

(b) Tara's aunt invests \$2000 for her when she is born.

The interest rate is 3.5% per year.

This rate does not change as long as the money stays invested.

The interest is added to the amount she has invested on her birthday each year.

The value of the investment after  $t$  years can be modelled by the equation

$$A = 2000 \times (1.035)^t$$

where the  $A$  is the value of the investment.

(i) How long would it take for the value of the investment to be \$2250?

(ii) Tara reaches her 18th birthday.

Calculate how much **extra** the investment will be worth if she leaves the money invested for another 3 years beyond her 18th birthday.

## Continued

(iii) Tara is calculating  $2000 \times 1.035^m (1.035^n - 1)$

With reference to the investment, explain what Tara is calculating.

(c) Solve  $9^n - (6 \times 3^n) - 27 = 0$  and explain why it has only one real solution.

Hint: let  $3^n = x$

- (c) Mark solves the equation  $\frac{x^2 - 5x + 6}{x^2 + x - 6} = 4$

His working is shown below.

$$x^2 - 5x + 6 = 4x^2 + 4x - 24$$

$$3x^2 + 9x - 30 = 0$$

$$3(x^2 + 3x - 10) = 0$$

$$3(x + 6)(x - 2) = 0$$

$$x = -6 \text{ or } x = 2$$

Is Mark's answer correct?

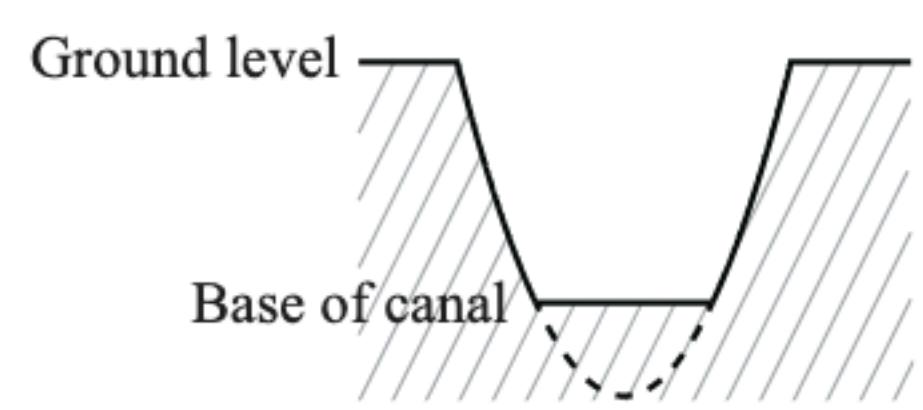
Fully justify your answer.

- (e) The width of a canal at ground level is 16 m.

The **sides** of the canal can be modelled by a quadratic expression that would give a maximum depth of 16 m.

However, the base of the canal is **flat** and has a width of 12 m.

What is the actual depth of the canal?



(b) (i) Mark is solving  $(2x - 3)(x + 4) = 13$  by using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Give the values of  $a$ ,  $b$  and  $c$  and hence solve the equation.

(ii) The equation  $(2x - 3)(x + 4) = k$  has only one real solution.

Find the value of  $k$ .

(c) Find the possible values of  $d$  if real solutions exist for  $x^2 + 5x - 1 - d(x^2 + 1) = 0$ .

(d) The equation  $(x+2) - 3\sqrt{(x+2)} - 4 = 0$  has only one real solution.

Find the value of  $x$ .

(Hint: Let  $a = \sqrt{(x+2)}$ )

(e) (i) Find expressions, in terms of  $m$  and  $n$ , for the roots of the equation:

$$\frac{x-m}{x-n} = \frac{2(x+m)}{x+n}$$

(ii) Give an inequality, in terms of  $m$  and  $n$ , so that the equation has two distinct roots.

(d) Rearrange the formula  $a^x = 5^{(x-1)}$  to make  $x$  the subject.

(e) The equation  $3x^2 + 4x - k = 0$  has two distinct real roots.

If 2 is a root of this equation, find the value of  $k$  and the second root.

- (c) Explain why the equation  $(3x + 1)^2 = -7$  does not have any real solutions, and explain what this means graphically.

(d) Solve the equation  $\log x = 2 \log(mx)$  for  $x$  in terms of  $m$ .

(b) One root of the equation  $x^2 + mx + 12 = 0$  is three times the other.

Find the values of  $m$ .

(c) The equation  $3x^2 - nx + 5 = 0$  has two distinct roots.

Find the values of  $n$ .

(d) Solve  $10x^4 - 13x^2 + 4 = 0$

*You must show algebraic working.*

(b) (i) Write as a single fraction  $\frac{3}{x-2} - \frac{4x}{x+1}$

(ii) Solve the equation  $\frac{x^2 + 2x - 8}{x^2 - x - 2} = 3$

*You must show algebraic working.*

- (c) (i) The height  $h$  metres of a tunnel is modelled by a function of the form

$$h = rx^2 - tx$$

where  $r$  and  $t$  are constants.

Make  $x$ , the distance in metres from the left side of the tunnel, the subject of the equation.

- (ii) The shape of the tunnel can be modelled by a parabola.

The maximum height of the tunnel is 6 m, and at ground level its width is 12 m.

Find the equation of the parabola.

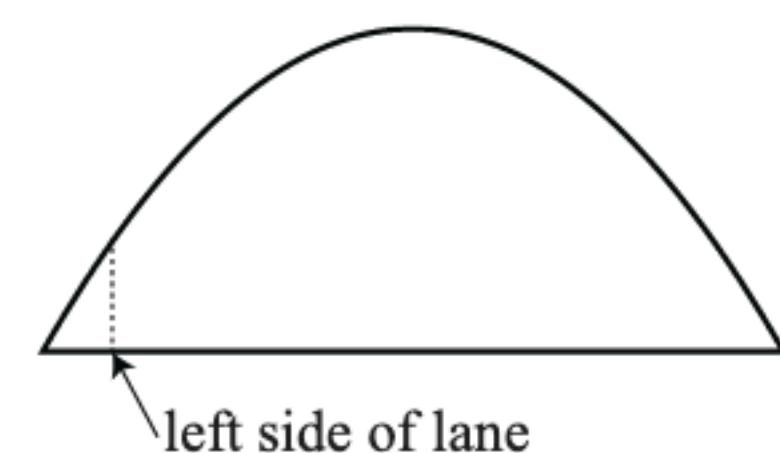
## Continued

- (iii) There are two lanes of equal width through the tunnel.

The outside edge of each lane is marked by a line so that a car of height 1.8 m would have a minimum clearance of 0.1 m vertically from the top of the car to the tunnel roof.

(Ignore the width of the line.)

Find the width of each lane.



(b) Solve for  $x$ :  $5^x \times 2^{-2x} = 15$

- (c) Thirty minutes after a patient is administered his first dose of a medication, the amount of medication in his blood stream reaches 224 mg.

The amount of the medication in the blood stream decreases continuously by 20% each hour. The amount of the medication  $M$  mg in the patient's blood stream after it is administered can be modelled by the function

$$M = 224 \times 0.8^{t-0.5}$$

where  $t$  is the time in hours since the drug was administered.

- (i) Explain what the 0.8 represents in this function.

- (ii) Find the amount of medication administered initially.

## Continued

- (iii) A second dose of the medication can be administered some time later, and again the amount of the medication in the patient's bloodstream from the second dose can be modelled by the same function as that for the first.

The total amount of the drug in the blood stream must never exceed 300 mg.

How long after administering the first dose can the second dose be administered?

(iii) Luka says that the equation  $\log_x(4x + 12) = 2$  has only one solution.

Is he correct?

Find the solution(s), justifying your answer.

(b) Make  $x$  the subject of the equation  $a^{2x} = b^{x+1}$ .

- (c) The market value of Sue's house has been increasing at a constant exponential rate of 3% per annum since she bought it sixteen years ago at the start of 1999. At the start of 2015 it was worth \$350 000.
- (i) Assuming the exponential growth is of the form  $y = A r^t$ , what was the value of the house at the start of 1999 when she bought it?

- (ii) A friend also bought a house at the start of 1999 that cost \$200 000.

Its market value also has been steadily increasing, but at a slightly higher exponential rate of 3.5%.

Its value,  $\$y$ ,  $t$  years after the start of 1999, is given by the function

$$y = 200\,000 \times (1.035)^t$$

If the houses continue to keep increasing in value at the original rates, in which year will the two houses be worth the same amount?

(c) Solve the equation  $2u^{\frac{2}{3}} + 7u^{\frac{1}{3}} = 4$

(d) Talia used timber to form the exterior sides of her rectangular garden. The length of the garden is  $x$  metres, and its area is  $50 \text{ m}^2$ .

(i) Show that the perimeter of the garden is given by  $2x + \frac{100}{x}$

(ii) If she uses 33 m of timber to build the sides, find the dimensions of the garden.

- (e) David and Sione are competing in a cycle race of 150 km.

Sione cycles on average 4 km per hour faster than David, and finishes half an hour earlier than David.

Find David's average speed.

*You MUST use algebra to solve this problem. (Hint: average speed =  $\frac{\text{distance}}{\text{time}}$ )*

(c) For what value(s) of  $k$  does the graph of the quadratic function

$$y = x^2 + (3k - 1)x + (2k + 10)$$

never touch the  $x$ -axis?

(d) The quadratic equation

$$mx^2 - (m + 2)x + 2 = 0$$

has two positive real roots.

Find the possible value(s) of  $m$ , and the roots of the equation.

