Climate Data Forecasting -

Atmospheric CO_2 Concentration / Temperature / Precipitation

Wolfgang Vollmer

2026-01-08

Contents

1 Forecasting of Cottbus - Temperature and Precipitation Climate Analysis

1.1 Stationarity and differencing

Stationary time series is one whose properties do not depend on the time at which the series is observed. Thus, time series with trends, or with seasonality, are not stationary — the trend and seasonality will affect the value of the time series at different times. On the other hand, a white noise series is stationary — it does not matter when you observe it, it should look much the same at any point in time.

Stationary time series will have no predictable patterns in the long-term. Time plots will show the series to be roughly horizontal (although some cyclic behaviour is possible), with constant variance.

If y_t is a *stationary* time series, then for all s, the distribution of
$(y_t, ..., y_{t+s})$ does not depend on t.

If Time Series data with seasonality are non-stationary

- => first take a seasonal difference
- if seasonally differenced data appear are still non-stationary
- => take an additional first seasonal difference

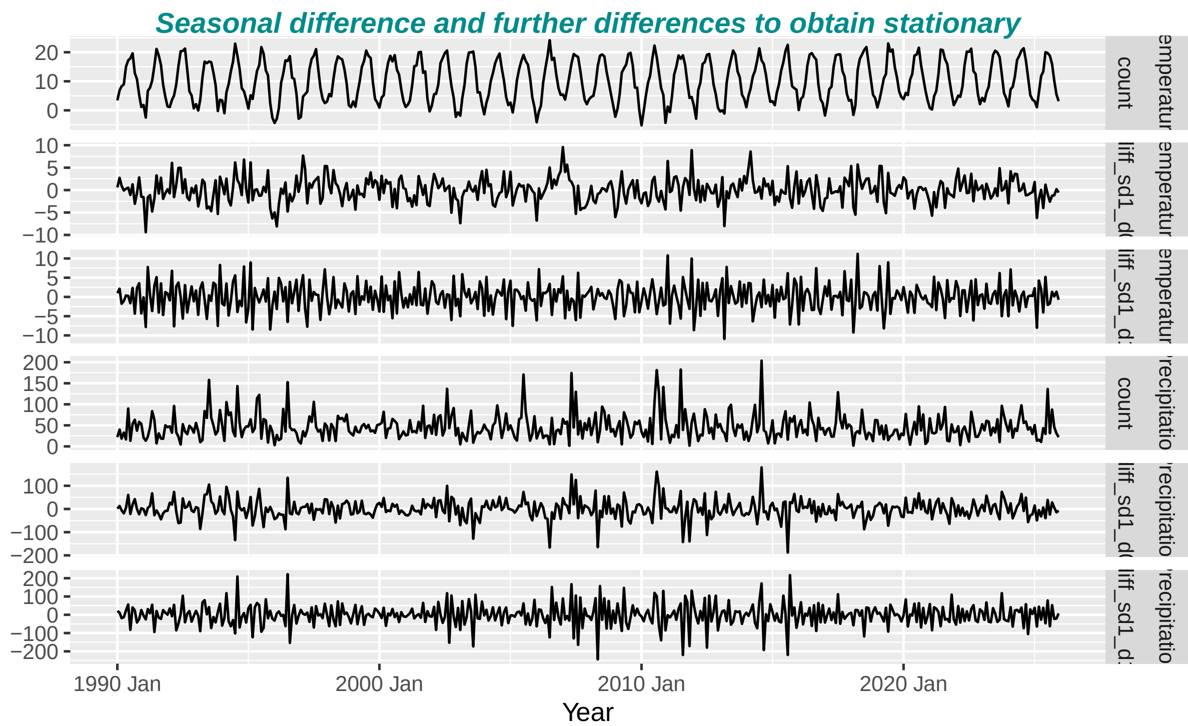
The model fit residuals have to be stationary. For good forecasting this has to be verified with residual diagnostics.

Essential:

- Residuals are uncorrelated
- The residuals have zero mean

Useful (but not necessary):

- The residuals have constant variance.
- The residuals are normally distributed.



1.1.1 Unitroot KPSS Test - fix number of seasonal differences/differences required

- For **ARIMA model** use Unitroot KPSS Test to fix number of differences
  - unitroot_nsdiffs() to determine D (the number of seasonal differences to use)
  - unitroot_ndiffs() to determine d (the number of ordinary differences to use)
  - The selection of the other model parameters (p,q,P and Q) are all determined by minimizing the AICc
- kpss test of stationary of the differentiated data (with seasonal and ordinary differences) used in the **ARIMA model**
  - stationary times series: the distribution of $(y_t, ..., y_{t-s})$ does not depend on t.

  - *Null Hypothesis* H_0: stationary is given in the time series: data are stationary and non seasonal
    - for $p < \alpha = 0.05$: reject H_0
    - for $p >> \alpha = 0.05$: conclude: differentiated data are stationary and non seasonal
- kpss test of stationary w/ unitroot_nsdiffs & unitroot_ndiff provides
  - minimum number of seasonal & ordinariel differences required for a stationary series
  - first fix required seasonal differences and then apply ndiffs to the seasonally differenced data
  - returns 1 => for stationarity one seasonal difference rsp. difference is required

```
#>   ndiffs gives the number of differences required and
#>  nsdiffs gives the number of seasonal differences required
#>  - to make a series stationary (test is based on the KPSS test
#> unitroot_kpss test to define seasonal (nsdiffs) and ordinary (ndiffs) differences
#> # A tibble: 2 x 5
#>   Measure       kpss_stat kpss_pvalue nsdiffs ndiffs
#>   <fct>             <dbl>       <dbl>   <int>  <int>
#> 1 Temperature      0.834         0.01       1      1
#> 2 Precipitation    0.0762        0.1        0      0
#> unitroot_kpss test on (difference(count, 12) + difference())
#> # A tibble: 2 x 3
#>   Measure       kpss_stat kpss_pvalue
#>   <fct>             <dbl>       <dbl>
#> 1 Temperature     0.00558         0.1
#> 2 Precipitation   0.00378         0.1
```

1.1.2 Ljung-Box Test - independence/white noise of the time series

The Ljung-Box Test becomes important when checking independence/white noise of the forecasts residuals of the fitted ETS rsp. ARIMA models. There we have to check whether the forecast errors are normally distributed with mean zero

