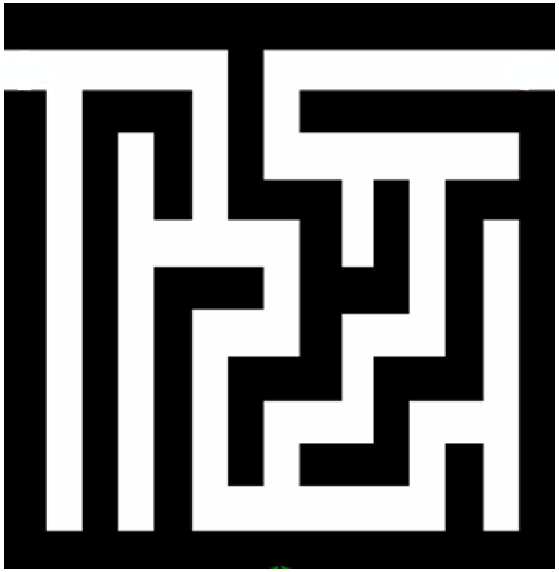
Backtracking

- N Queens
- Subset Sum

Backtracking

- Suppose you have to make a series of *decisions,* among various *choices,* where
  - You don't have enough information to know what to choose
  - Each decision leads to a new set of choices
  - Some sequence of choices (possibly more than one) may be a solution to your problem
- Backtracking is a methodical way of trying out various sequences of decisions, until you find one that "works"

Maze Problem



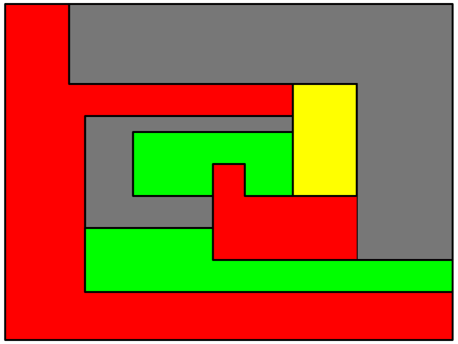
Problem. Given a maze, find a path from start to finish. At each intersection, you have to decide between three or fewer choices:

- Go straight
- Go left
- Go right

- You don't have enough information to choose correctly
- Each choice leads to another set of choices
- One or more sequences of choices may (or may not) lead to a solution

Coloring a Map

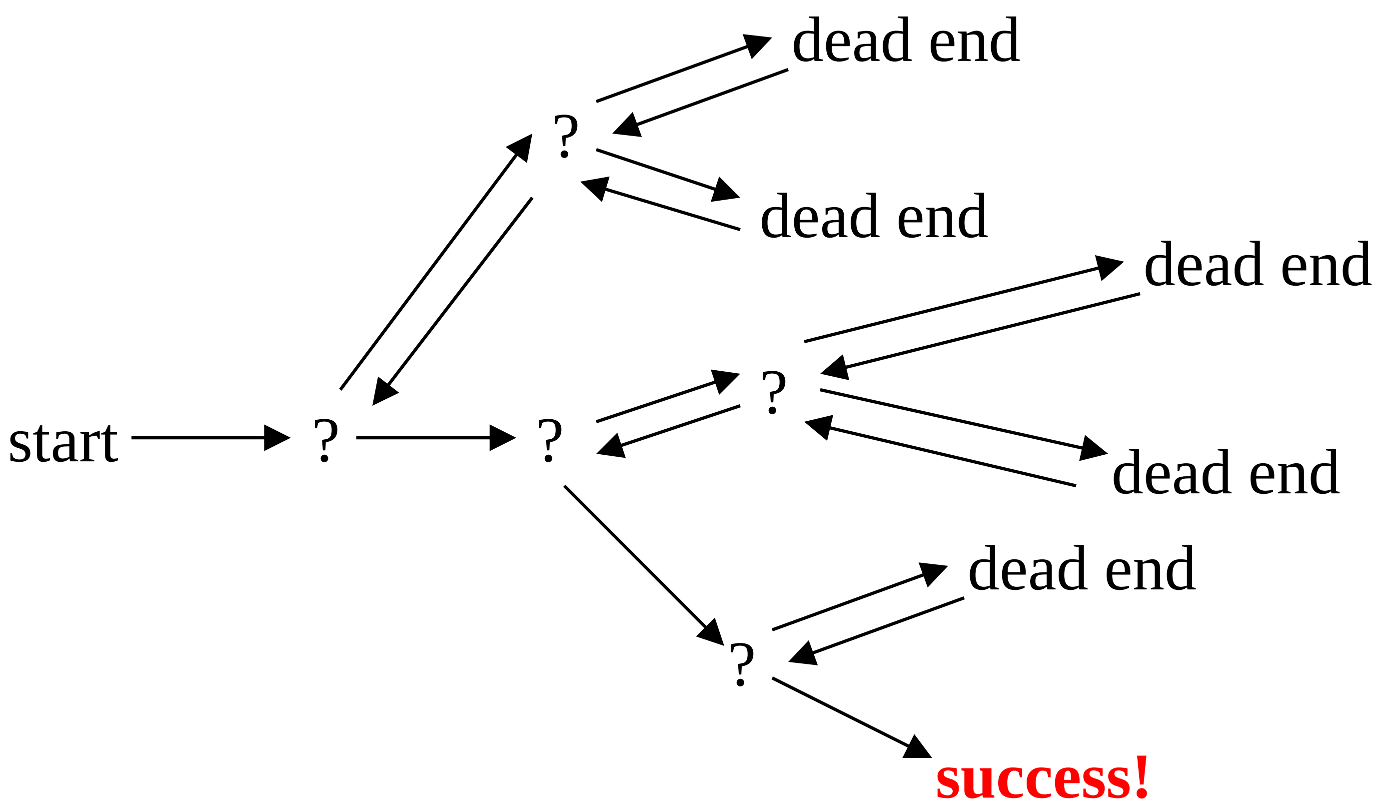
Problem. Color a map with not more than 4 colors so adjacent countries have different colors.



