

Climate Data Forecasting - Atmospheric CO_2 Concentration / Temperature / Precipitation

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1 Forecasting of Davos - Temperature Climate Analysis

1.1 Stationarity and differencing

Stationary time series is one whose properties do not depend on the time at which the series is observed. Thus, time series with trends, or with seasonality, are not stationary — the trend and seasonality will affect the value of the time series at different times. On the other hand, a white noise series is stationary — it does not matter when you observe it, it should look much the same at any point in time.

Stationary time series will have no predictable patterns in the long-term. Time plots will show the series to be roughly horizontal (although some cyclic behaviour is possible), with constant variance.

If Time Series data with seasonality are non-stationary

- => first take a seasonal difference
- if seasonally differenced data appear are still non-stationary
- => take an additional first seasonal difference

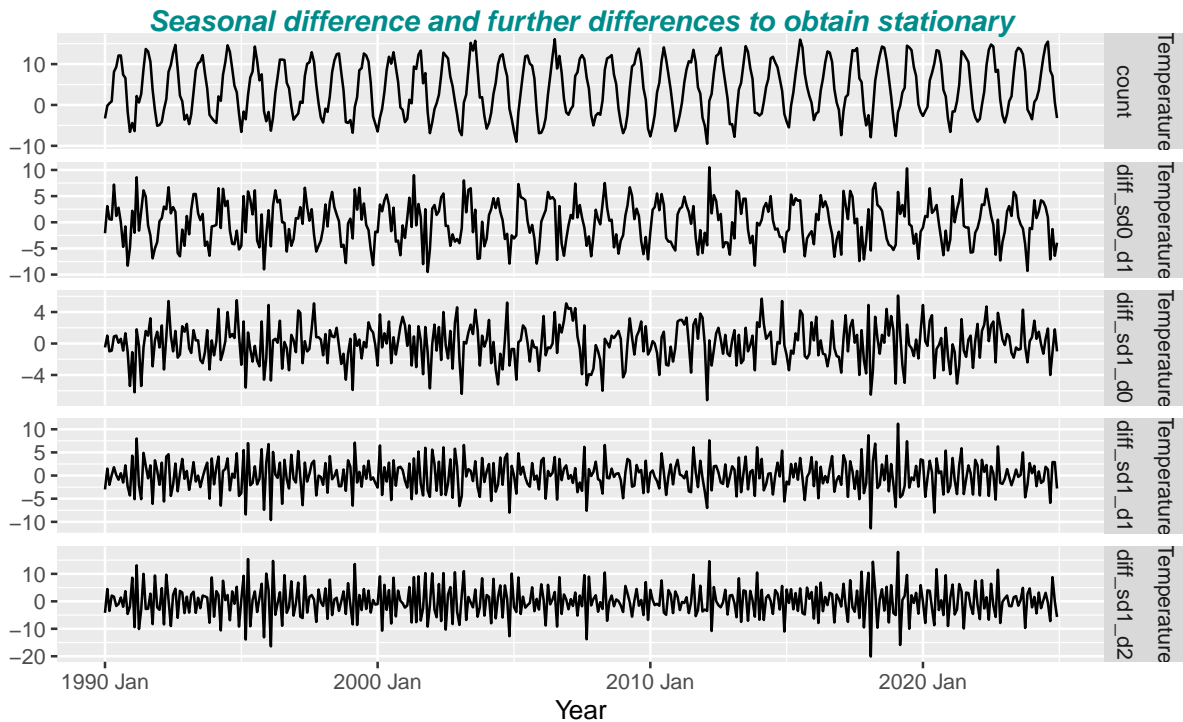
The model fit residuals have to be stationary. For good forecasting this has to be verified with residual diagnostics.

Essential:

- Residuals are uncorrelated
- The residuals have zero mean

Useful (but not necessary):

- The residuals have constant variance.
- The residuals are normally distributed.



1.1.1 Ljung-Box Test - independence/white noise of the time series

The Ljung-Box Test becomes important when checking independence/white noise of the forecasts residuals of the fitted ETS resp. ARIMA models. There we have to check whether the forecast errors are normally distributed with mean zero

Null Hypothesis of independence/white noise in a given time series

=> H_0 to be rejected for $p < \alpha = 0.05$

=> data in the given time series are dependent

=> even differenced data are dependent if $p < \alpha = 0.05$

=> independence/white noise of residuals of fitted models to be verified

```
#> Ljung-Box test with (count), w/o differences
#> # A tibble: 1 x 3
#>   Measure    lb_stat lb_pvalue
#>   <fct>      <dbl>    <dbl>
#> 1 Temperature 6785.      0
#> Ljung-Box test on (difference(count, 12))
#> # A tibble: 1 x 3
#>   Measure    lb_stat lb_pvalue
#>   <fct>      <dbl>    <dbl>
#> 1 Temperature 25.3    0.00487
#> Ljung-Box test on (difference(count, 12) + difference())
#> # A tibble: 1 x 3
#>   Measure    lb_stat lb_pvalue
#>   <fct>      <dbl>    <dbl>
#> 1 Temperature 419.     0
```

1.1.2 Unitroot KPSS Test - fix number of seasonal differences/differences required

kpss test of stationary

Null Hypothesis of stationary in a given time series

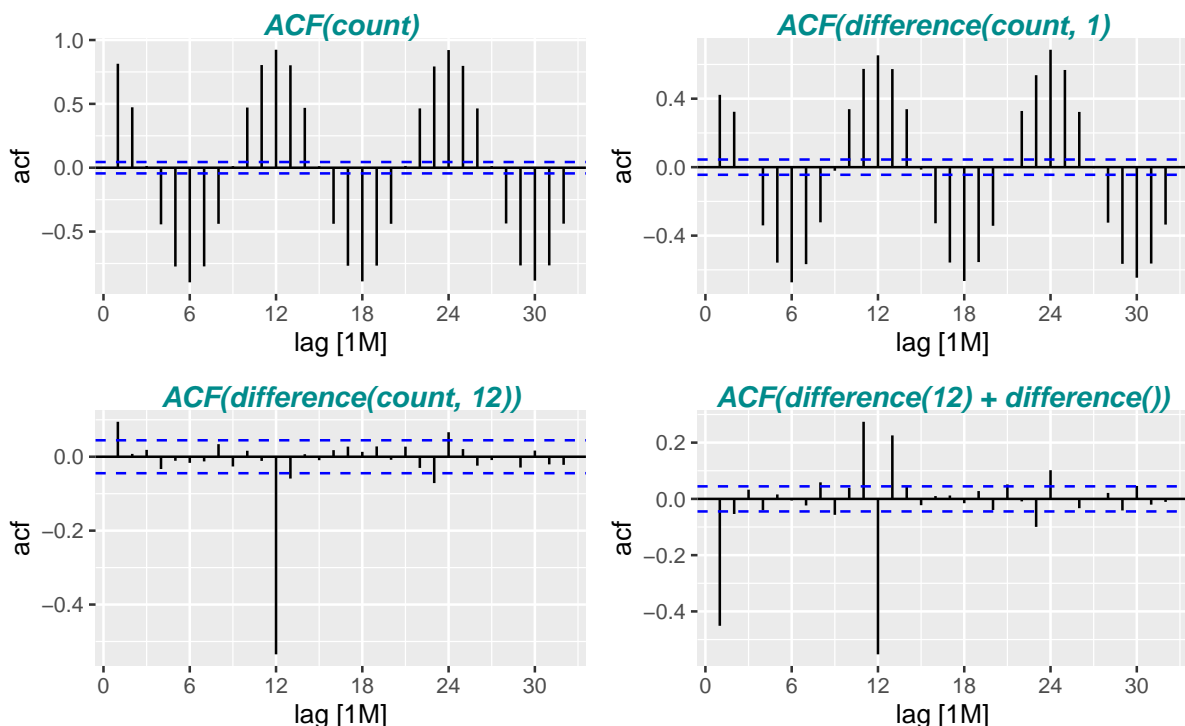
=> H_0 to be rejected for $p < \alpha = 0.05$

unitroot_nsdiffs/ndiff provides minimum number of seasonal differences/differences required for a stationary series. First fix required seasonal differences and then apply ndiffs to the seasonally differenced data.

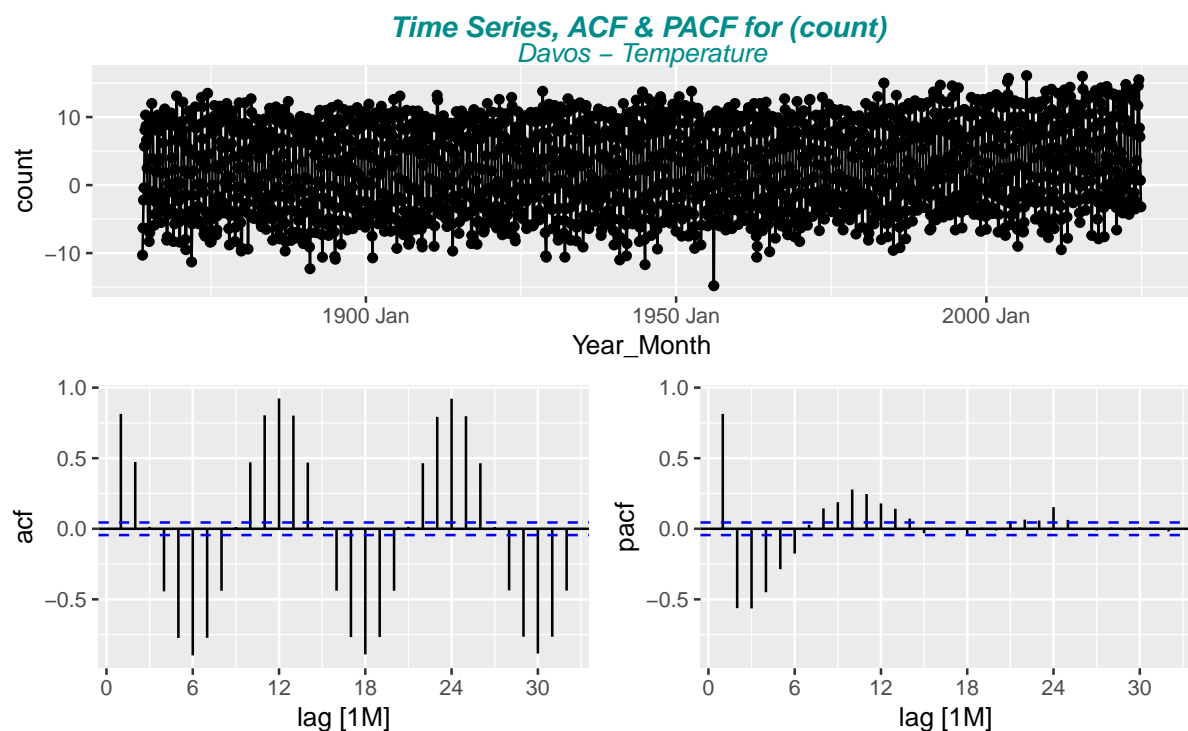
- returns 1 => for stationarity one seasonal difference resp. difference is required

```
#> ndiffs gives the number of differences required resp.
#> nsdiffs gives the number of seasonal differences required to make
#> a series stationary (test is based on the KPSS test
#> kpss test, nsdiffs & ndiffs on (count), w/o differences
#> # A tibble: 1 x 5
#>   Measure      kpss_stat kpss_pvalue nsdiffs ndiffs
#>   <fct>         <dbl>         <dbl>   <int>  <int>
#> 1 Temperature      2.55          0.01       1      1
#> kpss test, nsdiffs & ndiffs on (difference(count, 12))
#> # A tibble: 1 x 5
#>   Measure      kpss_stat kpss_pvalue nsdiffs ndiffs
#>   <fct>         <dbl>         <dbl>   <int>  <int>
#> 1 Temperature      0.0115          0.1       0      0
#> kpss test, nsdiffs & ndiffs on (difference(count, 12) %>% difference(1))
#> # A tibble: 1 x 5
#>   Measure      kpss_stat kpss_pvalue nsdiffs ndiffs
#>   <fct>         <dbl>         <dbl>   <int>  <int>
#> 1 Temperature      0.00395          0.1       0      0
```

