4.1 (1) $\vec{v} = -a\omega\sin\omega t\,\hat{i} + b\omega\cos\omega t\,\hat{j}$

$\vec{p} = m\vec{v} = -a\omega m\sin\omega t\,\hat{i} + b\omega m\cos\omega t\,\hat{j}$

(2) $\vec{I} = \vec{p}_2 - \vec{p}_1 = b\omega m\hat{j} - b\omega m\hat{j} = \vec{0}$

4.5. $F\Delta t_1 = (m_1+m_2)v_1$

$F\Delta t_2 = m_2v_2 - m_2v_1$

$\therefore v_1 = \frac{F\Delta t_1}{m_1+m_2}$

$v_2 = \frac{F\Delta t_2}{m_2} + \frac{F\Delta t_1}{m_1+m_2}$

4.6. (1) a最大时，F最大，应为外力刚撤去时

$k\Delta l = (m_1+m_2)a_C$

$\therefore a_C = \frac{k\Delta l}{m_1+m_2}$

(2) 当A刚脱离墙面时质心速度最大

$a_B = \frac{k(\Delta l - x)}{m_2}$

即 $\frac{dv_B}{dt} = \frac{k(\Delta l - x)}{m_2}$，$\frac{dv_B}{dx}\cdot\frac{dx}{dt} = \frac{k(\Delta l - x)}{m_2}$

即 $\frac{dv_B}{dx}v_B = \frac{k(\Delta l - x)}{m_2}$，~~$dv_B$~~ $\int_0^{v_2} v_B\,dv_B = \int_0^{\Delta l}\frac{k(\Delta l - x)}{m_2}dx$

故 $v_2 = \sqrt{\frac{k}{m_2}}\Delta l$

又 $(m_1+m_2)x_C = m_1x_1 + m_2x_2$

有 $(m_1+m_2)v_C = m_1\times 0 + m_2v_2$

$\therefore v_C = \frac{m_2}{m_1+m_2}\sqrt{\frac{k}{m_2}}\Delta l$

4.7. $0-q_m\Delta t\, v_1 = F_\perp \Delta t$

$q_m\Delta t\, v_2 - 0 = F_{/\!/}\Delta t$

$\therefore F_\perp = \cancel{80}\,kgm/s^2$, $F_{/\!/} = 40\ kgm/s^2$

故 $F = \sqrt{F_\perp^2 + F_{/\!/}^2} = 40\sqrt{5}\ N.$

$\tan\theta = \dfrac{|F_\perp|}{|F_{/\!/}|} = 2$. 即与B传送带成 arctan2 向左上

4.9. 以水平向左为正方向:

$\because m_1v_1 + m_2v_2 = 0$

$\therefore m_1x_1 + m_2x_2 = 0.$

故有 $m_1x + m_2(l\cos30^\circ - l\cos60^\circ + x) = 0$

$\therefore x = \dfrac{m_2 l(\cos60^\circ - \cos30^\circ)}{m_1+m_2} = -\dfrac{4\sqrt{3}-4}{11}m \approx -0.27m$

即右移 0.27m.

4.10 (1) $m_1v_0 = (m_1+m_2)v_1$

$\frac{1}{2}m_1v_0^2 = \frac{1}{2}(m_1+m_2)v_1^2 + m_2gh$

$\therefore h = \dfrac{m_1v_0^2}{2(m_1+m_2)g}$

(2) $m_1v_0 = m_2v_2' + m_1v_1'$

$\frac{1}{2}m_1v_0^2 = \frac{1}{2}m_2v_2'^2 + \frac{1}{2}m_1v_1'^2$

$\therefore v_1' = \dfrac{m_1-m_2}{m_1+m_2}v_0$

$v_2' = \dfrac{2m_1}{m_1+m_2}v_0$

系列化.

4.2 (1) $\vec{I} = \Delta\vec{p} = m\Delta\vec{v} = 0$

(2) $m\vec{g}\frac{2\pi}{\omega} + \vec{I}_T = 0$

$\therefore \vec{I}_T = -m\vec{g}\frac{2\pi}{\omega}$

大小：$mg\frac{2\pi}{\omega}$. 方向：竖直向上

4.8. x方向

$mv = m_A v_{Ax} + m_B v_{Bx} + m_C v_{Cx}$

y方向

$0 = m_A v_{Ay} + m_B v_{By} + m_C v_{Cy}$.

