170})^2 + (\frac{1}{170})^2$ . Therefore,  $r_i\bar{r}_i = (\frac{13}{170})^2 + (\frac{1}{170})^2 = \frac{169+1}{170^2} = \frac{170}{170^2} = \frac{1}{170}$ .

Step 4: Find the sum of  $\frac{1}{r_i\bar{r}_i}$ . Since there are 5 terms and each term is equal to  $\frac{1}{r_i\bar{r}_i} = \frac{170}{1}$ , the sum of these terms is  $5 \cdot \frac{170}{1} = 850$ .

Therefore, the final answer is: 850.

- **id52:** false positive

- Logical Error

Step 1: Rewrite the expression To analyze the given expression, let's rewrite it in a form that's easier to work with:  $\frac{(x+5)(x+2)}{x+1} = \frac{x^2+7x+10}{x+1}$ .

Step 2: Simplify the expression We can simplify this expression further by dividing the numerator by the denominator using polynomial division or synthetic division.  $\frac{x^2+7x+10}{x+1} = x + 6 + \frac{4}{x+1}$ .

Step 3: Find the minimum value Since  $x > 0$ , let  $y = x+1$  so that  $y > 1$ . The expression  $x + 6 + \frac{4}{x+1}$  becomes  $y + 5 + \frac{4}{y}$ . To minimize  $y + 5 + \frac{4}{y}$ , we should find the minimum value of the expression  $\frac{4}{y}$  and add 5, as  $y$  is positive. For  $y > 1$ , the minimum value of  $\frac{4}{y}$  occurs when  $y = 2$ , corresponding to  $x + 1 = 2$  and  $x = 1$ . Therefore, the minimum value of the expression  $y + 5 + \frac{4}{y}$  is  $2 + 5 + 2 = 9$ .

Step 4: Determine the minimum value of the original expression Now we need to find the minimum value of the original expression  $\frac{(x+5)(x+2)}{x+1}$  for  $x > 0$ . We found that the minimum value of the expression  $x + 6 + \frac{4}{x+1}$  is 9 when  $x = 1$ . Therefore, the minimum value of the original expression is also 9.

Therefore, the final answer is: 9.

- **id54:** false positive, 没讨论A、B为负数的情况

- Logical Error

Step 1: Expand the given expression  $(Ax - 8)(Bx + 3)$   $(Ax - 8)(Bx + 3) = ABx^2 + (3A - 8B)x - 24$ .

Step 2: Match the expanded expression to the original expression  $10x^2 - x - 24$ . We find that  $AB = 10$  and  $3A - 8B = -1$ .

Step 3: Solve for A and B using the matched values Since  $AB = 10$ , we have the following possibilities for (A, B): (10,1), (5,2), (2,5), (1,10). We can test each pair to see which pair satisfies  $3A - 8B = -1$ .

Step 4: Test each pair to find the correct values of A and B - For pair (10,1),  $3(10) - 8(1) = 30 - 8 = 22 \neq -1$  - For pair (5,2),  $3(5) - 8(2) = 15 - 16 = -1$ . Therefore,  $A = 5$  and  $B = 2$ .

- **id55:** false positive, 一开始就化简错误

- Calculation Error

Let  $a$ ,  $b$ , and  $c$  be distinct real numbers. Find the degree of the polynomial

$$p(x) = \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-a)(x-c)}{(b-a)(b-c)} + \frac{(x-a)(x-b)}{(c-a)(c-b)}.$$

Solution:

Step 1: Notice that the denominators in each fraction are the same but in the opposite order as the numerators in the other two fractions. This pattern suggests that combining the fractions and factoring may simplify the polynomial.

Step 2: Combine the fractions over a common denominator. When we combine the fractions, the common denominator is  $(a-b)(a-c)(b-a)(b-c)$ .

Step 3: Rewrite the polynomial with the common denominator.

$$p(x) = \frac{(x-b)(x-c)((b-a)(c-b)) + (x-a)(x-c)((a-b)(c-a)) + (x-a)(x-b)((a-c)(b-a))}{(a-b)(a-c)(b-a)(b-c)}.$$

- **id59: false positive, 理由稍微有点问题**

- Logical Error

Step 3: Expressing the right-hand side in terms of a continued fraction From the previous equation, we can express the right-hand side as a continued fraction:

$$\frac{1}{\sqrt[3]{2} - a} = b + \frac{1}{c + \frac{1}{d + \dots}}.$$

Step 4: Finding the value of  $a$ . Since  $a$ ,  $b$ ,  $c$ ,  $d$  are positive integers, the value of  $a$  must be an integer that makes  $\sqrt[3]{2}$  an integer when subtracted by  $a$ . We claim that  $a = 1$  is the solution since  $\sqrt[3]{2}$  is irrational.

- **id74: false positive, 少讨论一种情况**

- Jump in Reasoning

Step 17: Now we'll move on to calculate  $f^{-1}(6)$ , we need to find the value of  $x$  for which  $f(x) = 6$ . We'll first consider the case when  $x \leq 3$ , so we'll solve  $3 - x = 6$ .

Step 18: Solving the equation  $3 - x = 6$  gives us  $x = 3 - 6 = -3$ .

Step 19: Now, we need to check if  $x = -3$  satisfies the condition  $x \leq 3$ . Since  $-3 \leq 3$ , this value is valid for the first case.

Step 20: We have found the value of  $f^{-1}(0)$ , which is 3, and the value of  $f^{-1}(6)$ , which is  $-3$ .

Step 21: Finally, we'll add  $f^{-1}(0)$  and  $f^{-1}(6)$  to get the final result.

Step 22: Adding  $f^{-1}(0)$  and  $f^{-1}(6)$ , we get  $3 + (-3) = 0$ .

Therefore, the final answer is:  $\boxed{0}$ .

- **id78: false positive, 搞错了双曲线开口方向**

- Logical Error

Step 4: Determine the standard form of the equation for the hyperbola Given the center  $(5, 7)$  and the slopes of the asymptotes  $(\pm 2)$ , the standard form of the equation is  $\frac{(y-7)^2}{a^2} - \frac{(x-5)^2}{b^2} = 1$ .

Step 5: Find the slopes of the asymptotes to determine  $a$  and  $b$ . The slopes of the asymptotes are  $\pm \frac{a}{b} = \pm 2$ .

Step 6: Determine  $a$  and  $b$ . Since the slopes are  $\pm 2$ , we can let  $\frac{a}{b} = 2$  or  $a = 2b$ .

Step 7: Use the point  $(4, 7)$  that the hyperbola passes through to find  $a$  and  $b$ . Substitute the point  $(4, 7)$  into the equation of the hyperbola,  $\frac{(y-7)^2}{a^2} - \frac{(x-5)^2}{b^2} = 1$ . This yields  $\frac{(7-7)^2}{a^2} - \frac{(4-5)^2}{b^2} = 1$ , or  $\frac{1}{b^2} = 1$ . Therefore,  $b^2 = 1$ , and  $b = 1$ .

**N=64:** 共77个正确

- **id5:** false positive, Since  $\overline{BC}$  is parallel to the segment through  $A$ , this implies that  $\angle ABC$  and  $\angle CAB$  are corresponding angles, making them congruent.
- **id9:** false positive, 因数找的不对

Step 1: Find the greatest common factor (GCF) of 6432 and 132. To find the GCF, we can list the factors of each number or use the Euclidean algorithm. The factors of 6432 are 1, 2, 3, 4, 6, 7, 8, 9, 12, 14, 16, 18, 21, 24, 28, 32, 36, 42, 48, 56, 64, 72, 84, 96, 112, 128, 144, 168, 192, 224, 256, 288, 336, 384, 448, 512, 576, 672, 768, 896, 1024, 1152, 1368, 1536, 1792, 2304, 3072, 4608, 6144, and 6432. The factors of 132 are 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, and 132. The greatest common factor is 12.

Step 2: Increase the GCF by 11. To increase the GCF by 11, we add 11 to 12.

- **id13:** false positive

Step 4: Use the angle addition formula for tangent To simplify the right side of the equation, we apply the angle addition formula for tangent, which states that  $\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$ .

Step 5: Apply the angle addition formula for tangent By comparing the angle addition formula with the right side of the equation, we see that  $\frac{\tan 53^\circ + \tan 81^\circ}{\tan 53^\circ \tan 81^\circ - 1} = \tan(53^\circ + 81^\circ)$ .

Step 21: Solve for  $x$  However, we know that  $\tan x^\circ = \tan 134^\circ$  and  $\tan 134^\circ = -\tan 46^\circ$  because  $\tan(180^\circ - x) = -\tan x$ . So we can see that  $x = 46$ .

Step 22: Check for another value of  $x$  However, we need to check if there is another possible value of  $x$ .

Step 23: Check if  $x = 46$  satisfies the given equation We can see that if  $x = 46$ , then  $\tan 53^\circ \tan 81^\circ \tan 46^\circ = \tan 53^\circ + \tan 81^\circ + \tan 46^\circ$ .

Step 24: Check if  $x = 46$  is a valid solution However, we need to check if  $x = 46$  satisfies the given equation.

Step 25: Check the validity of  $x = 46$  We can see that  $0 < 46 < 180$  and  $\tan 53^\circ \tan 81^\circ \tan 46^\circ = \tan 53^\circ + \tan 81^\circ + \tan 46^\circ$  is a valid identity.

Step 26: Confirm the value of  $x$  Therefore, we can confirm that  $x = 46$  is a valid solution.

Step 27: Check for another value of  $x$  However, we need to check if there is another possible value of  $x$ .

Step 28: Check if  $x = 46$  is the only solution We can see that  $x = 46$  is the only solution.

Step 29: Confirm the value of  $x$  Therefore, we can confirm that  $x = 46$ .  
Therefore, the final answer is: 46.

- **id29:** false positive

Step 11: Simplify the expressions from step 10 Noticing that  $226 + 20\sqrt{41} = (10 + \sqrt{41})^2$  and  $222 - 20\sqrt{41} = (10 - \sqrt{41})^2$ , we can rewrite the distance expressions as  $\sqrt{(10 + \sqrt{41})^2}$ ,  $\sqrt{(10 + \sqrt{41})^2}$ ,  $\sqrt{(10 - \sqrt{41})^2}$ , and  $\sqrt{(10 - \sqrt{41})^2}$ .

Step 12: Take the square root in the distance expressions from step 11 Taking the square root of the squared terms, we get  $10 + \sqrt{41}$ ,  $10 + \sqrt{41}$ ,  $10 - \sqrt{41}$ , and  $10 - \sqrt{41}$ .