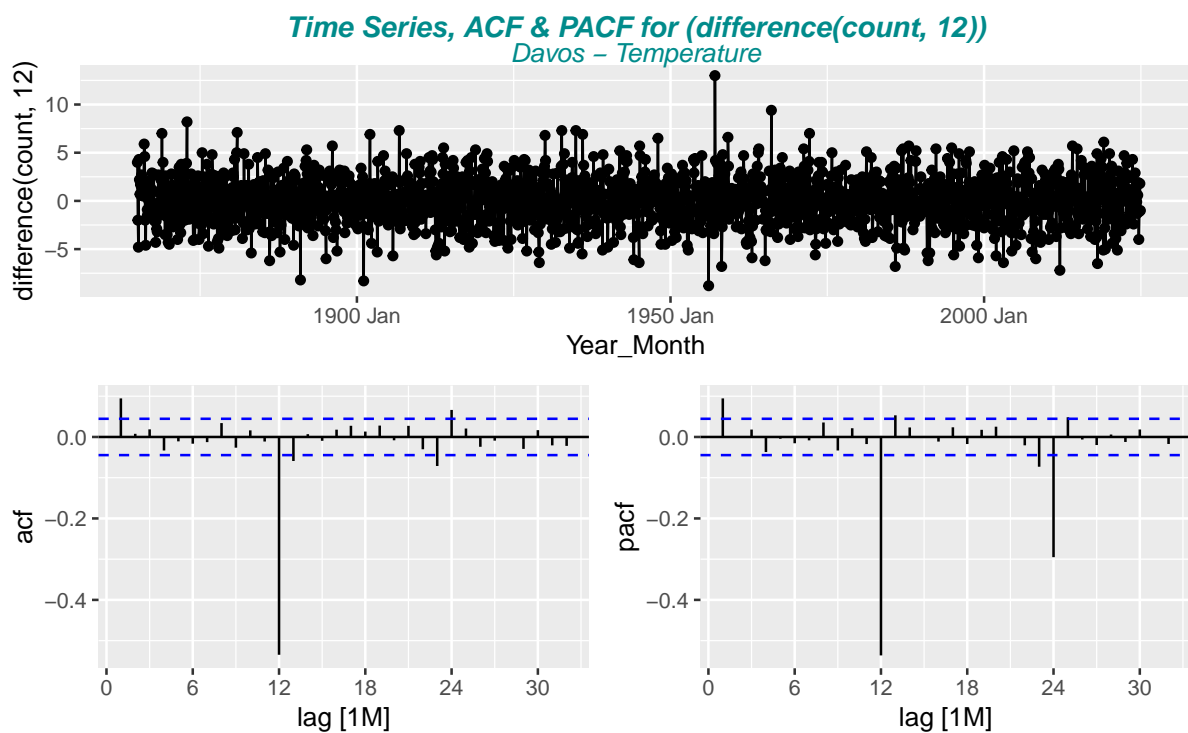
1.1.3 ACF Plots of Differences



1.1.4 Time Series, ACF and PACF Plots of Differences - for ARIMA p, q check

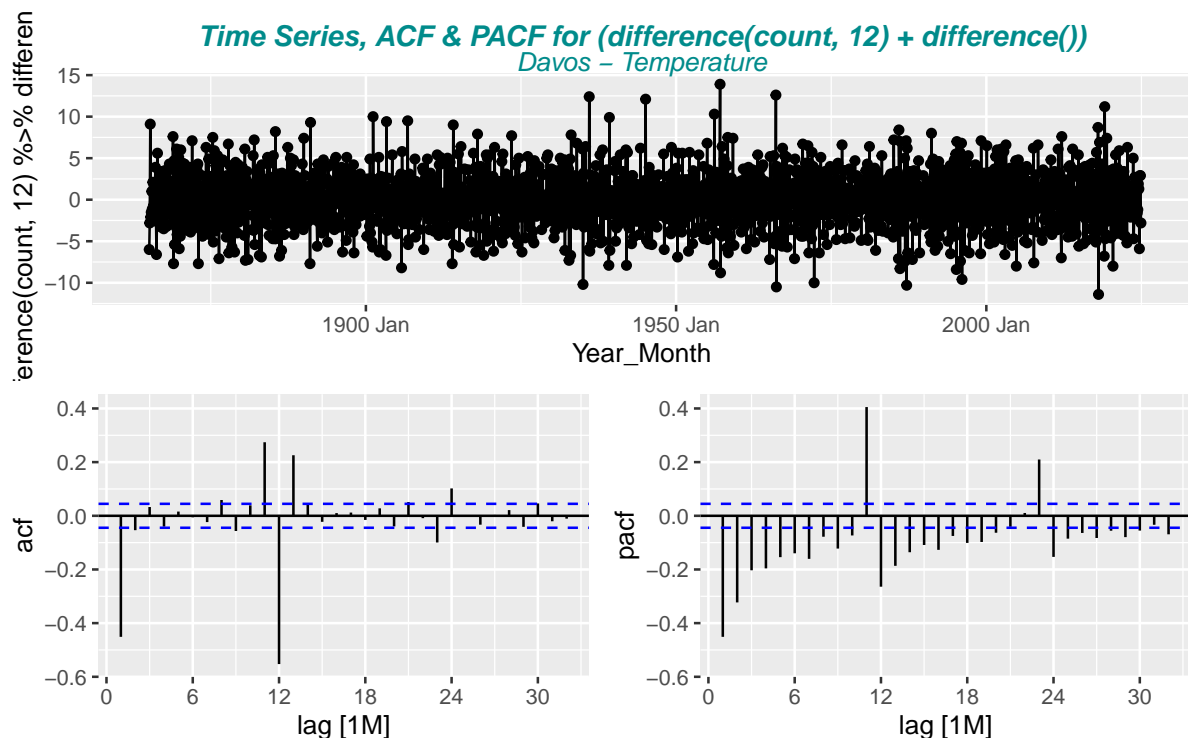


```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City Measure      Sum      Mean
#>   <chr> <fct>      <dbl>   <dbl>
#> 1 Davos Temperature  50.2  0.0261
```



```
#> # A tibble: 1 x 4
```

```
#> # Groups:   City [1]
#>   City Measure      Sum      Mean
#>   <chr> <fct>      <dbl>  <dbl>
#> 1 Davos Temperature  50.2  0.0261
```



```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City Measure      Sum      Mean
#>   <chr> <fct>      <dbl>  <dbl>
#> 1 Davos Temperature -5.00 -0.00261
```

2 ExponenTial Smoothing (ETS) Forecasting Models

Forecasts produced using exponential smoothing methods are weighted averages of past observations, with the weights decaying exponentially as the observations get older.

The parameters are estimated by maximising the “likelihood”. The likelihood is the probability of the data arising from the specified model. AIC, AICc and BIC can be used here to determine which of the ETS models is most appropriate for a given time series (see output `glance(fit_ets)`).

The model selection is based on recognising key components of the time series (trend and seasonal) and the way in which these enter the smoothing method (e.g., in an additive, damped or multiplicative manner).

- Mauna Loa CO_2 data best Models: ETS(M,A,A) & ETS(A,A,A)
- Basel Temperature data best Models: ETS(A,N,A), ETS(A,A,A), ETS(A,Ad,A) (close together). Best Forecast accuracy is with ETS(A,A,A), ETS(A,Ad,A).
- Basel Precipitation data best Models: ETS(A,N,A), ETS(A,Ad,A), ETS(A,A,A) (close together). Best Forecast accuracy is with ETS(A,A,A), ETS(A,Ad,A), ETS(A,N,A),

Trend term “N” for Basel Temperature/Precipitation correspondends to a “pure” exponential smoothing which results in a slope $\beta = 0$. This results in a forecast predicting a constant level. This does not fit to the result of the STL decomposition. Therefore best model choice is **ETS(A,A,A)**.

Method Selection

Error term: either additive (“A”) or multiplicative (“M”).

Both methods provide identical point forecasts, but different prediction intervals and different likelihoods. AIC & BIC are able to select between the error types because they are based on likelihood.

Nevertheless, difference is for

- Mauna Loa CO_2 not relevant and AIC/AICc/BIC values are only a little bit smaller for multiplicative errors. The prediction interval plots are fully overlapping.
- Basel Temperature AIC/AICc/BIC of additive error types are much better than the multiplicative ones.
- Basel Precipitation AIC/AICc/BIC of additive error types are much better than the multiplicative ones.

Note: For Basel Temperature and Precipitation Forecast plots the models ETS_MAdA, ETS_MMA, ETS_MMA, ETS_MNA are to be taken out since forecasts with multiplicative errors are exploding (forecast > 3 years impossible !!)

Therefore finally **Error term** = “A” is chosen in general.

Trend term: either none (“N”), additive (“A”), multiplicative (“M”) or damped variants (“Ad”, “Md”).

Note: Mauna Loa CO_2 model ETS(A,Ad,A) fit plot shows to strong damping. For Basel Temperature model ETS(A,N,A) and ETS(A,Ad,A) are providing more or less the same forecast. This means that forecast remains on constant level since Trend “N” means “pure” exponential smoothing without trend (see above).

Therefore finally **Trend term** = “A” is chosen in general.

Seasonal term: either none (“N”), additive (“A”) or multiplicative (“M”).

For CO2 and Temperature Data we have a clear seasonal pattern and seasonal term adds always a (more or less) fix amount on level and trend component. Therefore “A” additive term is chosen. For Precipitation the seasonal pattern is only slight. Instead, a multiplicative seasonal term results in “exploding” forecasts.

Since monthly data are strongly seasonal **seasonal term** “A” is chosen.

