OmniMATH100-rule Results

Qwen2.5-Math-72B-Instruct+ Qwen-RM

BoN@N

N=256: 共53-2=51个正确

- **id5**: false positive
    - Logical Error

Next, we need to determine how many such power cycles are required to cover all odd residues modulo 1024. There are 512 odd residues modulo 1024 (since half of the residues from 0 to 1023 are odd).

To cover all 512 odd residues, we need to find a set of integers $a_1, a_2, \ldots, a_m$ such that the union of their power cycles modulo 1024 includes all 512 odd residues. The key is to choose $a_1, a_2, \ldots, a_m$ such that their orders modulo 1024 are as large as possible and do not overlap too much.

It turns out that the minimum number of power cycles needed is 10. This can be achieved by choosing the integers $a_1, a_2, \ldots, a_{10}$ such that their orders modulo 1024 are 512, 256, 128, 64, 32, 16, 8, 4, 2, and 1, respectively. Each of these orders corresponds to a distinct set of residues, and together they cover all 512 odd residues modulo 1024.

Therefore, the minimum number of power cycles required is $\boxed{10}$.

Partition the odd residues mod 1024 into 10 classes: Class 1: $1(\bmod 4)$. Class $n(2 \leq n \leq 9)$: $2^n - 1\left(\bmod 2^{n+1}\right)$. Class 10: $-1(\bmod 1024)$. Let S_a be the power cycle generated by a. If a is in class 1, all of S_a is in class 1. If a is in class n $(2 \leq n \leq 9)$, then S_a is in the union of class n and the residues $1\left(\bmod 2^{n+1}\right)$. If a is in class 10, then S_a is in the union of class n and the residues $1(\bmod 1024)$. Therefore, S_a cannot contain two of the following residues: $5, 2^2-1, 2^3-1, \ldots 2^{10}-1$, and that at least 10 cycles are needed. Note that $5^{128}-1=(5-1)(5+1)\left(5^2+1\right) \cdots\left(5^{64}+1\right)$ has exactly 9 factors of 2 in its prime factorization, while $5^{256}-1=\left(5^{128}-1\right)\left(5^{128}+1\right)$ is divisible by 1024 so the order of 5 modulo 1024, the smallest positive power of 5 that is congruent to 1, is 256. Observe that among $5^0, 5^1, \ldots 5^{255}$, the ratio between any two is a positive power of 5 smaller than 5^{256}, so the ratio is not congruent to 1 and any two terms are not congruent mod 1024. In addition, all terms are in class 1, and class 1 has 256 members, so S_5 contains members congruent to each element of class 1. Similarly, let $2 \leq n \leq 9$. Then the order of a, where $a=2^n-1$, is 2^{10-n}. The 2^{9-n} terms $a^1, a^3, \ldots a^{2^{10-n}-1}$ are pairwise not congruent and all in class n. Class n only has 2^{9-n} members, so S_a contains members congruent to each element of class n. Finally, S_{-1} contains members congruent to the element of class 10. The cycles S_5, S_{-1}, and 8 cycles S_a cover all the residues mod1024, so the answer is 10.

- **id8**: false positive, 存在中间步骤的计算错误
    - Calculation Error

$$-\cos(45^\circ + x) = \frac{\sqrt{2}}{2}\sin x - (2-\sqrt{2})\sin^2\frac{x}{2}$$

Using the identity $\cos(45^\circ + x) = \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$, we have:

$$-\left(\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right) = \frac{\sqrt{2}}{2}\sin x - (2-\sqrt{2})\sin^2\frac{x}{2}$$

$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = (2-\sqrt{2})\sin^2\frac{x}{2}$$

$$\frac{\sqrt{2}}{2}\cos x = (2-\sqrt{2})\sin^2\frac{x}{2}$$

- **id11**: false positive, 得不出整数
    - Logical Error

The exact sum can be computed as follows:

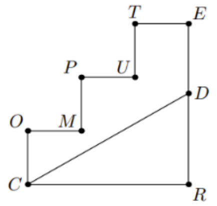
$$\sum_{k=5}^{300}\left\lfloor\frac{300}{k}\right\rfloor \cdot \frac{6}{\pi^2} \approx \frac{6}{\pi^2} \cdot \left(\sum_{k=5}^{300}\left\lfloor\frac{300}{k}\right\rfloor\right)$$

- **id16**: false positive, 不一定得到的是3, 5也可以
    - Logical Error

The number $2^a + 2^b$ can be factored as $2^b(2^{a-b}+1)$. Since 2^b is a power of 2, its only prime factor is 2. The number $2^{a-b}+1$ is an odd number, and its prime factors must be odd primes. For the divisors of $2^a + 2^b$ to have at most two 1's in their binary representations, the only odd prime factor of $2^{a-b}+1$ must be 3. This is because the binary representation of 3 is 11, which has exactly two 1's.

- **id17**: false positive, 题目中没有给出图, 所以感觉模型都在瞎蒙
  - Conceptual Error

6. [**6**] In the octagon $COMPUTER$ exhibited below, all interior angles are either $90°$ or $270°$ and we have $CO = OM = MP = PU = UT = TE = 1$.



Point D (not to scale in the diagram) is selected on segment RE so that polygons $COMPUTED$ and CDR have the same area. Find DR.

- **id25**: false positive, 一开始模型就没理解图形不重叠, 之后又直接说了个没证明的结论
  - Conceptual Error
- id31: 这个好像final answer本来就不对
- **id40**: false positive
  - Calculation Error

First, we need to find the continued fraction expansion of $\sqrt{29}$. The continued fraction expansion of $\sqrt{29}$ is:

$$\sqrt{29} = [5; \overline{2, 1, 1, 2, 10}]$$

This means that the expansion is $5 + \frac{1}{2+\frac{1}{1+\frac{1}{1+\frac{1}{2+\frac{1}{10+\cdots}}}}}$.