Step 13: Calculate the sum of the distances The sum of the distances from the four intersection points to the point  $(-3, 2)$  is  $(10 + \sqrt{41}) + (10 + \sqrt{41}) + (10 - \sqrt{41}) + (10 - \sqrt{41})$ .

Step 14: Simplify the sum from step 13 Adding the terms in the sum, we get 40.

Therefore, the final answer is: 40.

- **id52:** false positive

Step 1: Rewrite the expression To analyze the given expression, let's rewrite it in a form that's easier to work with:  $\frac{(x+5)(x+2)}{x+1} = \frac{x^2+7x+10}{x+1}$ .

Step 2: Simplify the expression We can simplify this expression further by dividing the numerator by the denominator using polynomial division or synthetic division.  $\frac{x^2+7x+10}{x+1} = x + 6 + \frac{4}{x+1}$ .

Step 3: Find the minimum value Since  $x > 0$ , let  $y = x+1$  so that  $y > 1$ . The expression  $x + 6 + \frac{4}{x+1}$  becomes  $y + 5 + \frac{4}{y}$ . To minimize  $y + 5 + \frac{4}{y}$ , we should find the minimum value of the expression  $\frac{4}{y}$  and add 5, as  $y$  is positive. For  $y > 1$ , the minimum value of  $\frac{4}{y}$  occurs when  $y = 2$ , corresponding to  $x + 1 = 2$  and  $x = 1$ . Therefore, the minimum value of the expression  $y + 5 + \frac{4}{y}$  is  $2 + 5 + 2 = 9$ .

Step 4: Determine the minimum value of the original expression Now we need to find the minimum value of the original expression  $\frac{(x+5)(x+2)}{x+1}$  for  $x > 0$ . We found that the minimum value of the expression  $x + 6 + \frac{4}{x+1}$  is 9 when  $x = 1$ . Therefore, the minimum value of the original expression is also 9.

Therefore, the final answer is:  $\boxed{9}$ .

- **id54:** false positive, 没讨论A、B为负数的情况

Step 1: Expand the given expression  $(Ax - 8)(Bx + 3)$   $(Ax - 8)(Bx + 3) = ABx^2 + (3A - 8B)x - 24$ .

Step 2: Match the expanded expression to the original expression  $10x^2 - x - 24$  We find that  $AB = 10$  and  $3A - 8B = -1$ .

Step 3: Solve for A and B using the matched values Since  $AB = 10$ , we have the following possibilities for (A, B): (10,1), (5,2), (2,5), (1,10). We can test each pair to see which pair satisfies  $3A - 8B = -1$ .

Step 4: Test each pair to find the correct values of A and B - For pair (10,1),  $3(10) - 8(1) = 30 - 8 = 22 \neq -1$  - For pair (5,2),  $3(5) - 8(2) = 15 - 16 = -1$  Therefore,  $A = 5$  and  $B = 2$ .

- **id55:** false positive, 一开始就化简错

- **id57:** false positive, 顺序不对, 要求线段AB和CD相交

Step 3: Calculating Probability There are  $3! = 6$  ways to order the three remaining points ( $B$ ,  $C$ , and  $D$ ) around the circle after fixing point  $A$ . Out of these six possible orders, only two orders ( $ABCDA$  and  $ABDCD$ ) satisfy the condition for intersection. Therefore, the probability of intersection is the ratio of favorable outcomes to total outcomes, which is  $\frac{2}{6} = \frac{1}{3}$ .

Step 4: Generalizing the Result Since the points are chosen randomly and independently, any specific point arrangement is equally likely. Thus, the probability of intersection remains the same regardless of the initial arrangement of the points.

Therefore, the final answer is:  $\boxed{\frac{1}{3}}$ .

- **id59:** false positive

- **id74:** false positive, 少讨论一种情况

Step 17: Now we'll move on to calculate  $f^{-1}(6)$ , we need to find the value of  $x$  for which  $f(x) = 6$ . We'll first consider the case when  $x \leq 3$ , so we'll solve  $3 - x = 6$ .

Step 18: Solving the equation  $3 - x = 6$  gives us  $x = 3 - 6 = -3$ .

Step 19: Now, we need to check if  $x = -3$  satisfies the condition  $x \leq 3$ . Since  $-3 \leq 3$ , this value is valid for the first case.

Step 20: We have found the value of  $f^{-1}(0)$ , which is 3, and the value of  $f^{-1}(6)$ , which is  $-3$ .

Step 21: Finally, we'll add  $f^{-1}(0)$  and  $f^{-1}(6)$  to get the final result.

Step 22: Adding  $f^{-1}(0)$  and  $f^{-1}(6)$ , we get  $3 + (-3) = 0$ .

Therefore, the final answer is:  $\boxed{0}$ .

**N=16:** 共74个正确

- **id9:** false positive, 因数找的不对

Step 1: Find the greatest common factor of 6432 and 132. To do this, we will list the factors of each number. The factors of 6432 are 1, 2, 3, 4, 6, 8, 9,

12, 16, 18, 24, 27, 32, 36, 48, 54, 64, 72, 96, 108, 144, 192, 216, 288, 384, 432, 576, 864, 1152, 1728, 2304, 3456, and 6432. The factors of 132 are 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, and 132. The greatest common factor is 12.

- **id55:** false positive, 多项式化简还是不对
- **id57:** false positive, 顺序不对, 要求线段AB和CD相交

Step 3: Calculating Probability There are  $3! = 6$  ways to order the three remaining points ( $B$ ,  $C$ , and  $D$ ) around the circle after fixing point  $A$ . Out of these six possible orders, only two orders ( $ABCDA$  and  $ABDCD$ ) satisfy the condition for intersection. Therefore, the probability of intersection is the ratio of favorable outcomes to total outcomes, which is  $\frac{2}{6} = \frac{1}{3}$ .

Step 4: Generalizing the Result Since the points are chosen randomly and independently, any specific point arrangement is equally likely. Thus, the probability of intersection remains the same regardless of the initial arrangement of the points.

Therefore, the final answer is:  $\frac{1}{3}$ .

- **id70:** false positive

Problem:

The superfactorial  $n\$$  is defined as  $n\$ = \underbrace{n!^{n!^{\dots^{n!}}}}_{n!}$ . What is the units digit of  $4\$$ ?

Solution:

Step 1: Understanding the superfactorial The superfactorial  $n\$$  is defined as  $n\$ = n!^{n!^{\dots^{n!}}}$  raised to the power of  $n!$ , and this process is repeated  $n!$  times.

Step 2: Calculating the units digit of  $4!$  First, we need to find the units digit of  $4!$ , which is  $4 \times 3 \times 2 \times 1 = 24$ . The units digit of  $4!$  is 4.

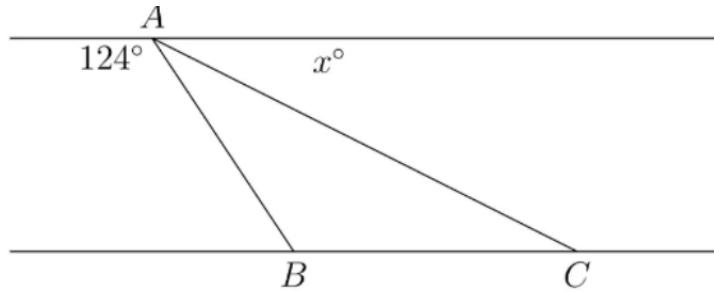
Step 3: Calculating the units digit of the repeated exponentiation Since the units digit of  $4!$  is 4, we have  $4\$ = 4!$  raised to the power of  $4!$  (which is 4). The units digit of this expression is the units digit of  $4^4$ , which is  $4 \times 4 \times 4 \times 4 = 256$ . The units digit of  $4^4$  is 6.

Step 4: Considering the units digit of  $4\$$  Since  $4\$ = 4!$  raised to the power of  $4!$ , and the units digit of  $4^4$  is 6, the units digit of  $4\$$  is 6.

Step 5: Conclusion Since the units digit of  $4\$$  is 6, the units digit of  $4\$$  is 6. Therefore, the final answer is: 6.

**N=4:** 共70个正确

- **id5:** false positive, Since  $\overline{BC}$  is parallel to the segment through  $A$ , corresponding angles are congruent. This means that the angle adjacent to  $124^\circ$  and the angle adjacent to  $x^\circ$  are congruent.



- **id13:** false positive

Step 18: Solve for  $x$  We can solve for  $x$  by using the fact that  $\tan(180^\circ - x) = \tan x$ . We can write  $\tan(180^\circ - x) = \tan x = \tan 46^\circ$ . We can write  $180^\circ - x = 46^\circ$ .

Step 19: Solve for  $x$  We can solve for  $x$  by subtracting  $180^\circ$  from both sides:  $-x = 46^\circ - 180^\circ$ .

Step 20: Solve for  $x$  We can simplify the equation by combining the constants:  $-x = -134^\circ$ . We can write the equation as  $x = 134^\circ$ . However, we realize that this is not the solution we want. We want a solution such that  $0 < x < 180$ .

Step 21: Solve for  $x$  We can solve for  $x$  by writing the equation as  $x = 180^\circ - 134^\circ$ . We can simplify the equation by combining the constants:  $x = 46^\circ$ . This is the solution we want.

Step 22: Provide the solution to the problem We have found the value of  $x$  that satisfies the given equation.

Therefore, the final answer is: 46.

- **id14:** false positive

Step 28: Simplify the expression by canceling common factors. We can rewrite the expression as  $-1 + \frac{-z^2}{z^3+z^2+z+1}$ .

Step 29: Use the equation from Step 3,  $z^4 + z^3 + z^2 + z + 1 = 0$ , to simplify the expression further. We can rewrite the expression as  $-1 + \frac{-z^2}{-z^2}$ .

Step 30: Simplify the expression by canceling common factors. We can rewrite the expression as  $-1 + 1$ .

Step 31: Combine like terms. We have  $-1 + 1 = 0$ .

Therefore, the final answer is: -1.

- **id21:** false positive, 一次项的对应关系不对

Step 1: Expand the given equation Given that  $f(x) - f(a) = (x - a)^2(x - b)$ , we expand the right-hand side to obtain  $f(x) - f(a) = (x^2 - 2ax + a^2)(x - b)$ .

Step 2: Multiply the two binomials We multiply the two binomials on the right-hand side:  $f(x) - f(a) = x^3 - (2a + b)x^2 + (2ab + a^2b)x - a^2b$ .