$m = m_A + m_B + m_C$

$m_A = m_B = 3m_C$

$v_{Ax} t = x_A$, $v_{Bx} t = x_B$, $v_{Cx} t = x_C$

$v_{Ay} t = y_A$, $v_{By} t = y_B$, $v_{Cy} t = y_C$

$\therefore x_B = 7m$. $y_B = 3m$.

4.9. (1) $T = \int_r^L \frac{m}{L} dr\, \omega^2 r = \frac{m\omega^2}{2L}(L^2 - r^2)$

(2) ∵质量均匀分布

∴ r以外绳的质心即几何中心在 $\frac{r+L}{2}$ 处

$T = \frac{m}{L}(L-r)\omega^2\frac{L+r}{2} = \frac{m\omega^2}{2L}(L^2-r^2)$

4、10. $(m_1+m_2)g = m_c a_c$

$m_c a_c = m_1 a_1 + m_2 a_2$

$a_2 = 0$

$\therefore a_1 = \frac{m_1+m_2}{m_1} g.$

4.12. $mgR = \frac{1}{2}mv_1^2 + \frac{1}{2}m'v_2^2$

$mv_1 = m'v_2$

$\therefore v_1 = \sqrt{\frac{2m'gR}{m'+m}}$.

$v_2 = \frac{m}{m'}\sqrt{\frac{2m'gR}{m'+m}}$

4.13. $m_1 g = kx_1$

$m_2 g h = \frac{1}{2}m_2 v^2$

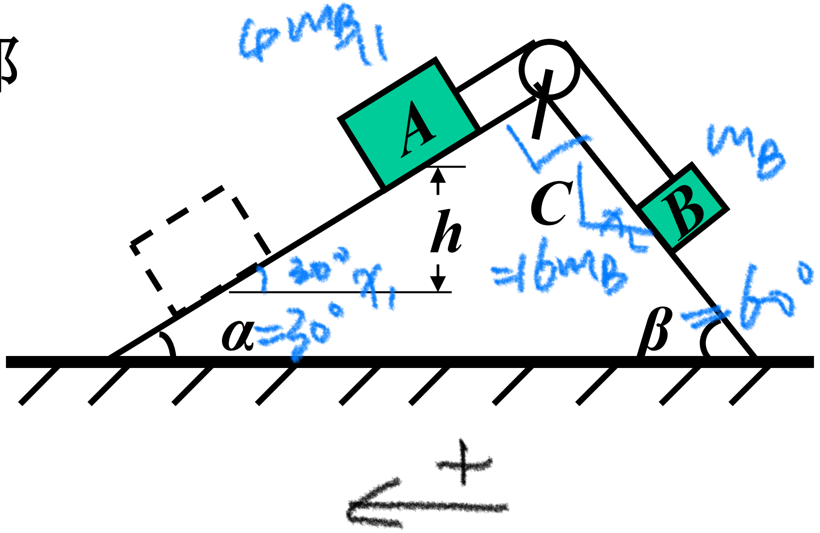
$m_2 v = (m_1+m_2)v_1$

$\frac{1}{2}(m_1+m_2)v_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}kx_2^2 + [-(m_1+m_2)g(x_2-x_1)]$

$\therefore x_2 = \cancel{0.2m}\ 0.4m$

$\Delta x = x_2 - x_1 = 0.3m$

补1、三棱体 C、滑块 A、B，开始都静止，各面均光滑。已知 $m_C=4m_A=16m_B$， $\alpha=30^0$，$\beta=60^0$。求A下降 h=10cm时三棱体 C 在水平方向的位移。



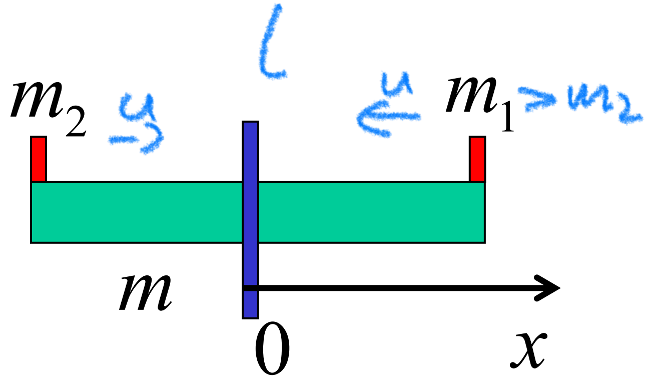
$$\frac{h}{\sin 30°}=\frac{h_B}{\sin 60°}$$