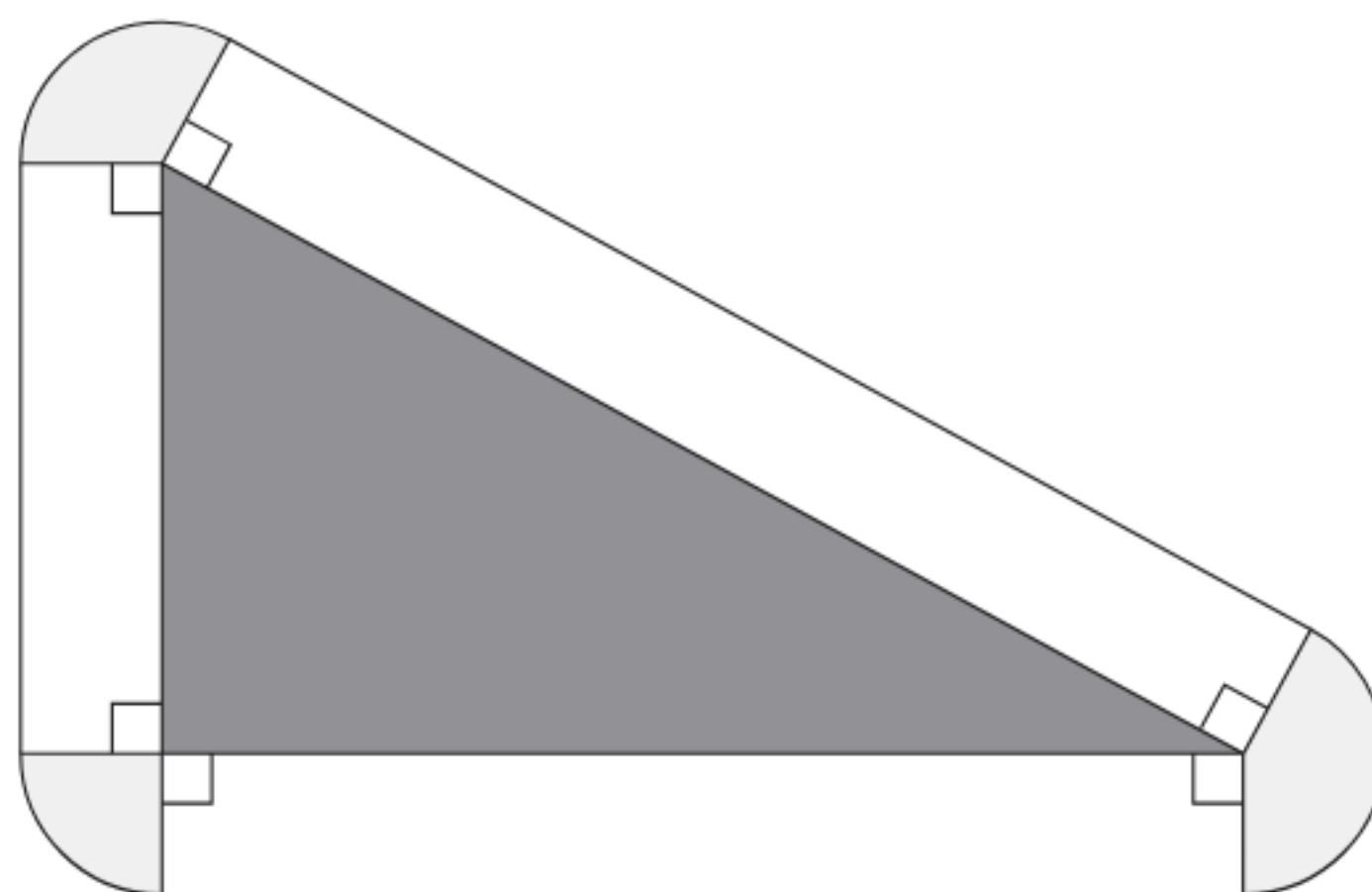
- (ii) Find the relationship between the solutions of the equation  $dx^2 + ex + f = 0$  and the solutions of the equation  $x^2 + ex + df = 0$ , where  $d$ ,  $e$ , and  $f$  are real numbers.

- (e) Find positive integer value(s) for  $k$  so that the quadratic equation  $2x^2 + 4kx + (2k^2 + 3k - 11) = 0$  has **real rational** solutions.

Justify your answer.

(ii) Solve the equation  $6(\log_8 x)^2 + 2\log_8 x - 4 = 0$ .

- (e) The diagram below shows a triangular garden with a path around it.



The triangular garden has sides with lengths in the ratio 3:4:5.

The path is 1 m wide.

At each corner of the garden, the path is a sector (part) of a circle with a radius of 1 m.

The difference between **twice** the **total** area of the path and the area of the garden is  $2\pi \text{ m}^2$ .

Find the length of the longest side of the garden.

(Area of circle =  $\pi r^2$ )

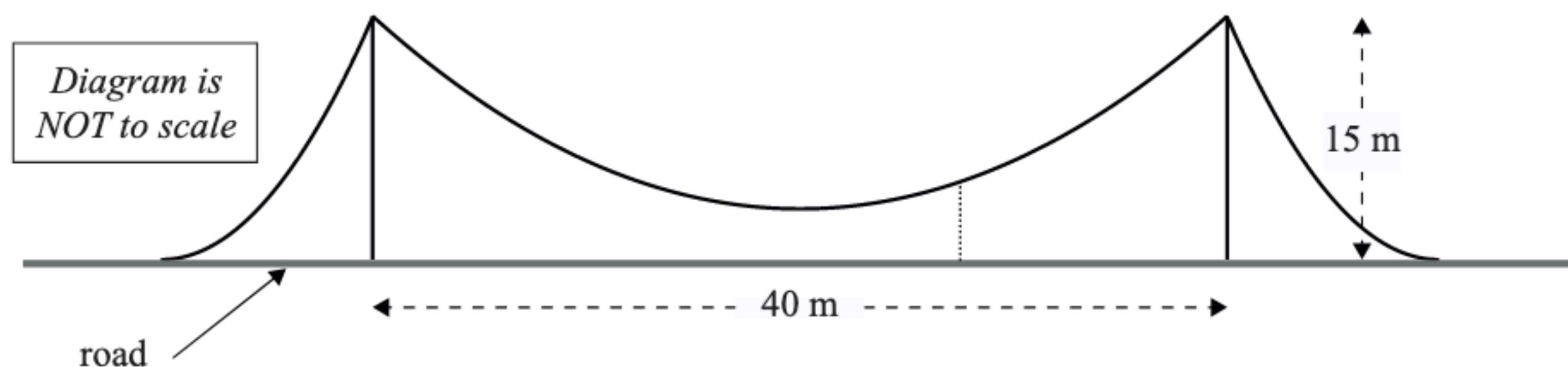
(d) Solve the equation  $9^{8n+6} = 27^{n^2-1} \times 3^{1-3n}$ .

- (e) A symmetrical bridge has its central cable in the shape of a parabola, as shown in the diagram below.

The towers supporting the cable are each 15 m high and 40 m apart.

At the point midway between the towers, the height of the cable above the road is 3 m.

A vertical post (shown dotted in the diagram) is placed 10 m from the centre of the bridge and just touches the cable.

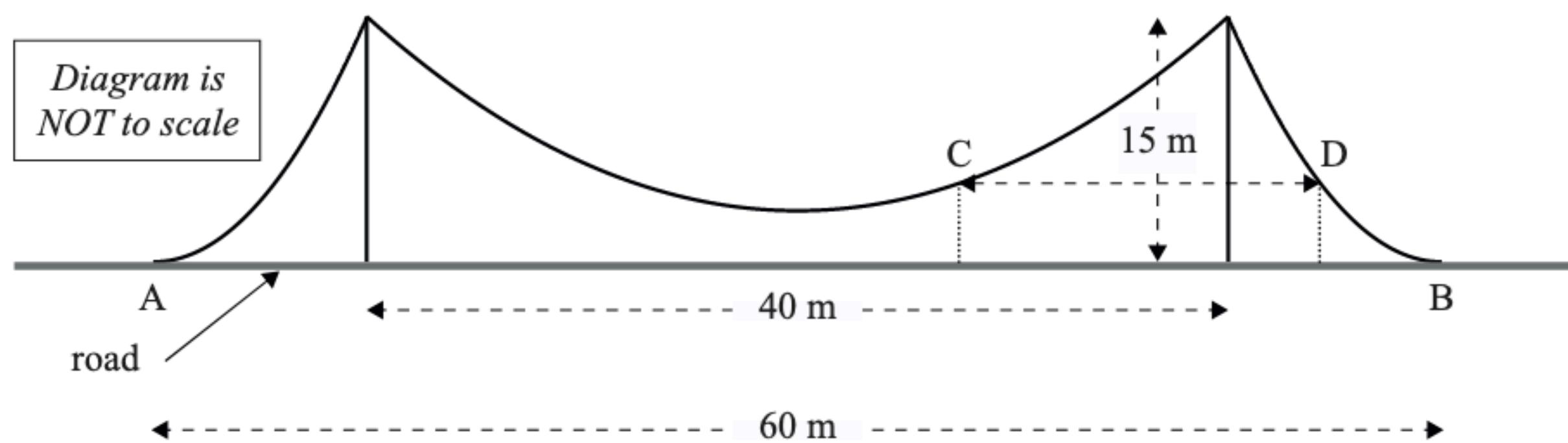


- (i) Use algebra to show that the post is 6 m high.

## Continued

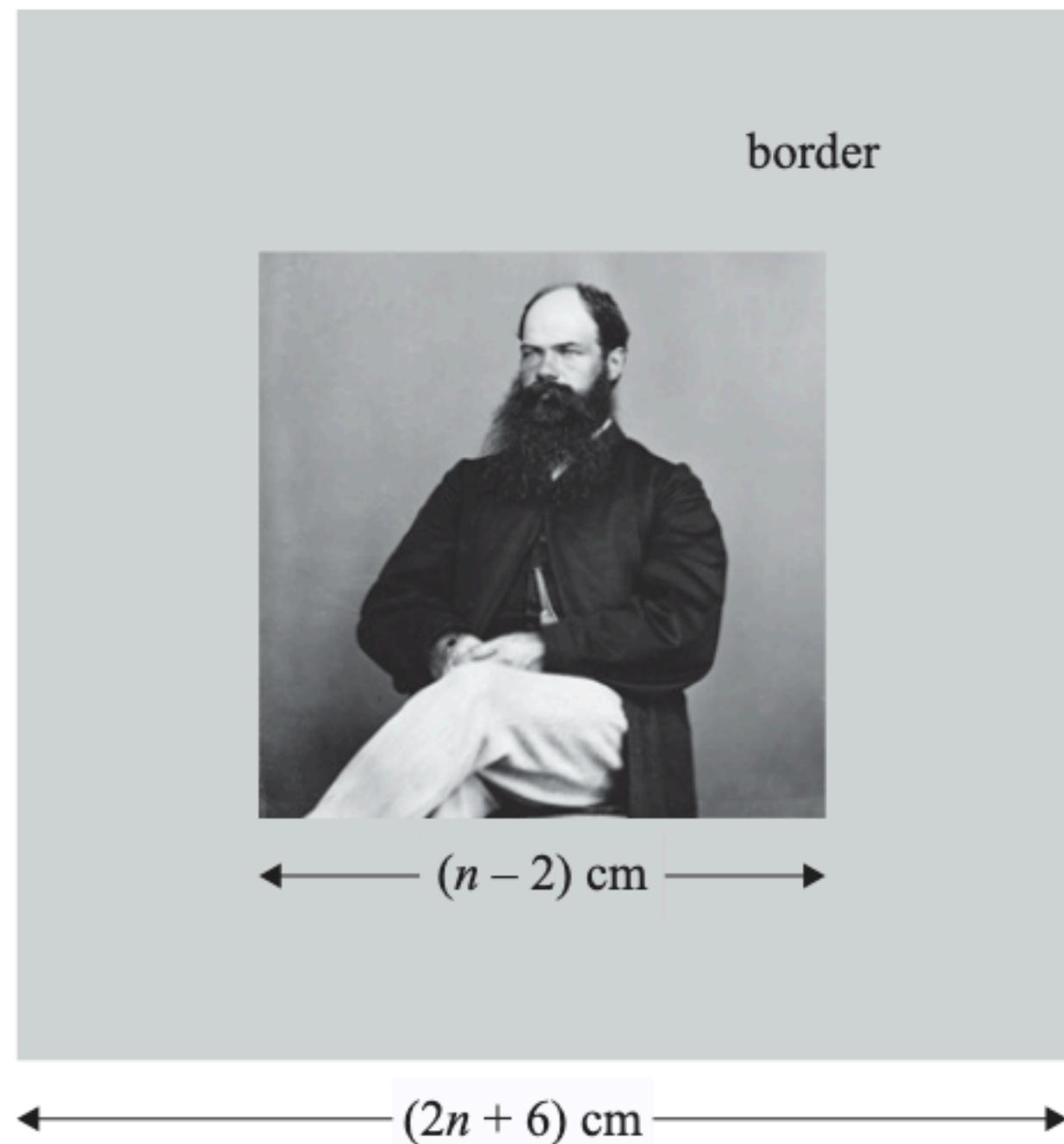
- (ii) The length of the bridge AB is 60 m.