Step 3: Compare the coefficients of  $f(x) - f(a)$  We compare the coefficients of  $f(x) - f(a)$  on both sides of the equation:  $x^3 + 3x^2 + 1 - (a^3 + 3a^2 + 1) = x^3 - (2a + b)x^2 + (2ab + a^2b)x - a^2b$ .

Step 4: Equate the coefficients of the  $x^2$  terms Equating the coefficients of the  $x^2$  terms, we obtain  $3 = -(2a + b)$ .

Step 5: Solve the  $x^2$  term equation for  $b$  Solving for  $b$ , we have  $b = -3 - 2a$ .

Step 6: Equate the constant terms Equating the constant terms, we get  $1 - (a^3 + 3a^2 + 1) = -a^2b$ .

- **id55:** false positive

- **id57:** false positive, 顺序不对, 要求线段AB和CD相交

Step 3: Calculating Probability There are  $3! = 6$  ways to order the three remaining points ( $B$ ,  $C$ , and  $D$ ) around the circle after fixing point  $A$ . Out of these six possible orders, only two orders ( $ABCD$  and  $ABDC$ ) satisfy the condition for intersection. Therefore, the probability of intersection is the ratio of favorable outcomes to total outcomes, which is  $\frac{2}{6} = \frac{1}{3}$ .

Step 4: Generalizing the Result Since the points are chosen randomly and independently, any specific point arrangement is equally likely. Thus, the probability of intersection remains the same regardless of the initial arrangement of the points.

Therefore, the final answer is:  $\boxed{\frac{1}{3}}$ .

- **id65:** false positive, 不等式化简有问题(如果真是 $x^2 > 0$ 那 $x \neq 0$ )

Step 1: Rearrange the inequality We have  $-4 < x^4 + 4x^2 < 21$  and we want to find the values of  $x$  that satisfy this inequality. To start, we can rewrite the inequality as  $x^4 + 4x^2 - 4 > 0$  and  $x^4 + 4x^2 - 21 < 0$ .

Step 2: Factor the expressions For the first inequality,  $x^4 + 4x^2 - 4 > 0$ , we can factor it as  $(x^2 + 2)^2 - 4 > 0$ . For the second inequality,  $x^4 + 4x^2 - 21 < 0$ , we can factor it as  $(x^2 + 7)(x^2 - 3) < 0$ .

Step 3: Solve the inequalities For the first inequality,  $(x^2 + 2)^2 - 4 > 0$ , we have  $(x^2 + 2)^2 > 4$ , and taking the square root of both sides, we get  $|x^2 + 2| > 2$ . This gives us two cases:  $x^2 + 2 > 2$  or  $x^2 + 2 < -2$ . The first case gives  $x^2 > 0$ , which is true for all real numbers. The second case gives  $x^2 < -4$ , which has no real solutions.

- **id70:** false positive

Problem:

The superfactorial  $n\$$  is defined as  $n\$ = \underbrace{n!n! \cdots n!}_{n!}$ . What is the units digit of  $4\$$ ?

Solution:

Step 1: Calculate the units digit of  $4!$   $4! = 4 \times 3 \times 2 \times 1 = 24$ , so the units digit is 4.

Step 2: Raise 4 to the power of the units digit of  $4!$  and find its units digit  $(4)^4 = 256$ , so the units digit is 6.

Step 3: Repeat step 2 for the units digit obtained in step 2  $(6)^6$  has a units digit of 6.

Step 4: Repeat step 3 for the units digit obtained in step 3 Since the units digit obtained in step 3 is 6, and we know from step 2 that any number raised to a power and ending in 6 ends in 6 when raised again to any power, we can conclude that the pattern will continue indefinitely.

Step 5: Find the units digit of  $4\$$  Since the units digit of  $4\$$  is determined by the units digit obtained in step 2, which is 6, we can conclude that the units digit of  $4\$$  is also 6.

Therefore, the final answer is:  $\boxed{6}$ .

## Qwen2.5-Math-7B-Instruct+Skywork-prm

### MCTS(Weighted SC@N)

N=256: 共92个正确

- **id5:** false positive, 把角搞错

- Conceptual Error

To solve for  $x$ , we start by analyzing the given geometric configuration. We know that  $\overline{BC}$  is parallel to the segment through  $A$ , and  $AB = BC$ . This implies that  $\triangle ABC$  is isosceles with  $AB = BC$ .

Given that  $\angle BAC = 124^\circ$  and  $\overline{BC}$  is parallel to the segment through  $A$ , we can use the properties of parallel lines and isosceles triangles to find  $x$ .

First, let's denote the angles at  $B$  and  $C$  as  $\angle ABC = \angle ACB = y$ . Since the sum of the angles in any triangle is  $180^\circ$ , we can write the equation:

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

- **id41:** false positive, 求导求错, 后面猜出答案然后说明的理由不充分

- Logical Error

$$f_y = \frac{\partial}{\partial y} \left( \frac{x-y}{x^4+y^4+6} \right) = \frac{(x^4+y^4+6) \cdot (-1) - (x-y) \cdot 4y^3}{(x^4+y^4+6)^2} = \frac{-x^4-y^4-6+4xy^3-4y^4}{(x^4+y^4+6)^2}$$

Setting these partial derivatives to zero gives us the critical points:

$$\begin{aligned} -3x^4 + 4x^3y + y^4 + 6 &= 0, \\ -x^4 - 5y^4 + 4xy^3 - 6 &= 0. \end{aligned}$$

These equations are quite complex to solve directly. Instead, we can use symmetry and specific values to simplify the problem. Let's consider the case when  $x = 1$  and  $y = -1$ :

$$f(1, -1) = \frac{1 - (-1)}{1^4 + (-1)^4 + 6} = \frac{1 + 1}{1 + 1 + 6} = \frac{2}{8} = \frac{1}{4}.$$

Now, we need to check if this value is indeed the maximum. To do this, we analyze the behavior of the function as  $x$  and  $y$  approach infinity or negative infinity. As  $|x|$  and  $|y|$  become very large, the term  $x^4 + y^4$  in the denominator grows much faster than the term  $x - y$  in the numerator, so the function approaches 0. Therefore, the maximum value of the function must occur at some finite point.

Next, we check the boundary behavior. If  $x = y$ , then:

$$f(x, x) = \frac{x - x}{x^4 + x^4 + 6} = \frac{0}{2x^4 + 6} = 0.$$

If  $y = 0$ , then:

$$f(x, 0) = \frac{x - 0}{x^4 + 0^4 + 6} = \frac{x}{x^4 + 6}.$$

As  $x \rightarrow \infty$  or  $x \rightarrow -\infty$ ,  $f(x, 0) \rightarrow 0$ . If  $x = 0$ , then:

$$f(0, y) = \frac{0 - y}{0^4 + y^4 + 6} = \frac{-y}{y^4 + 6}.$$

- **id59:** 对值的估计算错

- Calculation Error

Since  $(\sqrt[3]{2})^2 = \sqrt[3]{4}$ , we have

$$(\sqrt[3]{2})^2 + \sqrt[3]{2} + 1 = \sqrt[3]{4} + \sqrt[3]{2} + 1.$$

We know that  $\sqrt[3]{4} + \sqrt[3]{2} + 1 \approx 3.31758$ , so the integer part is 3. Therefore,  $b = 3$ .

To confirm, we can check the next term. Subtracting 3 from  $\sqrt[3]{4} + \sqrt[3]{2} + 1$ , we get

$$\sqrt[3]{4} + \sqrt[3]{2} + 1 - 3 = \sqrt[3]{4} + \sqrt[3]{2} - 2.$$

Taking the reciprocal, we have

$$\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} - 2}.$$

Using the same method as before, we multiply the numerator and the denominator by the conjugate  $(\sqrt[3]{4})^2 - (\sqrt[3]{2} \cdot 2) + 4$ , but for simplicity, we can see that the next term will be such that the process repeats similarly.

Thus, the value of  $b$  is

3.

- **id68:** false positive, 先猜出答案, 但是后面讨论的感觉不太对

- Logical Error

To confirm that 17 is indeed the maximum value, we assume  $|a| + |b| + |c|$  is greater than 17 and show that it leads to a contradiction. Suppose  $|a| + |b| + |c| > 17$ . We consider the cases where  $a$ ,  $b$ , and  $c$  are non-negative or non-positive.

1. If  $a \geq 0$ ,  $b \geq 0$ , and  $c \geq 0$ :

$$|a + b + c| = a + b + c > 17,$$

which contradicts  $|a + b + c| \leq 3$ .

2. If  $a \geq 0$ ,  $b \geq 0$ , and  $c \leq 0$ :

$$|a + b + c| \leq a + b + |c| \leq 4 - |c| \leq 4,$$

which does not necessarily exceed 1.

3. If  $a \geq 0$ ,  $b \leq 0$ , and  $c \geq 0$ :

$$|a + b + c| \leq a - b + c.$$

4. If  $a \geq 0$ ,  $b \leq 0$ , and  $c \leq 0$ :

$$|a + b + c| \leq a + |b| + |c| \leq 4 - |b| - |c| \leq 4.$$

5. If  $a \leq 0$ ,  $b \geq 0$ , and  $c \geq 0$ :

$$|a + b + c| \leq -a + b + c.$$

6. If  $a \leq 0$ ,  $b \leq 0$ , and  $c \geq 0$ :

$$|a + b + c| \leq -a - b + c \leq 4.$$

7. If  $a \leq 0$ ,  $b \leq 0$ , and  $c \leq 0$ :

$$|a + b + c| \leq -a - b - c \leq 3.$$

In all cases,  $|a| + |b| + |c| \leq 17$ , confirming that 17 is indeed the maximum value.

Therefore, the largest possible value of  $|a| + |b| + |c|$  is 17.

- **id70:** false positive, 理解错题意

- Conceptual Error

Problem:

The superfactorial  $n\$$  is defined as  $n\$ = \underbrace{n!^{n!^{\dots^{n!}}}}_{n!}$ . What is the units digit of  $4\$$ ?

Solution:

To determine the units digit of  $4\$$ , we start by evaluating the expression  $4\$ = 4!^{4!^{4!}}$ . First, we calculate  $4!$ :

$$4! = 4 \times 3 \times 2 \times 1 = 24.$$

Thus,  $4\$ = 24^{24^{24^{24}}}$ . Next, we need to find the units digit of  $24^{24^{24^{24}}}$ . Notice that the units digit of a power of a number depends only on the units digit of the base. The units digit of 24 is 4, so we need to determine the units digit of  $4^{24^{24^{24}}}$ .