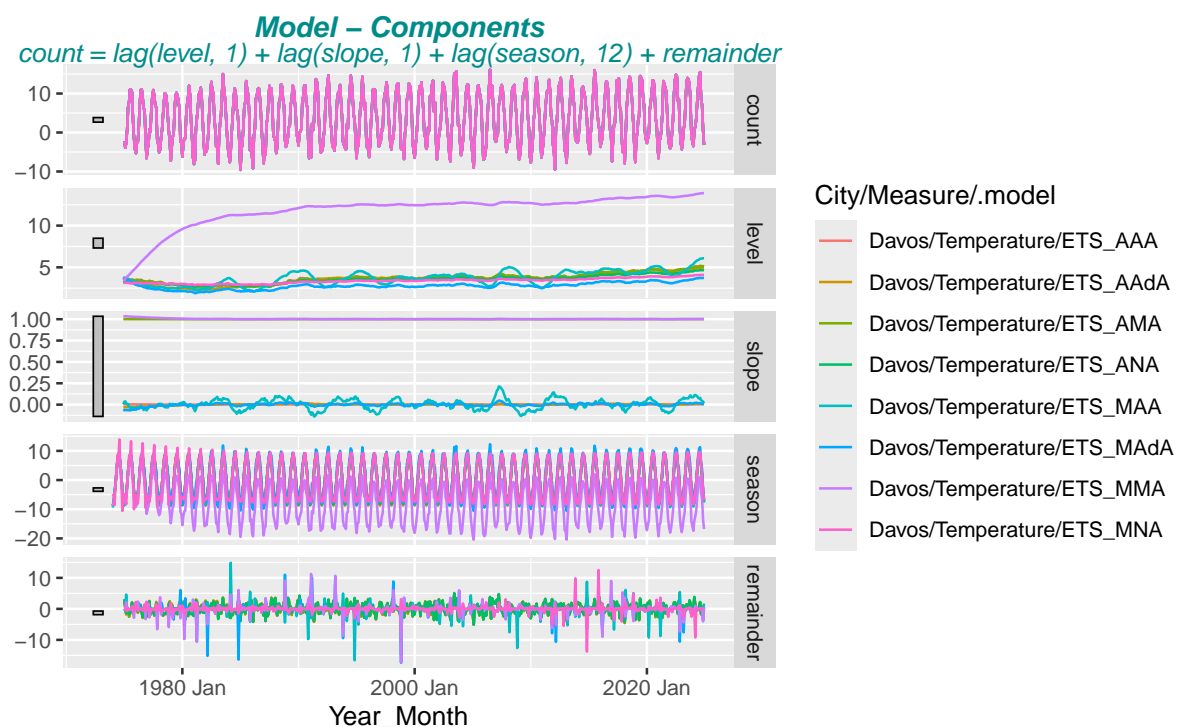
2.1 ETS Models and their components

```
#> [1] "model(ETS(count)) => provides best automatically chosen model"
#> # A tibble: 1 x 11
#>   City Measure .model      sigma2 log_lik  AIC  AICc  BIC  MSE  AMSE  MAE
#>   <chr> <fct>      <chr>      <dbl>  <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1 Davos Temperature ETS(coun~    2.94 -2234. 4504. 4506. 4584.  2.86  2.86  1.36
#> Series: count
#> Model: ETS(A,Ad,A)
#> Smoothing parameters:
#>   alpha = 0.0110662
#>   beta  = 0.0008325753
#>   gamma = 0.0001000112
#>   phi   = 0.9747746
#>
#> Initial states:
#>   l[0]      b[0]      s[0]      s[-1]  s[-2]      s[-3]      s[-4]      s[-5]
#> 3.608057 -0.03441282 -7.246684 -4.032501 1.49911 5.043996 8.368339 8.787725
#>   s[-6]      s[-7]      s[-8]      s[-9]  s[-10]      s[-11]
#> 6.780962 3.257533 -1.335596 -4.676189 -7.791509 -8.655187
#>
#> sigma^2: 2.9425
```

```

#>
#>      AIC      AICc      BIC
#> 4504.477 4505.654 4583.621
#> Model Selection by Information Criterion - lowest AIC, AICc, BIC
#> # A tibble: 8 x 11
#>   City Measure .model sigma2 log_lik AIC AICc BIC MSE AMSE MAE
#>   <chr> <fct>   <chr>   <dbl>   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1 Davos Temperature ETS_AAdA  2.94 -2234. 4504. 4506. 4584.  2.86  2.86  1.36
#> 2 Davos Temperature ETS_AMA  2.95 -2236. 4505. 4506. 4580.  2.87  2.88  1.37
#> 3 Davos Temperature ETS_AAA  2.95 -2236. 4506. 4507. 4580.  2.87  2.88  1.37
#> 4 Davos Temperature ETS_ANA  3.00 -2242. 4513. 4514. 4579.  2.93  2.93  1.37
#> 5 Davos Temperature ETS_MNA  1.60 -2961. 5953. 5953. 6018.  4.29  4.29  0.556
#> 6 Davos Temperature ETS_MMA  3.51 -3161. 6356. 6357. 6431.  4.79  4.80  0.815
#> 7 Davos Temperature ETS_MAA  3.90 -3192. 6418. 6419. 6493.  3.69  3.72  0.767
#> 8 Davos Temperature ETS_MAdA  3.94 -3198. 6431. 6432. 6510.  4.43  4.44  0.787

```



2.1.1 Residual Accuracy with one-step-ahead fitted residuals - check RMSE, MAE

Residual accuracy can be computed directly from models as the one-step-ahead fitted residuals are available. Select forecast models that minimises for lowest

- MAE (Mean absolute error, will lead to forecasts of the median) and
- RMSE (Root mean squared error, lead to forecasts of the mean)

```

#> # A tibble: 8 x 12
#>   City Measure .model .type ME RMSE MAE MPE MAPE MASE RMSSE
#>   <chr> <fct>   <chr>   <chr>   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1 Davos Temperature ETS_AAdA Trai~ 1.16e-1 1.69 1.36 Inf Inf 0.724 0.706
#> 2 Davos Temperature ETS_AMA Trai~ 1.60e-2 1.69 1.37 Inf Inf 0.727 0.707
#> 3 Davos Temperature ETS_AAA Trai~ 1.59e-2 1.70 1.37 Inf Inf 0.727 0.708
#> 4 Davos Temperature ETS_ANA Trai~ 1.30e-1 1.71 1.37 Inf Inf 0.731 0.715
#> 5 Davos Temperature ETS_MAA Trai~ 4.00e-4 1.92 1.55 -3.86 62.9 0.823 0.802

```

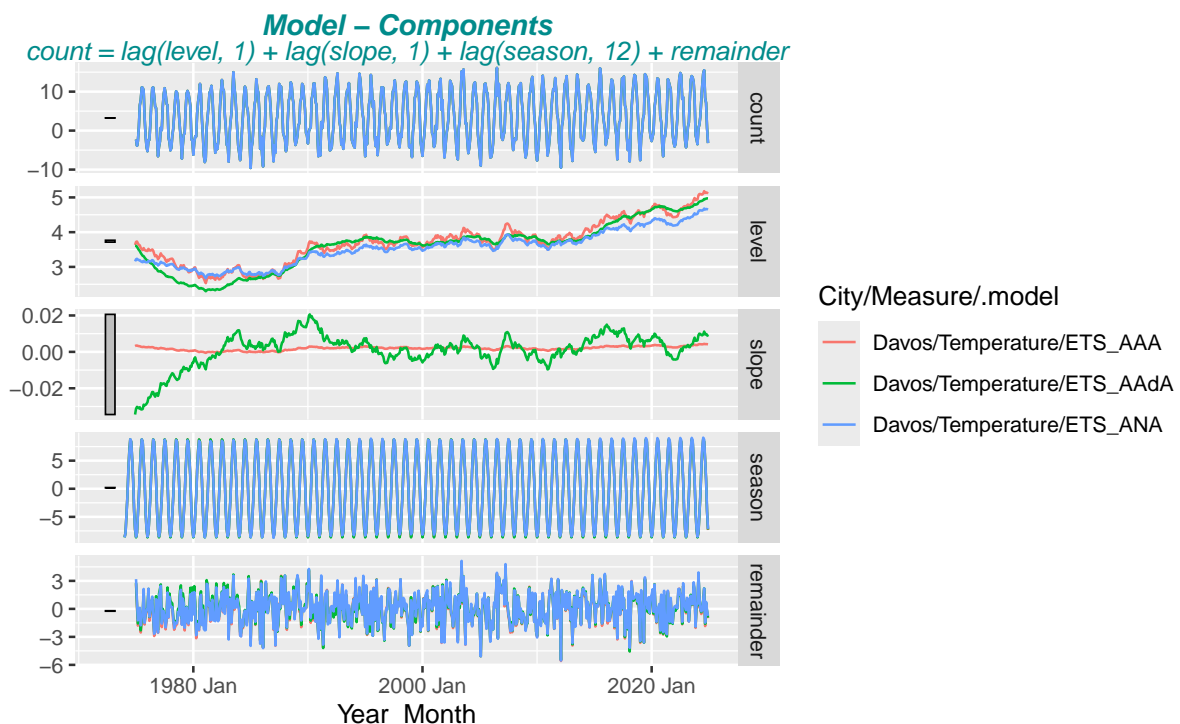


```
#> 6 Davos Temperature ETS_MNA Trai~ 1.54e-1 2.07 1.61 4.54 78.7 0.856 0.865
#> 7 Davos Temperature ETS_MAdA Trai~ 4.69e-2 2.11 1.64 -5.58 67.6 0.874 0.879
#> 8 Davos Temperature ETS_MMA Trai~ -2.44e-1 2.19 1.73 -11.9 75.1 0.920 0.914
#> # i 1 more variable: ACF1 <dbl>
```

2.1.2 Ljung-Box Test - independence/white noise of the forecasts residuals

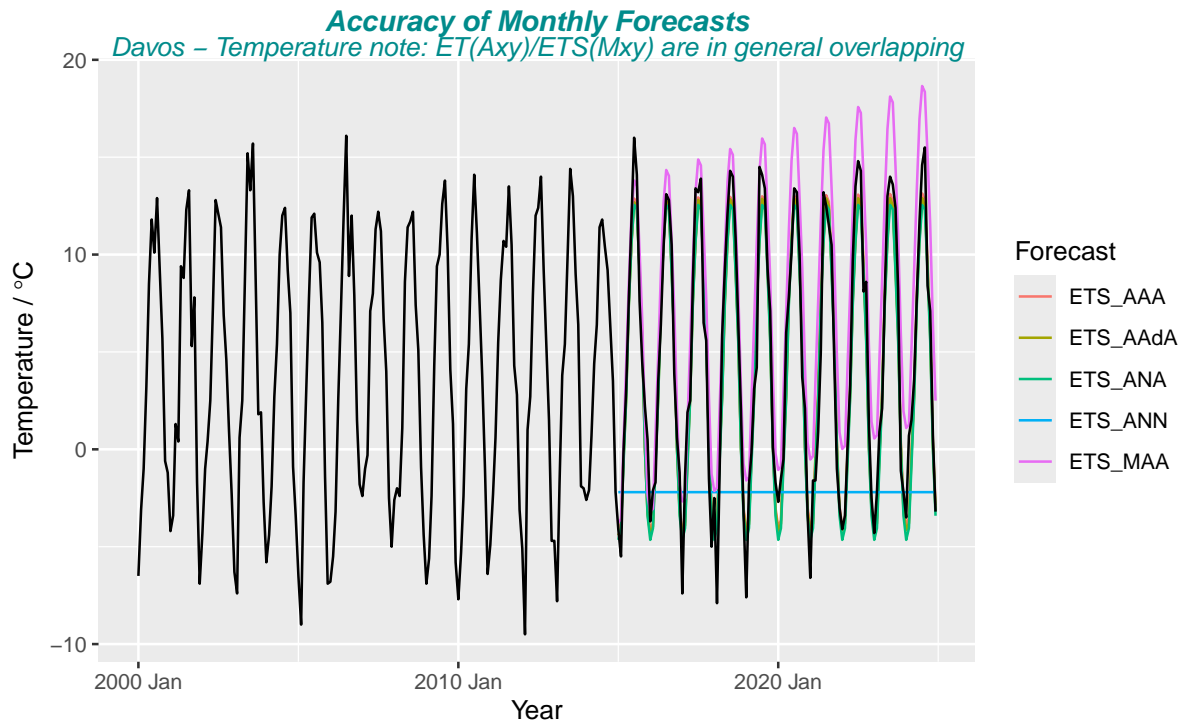
```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 8 x 5
#>   City Measure .model lb_stat lb_pvalue
#>   <chr> <fct> <chr> <dbl> <dbl>
#> 1 Davos Temperature ETS_ANA 49.8 0.0129
#> 2 Davos Temperature ETS_AMA 50.3 0.0116
#> 3 Davos Temperature ETS_AAA 50.6 0.0108
#> 4 Davos Temperature ETS_AAdA 52.5 0.00678
#> 5 Davos Temperature ETS_MAA 153. 0
#> 6 Davos Temperature ETS_MAdA 154. 0
#> 7 Davos Temperature ETS_MMA 164. 0
#> 8 Davos Temperature ETS_MNA 290. 0
```