The outside cables are also parabolic and symmetrical in shape, and touch the road at their vertices A and B.



Find the distance, CD, between the two parabolas at a height of 6 m above the road (the distance CD is shown in the diagram).

- (c) David has mounted a square photo on a square piece of card as shown below.



The border around the photo is of constant width.

The photo has sides of length  $(n - 2)$  cm while the card has sides of  $(2n + 6)$  cm.

If the total area of the border is  $200$   $\text{cm}^2$ , find the width of the border.

- (d) A teacher has hired a school bus for \$560 for a day trip with students. The cost of hiring the bus is to be shared equally between the students. At the last moment, three of the students were unable to go. As a result, the cost to each of those who did go was increased by \$1.50.

How many students finally went on the trip?

Justify your answer.

- (d) A computer depreciates continuously in value from \$4699 to \$1500 over a period of 4.25 years.

The value,  $\$y$ , of the computer  $t$  years after its value was \$4699 can be modelled by a function of the form

$$y = Ar^t, \text{ where } r \text{ is a constant.}$$

Find the computer's value after six years.

(e) Make  $p$  the subject of the formula:

$$81^{\left(\frac{px}{q}-3\right)} = 243$$

- (c) Find the value(s) for  $k$  for which the expression  $kx^2 - 12x + 5k$  is always greater than zero.

- (e) Find the value(s) of  $m$  for which the equation  $2^{mx-3} = 8^{x^2}$  has exactly one solution.

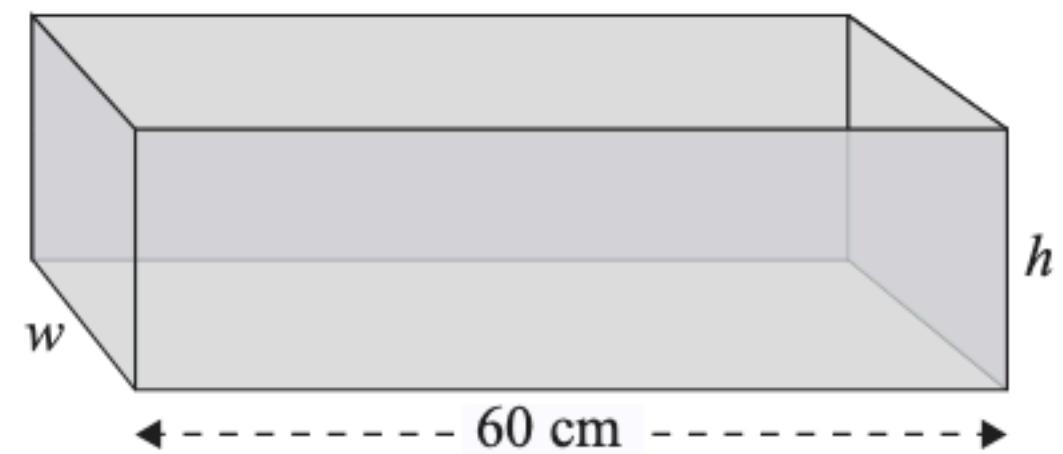
(e) A rectangular box has no lid.

The length of the base is 60 cm.

Its height is one quarter of the sum of its width and length.

The total area of the base **and** the four sides of the box is 7400 cm<sup>2</sup>.

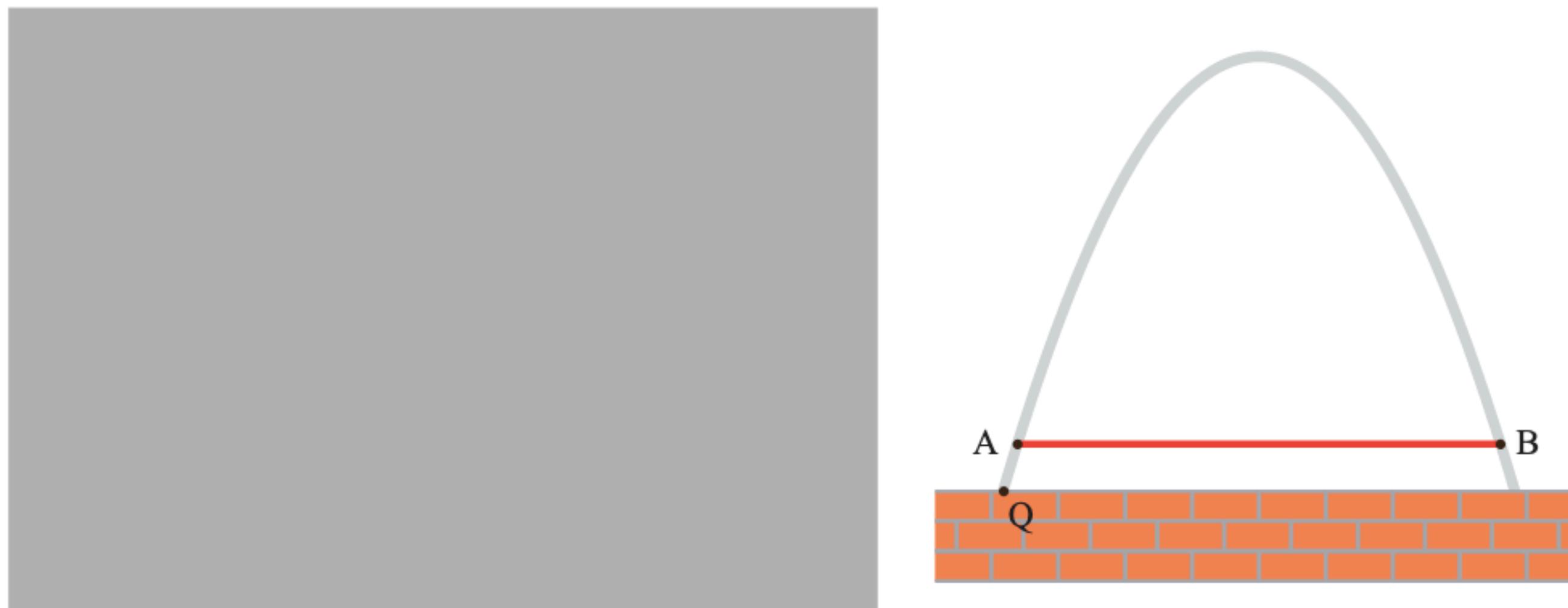
Find the height of the box.



(f)  $(3x + y)(x - 12y) - (2x + y)(x - 16y)$  can be written in the form  $(a + b)^2$ .

Find expressions for  $a$  and  $b$  in terms of  $x$  or  $y$ .

- (d) An equestrian jump has a parabolic arch mounted on a wall. Horses and riders jump through the arch.



Source: <http://luxequestrian.com/slideshow/incredible-jumps-brody-robertson>

The arch rises 2.43 metres **above the wall**.

The arch can be modelled by a function of the form  $h(x) = kx(3.6 - x)$ , where  $k$  is a constant,  $h$  metres is the height above the wall, and  $x$  metres is the horizontal distance from Q.

A rail AB can be placed above the wall and attached at each end to the arch. For one competition, the rail is placed 0.5 metres above the wall.

How long is the rail AB?

- (e) Interest is compounded on a principal investment,  $\$P$ , **at the end of each year**.

If the total amount of the investment after  $n$  years is  $\$A$  then  $A = P \left(1 + \frac{r}{100}\right)^n$

where  $r\%$  is the compound interest rate per year.

- (i) Anushka invests  $\$20\,000$  at an interest rate of  $3.85\%$  (so  $A = P(1.0385)^n$ ).

How many years will it take for her investment to be worth  $\$25\,000$ ?

- (ii) Semisi invests his money at a different interest rate than Anushka's investment.

His investment will double in value after twelve years.

What is the interest rate for Semisi's investment?

- (d) Show that the roots of the equation  $x^2 + 2(k + 1)x - (k^2 + 2k + 5) = 0$ , where  $k$  is a constant, can never be equal for any real number  $k$ .

(c) The polynomial  $p(x) = (2m - 1)x^2 + (m + 1)x + (m - 4)$  can be written as a **perfect square**.

Find the value(s) of  $m$ .

- (d) By factorising, find an expression in terms of  $p$  for the difference between the roots of the equation  $(px)^2 + 4px - 12 = 0$ .

- (e) Use algebra to show that the graph of the function  $y = (x - a)(x - b) - c^2$ , where  $c \neq 0$ , crosses the  $x$ -axis at two distinct points.

- (d) The shape below is divided into rectangles. All measurements are in cm.

This diagram has been corrected from that used in the examination.