To do this, we observe the pattern in the units digits of the powers of 4:

$$\begin{aligned} 4^1 &= 4 \quad (\text{units digit is } 4), \\ 4^2 &= 16 \quad (\text{units digit is } 6), \\ 4^3 &= 64 \quad (\text{units digit is } 4), \\ 4^4 &= 256 \quad (\text{units digit is } 6), \\ &\vdots \end{aligned}$$

We see that the units digits repeat every 2 numbers: 4, 6, 4, 6, ... . Therefore, the units digit of  $4^n$  depends on whether  $n$  is odd or even. If  $n$  is odd, the units digit is 4, and if  $n$  is even, the units digit is 6.

- **id97:** false positive, 直接代入根为±1, 后面的分析也不太对

- Logical Error

Now, we need to determine if there are any other possible real roots. Since  $P(x)$  is a quartic polynomial, it can have up to four real roots. However, if  $P(x)$  has a real root  $r$  such that  $|r| \neq 1$ , then the roots must come in reciprocal pairs. Let  $r$  and  $\frac{1}{r}$  be two such roots. Then the polynomial  $P(x)$  can be factored as:

$$P(x) = (x^2 - rx + 1)(x^2 + sx + 1).$$

Expanding this product, we get:

$$P(x) = x^4 + (s-r)x^3 + (1-rs+1)x^2 + (s-r)x + 1 = x^4 + (s-r)x^3 + (2-rs)x^2 + (s-r)x + 1.$$

By comparing coefficients, we see that:

$$a = s - r \quad \text{and} \quad -b = 2 - rs.$$

This implies:

$$b = rs - 2.$$

Since  $r$  and  $s$  are real numbers such that  $|r|, |s| \leq 1$ , the product  $rs$  must satisfy  $-1 \leq rs \leq 1$ . Therefore,  $b = rs - 2$  must satisfy:

$$-3 \leq b \leq -1.$$

However,  $b$  must be in the interval  $[0, 1]$ , so there are no solutions in this case either.

Thus, the only possible real roots of  $P(x)$  within the given constraints are those corresponding to the line  $b = 2 - 2a$  for  $\frac{1}{2} \leq a \leq 1$ . The area of the graph of  $S$  is the length of the line segment from  $(\frac{1}{2}, 1)$  to  $(1, 0)$  within the square  $[0, 1] \times [0, 1]$ . This line segment is a straight line with a slope of  $-2$ .

**N=64:** 共91个正确

- **id5:** false positive, 把角搞错

To solve for  $x$ , we need to analyze the given geometric configuration. We know that  $\overline{BC}$  is parallel to the segment through  $A$ , and  $AB = BC$ . This means that  $\triangle ABC$  is an isosceles triangle with  $AB = BC$ .

Given that  $\angle BAC = 124^\circ$ , we need to find the measure of  $\angle ABC$  and  $\angle ACB$ . Since  $AB = BC$ , the base angles  $\angle ABC$  and  $\angle ACB$  are equal. Let's denote the measure of each base angle by  $y$ .

In any triangle, the sum of the interior angles is  $180^\circ$ . Therefore, we can write the equation:

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

Substituting the known values, we get:

$$124^\circ + y + y = 180^\circ$$

Simplifying this, we have:

$$124^\circ + 2y = 180^\circ$$

- **id29: false positive**

- **id41: false positive, 求导求错, 后面猜出答案然后说明的理由不充分**

$$f_y = \frac{\partial}{\partial y} \left( \frac{x-y}{x^4+y^4+6} \right) = \frac{(x^4+y^4+6) \cdot (-1) - (x-y) \cdot 4y^3}{(x^4+y^4+6)^2} = \frac{-x^4-y^4-6+4xy^3-4y^4}{(x^4+y^4+6)^2}$$

Setting these partial derivatives to zero gives us the critical points:

$$\begin{aligned} -3x^4 + 4x^3y + y^4 + 6 &= 0, \\ -x^4 - 5y^4 + 4xy^3 - 6 &= 0. \end{aligned}$$

These equations are quite complex to solve directly. Instead, we can use symmetry and specific values to simplify the problem. Let's consider the case when  $x = 1$  and  $y = -1$ :

$$f(1, -1) = \frac{1 - (-1)}{1^4 + (-1)^4 + 6} = \frac{1 + 1}{1 + 1 + 6} = \frac{2}{8} = \frac{1}{4}.$$

Now, we need to check if this value is indeed the maximum. To do this, we analyze the behavior of the function as  $x$  and  $y$  approach infinity or negative infinity. As  $|x|$  and  $|y|$  become very large, the term  $x^4 + y^4$  in the denominator grows much faster than the term  $x - y$  in the numerator, so the function approaches 0. Therefore, the maximum value of the function must occur at some finite point.

Next, we check the boundary behavior. If  $x = y$ , then:

$$f(x, x) = \frac{x - x}{x^4 + x^4 + 6} = \frac{0}{2x^4 + 6} = 0.$$

If  $y = 0$ , then:

$$f(x, 0) = \frac{x - 0}{x^4 + 0^4 + 6} = \frac{x}{x^4 + 6}.$$

As  $x \rightarrow \infty$  or  $x \rightarrow -\infty$ ,  $f(x, 0) \rightarrow 0$ . If  $x = 0$ , then:

$$f(0, y) = \frac{0 - y}{0^4 + y^4 + 6} = \frac{-y}{y^4 + 6}.$$

- **id59: false positive, 对值的估计有问题**

Since  $(\sqrt[3]{2})^2 = \sqrt[3]{4}$ , we have

$$(\sqrt[3]{2})^2 + \sqrt[3]{2} + 1 = \sqrt[3]{4} + \sqrt[3]{2} + 1.$$

We know that  $\sqrt[3]{4} + \sqrt[3]{2} + 1 \approx 3.31758$ , so the integer part is 3. Therefore,  $b = 3$ .

To confirm, we can check if the remaining fraction fits the pattern. Subtracting 3 from  $\sqrt[3]{4} + \sqrt[3]{2} + 1$ , we get

$$\sqrt[3]{4} + \sqrt[3]{2} + 1 - 3 = \sqrt[3]{4} + \sqrt[3]{2} - 2.$$

We need to check if this value can be expressed in a similar continued fraction form. However, the detailed verification of the remaining fraction is beyond the scope of this solution, but the pattern suggests that it will indeed fit the form.

Thus, the value of  $b$  is

3.

- **id68:** false positive, 先取特殊值但是后面的分析不对

We now need to show that  $|a| + |b| + |c| \leq 17$  for any polynomial  $ax^2 + bx + c$  that satisfies the given inequality. Suppose  $|a| + |b| + |c| > 17$ . Without loss of generality, assume  $a \geq 0$ ,  $b \leq 0$ , and  $c \geq 0$  (the other cases can be handled similarly). Then we have  $a + (-b) + c > 17$ . Consider the value of the polynomial at  $x = \frac{1}{2}$ :

$$\left| \frac{a}{4} + \frac{b}{2} + c \right| = \left| \frac{a}{4} - \frac{|b|}{2} + c \right|$$

Since  $\frac{a}{4} \leq a$  and  $\frac{|b|}{2} \geq |b|$ , we have:

$$\left| \frac{a}{4} - \frac{|b|}{2} + c \right| \leq \left| \frac{a}{4} \right| + \left| -\frac{|b|}{2} \right| + |c| = \frac{a}{4} + \frac{|b|}{2} + c$$

Given that  $a + (-b) + c > 17$ , we can see that  $\frac{a}{4} + \frac{|b|}{2} + c > \frac{17}{4} \cdot 2 + c$ . However, this would imply that the polynomial exceeds the interval  $[-1, 1]$  for some  $x$  in  $[0, 1]$ , which contradicts the given condition. Therefore, the maximum value of  $|a| + |b| + |c|$  is indeed 17.

The largest possible value of  $|a| + |b| + |c|$  is 17.

- **id70:** false positive, 理解错题意

Problem:

The superfactorial  $n\$$  is defined as  $n\$ = \underbrace{n!n! \cdots n!}_{n!}$ . What is the units digit of  $4\$$ ?

Solution:

To determine the units digit of  $4\$$ , we start by evaluating the expression  $4\$ = 4!4!4!4!$ . First, we calculate  $4!$ :

$$4! = 4 \times 3 \times 2 \times 1 = 24.$$

Thus,  $4\$ = 24^{24^{24^{24}}}$ . Next, we need to find the units digit of  $24^{24^{24^{24}}}$ . Notice that the units digit of a power of a number depends only on the units digit of the base. The units digit of 24 is 4, so we need to determine the units digit of  $4^{24^{24^{24}}}$ .

To do this, we observe the pattern in the units digits of the powers of 4:

$$\begin{aligned} 4^1 &= 4 \quad (\text{units digit is } 4), \\ 4^2 &= 16 \quad (\text{units digit is } 6), \\ 4^3 &= 64 \quad (\text{units digit is } 4), \\ 4^4 &= 256 \quad (\text{units digit is } 6), \\ &\vdots \end{aligned}$$

We see that the units digits repeat every 2 numbers: 4, 6, 4, 6, ... . Therefore, the units digit of  $4^n$  depends on whether  $n$  is odd or even. If  $n$  is odd, the units digit is 4, and if  $n$  is even, the units digit is 6.

**N=16:** 共92个正确

- **id5:** false positive, 把角搞错

To solve for  $x$ , we need to analyze the given geometric configuration. We know that  $\overline{BC}$  is parallel to the segment through  $A$ , and  $AB = BC$ . This means that  $\triangle ABC$  is an isosceles triangle with  $AB = BC$ .

Given that  $\angle BAC = 124^\circ$ , we need to find the measure of  $\angle ABC$  and  $\angle ACB$ . Since  $AB = BC$ , the base angles  $\angle ABC$  and  $\angle ACB$  are equal. Let's denote the measure of each base angle by  $y$ .