2.1.3 ETS Models - components of ETS(A,N,A), ETS(A,A,A), ETS(A,Ad,A), models



2.1.4 Forecast Accuracy with Training/Test Data

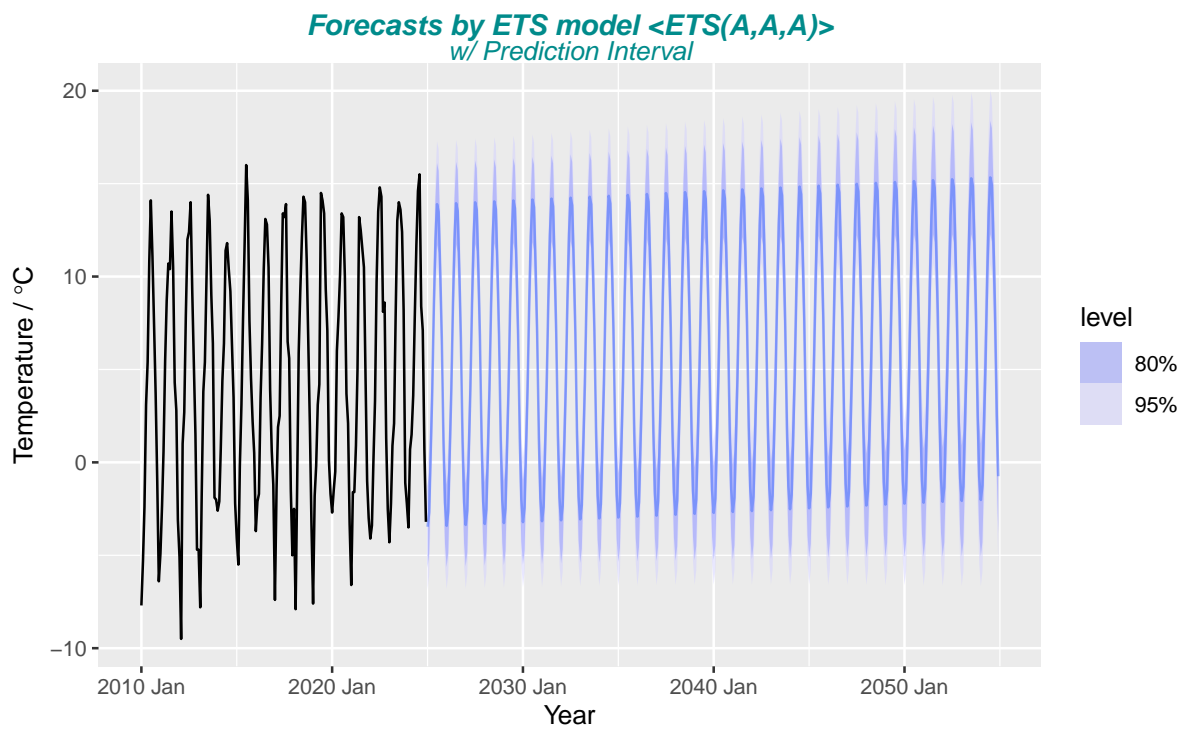
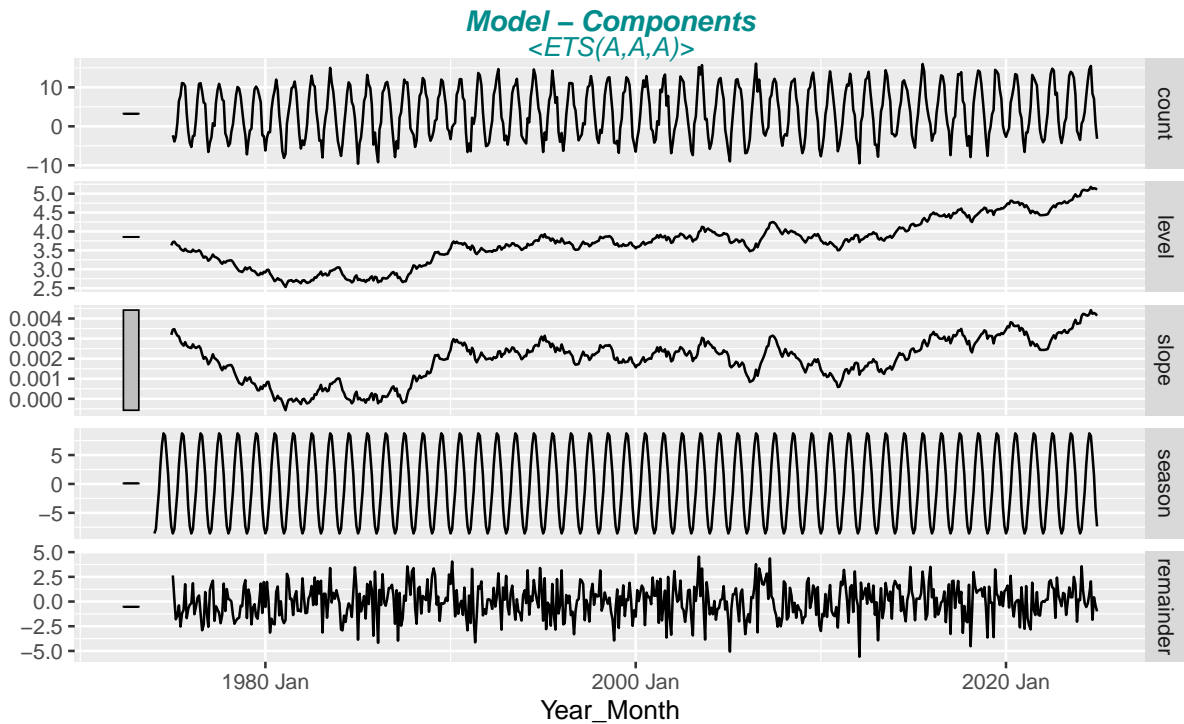
```
#> # A tibble: 5 x 12
#>   .model City Measure .type ME RMSE MAE MPE MAPE MASE RMSSE ACF1
#>   <chr> <chr> <fct> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1 ETS_A~ Davos Temper~ Test 0.398 1.68 1.34 Inf Inf 0.724 0.706 -0.0236
#> 2 ETS_A~ Davos Temper~ Test 0.594 1.73 1.41 Inf Inf 0.762 0.730 -0.0220
#> 3 ETS_A~ Davos Temper~ Test 0.801 1.81 1.51 Inf Inf 0.814 0.761 0.00193
#> 4 ETS_M~ Davos Temper~ Test -2.80 3.50 2.97 -Inf Inf 1.61 1.47 0.364
#> 5 ETS_A~ Davos Temper~ Test 6.93 9.56 7.63 Inf Inf 4.13 4.03 0.800
```



2.2 Forecasting with selected ETS model <ETS(A,A,A)>

2.2.1 Forecast Plot of selected ETS model

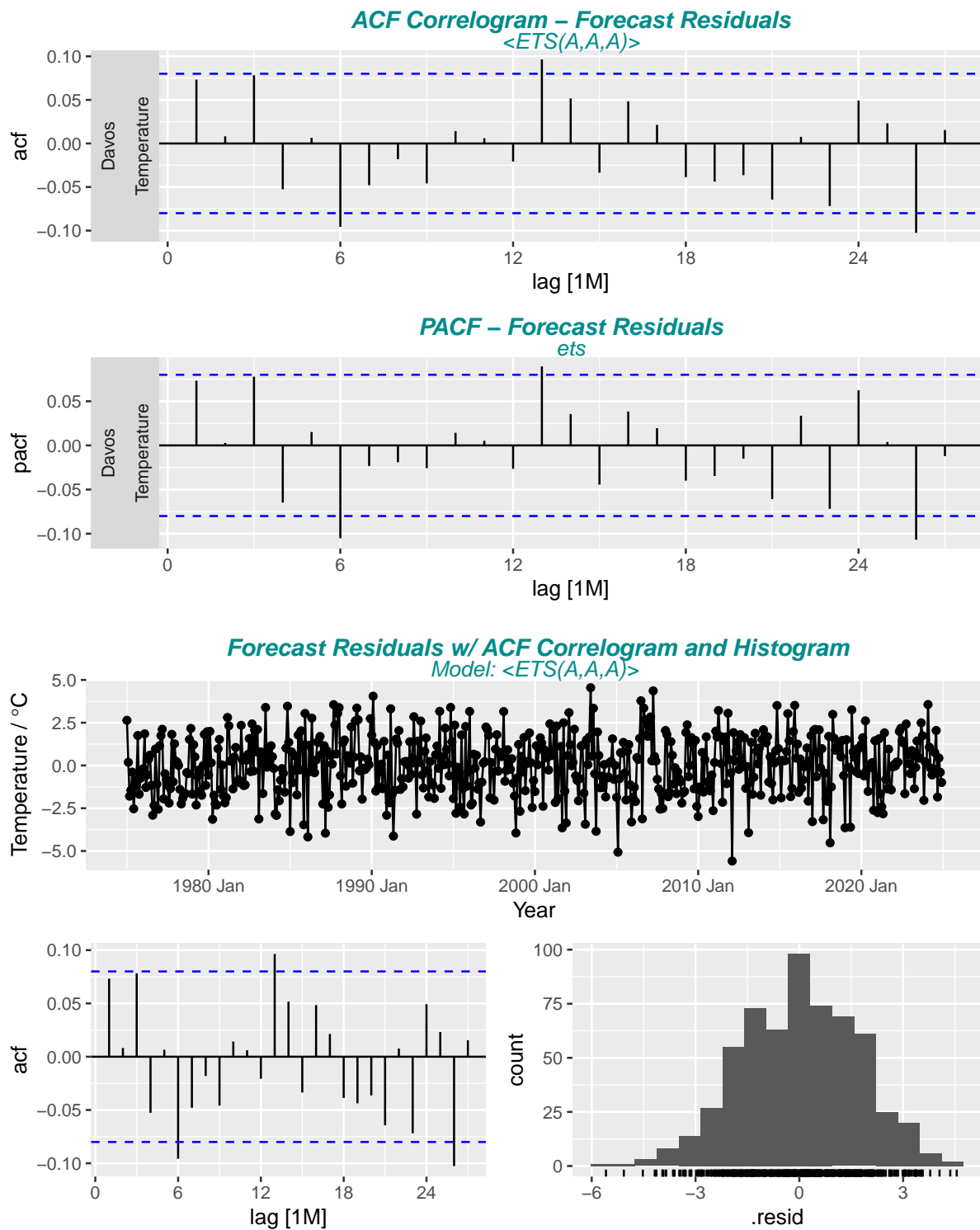
```
#> Provide model coefficients by report(fit_model)
#> Series: count
#> Model: ETS(A,A,A)
#> Smoothing parameters:
#>   alpha = 0.03233014
#>   beta  = 0.0001000124
#>   gamma = 0.0001170604
#>
#> Initial states:
#>   l[0]      b[0]      s[0]      s[-1]      s[-2]      s[-3]      s[-4]      s[-5]
#> 3.632366 0.003187921 -7.344464 -4.048129 1.534668 5.061042 8.339471 8.758817
#>   s[-6]      s[-7]      s[-8]      s[-9]      s[-10]      s[-11]
#> 6.807606 3.252448 -1.338723 -4.636271 -7.810098 -8.576367
#>
#> sigma^2: 2.9532
#>
#>   AIC   AICc   BIC
#> 4505.666 4506.717 4580.414
```