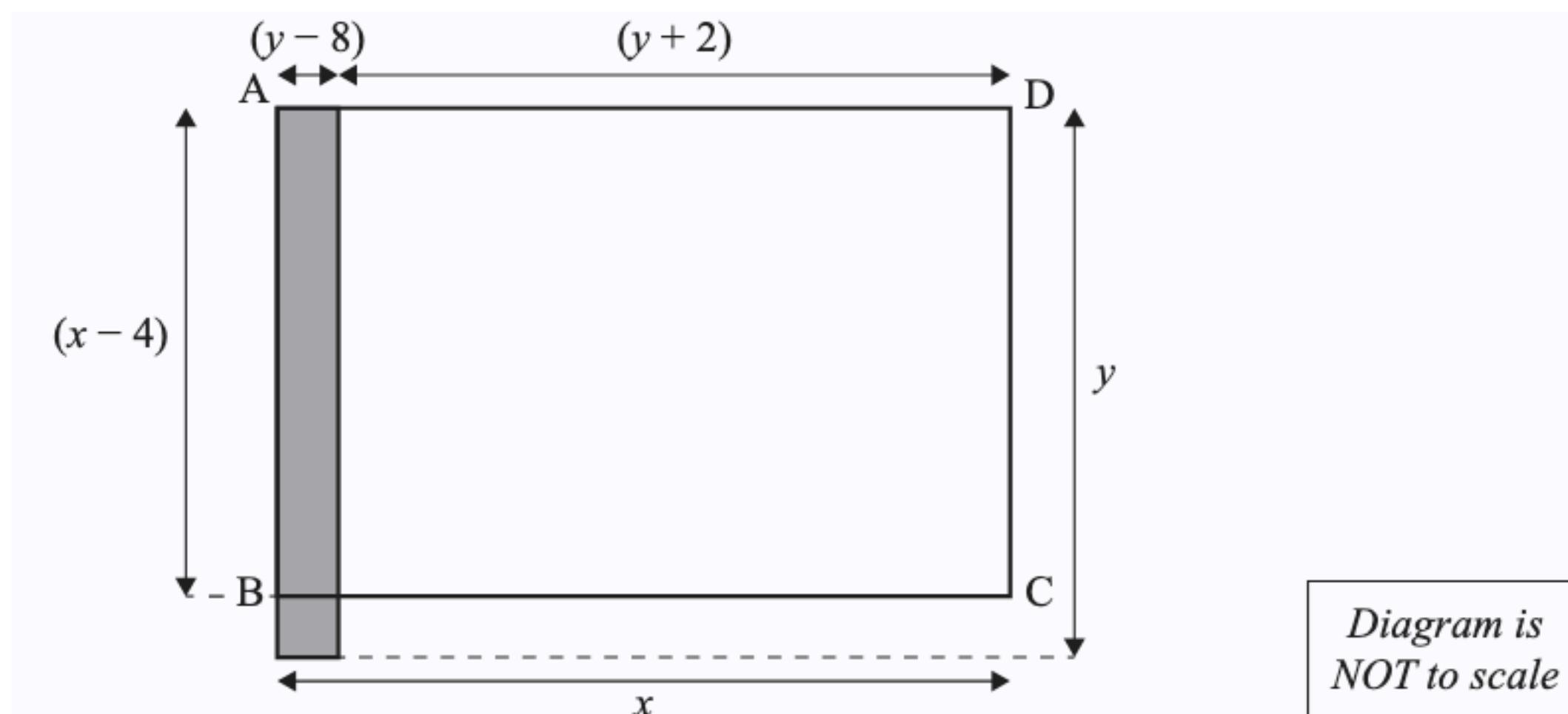


Diagram is  
NOT to scale

The shaded rectangle has an area of  $9 \text{ cm}^2$ .

Find the area of the rectangle ABCD.

(e) One root of the equation  $x^2 + px + q = 0$  is  $n$  times the other, where  $n \neq 0$ .

Show that  $qn^2 + (2q - p^2)n + q = 0$ .

(c) Fully simplify  $\frac{3^{2n-1} + 3^{2n+1}}{3^{2n} - 9^{n-2}}$

- (d) (i) The number of people  $N$  suffering from a contagious virus increases exponentially at a constant rate of 5.3% each week after the virus was initially diagnosed.

If  $N_0$  is the number of people initially diagnosed with the virus, then  $t$  weeks after the virus was initially diagnosed,  $N$  can be modelled by the function  $N = N_0 (1.053)^t$ .

How long will it take for the number of people diagnosed with the virus to be three times the number initially diagnosed?

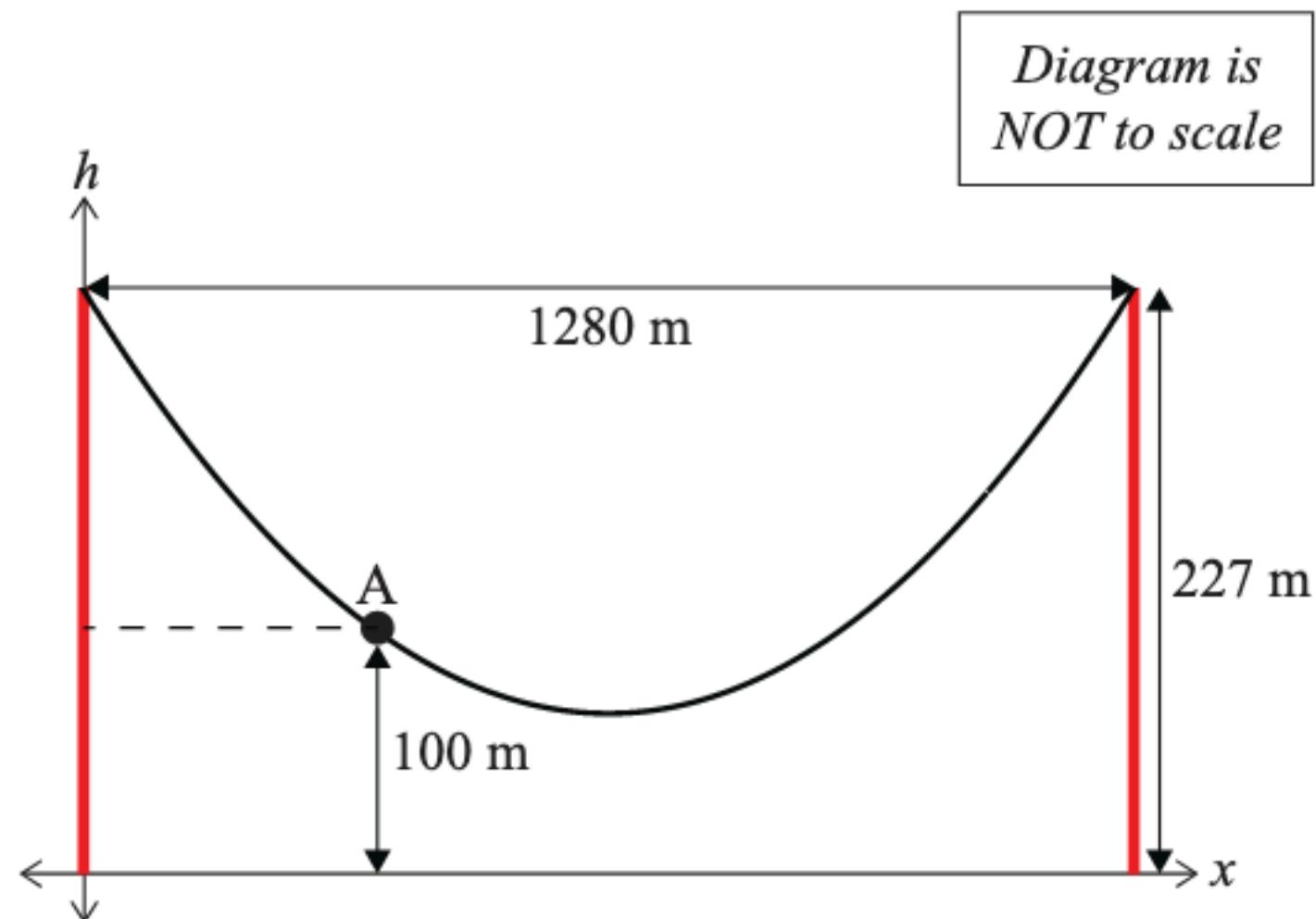
- (ii) The number of people  $N$  suffering from a different virus also increases exponentially at a constant rate of  $r\%$  each week. 2500 people were initially diagnosed with this virus. After 10 weeks, the number of people suffering from this virus had increased to 4250.

Find  $r$ , assuming the form of model in part (i) still applies.

- (e) The Golden Gate Bridge in San Francisco has two towers.

The height  $h$  in metres of the suspension cables above the mean water level, at a horizontal distance  $x$  metres from the base of the left tower, can be modelled by the function  $h = k(x - 640)^2 + 67$ .

At the mid-point between the two towers, the suspension cables are 67 metres above the mean water level. The distance between the towers is 1280 metres and the towers are 227 metres tall, measured from the mean water level.