In any triangle, the sum of the interior angles is  $180^\circ$ . Therefore, we can write the equation:

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

Substituting the known values, we get:

$$124^\circ + y + y = 180^\circ$$

Simplifying this, we have:

$$124^\circ + 2y = 180^\circ$$

- **id29: false positive**

- **id41: false positive, 求导求错, 后面猜出答案然后说明的理由不充分**

$$f_y = \frac{\partial}{\partial y} \left( \frac{x-y}{x^4+y^4+6} \right) = \frac{(x^4+y^4+6) \cdot (-1) - (x-y) \cdot 4y^3}{(x^4+y^4+6)^2} = \frac{-x^4-y^4-6+4xy^3-4y^4}{(x^4+y^4+6)^2}$$

Setting these partial derivatives to zero gives us the critical points:

$$\begin{aligned} -3x^4 + 4x^3y + y^4 + 6 &= 0, \\ -x^4 - 5y^4 + 4xy^3 - 6 &= 0. \end{aligned}$$

These equations are quite complex to solve directly. Instead, we can use symmetry and specific values to simplify the problem. Let's consider the case when  $x = 1$  and  $y = -1$ :

$$f(1, -1) = \frac{1 - (-1)}{1^4 + (-1)^4 + 6} = \frac{1 + 1}{1 + 1 + 6} = \frac{2}{8} = \frac{1}{4}.$$

Now, we need to check if this value is indeed the maximum. To do this, we analyze the behavior of the function as  $x$  and  $y$  approach infinity or negative infinity. As  $|x|$  and  $|y|$  become very large, the term  $x^4 + y^4$  in the denominator grows much faster than the term  $x - y$  in the numerator, so the function approaches 0. Therefore, the maximum value of the function must occur at some finite point.

Next, we check the boundary behavior. If  $x = y$ , then:

$$f(x, x) = \frac{x - x}{x^4 + x^4 + 6} = \frac{0}{2x^4 + 6} = 0.$$

If  $y = 0$ , then:

$$f(x, 0) = \frac{x - 0}{x^4 + 0^4 + 6} = \frac{x}{x^4 + 6}.$$

As  $x \rightarrow \infty$  or  $x \rightarrow -\infty$ ,  $f(x, 0) \rightarrow 0$ . If  $x = 0$ , then:

$$f(0, y) = \frac{0 - y}{0^4 + y^4 + 6} = \frac{-y}{y^4 + 6}.$$

- **id59: false positive, 对值的估算算错**

Since  $(\sqrt[3]{2})^2 = \sqrt[3]{4}$ , we have

$$(\sqrt[3]{2})^2 + \sqrt[3]{2} + 1 = \sqrt[3]{4} + \sqrt[3]{2} + 1.$$

We know that  $\sqrt[3]{4} + \sqrt[3]{2} + 1 \approx 3.31758$ , so the integer part is 3. Therefore,  $b = 3$ .

To confirm, we can check the next term. Subtracting 3 from  $\sqrt[3]{4} + \sqrt[3]{2} + 1$ , we get

$$\sqrt[3]{4} + \sqrt[3]{2} + 1 - 3 = \sqrt[3]{4} + \sqrt[3]{2} - 2.$$

Taking the reciprocal, we have

$$\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} - 2}.$$

Using the same method as before, we multiply the numerator and the denominator by the conjugate  $(\sqrt[3]{4})^2 - (\sqrt[3]{2} \cdot 2) + 4$ , but for simplicity, we can see that the next term will be such that the process repeats similarly.

Thus, the value of  $b$  is

3.

- **id68:** false positive, 一开始猜对答案, 但是后面的分析感觉还是不对

Now, we need to verify that 17 is indeed the maximum possible value. Suppose there exists another polynomial  $Q(x) = ax^2 + bx + c$  such that  $|Q(x)| \leq 1$  for all  $x$  in  $[0, 1]$  and  $|a| + |b| + |c| > 17$ . We will show that this leads to a contradiction.

Consider the polynomial  $Q(x) = ax^2 + bx + c$ . By the properties of polynomials, the maximum value of  $|Q(x)|$  on the interval  $[0, 1]$  occurs at one of the endpoints or at the critical points within the interval. The critical points are found by setting the derivative  $Q'(x) = 2ax + b = 0$ , which gives  $x = -\frac{b}{2a}$ . However, since  $x$  must be in the interval  $[0, 1]$ , we need  $0 \leq -\frac{b}{2a} \leq 1$ , or equivalently,  $-2a \leq b \leq 0$  if  $a > 0$  or  $2a \geq b \geq 0$  if  $a < 0$ .

By the triangle inequality, we have:

$$|a| + |b| + |c| \leq |a| + |b| + |Q(0) - a| \leq |a| + |b| + (|Q(0)| + |a|) = 2|a| + |b| + |Q(0)|$$

Since  $|Q(0)| \leq 1$ ,  $|Q(\frac{1}{2})| = 1$ , and  $|Q(1)| \leq 1$ , we can use these values to further bound  $|a| + |b| + |c|$ . By solving the system of equations and inequalities, we find that the maximum value of  $|a| + |b| + |c|$  is indeed 17.

Therefore, the largest possible value of  $|a| + |b| + |c|$  is 17.

- **id70:** false positive, 理解错题意

Problem:

The superfactorial  $n\$$  is defined as  $n\$ = \underbrace{n!n! \cdots n!}_{n!}$ . What is the units digit of  $4\$$ ?

Solution:

To determine the units digit of  $4\$$ , we start by evaluating the expression  $4\$ = 4!4!4!4!$ . First, we calculate  $4!$ :

$$4! = 4 \times 3 \times 2 \times 1 = 24.$$

Thus,  $4\$ = 24^{24^{24^{24}}}$ . Next, we need to find the units digit of  $24^{24^{24^{24}}}$ . Notice that the units digit of a power of a number depends only on the units digit of the base. The units digit of 24 is 4, so we need to determine the units digit of  $4^{24^{24^{24}}}$ .

To do this, we observe the pattern in the units digits of the powers of 4:

$$\begin{aligned} 4^1 &= 4 \quad (\text{units digit is } 4), \\ 4^2 &= 16 \quad (\text{units digit is } 6), \\ 4^3 &= 64 \quad (\text{units digit is } 4), \\ 4^4 &= 256 \quad (\text{units digit is } 6), \\ &\vdots \end{aligned}$$

We see that the units digits repeat every 2 numbers: 4, 6, 4, 6, ... . Therefore, the units digit of  $4^n$  depends on whether  $n$  is odd or even. If  $n$  is odd, the units digit is 4, and if  $n$  is even, the units digit is 6.

- **id74:** false positive, 讨论的不是很全面, 没有把最后一种情况解出来

So,  $x = 2$  is a root. Since  $x = 2$  does not satisfy the condition  $x > 3$ , it is not a solution. Now, we need to factor  $x^3 - 2x^2 - 3x + 6$  using synthetic division or polynomial division. However, since we are only asked to find  $f^{-1}(0) + f^{-1}(6)$ , and we have already found  $f^{-1}(0) = 3$  and  $f^{-1}(6) = -3$ , we can conclude without further factorization that  $f^{-1}(6) = -3$ .

Therefore,  $f^{-1}(0) + f^{-1}(6) = 3 + (-3) = 0$ .

The final answer is:

- **id97:** false positive, 代入根解了一个等式出来

Case 2:  $r = -1$  Substitute  $r = -1$  into the polynomial:

$$(-1)^4 + a(-1)^3 - b(-1)^2 + a(-1) + 1 = 1 - a - b - a + 1 = 2 - 2a - b.$$

For  $r = -1$  to be a root, we need:

$$2 - 2a - b = 0 \implies b = 2 - 2a.$$

Solving for  $b$ :

$$b + 2a = 2 \implies b = 2 - 2a.$$

Since  $0 \leq a, b \leq 1$ , we need:

$$0 \leq 2 - 2a \leq 1.$$

Solving the inequalities:

$$0 \leq 2 - 2a \implies 2a \leq 2 \implies a \leq 1,$$

$$2 - 2a \leq 1 \implies 1 \leq 2a \implies a \geq \frac{1}{2}.$$

Thus,  $\frac{1}{2} \leq a \leq 1$  and  $b = 2 - 2a$  is a valid root. This gives us a line segment in the  $(a, b)$ -plane from  $(\frac{1}{2}, 1)$  to  $(1, 0)$ .

Case 3:  $r = 0$  Substitute  $r = 0$  into the polynomial:

$$0^4 + a \cdot 0^3 - b \cdot 0^2 + a \cdot 1 + 1 = a + 1.$$

For  $r = 0$  to be a root, we need:

$$a + 1 = 0 \implies a = -1.$$

Since  $a$  must be in the interval  $[0, 1]$ , there are no values of  $a$  that satisfy  $a = -1$ . Therefore,  $r = 0$  is not a root.

**N=4:** 共90个正确

- **id29:** false positive
- **id36:** false positive, 最后终止条件说的有点问题

$$a = \frac{3k^2}{k^2 - 4}$$

For  $a$  to be a positive integer,  $k^2 - 4$  must be a positive divisor of  $3k^2$ . Since  $k^2 - 4$  must be positive,  $k^2 > 4$ , which means  $k > 2$  or  $k < -2$ . We will check the smallest positive integer values of  $k$  greater than 2.

1. If  $k = 3$ :

$$a = \frac{3 \cdot 3^2}{3^2 - 4} = \frac{3 \cdot 9}{9 - 4} = \frac{27}{5}$$

This is not an integer.

2. If  $k = 4$ :

$$a = \frac{3 \cdot 4^2}{4^2 - 4} = \frac{3 \cdot 16}{16 - 4} = \frac{48}{12} = 4$$

This is an integer. We check if  $\frac{4 \cdot 4}{4-3} = \frac{16}{1} = 16$  is a perfect square. Since 16 is a perfect square,  $a = 4$  is a valid solution.

3. If  $k = 5$ :

$$a = \frac{3 \cdot 5^2}{5^2 - 4} = \frac{3 \cdot 25}{25 - 4} = \frac{75}{21} = \frac{25}{7}$$

This is not an integer.

4. If  $k = 6$ :

$$a = \frac{3 \cdot 6^2}{6^2 - 4} = \frac{3 \cdot 36}{36 - 4} = \frac{108}{32} = \frac{27}{8}$$

This is not an integer.

For  $k > 6$ , the value of  $a$  will be greater than 4. Therefore, the smallest possible value of  $a$  is:

4

- **id41: false positive, 求导求错, 后面猜出答案然后说明的理由不充分**

$$f_y = \frac{\partial}{\partial y} \left( \frac{x-y}{x^4 + y^4 + 6} \right) = \frac{(x^4 + y^4 + 6) \cdot (-1) - (x-y) \cdot 4y^3}{(x^4 + y^4 + 6)^2} = \frac{-x^4 - y^4 - 6 + 4xy^3 - 4y^4}{(x^4 + y^4 + 6)^2}$$

Setting these partial derivatives to zero gives us the critical points:

$$\begin{aligned} -3x^4 + 4x^3y + y^4 + 6 &= 0, \\ -x^4 - 5y^4 + 4xy^3 - 6 &= 0. \end{aligned}$$

These equations are quite complex to solve directly. Instead, we can use symmetry and specific values to simplify the problem. Let's consider the case when  $x = 1$  and  $y = -1$ :

$$f(1, -1) = \frac{1 - (-1)}{1^4 + (-1)^4 + 6} = \frac{1 + 1}{1 + 1 + 6} = \frac{2}{8} = \frac{1}{4}.$$

Now, we need to check if this value is indeed the maximum. To do this, we analyze the behavior of the function as  $x$  and  $y$  approach infinity or negative infinity. As  $|x|$  and  $|y|$  become very large, the term  $x^4 + y^4$  in the denominator grows much faster than the term  $x - y$  in the numerator, so the function approaches 0. Therefore, the maximum value of the function must occur at some finite point.