2.2.2 Residual Stationarity

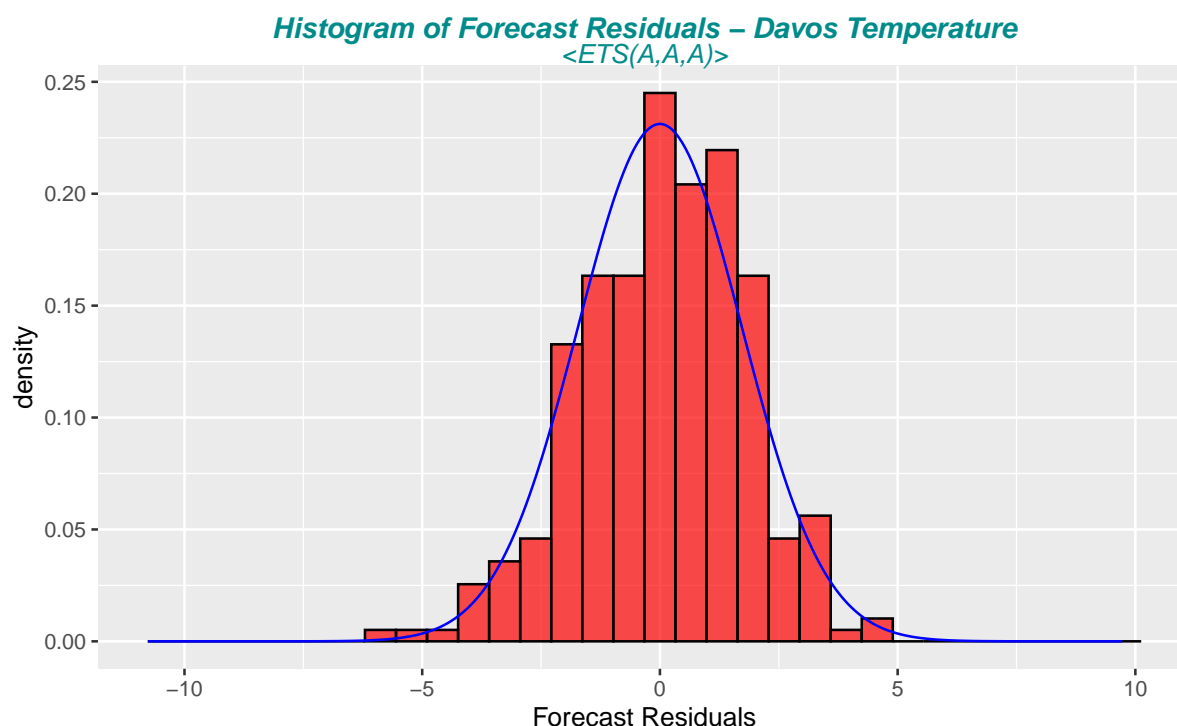
Required checks to be ready for forecasting:

- ACF Forecast Residual: all spikes are within the significance limits, so the residuals appear to be white noise
- The Ljung-Box test also shows that the residuals have no remaining autocorrelations
- Forecast Residuals are more or less normally distributed with roughly centred on zero



2.2.3 Histogram of forecast residuals with overlaid normal curve

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 1 x 5
#>   City Measure      .model lb_stat lb_pvalue
#>   <chr> <fct>      <chr>    <dbl>    <dbl>
#> 1 Davos Temperature ets        39.3    0.119
```



3 ARIMA Forecasting Models - AutoRegressive-Integrated Moving Average

Exponential smoothing and ARIMA (AutoRegressive-Integrated Moving Average) models are the two most widely used approaches to time series forecasting, and provide complementary approaches to the problem.

While exponential smoothing models are based on a description of the trend and seasonality in the data, ARIMA models aim to describe the autocorrelations in the data.

3.1 Seasonal ARIMA models

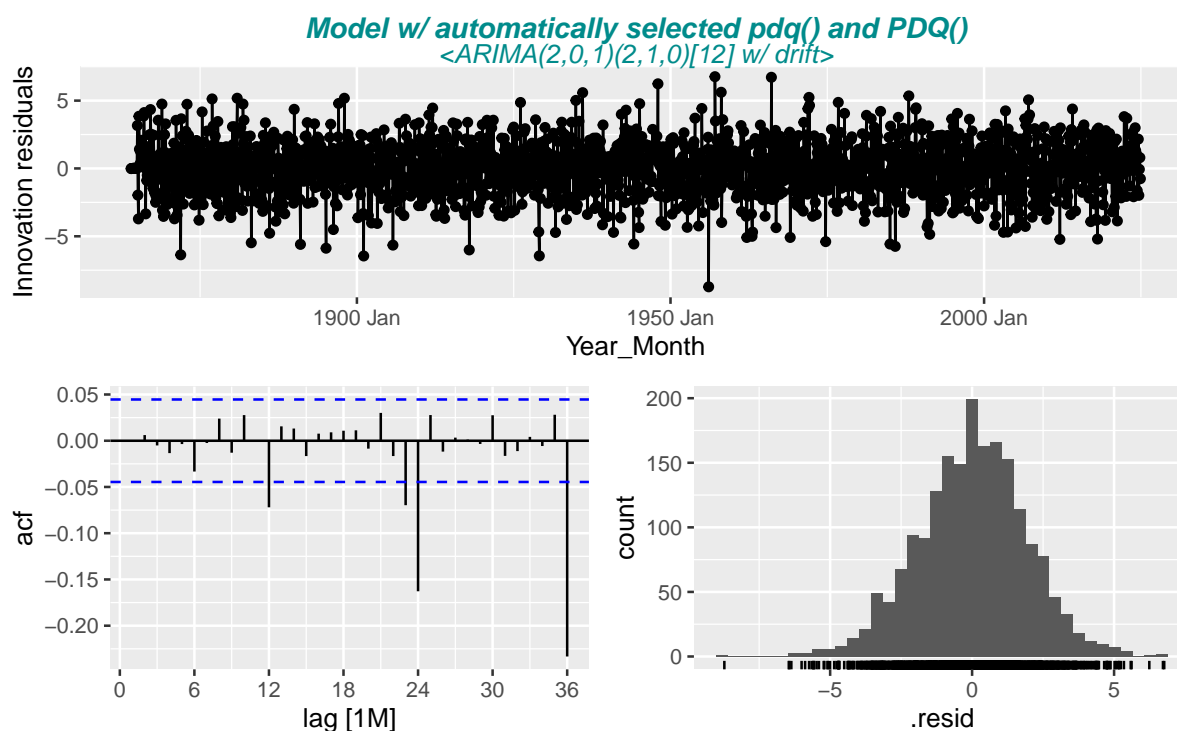
Non-seasonal ARIMA models are generally denoted $ARIMA(p,d,q)$ where parameters p , d , and q are non-negative integers, * p is the order (number of time lags) of the autoregressive model * d is the degree of differencing (number of times the data have had past values subtracted) * q is the order of the moving-average model of past forecast errors .

The value of d has an effect on the prediction intervals — the higher the value of d , the more rapidly the prediction intervals increase in size. For $d=0$, the point forecasts are equal to the mean of the data and the long-term forecast standard deviation will go to the standard deviation of the historical data, so the prediction intervals will all be essentially the same.

Seasonal ARIMA models are usually denoted $ARIMA(p,d,q)(P,D,Q)_m$, where m refers to the number of periods in each season, and the uppercase P,D,Q refer to the autoregressive, differencing, and moving average terms for the seasonal part of the ARIMA model.

```
#> # A tibble: 1 x 10
#>   City Measure   .model sigma2 log_lik   AIC   AICc   BIC ar_roots  ma_roots
#>   <chr> <fct>     <chr>   <dbl>   <dbl> <dbl> <dbl> <list>   <list>
#> 1 Davos Temperature arima     3.73 -3987. 7988. 7988. 8027. <cpl [26]> <cpl [1]>
#> Series: count
#> Model: ARIMA(2,0,1)(2,1,0)[12] w/ drift
```

```
#>
#> Coefficients:
#>      ar1      ar2      ma1      sar1      sar2  constant
#>      0.4523 -0.0271 -0.3445 -0.7072 -0.3076   0.0259
#> s.e.      NaN      NaN      NaN   0.0210   0.0217   0.0289
#>
#> sigma^2 estimated as 3.725:  log likelihood=-3987.14
#> AIC=7988.29  AICc=7988.35  BIC=8027.21
```



```
#> Model Selection by Information Criterion - lowest AIC, AICc, BIC
#> choose p, q parameter accordingly - but only for same d, D values
#> # A tibble: 13 x 10
#>   City Measure .model sigma2 log_lik AIC AICc BIC ar_roots ma_roots
#>   <chr> <fct>    <chr>   <dbl>  <dbl> <dbl> <dbl> <dbl> <list> <list>
#> 1 Davos Temperature ARIMA_0~  2.98 -1168. 2347. 2347. 2368. <cpl> <cpl>
#> 2 Davos Temperature ARIMA_1~  2.98 -1168. 2347. 2347. 2368. <cpl> <cpl>
#> 3 Davos Temperature ARIMA_2~  2.98 -1169. 2347. 2347. 2369. <cpl> <cpl>
#> 4 Davos Temperature ARIMA_1~  2.98 -1168. 2348. 2348. 2374. <cpl> <cpl>
#> 5 Davos Temperature ARIMA_1~  3.70 -1221. 2449. 2450. 2467. <cpl> <cpl>
#> 6 Davos Temperature ARIMA_1~  4.10 -1249. 2509. 2509. 2530. <cpl> <cpl>
#> 7 Davos Temperature ARIMA_2~  4.10 -1249. 2509. 2509. 2530. <cpl> <cpl>
#> 8 Davos Temperature ARIMA_3~  3.92 -1272. 2559. 2560. 2595. <cpl> <cpl>
#> 9 Davos Temperature ARIMA_2~  5.00 -1306. 2621. 2621. 2638. <cpl> <cpl>
#> 10 Davos Temperature ARIMA_0~  5.76 -1349. 2704. 2704. 2717. <cpl> <cpl>
#> 11 Davos Temperature ARIMA_1~  5.76 -1349. 2704. 2704. 2717. <cpl> <cpl>
#> 12 Davos Temperature ARIMA_0~  7.54 -1428. 2859. 2859. 2868. <cpl> <cpl>
#> 13 Davos Temperature ARIMA_1~  8.68 -1467. 2938. 2938. 2946. <cpl> <cpl>
```

Good models are obtained by minimising the AIC, AICc or BIC (see `glance(fit_arma)` output). The preference is to use the AICc to select p and q .

These information criteria tend not to be good guides to selecting the appropriate order of differencing (d) of a model, but only for selecting the values of p and q . This is because the differencing changes the data on which the likelihood is computed, making the AIC values between models with different orders of differencing not comparable.

3.1.1 Residual Accuracy with one-step-ahead fitted residuals - check RMSE, MAE

Residual accuracy can be computed directly from models as the one-step-ahead fitted residuals are available. Select forecast models that minimises for lowest

- MAE (Mean absolute error, will lead to forecasts of the median) and
- RMSE (Root mean squared error, lead to forecasts of the mean)

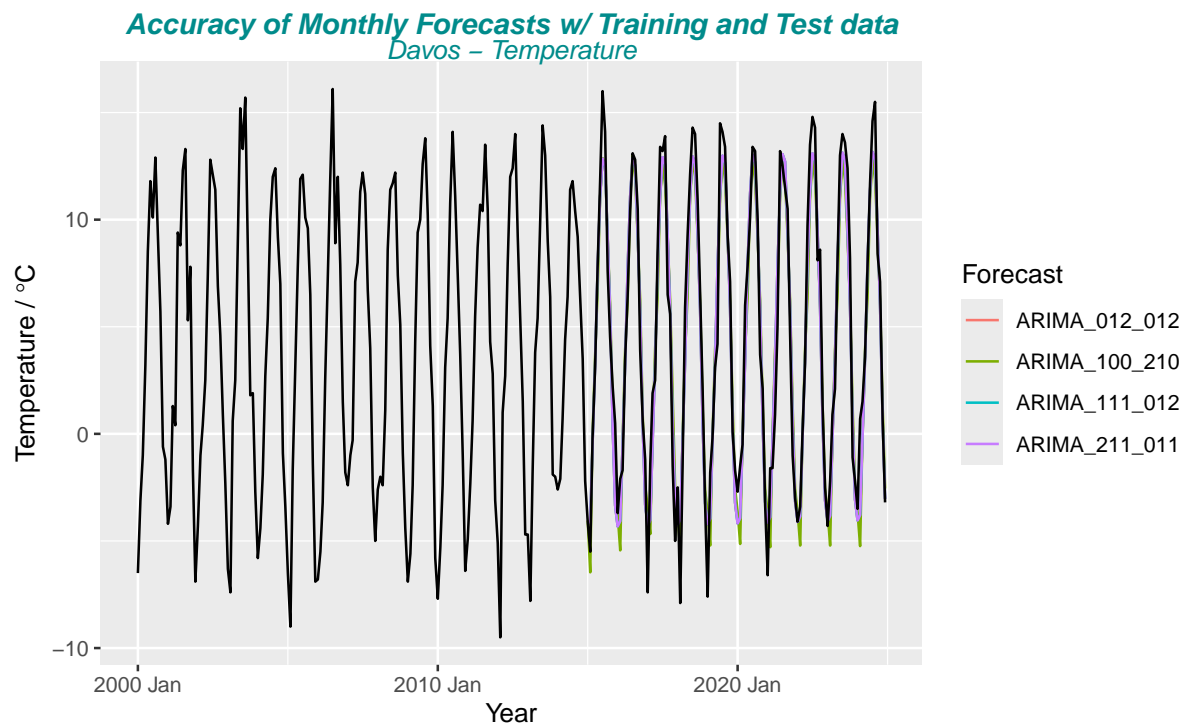
```
#> # A tibble: 14 x 12
#>   City Measure .model .type ME RMSE MAE MPE MAPE MASE
#>   <chr> <fct> <chr> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1 Davos Temperature ARIMA_11~ Trai~ 0.0726 1.70 1.34 Inf Inf 0.712
#> 2 Davos Temperature ARIMA_21~ Trai~ 0.0701 1.70 1.34 Inf Inf 0.711
#> 3 Davos Temperature ARIMA_11~ Trai~ 0.0733 1.70 1.34 Inf Inf 0.712
#> 4 Davos Temperature ARIMA_01~ Trai~ 0.0739 1.70 1.34 Inf Inf 0.712
#> 5 Davos Temperature ARIMA_10~ Trai~ 0.0854 1.90 1.50 Inf Inf 0.797
#> 6 Davos Temperature ARIMA_30~ Trai~ 0.0397 1.97 1.57 Inf Inf 0.836
#> 7 Davos Temperature ARIMA_10~ Trai~ -0.00227 2.00 1.56 Inf Inf 0.830
#> 8 Davos Temperature ARIMA_20~ Trai~ -0.00227 2.00 1.56 Inf Inf 0.830
#> 9 Davos Temperature ARIMA_21~ Trai~ 0.00588 2.21 1.74 Inf Inf 0.923
#> 10 Davos Temperature ARIMA_01~ Trai~ 0.0477 2.37 1.85 Inf Inf 0.981
#> 11 Davos Temperature ARIMA_11~ Trai~ 0.0477 2.37 1.84 Inf Inf 0.981
#> 12 Davos Temperature ARIMA_01~ Trai~ 0.00525 2.71 2.10 Inf Inf 1.12
#> 13 Davos Temperature ARIMA_11~ Trai~ 0.00626 2.91 2.29 Inf Inf 1.22
#> 14 Davos Temperature ARIMA_00~ Trai~ NaN NaN NaN NaN NaN
#> # i 2 more variables: RMSSE <dbl>, ACF1 <dbl>
```

3.1.2 Ljung-Box Test - independence/white noise of the forecasts residuals

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 14 x 5
#>   City Measure .model lb_stat lb_pvalue
#>   <chr> <fct> <chr> <dbl> <dbl>
#> 1 Davos Temperature ARIMA_111_112 45.9 3.16e- 2
#> 2 Davos Temperature ARIMA_012_012 46.8 2.63e- 2
#> 3 Davos Temperature ARIMA_111_012 46.8 2.62e- 2
#> 4 Davos Temperature ARIMA_211_011 47.5 2.21e- 2
#> 5 Davos Temperature ARIMA_301_200 90.8 4.96e- 8
#> 6 Davos Temperature ARIMA_100_210 107. 1.22e-10
#> 7 Davos Temperature ARIMA_100_110 121. 6.64e-13
#> 8 Davos Temperature ARIMA_200_110 121. 6.64e-13
#> 9 Davos Temperature ARIMA_210_110 140. 4.44e-16
#> 10 Davos Temperature ARIMA_010_110 312. 0
#> 11 Davos Temperature ARIMA_012_010 237. 0
#> 12 Davos Temperature ARIMA_110_010 458. 0
#> 13 Davos Temperature ARIMA_111_010 237. 0
#> 14 Davos Temperature ARIMA_002_200 NA NA
```