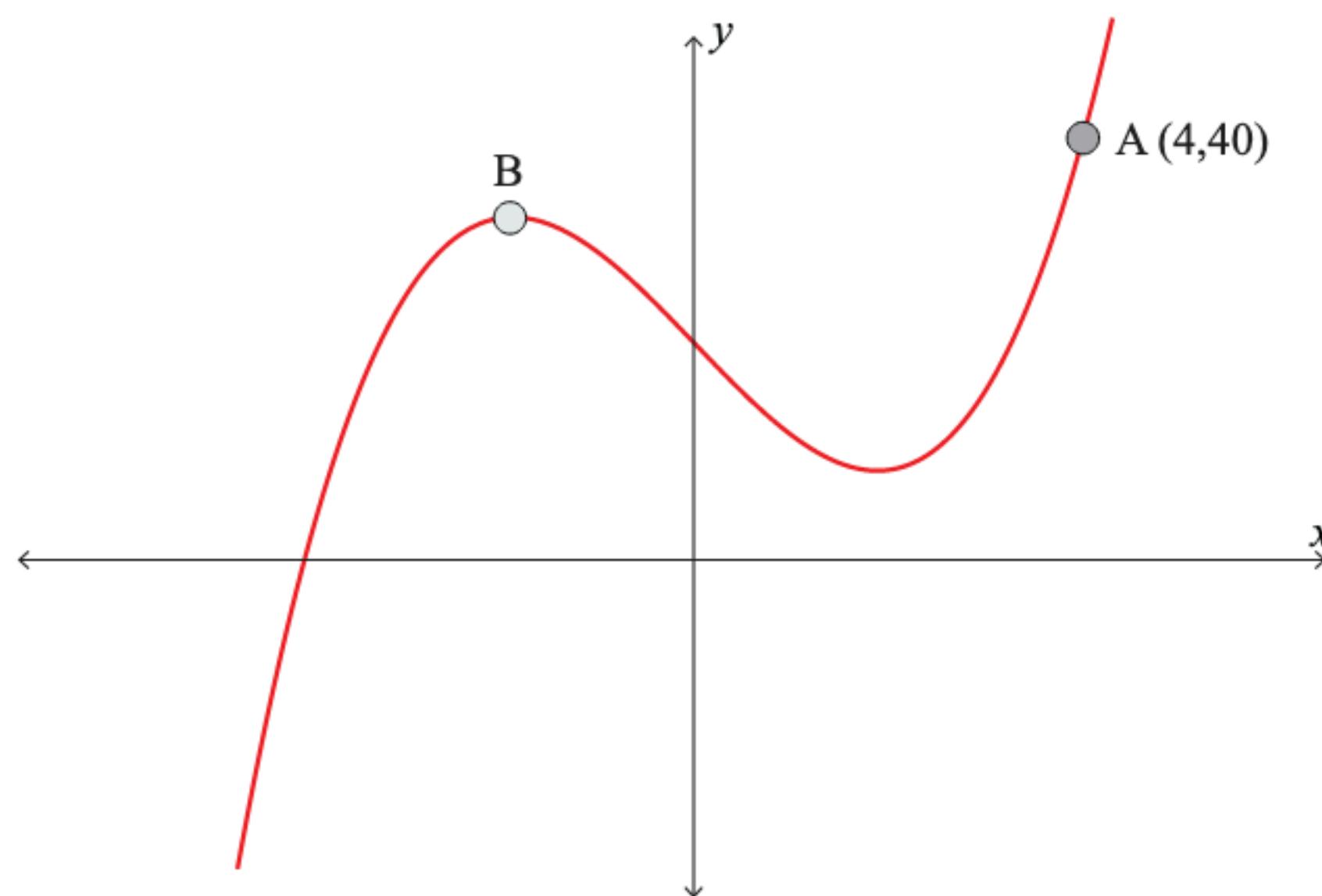


Rich Niewiroski Jr (<https://commons.wikimedia.org/wiki/File:GoldenGateBridge-001.jpg>), CC BY 2.5.

An anemometer (shown as A in the left-hand diagram above) to measure wind speed is placed on a cable at a height of 100 metres above the mean water level.

Find the horizontal distance of the anemometer from the left tower.

- (c) In the 16th century, mathematicians were developing a formula to solve any cubic equation. They used expressions in the form of  $y = x^3 - 12Px + R$ , where  $P$  and  $R$  are positive constants.
- (i) The graph of  $y = x^3 - 12Px + R$ , for some values of  $P$  and  $R$ , passes through the point A (4,40) and is sketched below.



Find an expression for  $P$  in terms of  $R$ .

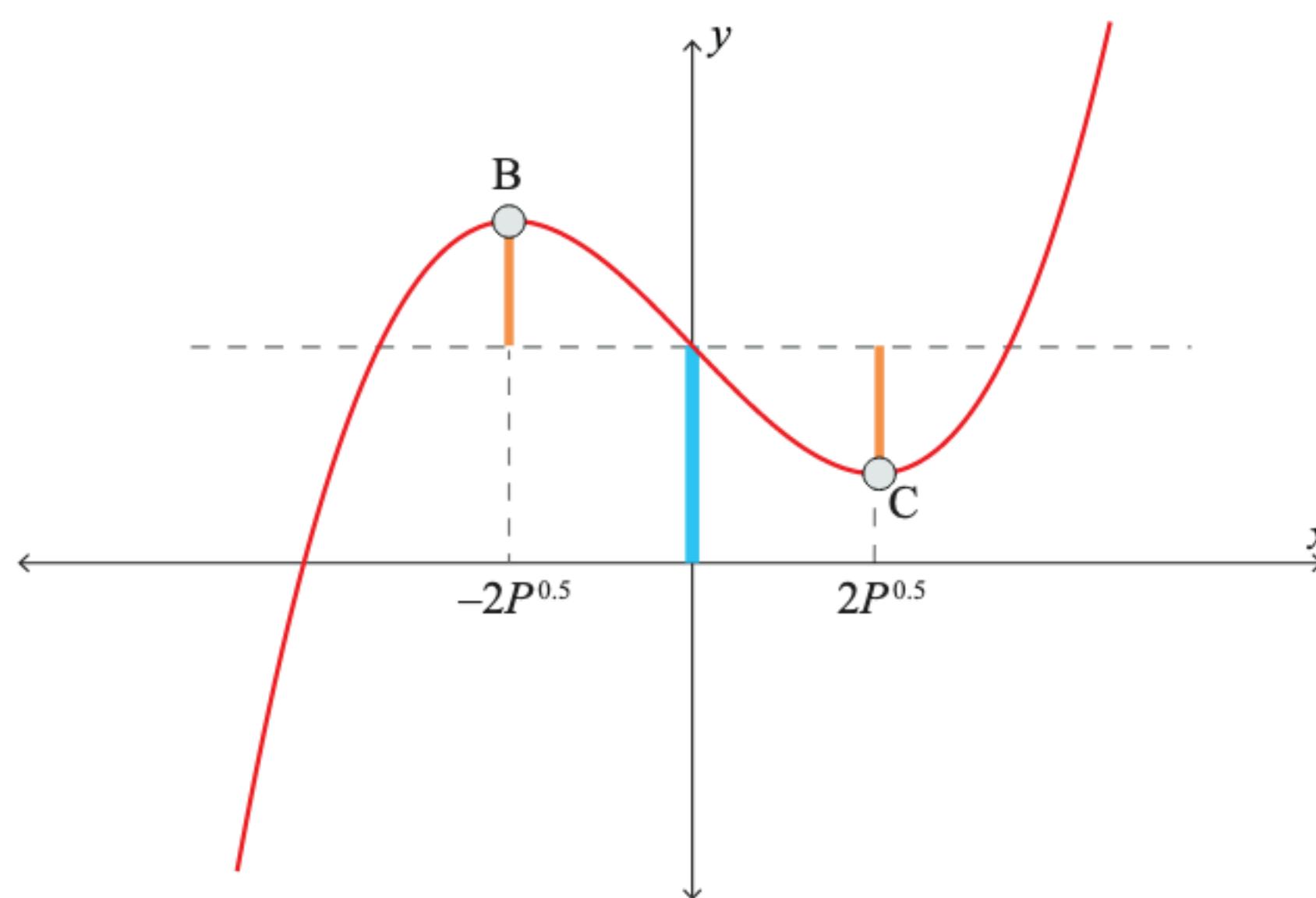
## Continued

(ii) At point B it is true that  $3x^2 - 12P = 0$ .

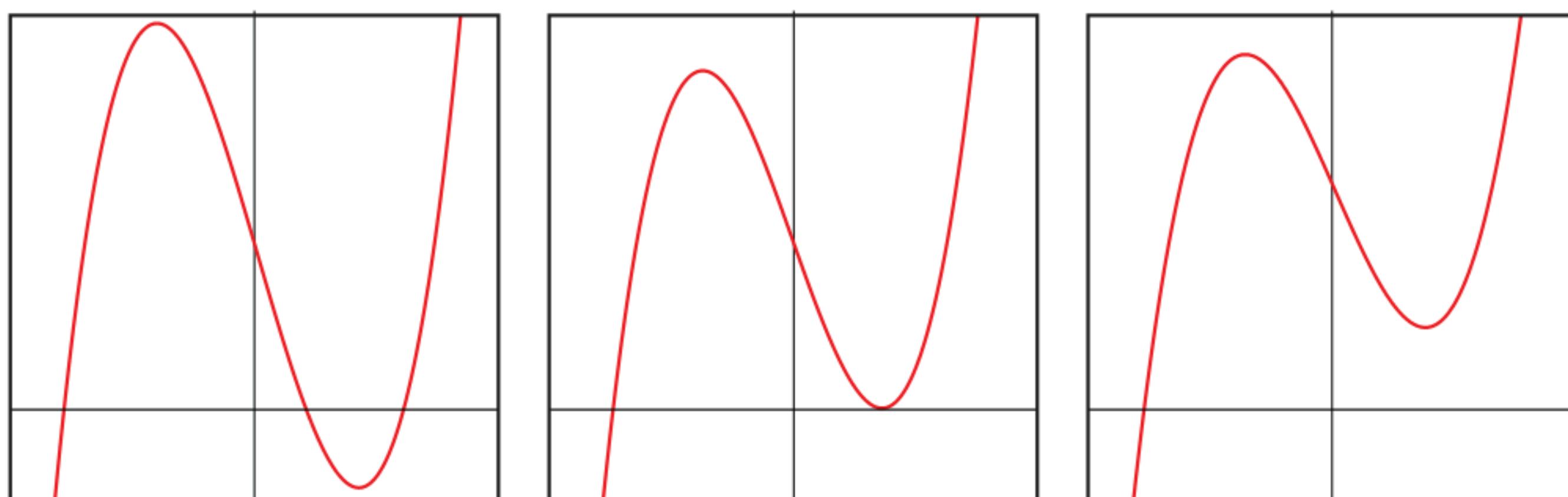
Using algebra, show that  $x = -2P^{0.5}$  at B.

## Continued

- (iii) Consider again the curve with the equation  $y = x^3 - 12Px + R$ . As the values of  $P$  and  $R$  vary, the shape of the curve changes, and the lengths of the orange lines and of the central blue line (below) vary. However, by symmetry, the two orange lines remain the same length as each other.



Some examples of the graphs obtained from various values of  $P$  and  $R$  are illustrated below.



For some combinations of  $P$  and  $R$ , the curve can intersect the  $x$ -axis three times. This will happen if each orange line is longer than the blue line.

Find the condition that  $P$  and  $R$  must satisfy for the graph to have three distinct  $x$ -intercepts.

Write your answer in a form where the exponents of  $P$  and  $R$  are both positive integers.

$$\text{(ii)} \quad \log_5(x) + \log_5(2x) = 4.$$

(d) Consider two parabolas:

- Parabola One given by  $y = ax^2 + bx + c$  and
- Parabola Two given by  $y = dx^2 + ex + f$ , where  $a, b, c, d$ , and  $e$  are constants.