Next, we check the boundary behavior. If  $x = y$ , then:

$$f(x, x) = \frac{x - x}{x^4 + x^4 + 6} = \frac{0}{2x^4 + 6} = 0.$$

If  $y = 0$ , then:

$$f(x, 0) = \frac{x - 0}{x^4 + 0^4 + 6} = \frac{x}{x^4 + 6}.$$

As  $x \rightarrow \infty$  or  $x \rightarrow -\infty$ ,  $f(x, 0) \rightarrow 0$ . If  $x = 0$ , then:

$$f(0, y) = \frac{0 - y}{0^4 + y^4 + 6} = \frac{-y}{y^4 + 6}.$$

- **id59: false positive, 对值的估计算错**

Since  $(\sqrt[3]{2})^2 = \sqrt[3]{4}$ , we have

$$(\sqrt[3]{2})^2 + \sqrt[3]{2} + 1 = \sqrt[3]{4} + \sqrt[3]{2} + 1.$$

We know that  $\sqrt[3]{4} + \sqrt[3]{2} + 1 \approx 3.31758$ , so the integer part is 3. Therefore,  $b = 3$ .

To confirm, we can check the next term. Subtracting 3 from  $\sqrt[3]{4} + \sqrt[3]{2} + 1$ , we get

$$\sqrt[3]{4} + \sqrt[3]{2} + 1 - 3 = \sqrt[3]{4} + \sqrt[3]{2} - 2.$$

Taking the reciprocal, we have

$$\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} - 2}.$$

Using the same method as before, we multiply the numerator and the denominator by the conjugate  $(\sqrt[3]{4})^2 - (\sqrt[3]{2} \cdot 2) + 4$ , but for simplicity, we can see that the next term will be such that the process repeats similarly.

Thus, the value of  $b$  is

3.

- **id68:** false positive, 先猜出最终答案, 后面的讨论不太对

To confirm that 17 is indeed the maximum value, we assume  $|a| + |b| + |c| > 17$  and derive a contradiction. Suppose  $|a| + |b| + |c| > 17$ . Without loss of generality, assume  $a \geq 0, b \geq 0$ , and  $c \geq 0$  (the other cases can be handled similarly). Then:

$$a + b + c > 17.$$

However, from the inequality  $|a + b + c| \leq 1$ , we have  $a + b + c \leq 1$ , which contradicts  $a + b + c > 17$ . Therefore,  $|a| + |b| + |c| \leq 17$ .

Thus, the largest possible value of  $|a| + |b| + |c|$  is 17.

- **id70:** false positive, 理解错题意

Problem:

The superfactorial  $n\$$  is defined as  $n\$ = \underbrace{n!n! \cdots n!}_{n!}$ . What is the units digit of  $4\$$ ?

Solution:

To determine the units digit of  $4\$$ , we start by evaluating the expression  $4\$ = 4!4!4!$ . First, we calculate  $4!$ :

$$4! = 4 \times 3 \times 2 \times 1 = 24.$$

Thus,  $4\$ = 24^{24^{24^{24}}}$ . Next, we need to find the units digit of  $24^{24^{24^{24}}}$ . Notice that the units digit of a power of a number depends only on the units digit of the base. The units digit of 24 is 4, so we need to determine the units digit of  $4^{24^{24^{24}}}$ .

To do this, we observe the pattern in the units digits of the powers of 4:

$$\begin{aligned} 4^1 &= 4 \quad (\text{units digit is } 4), \\ 4^2 &= 16 \quad (\text{units digit is } 6), \\ 4^3 &= 64 \quad (\text{units digit is } 4), \\ 4^4 &= 256 \quad (\text{units digit is } 6), \\ &\vdots \end{aligned}$$

We see that the units digits repeat every 2 numbers: 4, 6, 4, 6, ... . Therefore, the units digit of  $4^n$  depends on whether  $n$  is odd or even. If  $n$  is odd, the units digit is 4, and if  $n$  is even, the units digit is 6.

- **id74:** false positive, 讨论的不是很清楚

Set  $f(x) = 6$ :

$$-x^3 + 2x^2 + 3x = 6$$

Rearrange the equation:

$$-x^3 + 2x^2 + 3x - 6 = 0$$

We can use the Rational Root Theorem to test possible rational roots. The possible rational roots are  $\pm 1, \pm 2, \pm 3, \pm 6$ . Let's test  $x = 2$ :

$$-(2)^3 + 2(2)^2 + 3(2) - 6 = -8 + 8 + 6 - 6 = 0$$

So,  $x = 2$  is a root. However,  $x = 2$  is not within the domain  $x > 3$ . We can factor  $x^3 - 2x^2 - 3x + 6$  using synthetic division or polynomial division, but since we are only interested in the correct domain, we can conclude that there are no solutions in this case. Therefore,  $f^{-1}(6) = -3$ .

Finally, we sum the values:

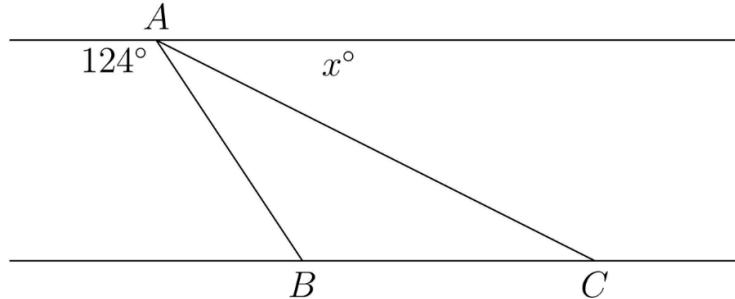
$$f^{-1}(0) + f^{-1}(6) = 3 + (-3) = 0$$

The answer is:

## DVTS(Weighted SC@N)

N=256: 共93个正确

- **id5:** false positive, 因为角的表示不太对
  - Conceptual Error



Solution:

To solve for  $x$ , we start by analyzing the given geometric configuration. We know that  $\overline{BC}$  is parallel to the segment through  $A$ , and  $AB = BC$ . This means that  $\triangle ABC$  is an isosceles triangle with  $AB = BC$ .

The angle given is  $124^\circ$ , which is the angle at vertex  $A$ . Since  $AB = BC$ , the base angles  $\angle ABC$  and  $\angle ACB$  are equal. Let's denote the measure of each

base angle by  $y$ . Therefore, we can write the equation for the sum of the angles in  $\triangle ABC$  as:

$$124^\circ + y + y = 180^\circ$$

- **id10:** false positive, 解的转换不太对
  - Calculation Error

Therefore, the solutions for  $z$  are:

$$z = \pm e^{\pi i/3} \quad \text{and} \quad z = \pm e^{-\pi i/3}.$$

These roots can also be written as:

$$z = e^{\pi i/3}, \quad z = e^{-\pi i/3}, \quad z = e^{5\pi i/3}, \quad z = e^{-5\pi i/3}.$$

These roots are all 6th roots of unity because they can be expressed as  $e^{2k\pi i/6}$  for  $k = 1, -1, 5, -5$ . To confirm, we note that:

$$e^{2\pi i/6} = e^{\pi i/3}, \quad e^{-2\pi i/6} = e^{-\pi i/3}, \quad e^{10\pi i/6} = e^{5\pi i/3}, \quad e^{-10\pi i/6} = e^{-5\pi i/3}.$$

Thus, the smallest positive integer  $n$  such that all the roots of  $z^4 + z^2 + 1 = 0$  are  $n$ -th roots of unity is  $n = 6$ .

The answer is: 6

- **id29:** false positive, formulate的有问题, 第二个式子不是圆

- Logical Error

First, consider the equation  $x^2 + y^2 + 6x - 24y + 72 = 0$ . We complete the square for both  $x$  and  $y$ :

$$x^2 + 6x + y^2 - 24y + 72 = 0 \implies (x+3)^2 - 9 + (y-12)^2 - 144 + 72 = 0 \implies (x+3)^2 + (y-12)^2 = 81.$$

So, the first circle has center  $(-3, 12)$  and radius 9.

Next, consider the equation  $x^2 - y^2 + 6x + 16y - 46 = 0$ . We complete the square for both  $x$  and  $y$ :

$$x^2 + 6x - y^2 + 16y - 46 = 0 \implies (x+3)^2 - 9 - (y-8)^2 + 64 - 46 = 0 \implies (x+3)^2 - (y-8)^2 = 9.$$

So, the second circle has center  $(-3, 8)$  and radius 3.

The centers of the circles are  $(-3, 12)$  and  $(-3, 8)$ , and the distance between them is:

$$\sqrt{(-3+3)^2 + (12-8)^2} = \sqrt{0+16} = 4.$$

Since the distance between the centers is 4, which is less than the sum of the radii ( $9 + 3 = 12$ ) and greater than the difference of the radii ( $9 - 3 = 6$ ), the circles intersect at two points.

- **id68:** false positive, 做法不严谨, 直接试出来的结果

- Logical Error

We want to maximize  $|a| + |b| + |c|$ . To do this, we consider the system of equations formed by setting  $f(x)$  to its boundary values at these points. Let's assume  $f(0) = 1$ ,  $f(1) = 1$ , and  $f\left(\frac{1}{2}\right) = -1$ . This gives us the system:

$$c = 1,$$

$$a + b + c = 1,$$

$$\frac{a}{4} + \frac{b}{2} + c = -1.$$

Substituting  $c = 1$  into the other two equations, we get:

$$a + b + 1 = 1 \implies a + b = 0,$$

$$\frac{a}{4} + \frac{b}{2} + 1 = -1 \implies \frac{a}{4} + \frac{b}{2} = -2.$$

Since  $a + b = 0$ , we have  $b = -a$ . Substituting  $b = -a$  into the second equation, we get:

$$\frac{a}{4} + \frac{-a}{2} = -2 \implies \frac{a}{4} - \frac{2a}{4} = -2 \implies -\frac{a}{4} = -2 \implies a = 8.$$

Then,  $b = -a = -8$  and  $c = 1$ . Therefore,  $|a| + |b| + |c| = 8 + 8 + 1 = 17$ .