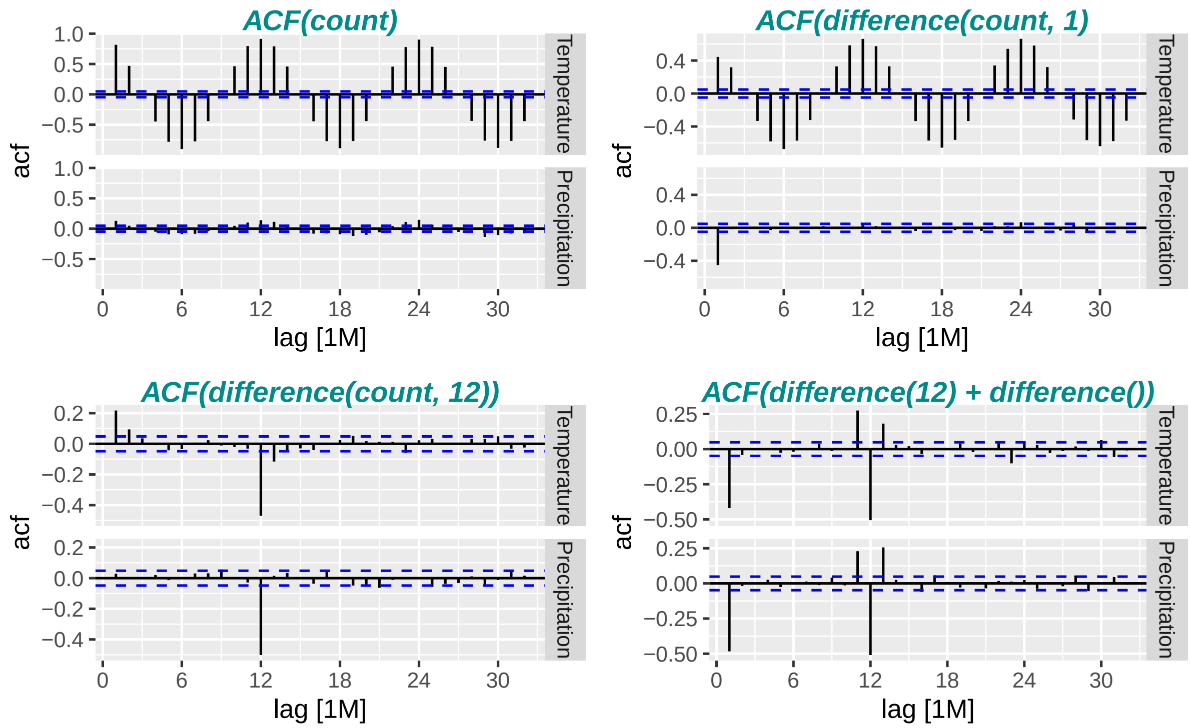
- augment(fit) |> features(.innov, ljung_box, lag=x, dof=y) (Ch. 5.4 Residaul diagnostics)
  - portmanteau test suggesting that the residuals are white noise
  - *Null Hypothesis* H_0: independence/white noise for residuals, i.e. each autocorrelation in the time series residuals of the model for lag l is close to zero.
    - for $p < \alpha = 0.05$: reject H_0
    - for $p >> \alpha = 0.05$: conclude: the residuals are not distinguishable from a white noise series
  - lag = 2*m (period of season, e.g. m=12 for monthly season) | no season: lag=10
  - dof = p + q + P + Q (for ARIMA models only, degree of freedom)

Time series with trend and/or seasonality is not stationary. Tests are to be taken on the residuals of the fitted models.

```
#> Ljung-Box test with (count), w/o differences
#> # A tibble: 2 x 3
#>   Measure       lb_stat lb_pvalue
#>   <fct>           <dbl>     <dbl>
#> 1 Temperature    5813.   0
#> 2 Precipitation    83.0  1.27e-13
#> Ljung-Box test on (difference(count, 12))
```

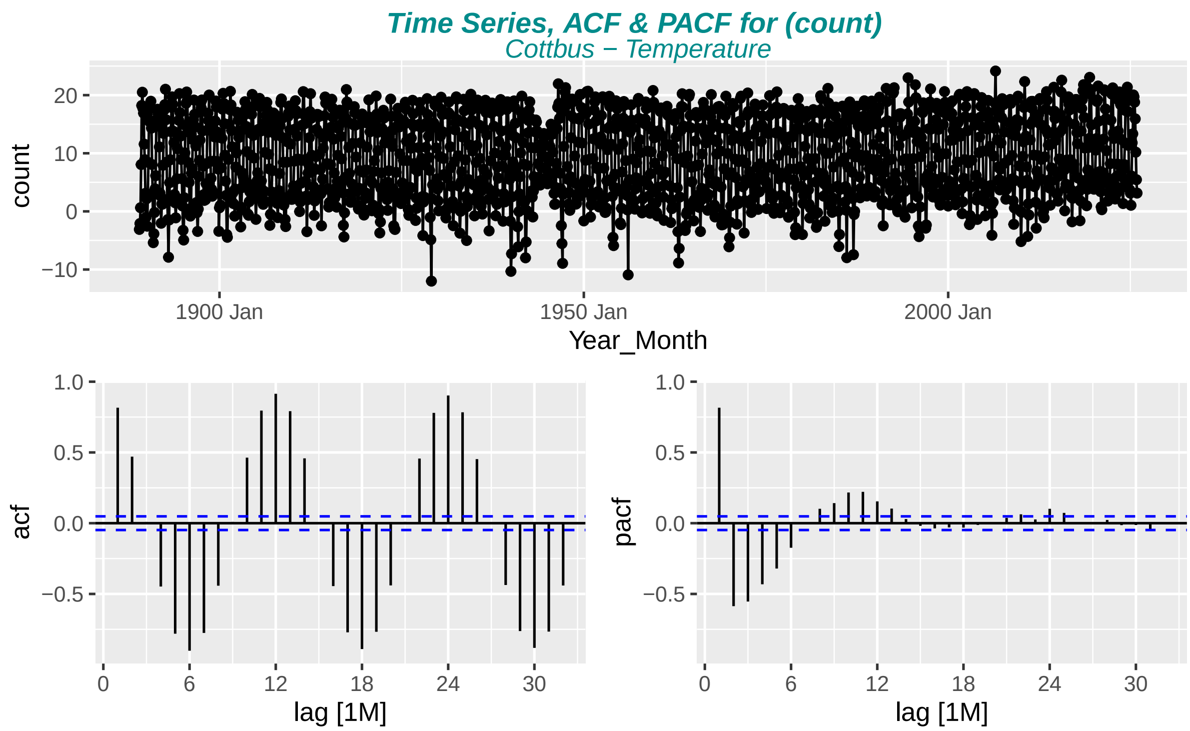
```
#> # A tibble: 2 x 3
#>   Measure       lb_stat lb_pvalue
#>   <fct>           <dbl>     <dbl>
#> 1 Temperature     100.      0
#> 2 Precipitation    10.6     0.386
#> Ljung-Box test on (difference(count, 12) + difference())
#> # A tibble: 2 x 3
#>   Measure       lb_stat lb_pvalue
#>   <fct>           <dbl>     <dbl>
#> 1 Temperature      296.         0
#> 2 Precipitation    388.         0
```