3.1.3 Forecast Accuracy with Training/Test Data

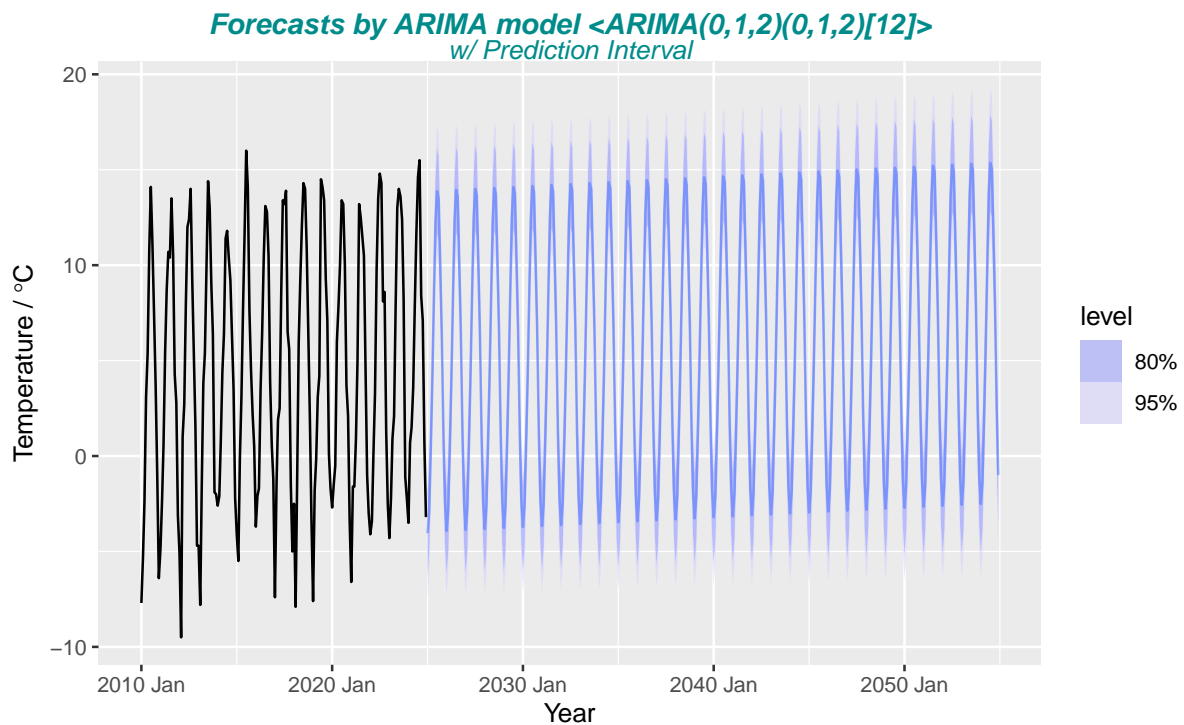
```
#> # A tibble: 4 x 12
#>   .model City Measure .type ME RMSE MAE MPE MAPE MASE RMSSE ACF1
#>   <chr> <chr> <fct> <chr> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1 ARIMA_~ Davos Temper~ Test 0.397 1.66 1.33 Inf Inf 0.720 0.701 -0.0243
#> 2 ARIMA_~ Davos Temper~ Test 0.395 1.66 1.33 Inf Inf 0.720 0.701 -0.0242
#> 3 ARIMA_~ Davos Temper~ Test 0.402 1.67 1.33 Inf Inf 0.721 0.702 -0.0243
#> 4 ARIMA_~ Davos Temper~ Test 0.496 1.91 1.56 Inf Inf 0.842 0.806 0.00164
```



3.2 Temperature - Forecasting with selected ARIMA model <ARIMA(0,1,2)(0,1,2)[12]>

3.2.1 Forecast Plot of selected ARIMA model

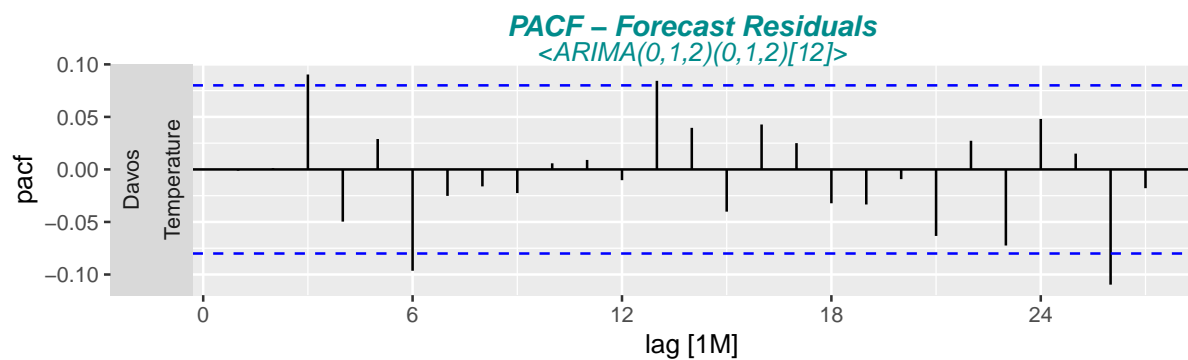
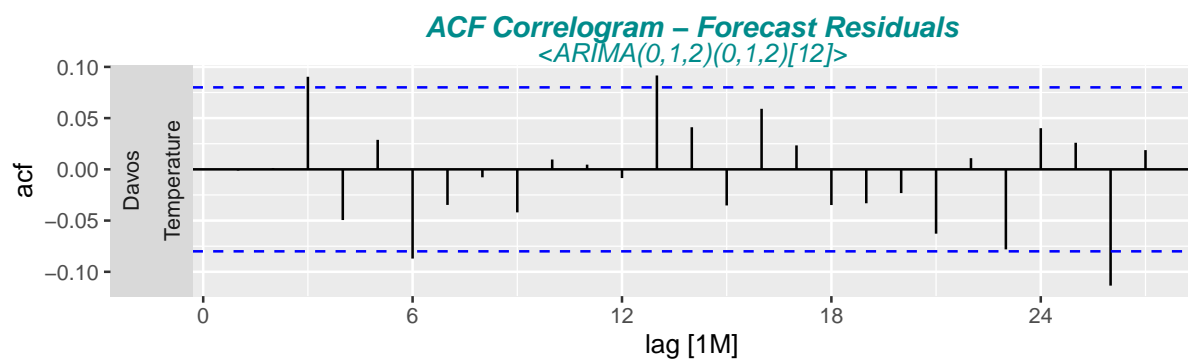
```
#> Provide model coefficients by report(fit_model)
#> Series: count
#> Model: ARIMA(0,1,2)(0,1,2)[12]
#>
#> Coefficients:
#>      ma1      ma2      sma1      sma2
#>    -0.9111 -0.0789 -0.9703  0.0300
#> s.e.    0.0426  0.0426  0.0418  0.0405
#>
#> sigma^2 estimated as 2.98:  log likelihood=-1168.28
#> AIC=2346.55  AICc=2346.66  BIC=2368.43
```