Use algebra to determine the restrictions on the values of  $a, b, c, d$ , and  $e$  that would ensure that the parabolas meet at two distinct points.

- (c) Zahra sells zips. Zahra notices that the higher the price of a zip, the fewer zips are sold. As an experiment, Zahra increases the price of a zip by \$2 each day (starting at \$7) and keeps a record of how many zips are sold each day. She does this for 6 days and finds that the number of zips sold each day started at 98 and is dropping by 3 each day.

The total amount of money Zahra received each day for zips, the turnover, is also recorded in the table below.

Day, $d$	1	2	3	4	5	6
Price of a zip (\$) = $2d + 5$	7	9	11	13	15	17
Number of zips sold = $101 - 3d$	98	95	92	89	86	83
Turnover (\$)	686	855	1012	1157	1290	1411

- (i) If all the patterns continue to be valid, is there any day on which the turnover is exactly \$445? Use algebra to justify your answer and explain your conclusions.

## Continued

- (ii) Zahra realises that not every whole number is a possible turnover value for a given day.

Using algebra, find at least three conditions a whole number  $k$  must satisfy for it to be a possible turnover for a given day.

(d) Consider the following two curves:

$$x^2 = y^2 + 1 \text{ and } y = (x - 1)(x + 1) - 2$$

Find the co-ordinates of each intersection point of the two curves.

- (c) Jessica is investigating a compounding investment. She wants to know how long it would take for an investment of \$1000 to double in value to \$2000. She forms the following equation:

$$2000 = 1000 \left(1 + \frac{R}{100}\right)^D$$

where  $R$  is the rate of return on the investment, as a percentage, and  
 $D$  is the time that the investment would take to double in value, in years.

- (i) If an investment takes 11 years to double in value, what is its rate of return?

## Continued

- (ii) By making  $D$  the subject of the expression:  $2000 = 1000 \left(1 + \frac{R}{100}\right)^D$ ,  
show that  $D = \frac{\log(2)}{\log\left(1 + \frac{R}{100}\right)}$

## Continued

In her research, Jessica comes across a simple but **approximate** rule for calculating  $D$ , the time that the investment would take to double in value. It is commonly called the ‘Rule of 72’, and it states that:

$$D = \frac{72}{R}$$

Jessica wonders how close the values of  $D$  from the ‘Rule of 72’ are to those calculated using the actual expression, which is:

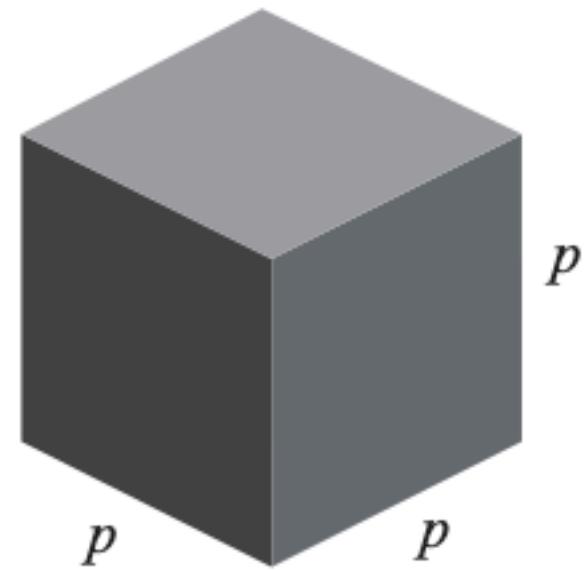
$$D = \frac{\log(2)}{\log\left(1 + \frac{R}{100}\right)}$$

- (iii) Show clearly that the value of  $R$  for which the ‘Rule of 72’ exactly calculates  $D$ , is the solution to the equation:

$$2^R - \left(1 + \frac{R}{100}\right)^{72} = 0$$

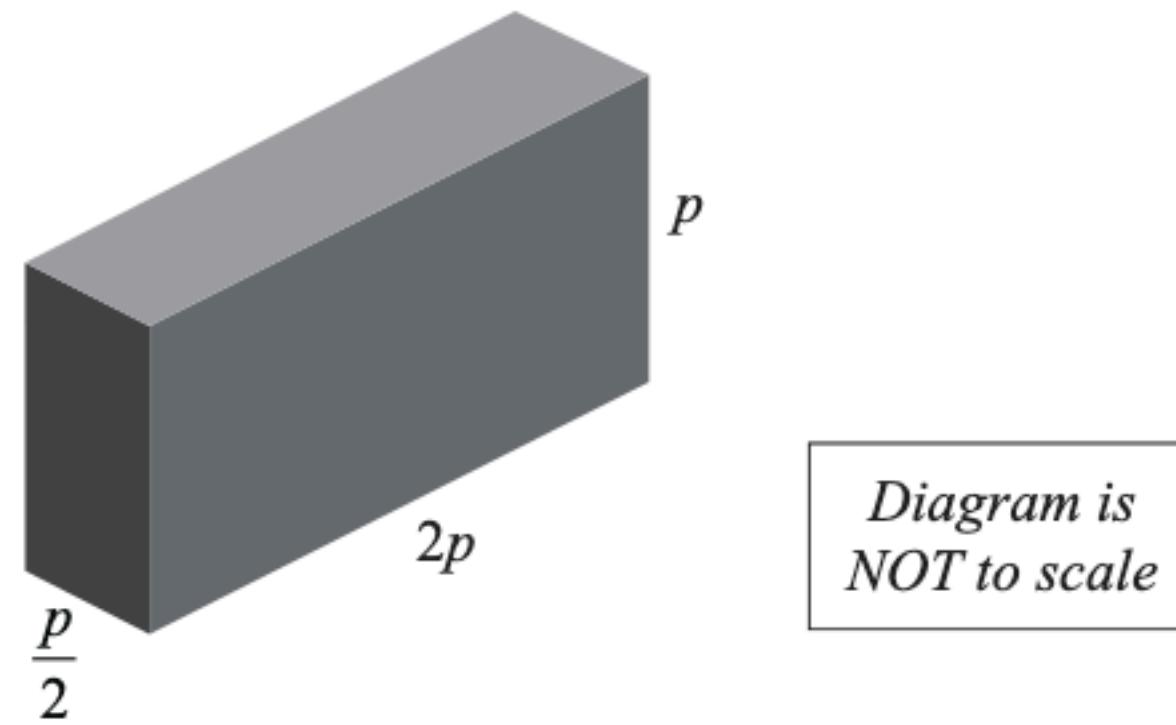
You do not need to solve this equation.

Consider a cube with sides of  $p$  cm (where  $p \neq 0$ ). The volume of the cube would be  $p^3$  cm<sup>3</sup>, and the surface area of the cube would be  $6p^2$  cm<sup>2</sup>.



Junyang wonders if it is possible to change the dimensions of the cube to make a cuboid that still has the same volume.

- (a) First, he tries doubling the length, halving the width, and keeping the height as  $p$ , as sketched below.



Using algebra, find the volume of this cuboid.

## Continued

- (b) The volume of the cuboid below is given by the expression:  $(p - 4)(p + 5)(p - 3)$ .

Expand and simplify this expression.

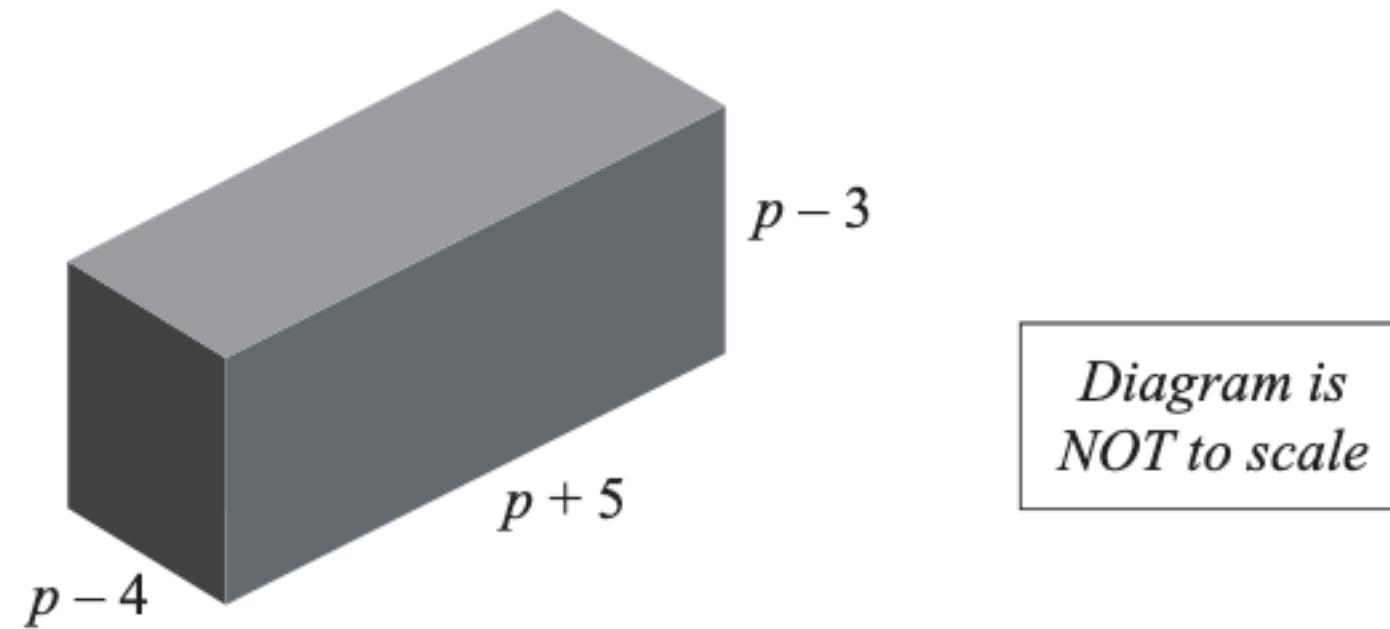
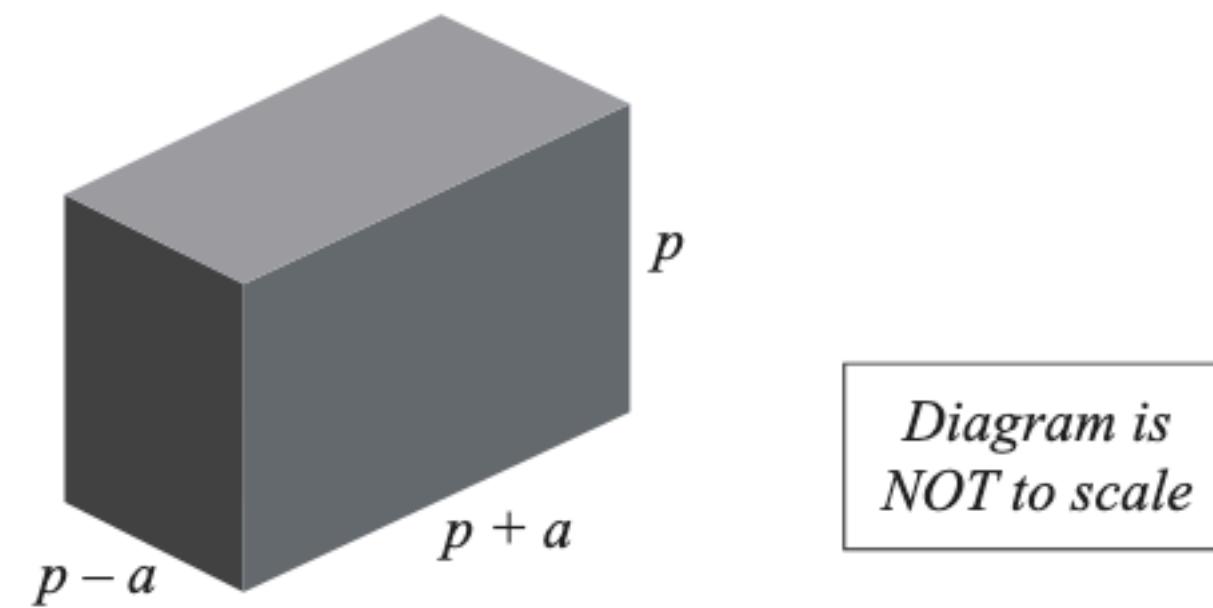