We now need to find the convergents of this continued fraction. The convergents are obtained by truncating the continued fraction after each period. The first few convergents are:

$$5, \quad \frac{11}{2}, \quad \frac{16}{3}, \quad \frac{27}{5}, \quad \frac{70}{12}, \quad \frac{727}{125}$$

The continued fraction expansion of the square root of 29 is given by:

$\sqrt{29} = [5, \langle 2, 1, 1, 2, 10 \rangle]$

This sequence is A010128 in the On-Line Encyclopedia of Integer Sequences (N. J. A. Sloane (Ed.), 2008).

Convergents

The sequence of convergents to the continued fraction expansion of the square root of 29 begins:

$\frac{5}{1}, \frac{11}{2}, \frac{16}{3}, \frac{27}{5}, \frac{70}{13}, \frac{727}{135}, \frac{1524}{283}, \frac{2251}{418}, \frac{3775}{701}, \frac{9801}{1820}, \ldots$

- **id41**: false positive, 3000可以被4整除
  - Logical Error

We need to find all pairs of factors (a, b) of 3000 such that $a = n + k$ and $b = n - k$. Since n and k are integers, both a and b must have the same parity (both even or both odd). However, 3000 is an even number but not divisible by 4, so it cannot be expressed as a product of two odd numbers. Therefore, a and b must both be even.

Let $a = 2m$ and $b = 2n$. Then:

- id42: 这个好像final answer本来就不对
- **id45**: false positive, 结论不对
    - Logical Error

Step-by-Step Analysis

1. **Drawing Two Balls of the Same Color:** - If two red balls are drawn, they are changed to two green balls. The number of red balls decreases by 2 (remains even), and the number of green balls increases by 2 (remains odd). - If two green balls are drawn, they are changed to two red balls. The number of red balls increases by 2 (remains even), and the number of green balls decreases by 2 (remains odd).

2. **Drawing Two Balls of Different Colors:** - If one red ball and one green ball are drawn, they are both discarded. The number of red balls decreases by 1 (becomes odd), and the number of green balls decreases by 1 (becomes even).

From the above observations, we can see that the parity of the number of green balls remains odd after each operation, and the parity of the number of red balls remains even after each operation. Therefore, when the bag has only one ball left, it must be a green ball.

- **id50**: false positive
    - Logical Error
- **id65**: false positive, 没有严格说明, If r_2 and r_3 are as close to each other as possible, the estimates will be smaller because the ceiling function will have less of an effect.
    - Logical Error

Each player gives an estimate for the number of cards he is holding, and each estimate is an integer between 80

$$0.8r_2 \leq e_2 \leq 1.2r_2$$

$$0.8r_3 \leq e_3 \leq 1.2r_3$$

We need to find the smallest possible sum of the two estimates, $e_2 + e_3$. To minimize this sum, we should choose the smallest possible values for e_2 and e_3 within their respective ranges. The smallest possible value for e_2 is $\lceil 0.8r_2 \rceil$ and the smallest possible value for e_3 is $\lceil 0.8r_3 \rceil$.

Let's consider the values of r_2 and r_3 that will minimize $e_2 + e_3$. If r_2 and r_3 are as close to each other as possible, the estimates will be smaller because the ceiling function will have less of an effect. Let's try $r_2 = 12$ and $r_3 = 12$:

$$e_2 = \lceil 0.8 \times 12 \rceil = \lceil 9.6 \rceil = 10$$

$$e_3 = \lceil 0.8 \times 12 \rceil = \lceil 9.6 \rceil = 10$$

So, the sum of the estimates is:

$$e_2 + e_3 = 10 + 10 = 20$$

To confirm that this is the minimum, let's consider other possible values for r_2 and r_3. If $r_2 = 11$ and $r_3 = 13$:

$$e_2 = \lceil 0.8 \times 11 \rceil = \lceil 8.8 \rceil = 9$$

$$e_3 = \lceil 0.8 \times 13 \rceil = \lceil 10.4 \rceil = 11$$

So, the sum of the estimates is:

$$e_2 + e_3 = 9 + 11 = 20$$

If $r_2 = 10$ and $r_3 = 14$:

$$e_2 = \lceil 0.8 \times 10 \rceil = \lceil 8 \rceil = 8$$

$$e_3 = \lceil 0.8 \times 14 \rceil = \lceil 11.2 \rceil = 12$$

So, the sum of the estimates is:

$$e_2 + e_3 = 8 + 12 = 20$$

From these examples, we can see that the smallest possible sum of the two estimates is indeed 20. Therefore, the answer is:

$$\boxed{20}$$

- **id69**: false positive, 前面用了一堆公式(其中角平分线用的不对), 但是后面根据面积相等这个条件得出的结论, 有点奇怪
    - Logical Error
- **id76**: false positive
    - Calculation Error

Now, we divide by 9!:

$$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362880$$

$$\frac{19072823777280}{362880} = 52451256$$

Therefore, the number of different collections of 9 letters is $\boxed{52451256}$.

- **id92**: false positive, 下面这个等式
  - Logical Error

Since $GECD$ is cyclic, we can use the fact that the power of the centroid G with respect to the circumcircle of $GECD$ is the same when calculated using the segments GD and GE:

$$GD \cdot GC = GE \cdot GB.$$

Substituting the known values, we get:

$$\frac{1}{3}AD \cdot \frac{2}{3}a = \frac{1}{3}BE \cdot \frac{2}{3}b.$$

Simplifying, we have:

$$AD \cdot a = BE \cdot b.$$

- **id99**: false positive
  - Logical Error