1.1.3 ACF (Autocorrelation Function) Plots of Differences

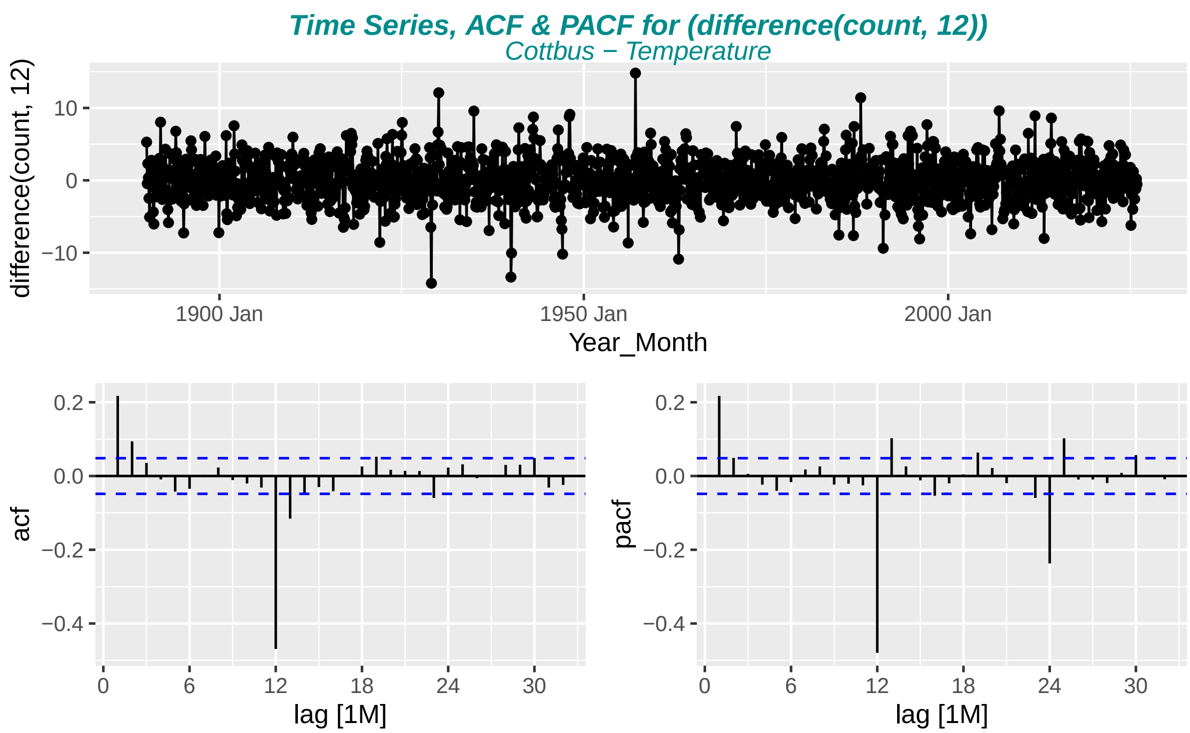


1.1.4 Time Series, ACF and PACF (Partial) Plots of Differences - for ARIMA p, q check

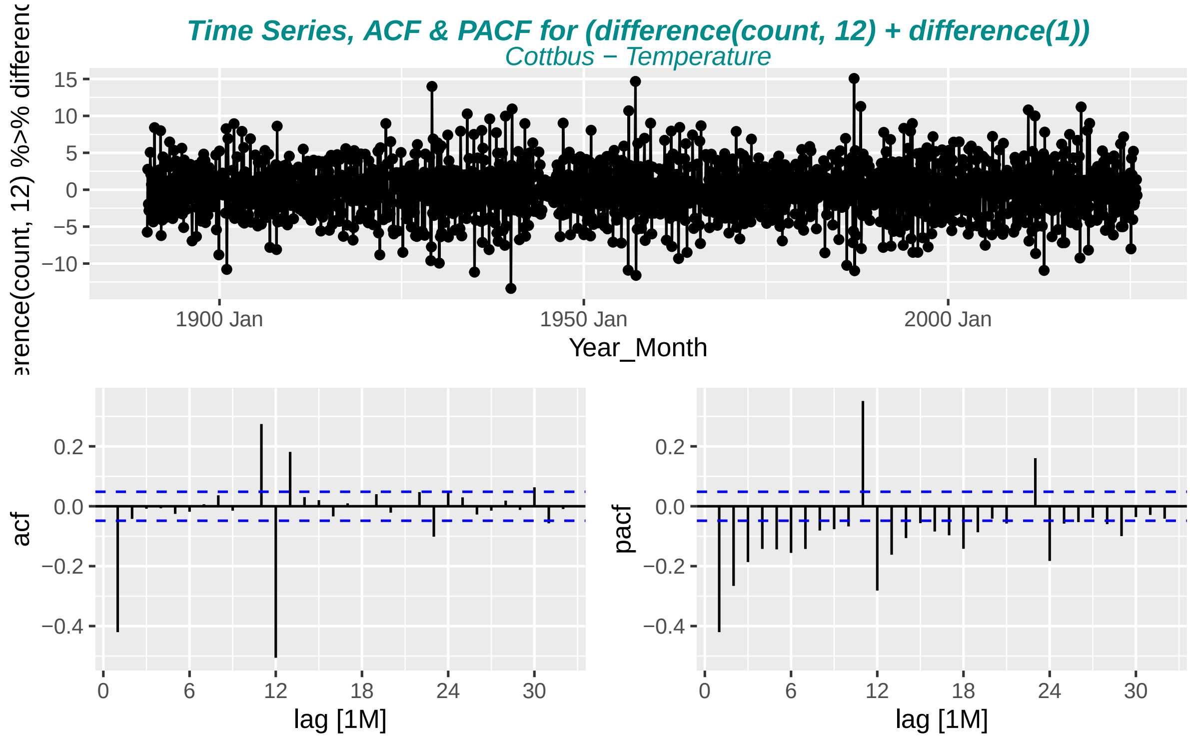
```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City    Measure         Sum  Mean
#>   <chr>   <fct>         <dbl> <dbl>
#> 1 Cottbus Temperature 15182.   9.23
```