Diagram is  
NOT to scale

## Continued

- (c) Next, Junyang tries adding an amount,  $a$ , to the length and then taking off the same amount from the width, keeping the height the same (see below).

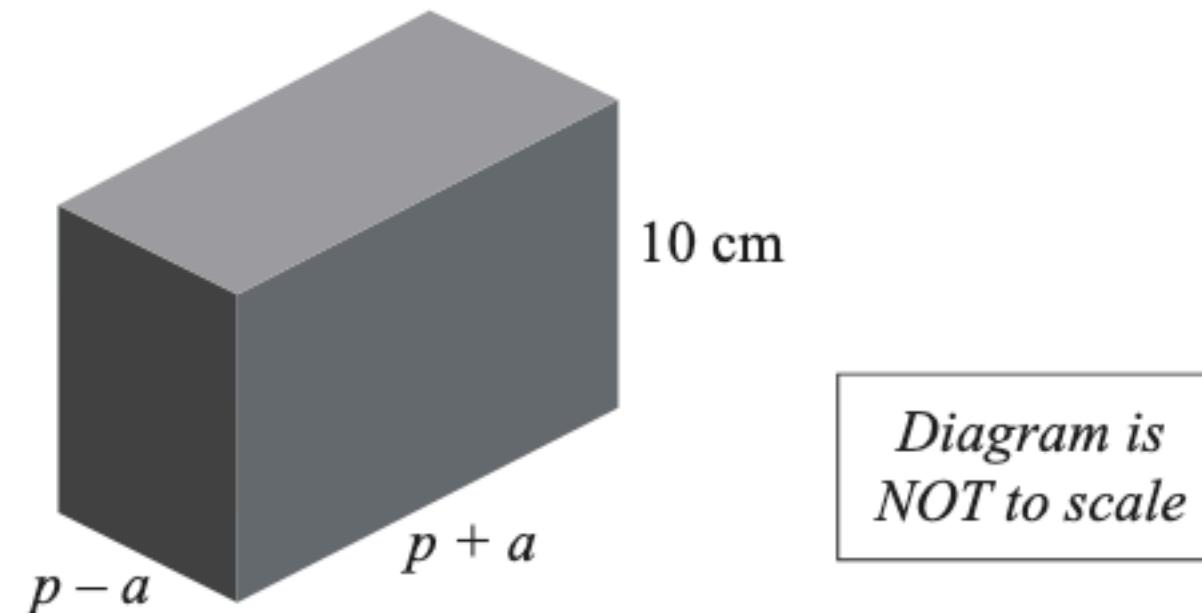


Are there any values of  $a$  for which the volume of the cuboid will be the same as the volume of the cube?

## Continued

- (d) Junyang realises that the **surface area** of the cuboid will not be the same as the surface area of the cube unless he also changes the height. He decides to make the height of the cuboid 10 cm.

He wants to find out which value of  $p$  would result in the cube having the same surface area as the cuboid. To do this, he needs to form and solve an equation for  $p$ .



- (i) If the surface area of the cube is the same as the surface area of the cuboid, show that  $2p^2 - 20p + a^2 = 0$ .

Remember that the surface area of the cube is  $6p^2 \text{ cm}^2$ .

## Continued

- (ii) As mentioned in part (i), if the surface area of the cube is the same as the surface area of the cuboid, then  $2p^2 - 20p + a^2 = 0$ .

By using the discriminant ( $\Delta$ ), find the largest possible **whole** number value that  $a$  could take in this context.

Use this value of  $a$  to find the dimensions of both the cube and the cuboid.

Explain your reasoning clearly.

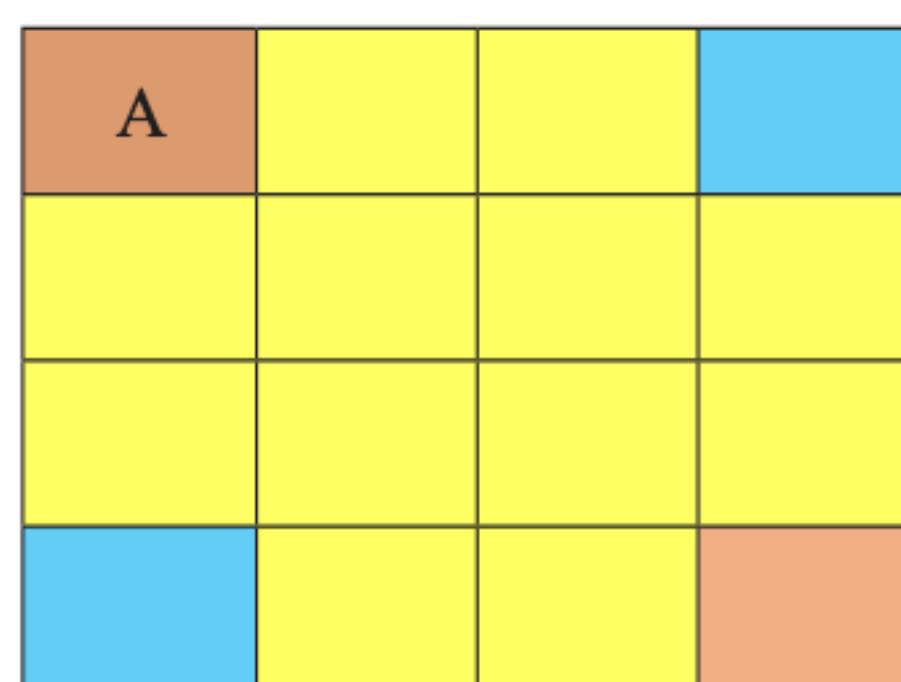
- (c) A calendar can be presented in the following way, where each day is given a number from 1 to 365. This is the beginning of a year's calendar:

M	T	W	TH	F	SA	SU
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
32	33	34	35	36	37	38
39	40	41	42	43	44	45
46	47	48	49	50	51	52
53	54	55	56	57	58	59
60	61	62	63	64	65	66
67	68	69	70	71	72	73

Jo draws a 4-by-4 square on the calendar to check a claim that she heard:

*“the sums of the diagonally opposite corners are always the same, no matter where you make your square”*. In other words, when you add the numbers in the orange corners, it is the same as when you add the numbers in the blue corners.

Jo wonders if the claim will still be true no matter where she starts the square, so she begins an investigation using algebra:



- (i) Use algebra to prove that, no matter where the 4-by-4 square is drawn on the calendar, **the sum of the orange corners must be the same as the sum of the blue corners**.

## Continued

(ii) Jo wonders about the **products** of the diagonally opposite corners: could they be the same?

Use algebra to prove that it is **not** possible to draw any 4-by-4 square for which the products of the diagonally opposite corners are the same.

## Continued

- (d) Will it always be true that the **sum** of the orange corners must be the same as the sum of the blue corners, regardless of the size or shape of the rectangle Jo draws?

Use algebra to support your answer by considering an  $m$ -by- $n$  rectangle drawn on the calendar below (where  $m$  and  $n$  are whole numbers greater than 1, and  $m \neq n$ ).

You may wish to draw a diagram on the calendar, or beside it, to help explain your reasoning.

M	T	W	TH	F	SA	SU
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31
32	33	34	35	36	37	38
39	40	41	42	43	44	45
46	47	48	49	50	51	52
53	54	55	56	57	58	59
60	61	62	63	64	65	66
67	68	69	70	71	72	73

- (d) Suppose  $Q(x) = fx^2 + gx + h$ , where  $f$ ,  $g$ , and  $h$  are constants. The “reciprocal polynomial” of  $Q(x)$  is defined as  $Q^*(x) = hx^2 + gx + f$ , where the coefficients are in the reverse order.
- (i) Find the solutions of the equation  $Q(x) = Q^*(x)$ .

## Continued

(ii) Suppose that  $Q(x) = 0$  has 2 different roots, A and B.

The roots of  $Q^*(x) = 0$  are multiples of A and of B, i.e. the roots are  $kA$  and  $kB$  for some constant  $k$ .

Find an expression for  $k$  in terms of  $f$ ,  $g$ , and/or  $h$ .

(c) Consider the equation  $\log_2(x - a) - \log_2(x + a) = c$ , where  $a$  and  $c$  are constants.

(i) Show that when  $x$  is made the subject of this equation,  $x = a \frac{1+2^c}{1-2^c}$ .

Ensure that you use correct mathematical statements in your reasoning.

- (ii) The equation  $\log_2(x - a) - \log_2(x + a) = c$ , is only possible to solve for some values of  $a$  and for some values of  $c$ .

Explaining your reasoning clearly, describe which values of  $a$  and of  $c$  will make the equation possible to solve.

You may find it useful to recall that, when  $x$  is made the subject of this equation,  $x = a \frac{1+2^c}{1-2^c}$ .