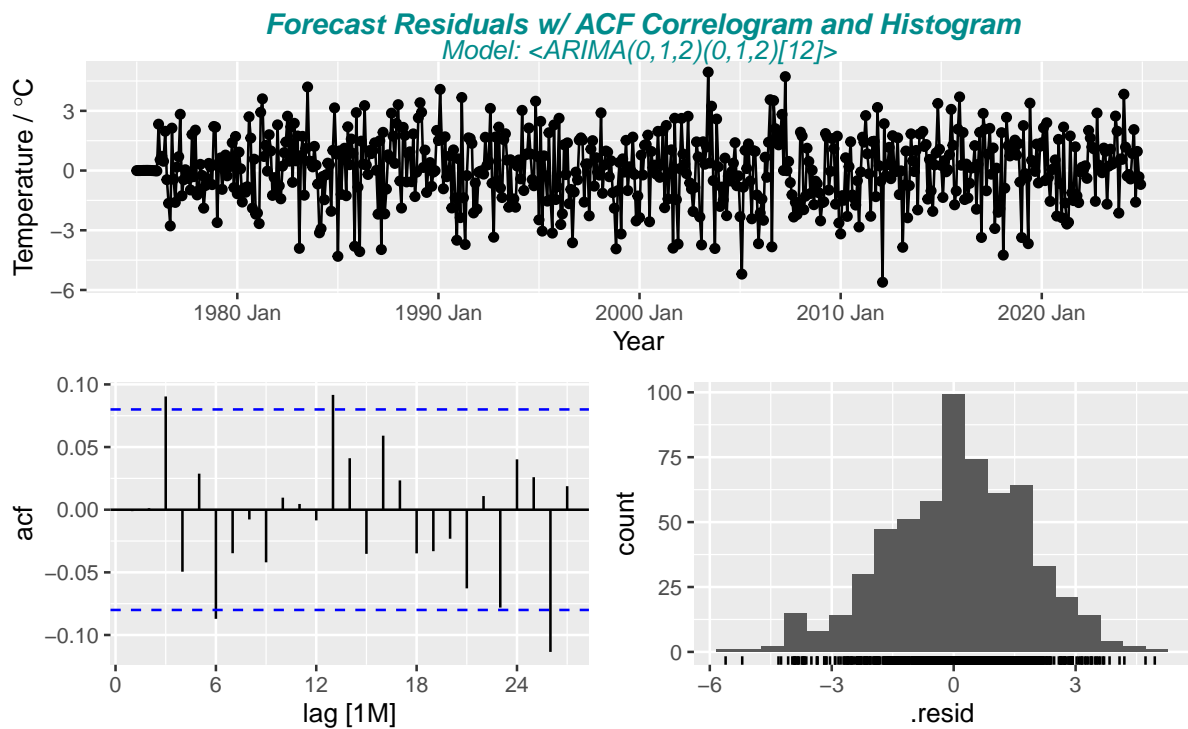



3.2.2 Residual Stationarity

Required checks to be ready for forecasting:

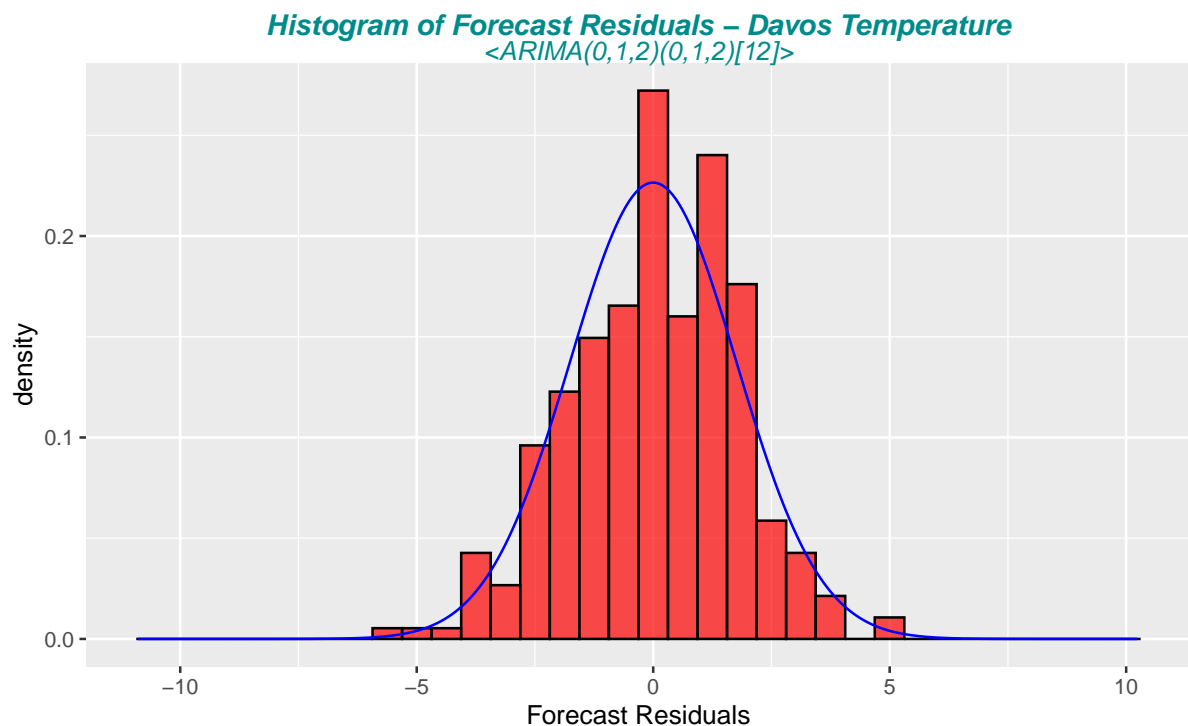
- ACF Forecast Residual: all spikes are within the significance limits, so the residuals appear to be white noise
- The Ljung-Box test also shows that the residuals have no remaining autocorrelations
- Forecast Residuals are more or less normally distributed with roughly centred on zero





3.2.3 Histogram of forecast residuals with overlaid normal curve

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 1 x 5
#>   City Measure      .model lb_stat lb_pvalue
#>   <chr> <fct>      <chr>    <dbl>    <dbl>
#> 1 Davos Temperature arima      34.4      0.265
```



4 ARIMA vs ETS

In particular, all ETS models are non-stationary, while some ARIMA models are stationary.

The ETS models with seasonality or non-damped trend or both have two unit roots (i.e., they need two levels of differencing to make them stationary). All other ETS models have one unit root (they need one level of differencing to make them stationary).

We compare for the chosen ETS resp. ARIMA model the RMSE / MAE values. Lower values indicate a more accurate model based on the test set RMSE, ..., MASE.

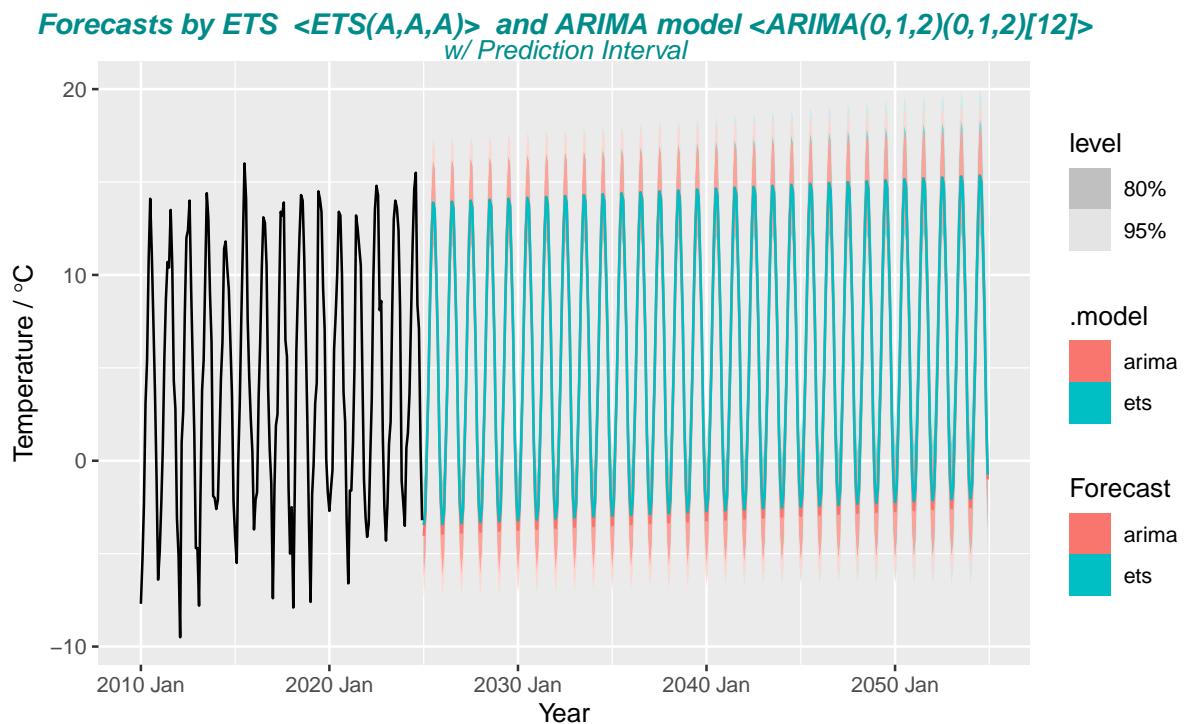
- Residual Accuracy with one-step-ahead fitted residuals
- Forecast Accuracy with Training/Test Data

Note: a good fit to training data is never an indication that the model will forecast well. Therefore the values of the Forecast Accuracy are the more relevant one.

4.0.1 Comparing Residual and Forecast Accuracy of selected ETS and ARIMA model

```
#> # A tibble: 4 x 12
#>   City Measure .model .type      ME  RMSE  MAE  MPE  MAPE  MASE RMSSE  ACF1
#>   <chr> <fct>   <chr> <chr>   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1 Davos Temper~ ets    Trai~ 0.0159 1.70 1.37  Inf  Inf  0.727 0.708 0.0732
#> 2 Davos Temper~ arima  Trai~ 0.0739 1.70 1.34  Inf  Inf  0.712 0.710 -0.00115
#> 3 Davos Temper~ ETS_A~ Test  0.398 1.68 1.34  Inf  Inf  0.724 0.706 -0.0236
#> 4 Davos Temper~ ARIMA~ Test  0.402 1.67 1.33  Inf  Inf  0.721 0.702 -0.0243
```

4.0.2 Forecast Plot of selected ETS and ARIMA model



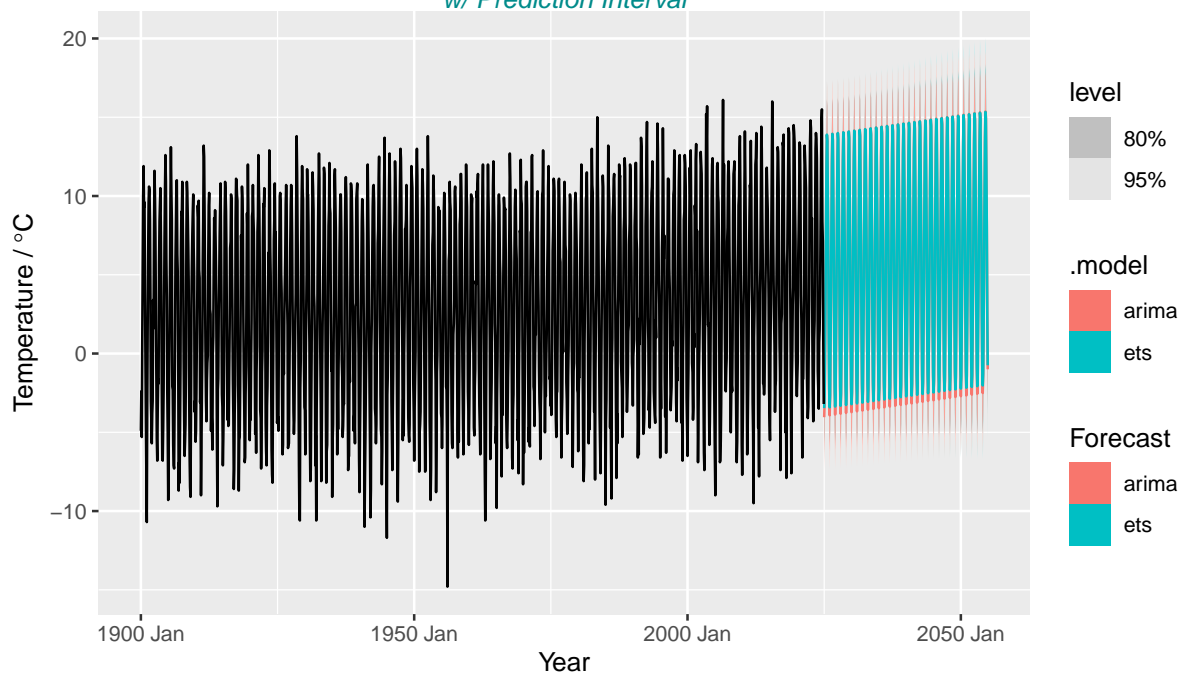
```
#> # A tsibble: 6 x 8 [1M]
#> # Key:      City, Measure, .model [2]
#> # Groups:   City, Measure, .model [2]
```