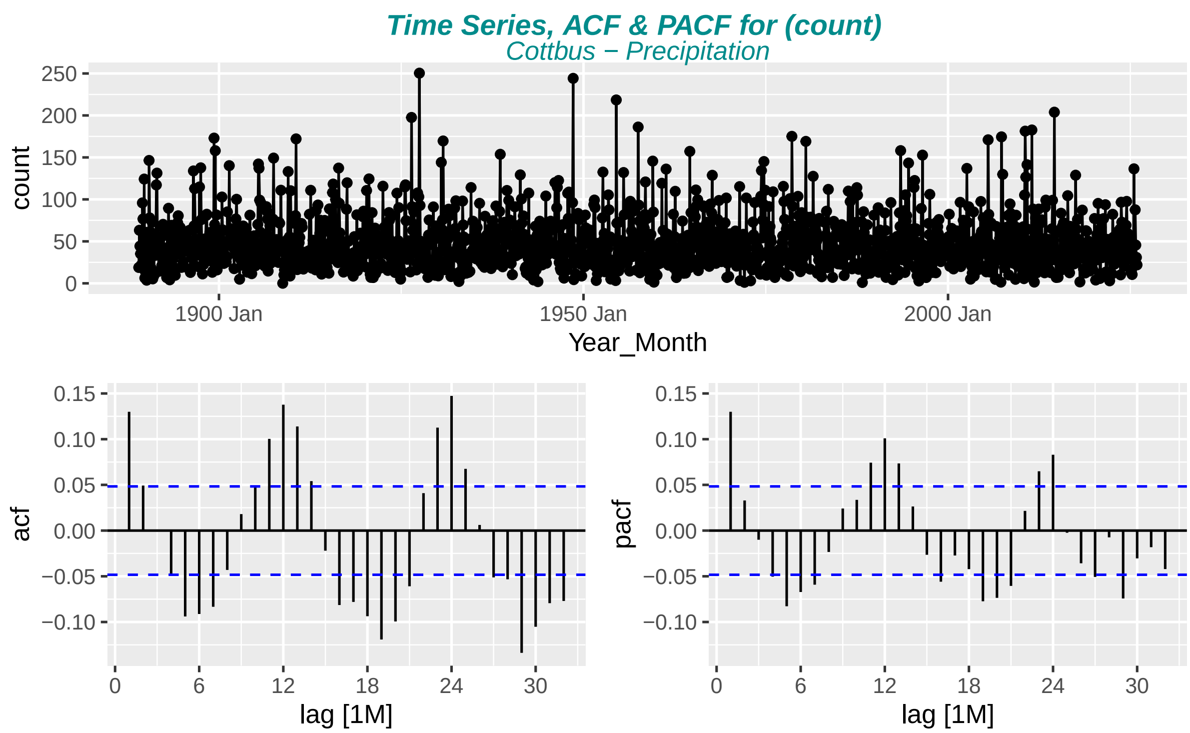

```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City     Measure       Sum   Mean
#>   <chr>    <fct>       <dbl>  <dbl>
#> 1 Cottbus Temperature  29.3 0.0180
```



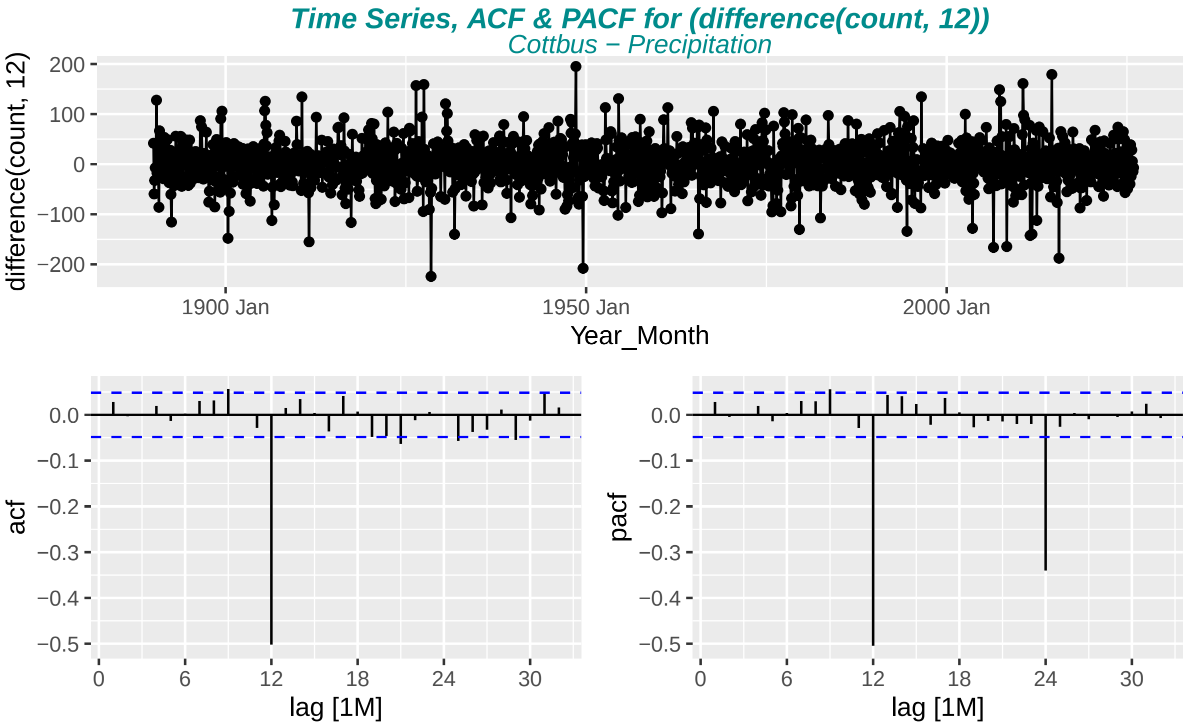

```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City     Measure       Sum     Mean
#>   <chr>    <fct>       <dbl>    <dbl>
#> 1 Cottbus Temperature -5.83 -0.00357
```