To confirm that this is indeed the maximum value, we check if the polynomial  $f(x) = 8x^2 - 8x + 1$  satisfies the condition  $|f(x)| \leq 1$  for all  $0 \leq x \leq 1$ . The vertex of the parabola  $f(x) = 8x^2 - 8x + 1$  is at  $x = -\frac{-8}{2 \cdot 8} = \frac{1}{2}$ . Evaluating  $f(x)$  at the vertex, we get:

$$f\left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right)^2 - 8\left(\frac{1}{2}\right) + 1 = 8 \cdot \frac{1}{4} - 4 + 1 = 2 - 4 + 1 = -1.$$

- **id70: false positive, 理解错题意**

- Conceptual Error

The superfactorial  $n\$$  is defined as  $n\$ = \underbrace{n!^{n!^{\dots^{n!}}}}_{n!}$ . What is the units digit of  $4\$$ ?

Solution:

To find the units digit of  $4\$$ , we start by understanding the definition of the superfactorial. The superfactorial  $n\$$  is defined as a power tower of  $n!$  where the height of the tower is  $n!$ . Specifically, for  $n = 4$ , we have:

$$4\$ = 4!^{4!^{4!}}$$

First, we calculate  $4!$ :

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

So, we need to find the units digit of  $24^{24^{24^{24}}}$ . Since the units digit of a number depends only on the units digit of its base, we focus on the units digit of 24, which is 4. Therefore, we need to find the units digit of  $4^{24^{24^{24}}}$ .

- **id74: false positive, 讨论的不是很完善**

- Logical Error

Rearranging the equation:

$$-x^3 + 2x^2 + 3x - 6 = 0$$

We can use the Rational Root Theorem to test possible rational roots. The possible rational roots are  $\pm 1, \pm 2, \pm 3, \pm 6$ . Testing  $x = 2$ :

$$-(2)^3 + 2(2)^2 + 3(2) - 6 = -8 + 8 + 6 - 6 = 0$$

So,  $x = 2$  is a root. However,  $x = 2$  is not within the domain  $x > 3$ . Therefore, there is no solution for  $f(x) = 6$  when  $x > 3$ .

Thus,  $f^{-1}(6) = -3$ .

Finally, we sum  $f^{-1}(0)$  and  $f^{-1}(6)$ :

$$f^{-1}(0) + f^{-1}(6) = 3 + (-3) = 0$$

The answer is:

- **id95:** false positive, 下面的式子没有 $(-1)^n$ 才对

- Logical Error

We need to find the distinct integer roots  $r_1, r_2, \dots, r_n$  such that their product is  $\pm 33$  and the coefficient  $a_{n-1}$  is minimized in absolute value. The coefficient  $a_{n-1}$  is given by:

$$a_{n-1} = 2 \left( (-1)^{n-1} \sum_{i=1}^n r_i \right) = -2(-1)^n \sum_{i=1}^n r_i.$$

We will consider the factorizations of 33 into distinct integers and calculate  $|a_{n-1}|$  for each case.

**N=64:** 共93个正确

- **id2:** false positive, 分解的时候少了个+1, 最后结果又没算进去

7.  $2 \cdot ((3 \cdot 4) \cdot 5)$

$$3 \cdot 4 = 12, \quad 12 \cdot 5 = 60, \quad 2 \cdot 60 = 120$$

Value: 120

- **id5:** false positive, 模型没有搞对哪个角相同

The angle given is  $124^\circ$ , which is the angle at vertex  $A$ . Since  $AB = BC$ , the base angles  $\angle ABC$  and  $\angle ACB$  are equal. Let's denote the measure of each base angle by  $y$ . Therefore, we can write the equation for the sum of the angles in  $\triangle ABC$  as:

$$124^\circ + y + y = 180^\circ$$

Simplifying this equation, we get:

$$124^\circ + 2y = 180^\circ$$

Subtracting  $124^\circ$  from both sides, we obtain:

$$2y = 56^\circ$$

Dividing both sides by 2, we find:

$$y = 28^\circ$$

- **id10:** false positive, 解的转换不太对

Therefore, the solutions for  $z$  are:

$$z = \pm e^{\pi i/3} \quad \text{and} \quad z = \pm e^{-\pi i/3}.$$

These roots can also be written as:

$$z = e^{\pi i/3}, \quad z = e^{-\pi i/3}, \quad z = e^{5\pi i/3}, \quad z = e^{-5\pi i/3}.$$

These roots are all 6th roots of unity because they can be expressed as  $e^{2k\pi i/6}$  for  $k = 1, -1, 5, -5$ . To confirm, we note that:

$$e^{2\pi i/6} = e^{\pi i/3}, \quad e^{-2\pi i/6} = e^{-\pi i/3}, \quad e^{10\pi i/6} = e^{5\pi i/3}, \quad e^{-10\pi i/6} = e^{-5\pi i/3}.$$

Thus, the smallest positive integer  $n$  such that all the roots of  $z^4 + z^2 + 1 = 0$  are  $n$ -th roots of unity is  $n = 6$ .

The answer is:

6

- **id29:** false positive
- **id41:** false positive, 做法不严谨

instead, we consider the function along the line  $y = x$ :

$$f(x, x) = \frac{x - x}{x^4 + x^4 + 6} = 0.$$

And along the line  $y = -x$ :

$$f(x, -x) = \frac{x - (-x)}{x^4 + (-x)^4 + 6} = \frac{2x}{2x^4 + 6} = \frac{x}{x^4 + 3}.$$

We need to maximize  $g(x) = \frac{x}{x^4 + 3}$ . take the derivative and set it to zero:

$$g'(x) = \frac{(x^4 + 3) - x(4x^3)}{(x^4 + 3)^2} = \frac{x^4 + 3 - 4x^4}{(x^4 + 3)^2} = \frac{-3x^4 + 3}{(x^4 + 3)^2} = \frac{-3(x^4 - 1)}{(x^4 + 3)^2}.$$

Setting  $g'(x) = 0$ , we get:

$$-3(x^4 - 1) = 0 \implies x^4 = 1 \implies x = \pm 1.$$

For  $x = 1$ ,  $g(1) = \frac{1}{1^4 + 3} = \frac{1}{4}$ . For  $x = -1$ ,  $g(-1) = \frac{-1}{(-1)^4 + 3} = \frac{-1}{4}$ .

Thus, the maximum value of  $g(x)$  is  $\frac{1}{4}$ , and therefore the maximum value of  $f(x, y)$  is  $\frac{1}{4}$ .

The maximum value is:

$\frac{1}{4}$

- **id68:** false positive, 直接试出来的结果

We want to maximize  $|a| + |b| + |c|$ . To do this, we will consider the case where  $f(x)$  takes on the values 1 and  $-1$  at specific points. Let's assume:

$$f(0) = c = 1, \quad f\left(\frac{1}{2}\right) = -1, \quad f(1) = 1.$$

This gives us the system of equations:

$$c = 1, \quad \frac{a}{4} + \frac{b}{2} + c = -1, \quad a + b + c = 1.$$

Substituting  $c = 1$  into the other equations, we get:

$$\begin{aligned} \frac{a}{4} + \frac{b}{2} + 1 &= -1 \implies \frac{a}{4} + \frac{b}{2} = -2 \implies a + 2b = -8, \\ a + b + 1 &= 1 \implies a + b = 0. \end{aligned}$$

We now have the system:

$$a + 2b = -8, \quad a + b = 0.$$

Subtracting the second equation from the first, we get:

$$b = -8.$$

Substituting  $b = -8$  back into  $a + b = 0$ , we get:

$$a - 8 = 0 \implies a = 8.$$

- **id70: false positive, 理解错题意**

### id70

Problem:

The superfactorial  $n\$$  is defined as  $n\$ = \underbrace{n!n! \cdots n!}_{n!}$ . What is the units digit of  $4\$$ ?

Solution:

To determine the units digit of  $4\%$ , we start by understanding the definition of the superfactorial. The superfactorial  $n\%$ , for  $n = 4$ , is given by:

$$4\% = 4!4!4!$$

First, we calculate  $4!$ :

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

Thus, we need to find the units digit of  $24^{24^{24}}$ . Since the units digit of a power of a number depends only on the units digit of the base, we focus on the units digit of 24, which is 4. Therefore, we need to determine the units digit of  $4^{24^{24}}$ .

- **id74: false positive, 有一种情况讨论的时候式子是错的**

This gives us:

$$-x^3 + 2x^2 + 3x - 6 = -(x-2)(x^2 + 3)$$

The quadratic equation  $x^2 + 3 = 0$  has no real solutions, so there are no solutions for  $f(x) = 6$  in the interval  $x > 3$ .

Therefore, the only solution is  $f^{-1}(6) = -3$ .

Finally, we sum the values of  $f^{-1}(0)$  and  $f^{-1}(6)$ :

$$f^{-1}(0) + f^{-1}(6) = 3 + (-3) = 0$$

The answer is:

- **id95: false positive, 下面的式子没有 $(-1)^n$ 才对**

We need to find the distinct integer roots  $r_1, r_2, \dots, r_n$  such that their product is  $\pm 33$  and the coefficient  $a_{n-1}$  is minimized in absolute value. The coefficient  $a_{n-1}$  is given by:

$$a_{n-1} = 2 \left( (-1)^{n-1} \sum_{i=1}^n r_i \right) = -2(-1)^n \sum_{i=1}^n r_i.$$

We will consider the factorizations of 33 into distinct integers and calculate  $|a_{n-1}|$  for each case.

## N=16: 共92个正确

- **id5:** false positive, 模型没有搞对哪个角相同
- **id10:** false positive, 解的转换不太对

Therefore, the solutions for  $z$  are:

$$z = \pm e^{\pi i/3} \quad \text{and} \quad z = \pm e^{-\pi i/3}.$$

These roots can also be written as:

$$z = e^{\pi i/3}, \quad z = e^{-\pi i/3}, \quad z = e^{5\pi i/3}, \quad z = e^{-5\pi i/3}.$$

These roots are all 6th roots of unity because they can be expressed as  $e^{2k\pi i/6}$  for  $k = 1, -1, 5, -5$ . To confirm, we note that:

$$e^{2\pi i/6} = e^{\pi i/3}, \quad e^{-2\pi i/6} = e^{-\pi i/3}, \quad e^{10\pi i/6} = e^{5\pi i/3}, \quad e^{-10\pi i/6} = e^{-5\pi i/3}.$$

Thus, the smallest positive integer  $n$  such that all the roots of  $z^4 + z^2 + 1 = 0$  are  $n$ -th roots of unity is  $n = 6$ .