以向左为正.

$$m_A\left(\frac{h}{\tan 30°}+x_C\right)+m_B\left(\frac{h_B}{\tan 60°}+x_C\right)+m_C x_C=0.$$

$$\therefore x_C=-\frac{40\sqrt{3}+10}{21}\ m$$

补2、质量为 m，长度为 l的小船静浮于河中，小船的两头分别站着质量为 m_1和 m_2的两个人（$m_1>m_2$）,他们同时以相同的速率u走向原位于船中、但固定于河中的木桩，如图所示。如果忽略水对船的阻力作用，问（1）谁先走到木桩处？（2）他用了多少时间？



解：(1) 以向右为正

$m_2(u+v)+mv+m_1(-u+v)=0$.

∴ $v=\frac{m_1-m_2}{m_1+m_2+m}u$

∵ $m_1>m_2$ ∴ $v>0$. 故船向右.

原先两人距柱均为$\frac{l}{2}$，而 m_2速度为 $u+v$，m_1速度为 $u-v$ (向左).

故 m_2速度比 m_1大，m_2先走到木桩处.

(2) $t=\dfrac{\frac{l}{2}}{u+\frac{m_1-m_2}{m_1+m_2+m}u}=\dfrac{m_1+m_2+m}{2(2m_1+m)}\dfrac{l}{u}$

附加题（选做，最多加2分）

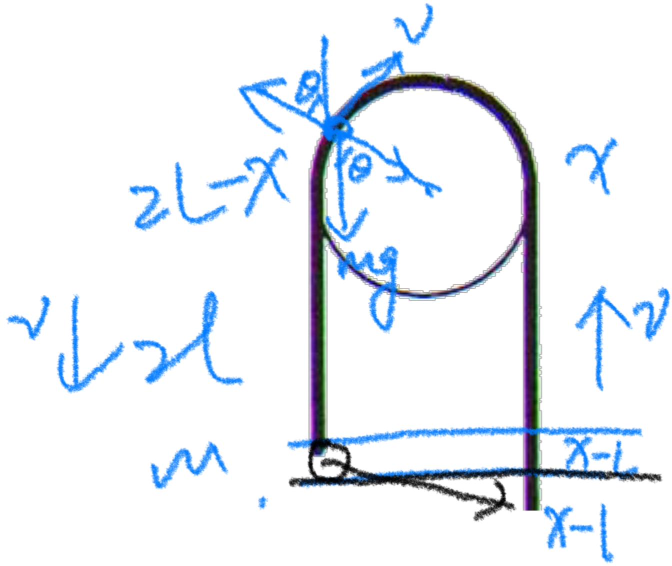
1、一条长为 $2l$，质量为 m 的柔软细绳，挂在一光滑的水平轴钉(粗细可忽略)上。当两边的绳长均为 l 时，绳索处于平衡状态。若给其一端加一个竖直方向的微小扰动，则细绳就从轴钉上滑落，不考虑过程中系统机械能的损耗。当较长的一边细绳的长度为 x 时（$l<x<=2l$），试求：①细绳的速度与加速度；②轴钉上所受的力，并对结果的合理性进行必要的讨论。

解：(1) $\frac{m}{2l}\cdot xg-\frac{m}{2l}(2l-x)g=ma$

$\therefore a=\frac{x-l}{l}g$

$\frac{m}{2l}(x-l)g(x-l)=\frac{1}{2}mv^2$

$\therefore v=(x-l)\sqrt{\frac{g}{l}}$



(2) $(mg-F)dt = \frac{m}{2l}(x+dx)(x+dx-l)\sqrt{\frac{g}{l}} - \frac{m}{2l}(2l-x-dx)(x+dx-l)\sqrt{\frac{g}{l}} - \left[\frac{m}{2l}x(x-l)\sqrt{\frac{g}{l}} - \frac{m}{2l}(2l-x)(x-l)\sqrt{\frac{g}{l}}\right]$

$\therefore F = \frac{mg}{l^2}(4xl-2x^2-l^2)$

由牛三： $F' = -F = \frac{mg}{l^2}(2x^2+l^2-4xl)$