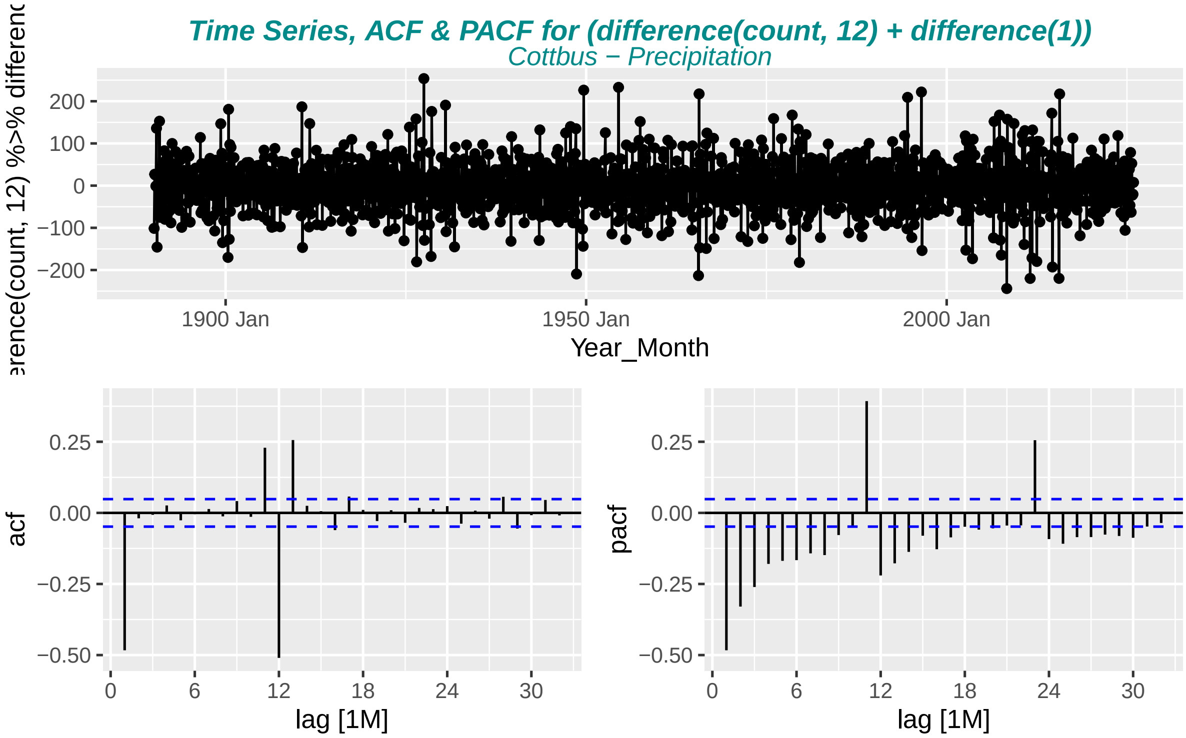

```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City    Measure          Sum  Mean
#>   <chr>   <fct>          <dbl> <dbl>
#> 1 Cottbus Precipitation 78260.  47.6
```




```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City    Measure         Sum    Mean
#>   <chr>   <fct>         <dbl>   <dbl>
#> 1 Cottbus Precipitation -43.6 -0.0267
```




```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City     Measure          Sum    Mean
#>   <chr>    <fct>          <dbl>   <dbl>
#> 1 Cottbus Precipitation -48.9 -0.0300
```



2 ExponenTial Smoothing (ETS) Forecasting Models

Forecasts produced using exponential smoothing methods are weighted averages of past observations, with the weights decaying exponentially as the observations get older.

The parameters are estimated by maximising the "likelihood". The likelihood is the probability of the data arising from the specified model. AIC, AICc and BIC can be used here to determine which of the ETS models is most appropriate for a given time series (see output glance(fit_ets)).

The model selection is based on recognising key components of the time series (trend and seasonal) and the way in which these enter the smoothing method (e.g., in an additive, damped or multiplicative manner).

- Mauna Loa CO_2 data best Models: ETS(M,A,A) & ETS(A,A,A)
- Basel Temperature data best Models: ETS(A,N,A), ETS(A,A,A), ETS(A,Ad,A) (close togehter). Best Forecast accuracy is with ETS(A,A,A), ETS(A,Ad,A).
- Basel Precipitation data best Models: ETS(A,N,A), ETS(A,Ad,A), ETS(A,A,A) (close togehter). Best Forecast accuracy is with ETS(A,A,A), ETS(A,Ad,A), ETS(A,N,A),

Trend term "N" for Basel Temperature/Precipitation correspondends to a "pure" exponential smooothing which results in a slope $\beta = 0$. This results in a forecast predicting a constant level. This does not fit to the result of the STL decomposition. Therefore best model choice is **ETS(A,A,A)**.

Method Selection

Error term: either additive ("A") or multiplicative ("M").

Both methods provide identical point forecasts, but different prediction intervals and different likelihoods. AIC & BIC are able to select between the error types because they are based on likelihood.

Nevertheless, difference is for

- Mauna Loa CO_2 not relevant and AIC/AICc/BIC values are only a little bit smaller for multiplicative errors. The prediction intervall plots are fully overlapping.
- Basel Temperature AIC/AICc/BIC of additive error types are much better than the multiplicative ones.
- Basel Precipitation AIC/AICc/BIC of additive error types are much better than the multiplicative ones.

Note: For Basel Temperature and Precipitation Forecast plots the models ETS_MAdA, ETS_MMA, ETS_MMA, ETS_MNA are to be taken out since forecasts with multiplicative errors are exploding (forecast > 3 years impossible !!)

Therefore finally **Error term = "A"** is chosen in general.

Trend term: either none ("N"), additive ("A"), multiplicative ("M") or damped variants ("Ad", "Md").

Note: Mauna Loa CO_2 model ETS(A,Ad,A) fit plot shows to strong damping. For Basel Temperature model ETS(A,N,A) and ETS(A,Ad,A) are providing more or less the same forecast. This means that forecast remains on constant level since Trend "N" means "pure" exponentiell smoothing without trend (see above).

Therefore finally **Trend term = "A"** is chosen in general.

Seasonal term: either none ("N"), additive ("A") or multiplicative ("M").