- (c) The value,  $V$ , in dollars (\$), of a laptop can be modelled by  $V = 40 + ke^{-0.5t}$ ,  $t \geq 0$ , where  $t$  is the time in years since the laptop was purchased. The original price of the laptop was \$900.

How long does it take for the laptop's value to be reduced to 50% of the original value?

(d)  $2y = 2x + 29$  is a tangent to the quadratic  $x^2 - 2ky + 32k = 0$ , where  $k$  is a non-zero constant.

Find the value of  $k$  and determine where the quadratic crosses the  $y$ -axis.

(c) Find the value of  $6 + \log_b\left(\frac{1}{b^3}\right) + \log_b(\sqrt{b})$ .

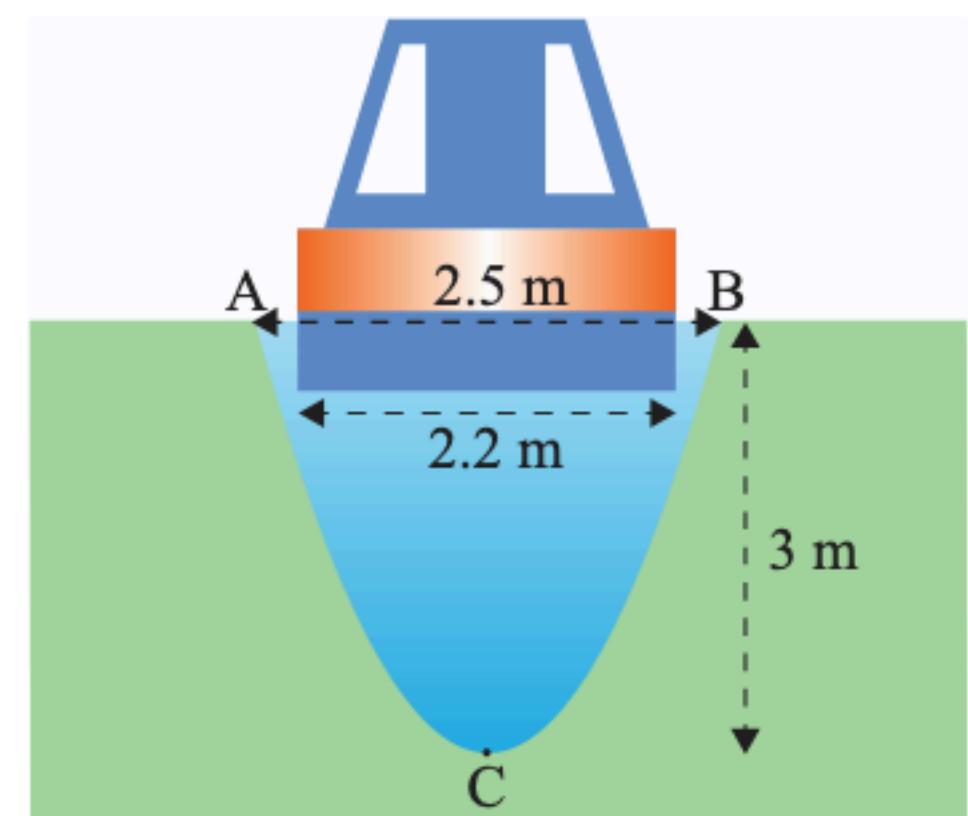
(d) Using an algebraic method, solve  $4^x - \frac{10}{4^x} = 3$ .

- (d) The diagram below shows the cross-section of a canal and narrowboat floating in the canal.
- The surface of the water (between points A and B) measures 2.5 m across.
  - The canal is 3 m deep, at the deepest point C.
  - The cross-section of the canal can be modelled as a quadratic curve ACB.
  - The cross-section of a narrowboat on the canal can be modelled as a rectangle with a width of 2.2 m.
  - The narrowboat must maintain a constant depth below the water of 1 m in order to float.

Will the narrowboat be able to float in this canal?



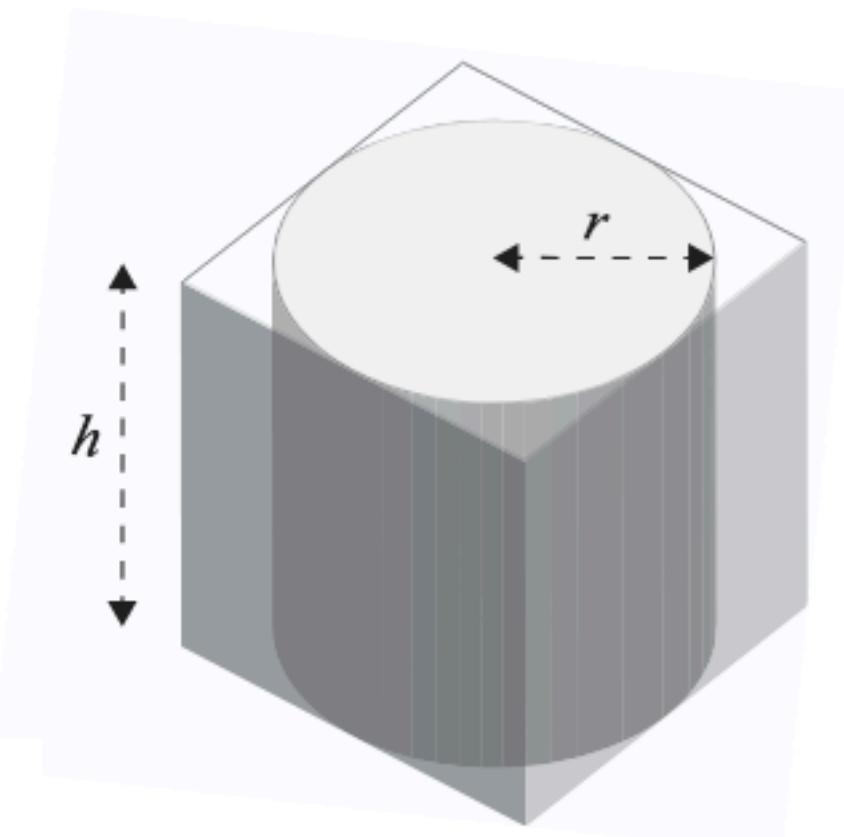
Source: www.pxfuel.com/en/free-photo-xerdj



- (e) An open box (i.e. with a base but no lid) has been designed to tightly fit a cylindrical candle. The surface area of the five surfaces of the box is equal to the total surface area of the candle.

Write an expression for the height,  $h$ , in terms of the radius,  $r$ , and  $\pi$ .

(Surface area of a cylinder =  $2\pi r^2 + 2\pi r h$ .)



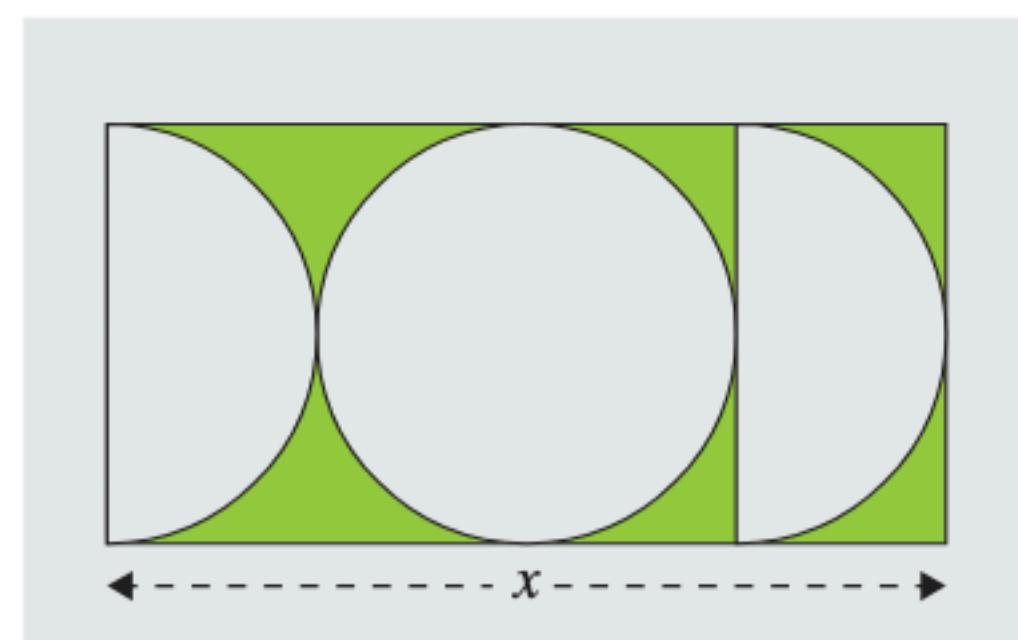
(e) Show that  $\frac{\left(\frac{3}{x^2} + \frac{1}{x^2}\right)\left(\frac{1}{x^2} - \frac{-1}{x^2}\right)}{\left(\frac{3}{x^2} - \frac{1}{x^2}\right)^2}$  can be simplified to  $\frac{x+1}{x(x-1)}$ .

- (c) Find the value of  $k$  for which the equation  $\frac{x^2 - 2x}{4x - 1} = \frac{k - 1}{k + 1}$  has roots numerically equal but opposite signs (for example, 2 and -2).

- (d) The logo for an IT company, *Doctor of Data*, is shown in the diagram to the right.

The green background of the logo is the shape of a rectangle with length  $x$ . The letters D, O, D are formed using semi-circles for both Ds and a circle for the O. Each has the same radius.

$$\text{Area of a circle} = \pi r^2.$$



- (i) Find an expression in terms of  $x$  for the area of the circle in the middle of the logo (i.e. the letter 'O').
- (ii) If the green background has an area of  $10 \text{ cm}^2$ , find the length of the rectangle.

- (d) Consider the equation  $3x^2 - 4kx + k^2 = 0$ , where  $k$  is a constant.

**Using the quadratic formula**, find the fully simplified solutions to the quadratic equation in terms of  $k$ .

- (e) The level of sound (intensity) is measured on a logarithmic scale using a unit called a decibel. The formula for the decibel level,  $d$ , is given by

$$d = 10 \log_{10} \left( \frac{P}{P_0} \right)$$

where  $P$  is the intensity of the sound and  $P_0$  is the weakest sound that the human ear can hear.

A cooling fan has a decibel level of 38 decibels. A heat pump has a decibel level of 30 decibels.

Show that the cooling fan sound intensity is more than six times that of the heat pump sound intensity.