The answer is:

6

- **id28:** false positive,  $x$ 解的不对

The solutions to  $-1 = e^{i\pi(2k+1)}$  for  $k = 0, 1, 2, \dots, 9$  are the 10th roots of  $-1$ . Therefore, we have:

$$\frac{x}{13x - 1} = e^{i\pi(2k+1)/10} \quad \text{for } k = 0, 1, 2, \dots, 9.$$

Solving for  $x$ , we get:

$$x = \frac{e^{i\pi(2k+1)/10}}{1 + 13e^{i\pi(2k+1)/10}}.$$

- **id29:** false positive
- **id68:** false positive, 后面给的理由不太准确

Since  $f(x) = 8x^2 - 8x + 1$  satisfies the inequality  $|ax^2 + bx + c| \leq 1$  for all  $0 \leq x \leq 1$ , we have  $a = 8$ ,  $b = -8$ , and  $c = 1$ . Therefore,

$$|a| + |b| + |c| = |8| + |-8| + |1| = 8 + 8 + 1 = 17.$$

To confirm that 17 is indeed the maximum, we note that if  $|a| + |b| + |c| > 17$ , then at least one of  $|a|$ ,  $|b|$ , or  $|c|$  would be greater than 8, and the polynomial  $ax^2 + bx + c$  would exceed 1 in absolute value for some  $x$  in the interval  $[0, 1]$ . Thus, the maximum value of  $|a| + |b| + |c|$  is indeed 17.

The answer is:

17

- **id70: false positive, 理解错题意**

Problem:

The superfactorial  $n\$$  is defined as  $n\$ = \underbrace{n!n!\cdots n!}_{n!}$ . What is the units digit of  $4\$$ ?

Solution:

To find the units digit of the superfactorial 4, we start by understanding the definition:  $4\$ = 4!^{4!^{4!^4}}$ . First, we calculate  $4!$ :

$$4! = 4 \times 3 \times 2 \times 1 = 24.$$

So,  $4\$ = 24^{24^{24^{24}}}$ . To determine the units digit of  $24^{24^{24^{24}}}$ , we only need to consider the units digit of the base, which is 4. Therefore, we need to find the units digit of  $4^{24^{24^{24}}}$ .

Next, we observe the pattern in the units digits of the powers of 4:

$$4^1 = 4 \quad (\text{units digit is } 4),$$

$$4^2 = 16 \quad (\text{units digit is } 6),$$

$$4^3 = 64 \quad (\text{units digit is } 4),$$

$$4^4 = 256 \quad (\text{units digit is } 6),$$

⋮

We see that the units digits alternate between 4 and 6. To determine the units digit of  $4^{24^{24^{24}}}$ , we need to find the parity (odd or even) of the exponent  $24^{24^{24}}$ . Since 24 is even, any power of 24 is also even. Therefore,  $24^{24^{24}}$  is even.

Since the exponent  $24^{24^{24}}$  is even, the units digit of  $4^{24^{24^{24}}}$  is the same as the units digit of  $4^2$ , which is 6.

Thus, the units digit of 4 is 6.

- **id74: false positive, 没有讨论全**

We can use the Rational Root Theorem to test possible rational roots. The possible rational roots are  $\pm 1, \pm 2, \pm 3, \pm 6$ . Testing  $x = 2$ :

$$-(2)^3 + 2(2)^2 + 3(2) - 6 = -8 + 8 + 6 - 6 = 0.$$

So,  $x = 2$  is a root. Since  $x = 2$  is not in the domain  $x > 3$ , it is not a valid solution. We need to check if there are any other real roots. However, since we are only asked to find  $f^{-1}(6)$  and we already have  $f^{-1}(6) = -3$  from the domain  $x \leq 3$ , we do not need to find any other roots.

So, the only solution to  $f(x) = 6$  is  $x = -3$ , and thus  $f^{-1}(6) = -3$ .

Finally, we sum the values of  $f^{-1}(0)$  and  $f^{-1}(6)$ :

$$f^{-1}(0) + f^{-1}(6) = 3 + (-3) = 0.$$

The answer is:

0.

- **id95: false positive, 对最后解的情况讨论的没有特别充分, 根的乘积也不太对.....**

Therefore,  $a_{n-1} = -2(r_1 + r_2 + \dots + r_n)$ , and we need to find the combination of factors of 33 that minimizes the absolute value of their sum.

Let's consider the possible combinations of factors: 1. 1, 3, 11: The sum is  $1+3+11 = 15$ , so  $|a_{n-1}| = 2 \times 15 = 30$ . 2.  $-1, 3, 11$ : The sum is  $-1+3+11 = 13$ , so  $|a_{n-1}| = 2 \times 13 = 26$ . 3.  $1, -3, 11$ : The sum is  $1-3+11 = 9$ , so  $|a_{n-1}| = 2 \times 9 = 18$ . 4.  $-1, -3, 11$ : The sum is  $-1-3+11 = 7$ , so  $|a_{n-1}| = 2 \times 7 = 14$ . 5.  $1, 3, -11$ : The sum is  $1+3-11 = -7$ , so  $|a_{n-1}| = 2 \times 7 = 14$ . 6.  $-1, 3, -11$ : The sum is  $-1+3-11 = -9$ , so  $|a_{n-1}| = 2 \times 9 = 18$ . 7.  $1, -3, -11$ : The sum is  $1-3-11 = -13$ , so  $|a_{n-1}| = 2 \times 13 = 26$ . 8.  $-1, -3, -11$ : The sum is  $-1-3-11 = -15$ , so  $|a_{n-1}| = 2 \times 15 = 30$ . 9.  $1, 1, 3, 11$ : The sum is  $1+1+3+11 = 16$ , so  $|a_{n-1}| = 2 \times 11 = 22$  (not possible since the roots must be distinct). 10.  $1, 1, -3, 11$ : The sum is  $1+1-3+11 = 10$ , so  $|a_{n-1}| = 2 \times 13 = 26$  (not possible since the roots must be distinct).

From the above combinations, the smallest value of  $|a_{n-1}|$  is 14, which occurs when the roots are  $-1, -3, 11$  or  $1, 3, -11$ .

Therefore, the least possible value of  $|a_{n-1}|$  is:

14.

**N=4:** 共85个正确

- **id5:** false positive

Solution:

To solve for  $x$ , we start by analyzing the given geometric configuration. We know that  $\overline{BC}$  is parallel to the segment through  $A$ , and  $AB = BC$ . This means that  $\triangle ABC$  is isosceles with  $AB = BC$ .

The angle at  $A$  is given as  $124^\circ$ . Since  $\overline{BC}$  is parallel to the segment through  $A$ , the angle formed by  $AB$  and the segment through  $A$  is the same as the angle formed by  $AB$  and  $BC$  (alternate interior angles). This angle is  $124^\circ$ .

In an isosceles triangle, the base angles are equal. Let the base angles at  $B$  and  $C$  be  $\theta$ . The sum of the interior angles of a triangle is always  $180^\circ$ . Therefore, we can write the equation for the sum of the angles in  $\triangle ABC$  as:

$$124^\circ + \theta + \theta = 180^\circ$$

- **id28:** false positive

The sum of the cosines of the angles that are multiples of  $\frac{\pi}{5}$  (excluding 0 and  $2\pi$ ) is zero. This is a known result from the properties of roots of unity. Therefore, we have:

$$\sum_{k=1}^5 \cos\left(\frac{k\pi}{5}\right) = 0$$

Thus, the sum simplifies to:

$$5 \cdot 170 + 26 \cdot 0 = 850$$

So, the value is:

850

- **id29:** false positive
- **id68:** false positive, 直接猜出二次函数的式子(也就是猜出a, b, c)
- **id70:** false positive, 理解错题意

Problem:

The superfactorial  $n\$$  is defined as  $n\$ = \underbrace{n!^{n!^{\dots^n!}}}_{n!}$ . What is the units digit of  $4\$$ ?

Solution:

To find the units digit of the superfactorial 4, we start by understanding the definition:  $4\$ = 4!^{4!^{4!^4}}$ . First, we calculate  $4!$ :

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Next, we observe the pattern in the units digits of the powers of 4:

$$\begin{aligned}4^1 &= 4 \quad (\text{units digit is } 4), \\4^2 &= 16 \quad (\text{units digit is } 6), \\4^3 &= 64 \quad (\text{units digit is } 4), \\4^4 &= 256 \quad (\text{units digit is } 6), \\&\vdots\end{aligned}$$

We see that the units digits alternate between 4 and 6. To determine the units digit of  $4^{24^{24^{24}}}$ , we need to find the parity (odd or even) of the exponent  $24^{24^{24}}$ . Since 24 is even, any power of 24 is also even. Therefore,  $24^{24^{24}}$  is even.

Since the exponent  $24^{24^{24}}$  is even, the units digit of  $4^{24^{24^{24}}}$  is the same as the units digit of  $4^2$ , which is 6.

Thus, the units digit of 4 is 6.

- **id95: false positive, 结果讨论的有点问题**

1. For the factorization  $33 = 33 \cdot 1$ , the roots are 33 and 1, and the sum is  $33 + 1 = 34$ .
2. For the factorization  $33 = 11 \cdot 3$ , the roots are 11 and 3, and the sum is  $11 + 3 = 14$ .
3. For the factorization  $33 = 11 \cdot 1 \cdot (-3)$ , the roots are 11, 1, and -3, and the sum is  $11 + 1 - 3 = 9$ .
4. For the factorization  $33 = 3 \cdot 1 \cdot (-11)$ , the roots are 3, 1, and -11, and the sum is  $3 + 1 - 11 = -7$ .
5. For the factorization  $33 = 1 \cdot (-1) \cdot (-11)$ , the roots are 1, -1, and -11, and the sum is  $1 - 1 - 11 = -11$ .
6. For the factorization  $33 = (-1) \cdot (-3) \cdot (-11)$ , the roots are -1, -3, and -11, and the sum is  $-1 - 3 - 11 = -15$ .

The smallest absolute value of the sum of the roots is 7, which corresponds to the factorization  $33 = 3 \cdot 1 \cdot (-11)$ . Therefore,  $|a_{n-1}| = |-2 \cdot (-7)| = 14$ .

The least possible value of  $|a_{n-1}|$  is:

14.