For CO2 and Temperature Data we have a clear seasonal pattern and seasonal term adds always a (more or less) fix amount on level and trend component. Therefore "A" additve term is chosen. For Precipitation the seasonal patttern is only slight. Indead, a multiplicative seasonal term results in "exploding" forecasts.

Since monthly data are strongly seasonal **seasonal term "A"** is chosen.

2.1 ETS Models and their componentes

ETS model with automatically selected $ETS(A|M, N|A|M, N|A|M)$

```
#> [1] "model(ETS(count)) => provides default / best automatically chosen model"
#> # A mable: 2 x 3
#> # Key:     City, Measure [2]
#>   City    Measure                 ETS
#>   <chr>   <fct>               <model>
#> 1 Cottbus Temperature   <ETS(A,N,A)>
#> 2 Cottbus Precipitation <ETS(M,N,M)>
#> [1] "Cottbus Temperature"
#> Series: count
#> Model: ETS(A,N,A)
#>   Smoothing parameters:
#>     alpha = 0.07133338
#>     gamma = 0.0001001933
#>
#>   Initial states:
#>      l[0]    s[0]     s[-1]   s[-2]    s[-3]    s[-4]    s[-5]    s[-6]
#>  9.916294 -7.9282 -4.773858 0.20368 4.739316 9.074699 9.623185 7.850016
#>     s[-7]      s[-8]     s[-9]   s[-10]    s[-11]
#>  4.496063 -0.6140388 -5.055738 -8.34123 -9.273894
#>
#>   sigma^2:  3.7136
#>
#>      AIC     AICc      BIC
#> 5697.569 5698.251 5766.258
#> [1] "Cottbus Precipitation"
#> Series: count
#> Model: ETS(M,N,M)
#>   Smoothing parameters:
#>     alpha = 0.009828973
#>     gamma = 0.0001003345
#>
#>   Initial states:
#>      l[0]      s[0]     s[-1]     s[-2]    s[-3]    s[-4]    s[-5]    s[-6]
#>  57.63916 0.8651934 0.8677062 0.8880489 0.970293 1.480015 1.488201 1.180102
#>     s[-7]     s[-8]     s[-9]    s[-10]    s[-11]
#>  1.138445 0.7754194 0.8165443 0.7156538 0.8143788
#>
#>   sigma^2:  0.3132
#>
#>      AIC     AICc      BIC
#> 9474.272 9474.954 9542.961
#> # A tibble: 2 x 8
#>   City    Measure       .model   AIC  AICc   BIC    MSE   MAE
#>   <chr>   <fct>         <chr>  <dbl> <dbl> <dbl>  <dbl> <dbl>
#> 1 Cottbus Temperature   ETS    5698. 5698. 5766.   3.64 1.48
#> 2 Cottbus Precipitation ETS    9474. 9475. 9543. 815.   0.436
```

Fit of different pre-defined $ETS(A|M, N|A|M, N|A|M)$ models

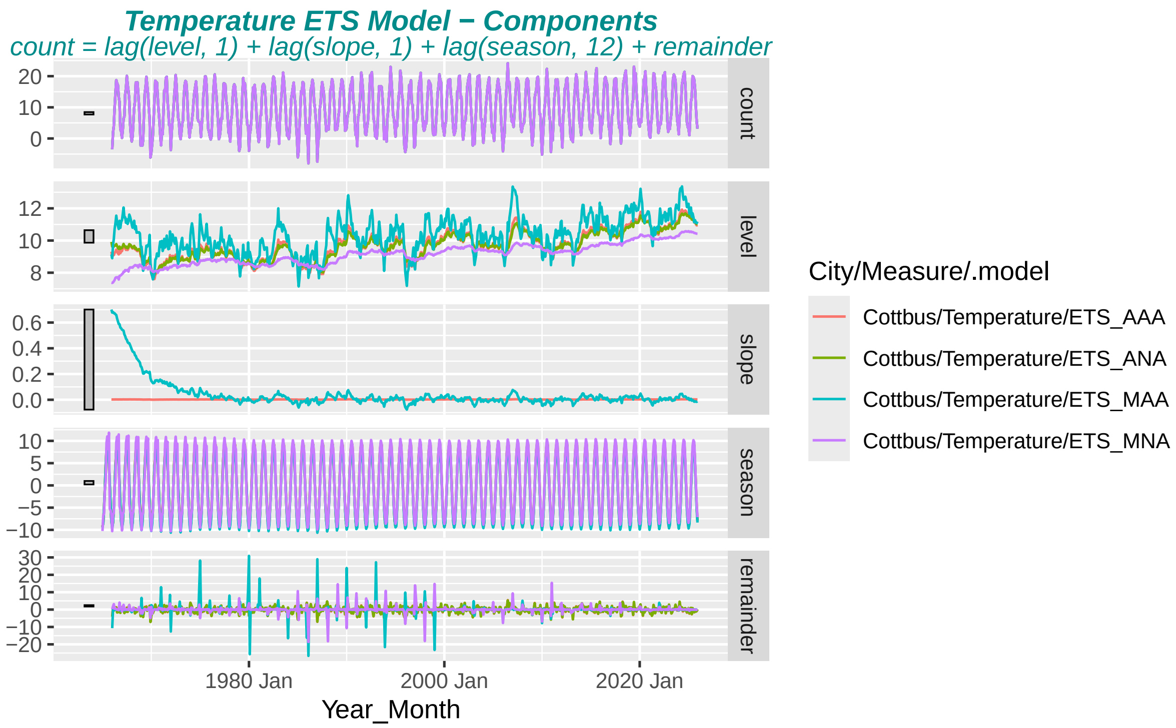
Model Selection by Information Criterion - lowest AIC, AICc, BIC