- You don't have enough information to choose colors
- Each choice leads to another set of choices
- One or more sequences of choices may (or may not) lead to a solution

Backtracking (Animation)

Backtracking can be thought of as searching a tree for a particular "goal" leaf node

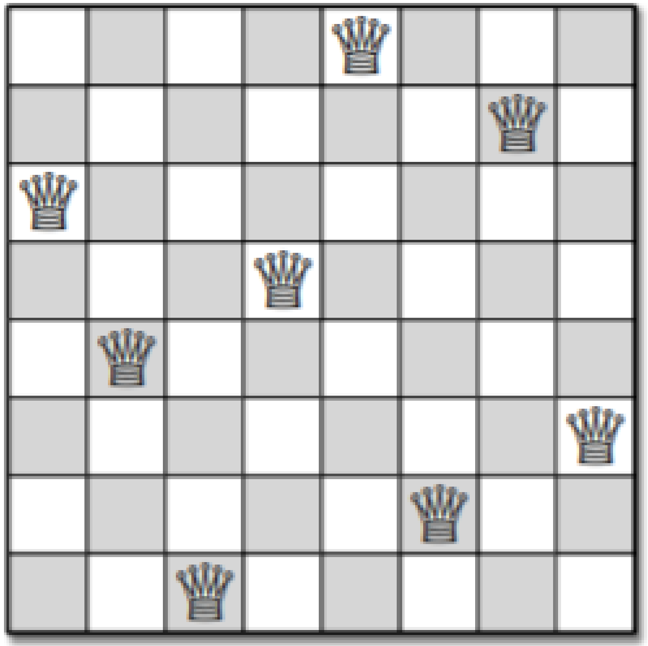


N Queen

N Queens

N Queens.

- The problem is to place n queens on an n X n chessboard, so that no two queens are attacking each other, i.e., no two queens are in the same row, the same column, or the same diagonal.