```

#>   City Measure      .model Year_Month
#>   <chr> <fct>      <chr>      <mth>
#> 1 Davos Temperature arima    2025 Jan
#> 2 Davos Temperature arima    2025 Feb
#> 3 Davos Temperature arima    2025 Mrz
#> 4 Davos Temperature ets      2025 Jan
#> 5 Davos Temperature ets      2025 Feb
#> 6 Davos Temperature ets      2025 Mrz
#> # i 4 more variables: count <dbl>, .mean <dbl>, '80%' <hilo>, '95%' <hilo>
#> # A tibble: 6 x 8 [1M]
#> # Key:      City, Measure, .model [2]
#> # Groups:   City, Measure, .model [2]
#>   City Measure      .model Year_Month
#>   <chr> <fct>      <chr>      <mth>
#> 1 Davos Temperature arima    2054 Okt
#> 2 Davos Temperature arima    2054 Nov
#> 3 Davos Temperature arima    2054 Dez
#> 4 Davos Temperature ets      2054 Okt
#> 5 Davos Temperature ets      2054 Nov
#> 6 Davos Temperature ets      2054 Dez
#> # i 4 more variables: count <dbl>, .mean <dbl>, '80%' <hilo>, '95%' <hilo>

```

Forecasts by ETS <ETS(A,A,A)> and ARIMA model <ARIMA(0,1,2)(0,1,2)[12]> w/ Prediction Interval



```

#> # A tibble: 180 x 5
#> # Groups:   City, Measure, .model, Year [60]
#>   City Measure      .model Year Year_avg
#>   <chr> <fct>      <chr> <dbl> <dbl>
#> 1 Davos Temperature arima    2025 -4.04
#> 2 Davos Temperature arima    2025 -2.93
#> 3 Davos Temperature arima    2025  0.157
#> 4 Davos Temperature arima    2026 -3.92
#> 5 Davos Temperature arima    2026 -2.76
#> 6 Davos Temperature arima    2026  0.252
#> 7 Davos Temperature arima    2027 -3.87

```

```

#> 8 Davos Temperature arima 2027 -2.71
#> 9 Davos Temperature arima 2027 0.302
#> 10 Davos Temperature arima 2028 -3.82
#> # i 170 more rows
#> # A tibble: 180 x 5
#> # Groups:   City, Measure, .model, Year [60]
#>   City Measure .model Year Year_avg
#>   <chr> <fct>   <chr> <dbl> <dbl>
#> 1 Davos Temperature arima 2025 6.40
#> 2 Davos Temperature arima 2025 0.910
#> 3 Davos Temperature arima 2025 -2.44
#> 4 Davos Temperature arima 2026 6.48
#> 5 Davos Temperature arima 2026 0.954
#> 6 Davos Temperature arima 2026 -2.42
#> 7 Davos Temperature arima 2027 6.53
#> 8 Davos Temperature arima 2027 1.00
#> 9 Davos Temperature arima 2027 -2.37
#> 10 Davos Temperature arima 2028 6.58
#> # i 170 more rows

```

4.0.3 Ljung-Box Test - independence/white noise of the forecasts residuals

```

#> # A tibble: 2 x 5
#>   City Measure .model lb_stat lb_pvalue
#>   <chr> <fct>   <chr>   <dbl>   <dbl>
#> 1 Davos Temperature arima 46.8 0.0263
#> 2 Davos Temperature ets 50.6 0.0108

```

5 Yearly Data Forecasts with ARIMA and ETS

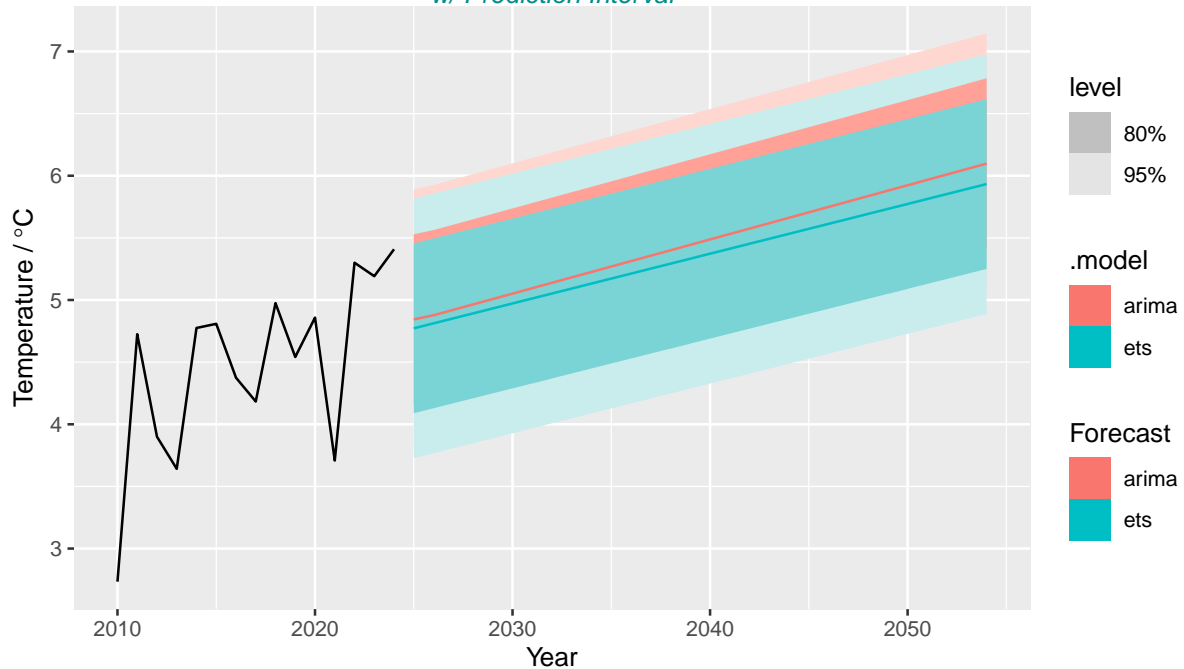
For yearly data the seasonal monthly data are replaced by the yearly average data. Therefore the seasonal component of the ETS and ARIMA model are to be taken out.

The ETS model $\langle ETS(A, A, N) \rangle$ with seasonal term change “A” -> “N” is chosen. For ARIMA models the seasonal term (P,D,Q)_m has to be taken out and an optimal ARIMA(p,1,q) with one differencing (d=1) is selected. However, for Mauna Loa two times differencing had to be selected $\$CO_2 \langle ARIMA(0,2,1) \text{ w/ poly} \rangle$. For Temperature and Precipitation the same model as for monthly data can be taken by leaving out the seasonal term $\langle ARIMA(0,1,2)w/drift \rangle$.

5.0.1 Comparing Residual and Forecast Accuracy of selected ETS and ARIMA model

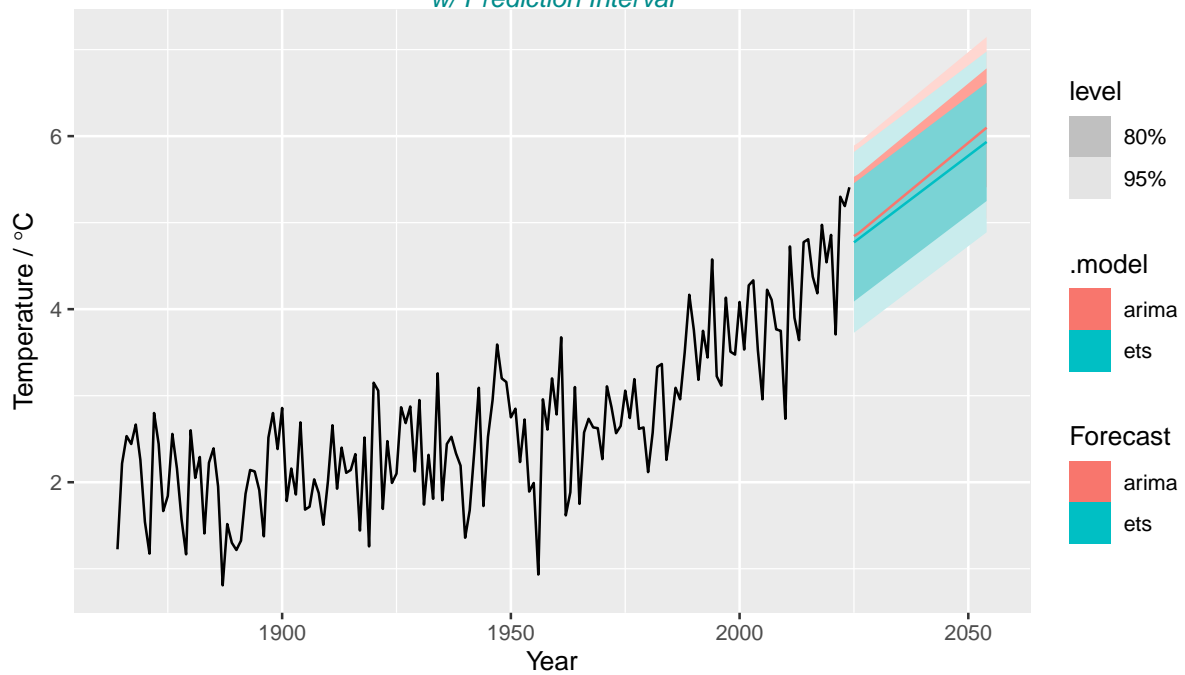
5.0.2 Forecast Plot of selected ETS and ARIMA model

Early Data Forecasts by ETS <ETS(A,A,N)> and ARIMA model <ARIMA(0,1,2) w/ drift>
w/ Prediction Interval



```
#> # A tsibble: 6 x 8 [1Y]
#> # Key:      City, Measure, .model [2]
#> # Groups:   City, Measure, .model [2]
#>   City Measure .model Year
#>   <chr> <fct>   <chr> <dbl>
#> 1 Davos Temperature arima  2025
#> 2 Davos Temperature arima  2026
#> 3 Davos Temperature arima  2027
#> 4 Davos Temperature ets    2025
#> 5 Davos Temperature ets    2026
#> 6 Davos Temperature ets    2027
#> # i 4 more variables: Year_avg <dist>, .mean <dbl>, '80%' <hilo>, '95%' <hilo>
#> # A tsibble: 6 x 8 [1Y]
#> # Key:      City, Measure, .model [2]
#> # Groups:   City, Measure, .model [2]
#>   City Measure .model Year
#>   <chr> <fct>   <chr> <dbl>
#> 1 Davos Temperature arima  2052
#> 2 Davos Temperature arima  2053
#> 3 Davos Temperature arima  2054
#> 4 Davos Temperature ets    2052
#> 5 Davos Temperature ets    2053
#> 6 Davos Temperature ets    2054
#> # i 4 more variables: Year_avg <dist>, .mean <dbl>, '80%' <hilo>, '95%' <hilo>
```

Early Data Forecasts by ETS <ETS(A,A,N)> and ARIMA model <ARIMA(0,1,2) w/ drift> w/ Prediction Interval



5.0.3 Ljung-Box Test - independence/white noise of the forecasts residuals

```
#> # A tibble: 2 x 5
#>   City Measure      .model lb_stat lb_pvalue
#>   <chr> <fct>      <chr>   <dbl>   <dbl>
#> 1 Davos Temperature arima     39.3     0.119
#> 2 Davos Temperature ets       35.2     0.234
```

6 Backup