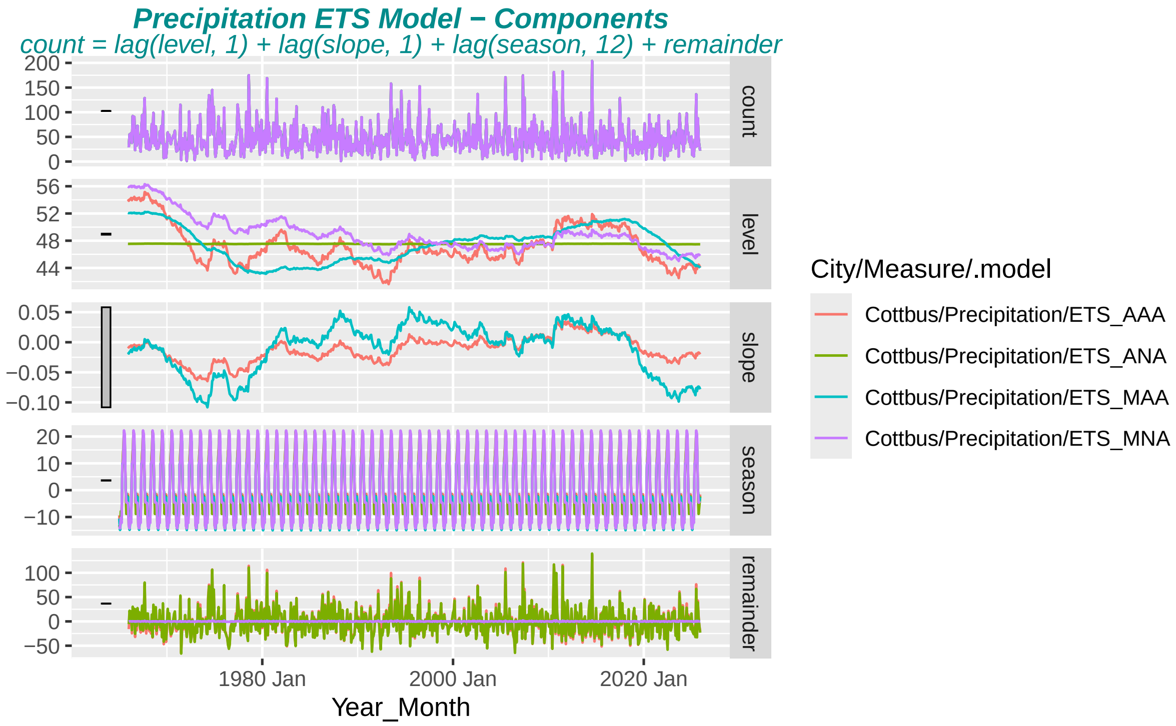
Best model fit with

- CV, AIC, AICc and BIC with the lowest values
- Adjusted R^2 the model with the highest value.

```
#> # A tibble: 16 x 9
#>    City    Measure       .model     AIC  AICc   BIC    MSE   AMSE    MAE
```

```
#>    <chr>   <fct>         <chr>    <dbl> <dbl> <dbl>  <dbl>  <dbl>  <dbl>
#>  1 Cottbus Temperature   ETS_ANA  5698. 5698. 5766.   3.64   3.69  1.48
#>  2 Cottbus Temperature   ETS_AAA  5703. 5703. 5780.   3.65   3.72  1.48
#>  3 Cottbus Temperature   ETS_AAdA 5703. 5704. 5786.   3.64   3.70  1.48
#>  4 Cottbus Temperature   ETS_AMA  5705. 5706. 5783.   3.66   3.73  1.49
#>  5 Cottbus Temperature   ETS_MNA  8528. 8529. 8597.   5.14   5.16  0.789
#>  6 Cottbus Temperature   ETS_MAdA 8621. 8622. 8703.   5.05   5.53  0.792
#>  7 Cottbus Temperature   ETS_MAA  9251. 9252. 9329.   4.28   4.62  1.05
#>  8 Cottbus Temperature   ETS_MMA  9431. 9432. 9509.   4.31   4.35  1.12
#>  9 Cottbus Precipitation ETS_MNA  9476. 9477. 9545. 810.   811.    0.438
#> 10 Cottbus Precipitation ETS_MMA  9489. 9490. 9567. 808.   809.    0.447
#> 11 Cottbus Precipitation ETS_MAA  9490. 9491. 9568. 810.   811.    0.447
#> 12 Cottbus Precipitation ETS_MAdA 9492. 9493. 9574. 809.   810.    0.438
#> 13 Cottbus Precipitation ETS_ANA  9580. 9580. 9648. 800.   800.   21.3
#> 14 Cottbus Precipitation ETS_AAdA 9585. 9586. 9667. 798.   799.   21.1
#> 15 Cottbus Precipitation ETS_AMA  9589. 9590. 9667. 805.   806.   21.2
#> 16 Cottbus Precipitation ETS_AAA  9590. 9591. 9668. 807.   808.   21.2
```





2.1.1 Residual Accuracy with one-step-ahead fitted residuals - check RMSE, MAE

Residual accuracy can be computed directly from models as the one-step-ahead fitted residuals are available. Select forecast models that minimises for lowest

- MAE (Mean absolute error, will lead to forecasts of the median) and
- RMSE (Root mean squared error, lead to forecasts of the mean)