Backtracking Solution

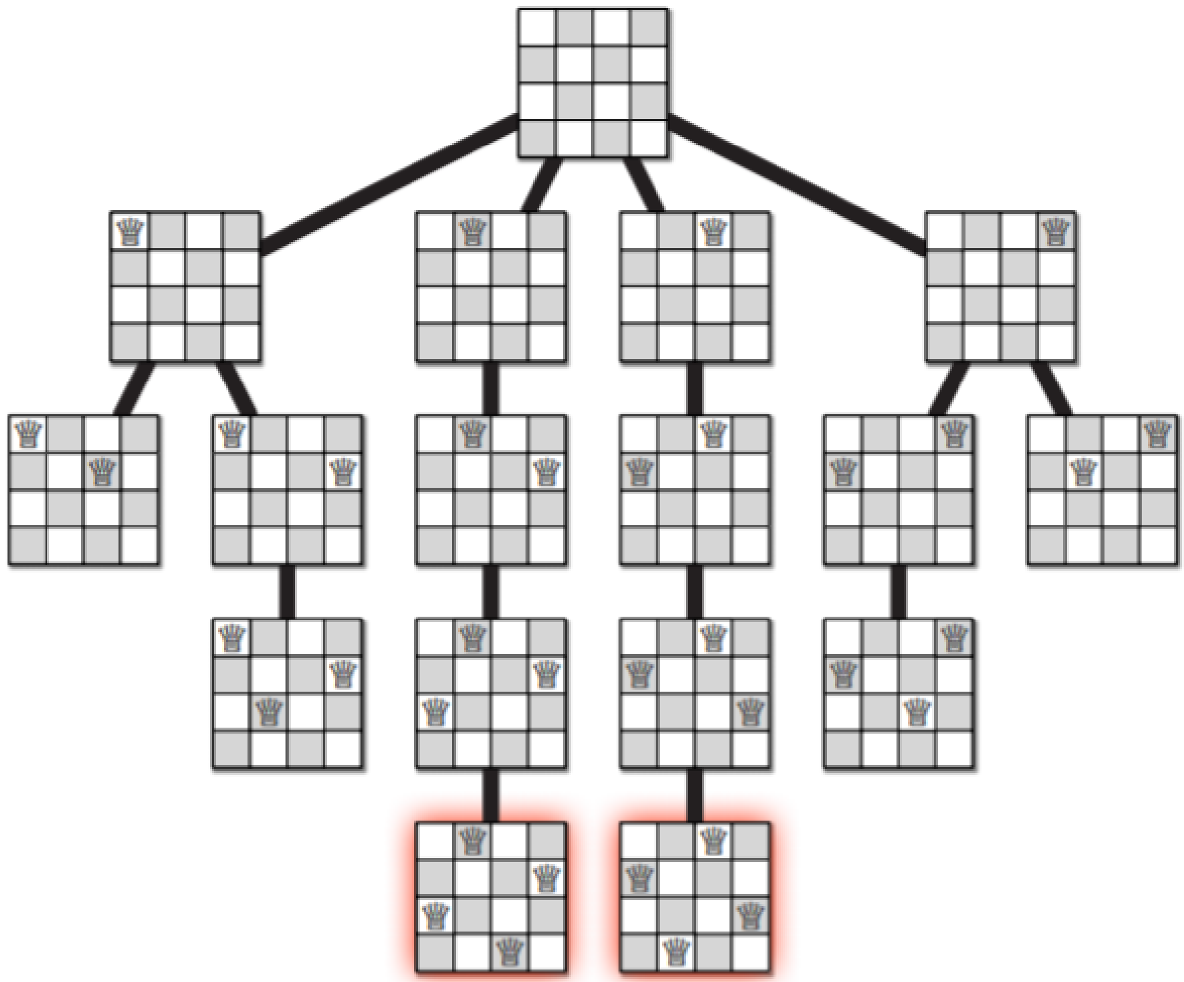
Q[i] indicates which square in row i contains a queen

```
PlaceQueens(Q[1..n],r)

if r = n+1 then
   Print Q[1..n]
else
  for j = 1 to n do
      legal = true
      for i = 1 to r-1 do
         if (Q[i] = j) or (Q[i]=j+r-i) or (Q[i]=j-+i+r) then
            legal = false
         if legal then
            Q[r] = j
            PlaceQueens(Q[1..n],r+1)
```

See here for the demo

Recursion Tree



Subset Sum

Subset Sum

Subset Sum.

- Given a set X of positive integers and target integer T, is there a subset of elements in X that add up to T?

Example.

- If X = {8, 6, 7, 5, 3, 10, 9} and T = 15, the answer is T, because the subsets {8, 7} and {7, 5, 3} and {6, 9} and {5, 10} all sum to 15.
- if X = {11, 6, 5, 1, 7, 13, 12} and T = 15, the answer is false.

Backtracking Solution

```
SubsetSum(X,T)
  if T = 0 then
     return True
  else if (T < 0) or (X is empty) then
     return false
  else
     x = any element of X
     with = SubsetSum(X\{x}, T-x)
     wout = SubsetSum(X \{x}, T)
  return (with or wout)
```

Avoiding passing the whole set (X: a global array)

```
SubsetSum(i,T)
  if T = 0 then
     return True
  else if (T < 0) or (i = 0) then
     return false
  else
     with = SubsetSum(i-1, T-X[i])
     wout = SubsetSum(i-1, T)
  return (with or wout)
```

General Pattern

General Pattern

```
Process at a node u of the recursion tree

if (u is a leaf) and (a solution is found) then
   return solution
else
   for any child v of u (i.e. any possible choice) do
      if no evidence that a solution does not exist at
         the subtree rooted at v then
         process v
backtrack to the parent of u
```

References

References

- Section 2.1 and 2.3 of the text book "algorithms" by Jeff Erikson