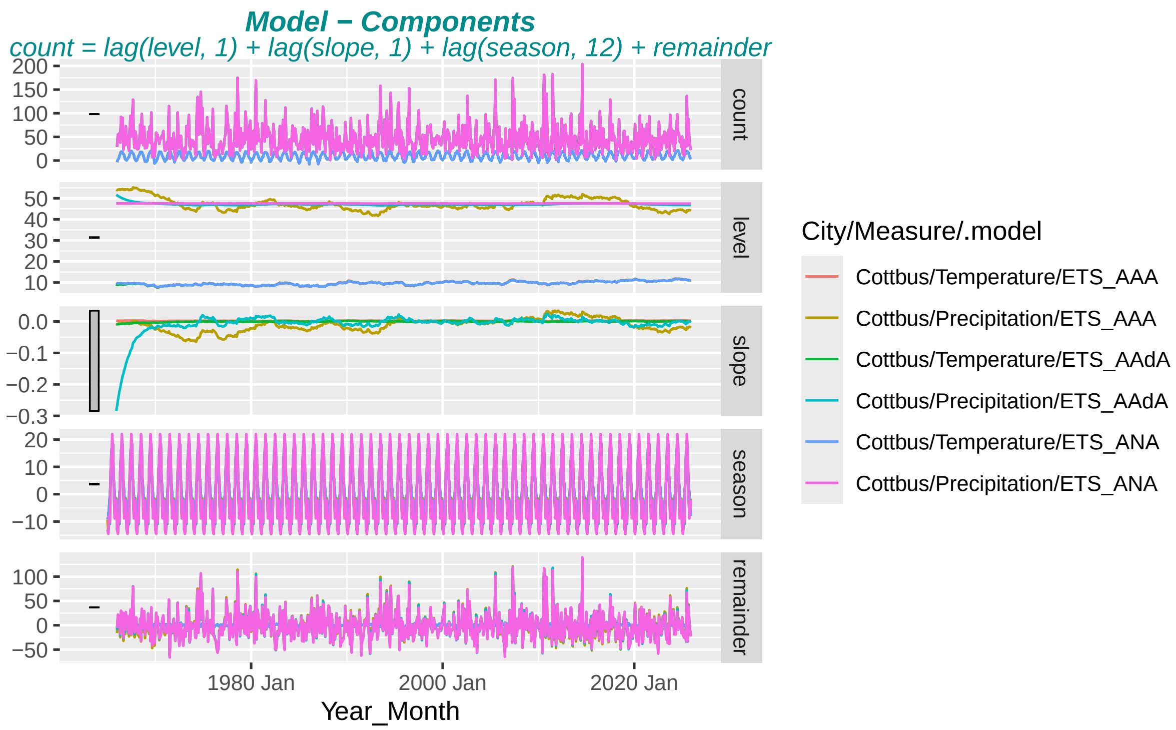
```
#> # A tibble: 16 x 7
#>    City    Measure       .model   .type          ME  RMSE   MAE
#>    <chr>   <fct>         <chr>    <chr>       <dbl> <dbl> <dbl>
#>  1 Cottbus Temperature   ETS_AAdA Training  0.0380   1.91  1.48
#>  2 Cottbus Temperature   ETS_ANA  Training  0.0216   1.91  1.48
#>  3 Cottbus Temperature   ETS_AAA  Training  0.00122  1.91  1.48
#>  4 Cottbus Temperature   ETS_AMA  Training  0.00294  1.91  1.49
#>  5 Cottbus Temperature   ETS_MAA  Training -0.158    2.07  1.62
#>  6 Cottbus Temperature   ETS_MMA  Training -0.0871   2.08  1.59
#>  7 Cottbus Temperature   ETS_MAdA Training  0.00541  2.25  1.75
#>  8 Cottbus Temperature   ETS_MNA  Training  0.175    2.27  1.73
#>  9 Cottbus Precipitation ETS_AAdA Training -0.190   28.3  21.1
#> 10 Cottbus Precipitation ETS_ANA  Training -0.568   28.3  21.3
#> 11 Cottbus Precipitation ETS_AMA  Training -0.180   28.4  21.2
#> 12 Cottbus Precipitation ETS_AAA  Training -0.180   28.4  21.2
#> 13 Cottbus Precipitation ETS_MMA  Training -0.403   28.4  21.3
#> 14 Cottbus Precipitation ETS_MAdA Training -1.62    28.4  21.5
#> 15 Cottbus Precipitation ETS_MAA  Training -0.533   28.5  21.4
#> 16 Cottbus Precipitation ETS_MNA  Training -2.01    28.5  21.6
```

2.1.2 Ljung-Box Test - independence/white noise of the forecasts residuals

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 16 x 5
#>    City    Measure       .model   lb_stat lb_pvalue
#>    <chr>   <fct>         <chr>      <dbl>     <dbl>
#>  1 Cottbus Precipitation ETS_MNA     24.9 0.252
```

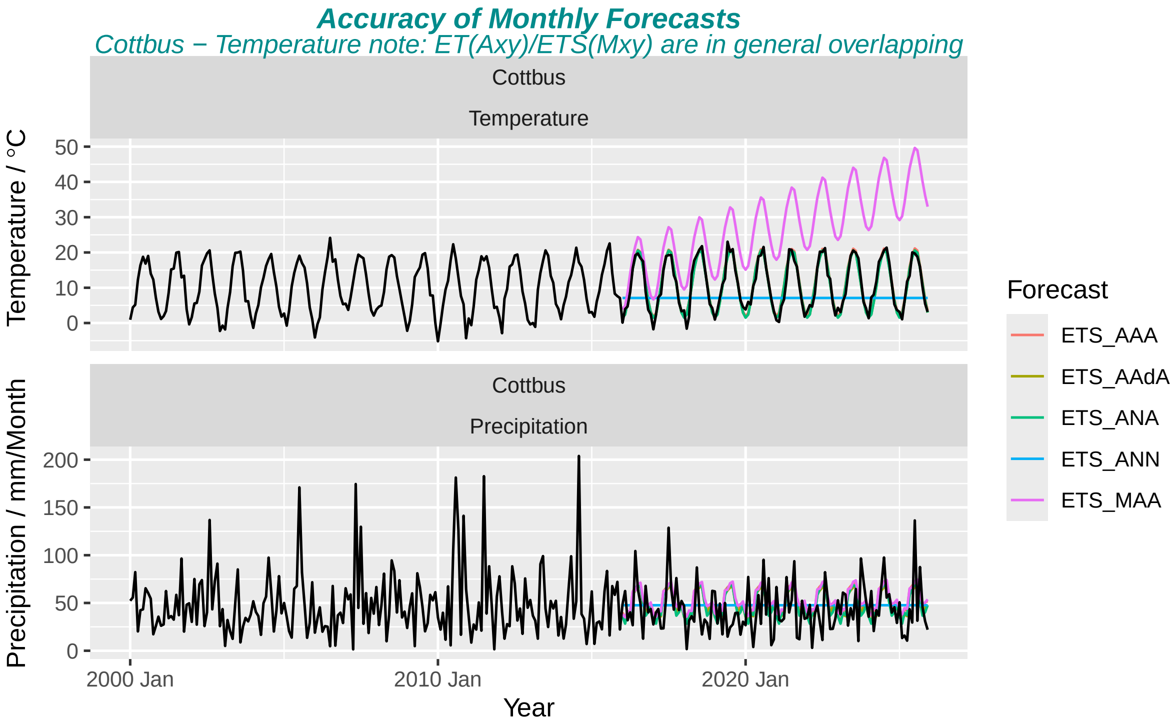
```
#>   2 Cottbus Precipitation ETS_ANA     25.0 0.246
#>   3 Cottbus Precipitation ETS_AAdA    25.6 0.224
#>   4 Cottbus Precipitation ETS_AAA     26.2 0.199
#>   5 Cottbus Precipitation ETS_AMA     26.3 0.197
#>   6 Cottbus Precipitation ETS_MMA     26.3 0.195
#>   7 Cottbus Precipitation ETS_MAdA    27.7 0.148
#>   8 Cottbus Precipitation ETS_MAA     27.9 0.143
#>   9 Cottbus Temperature   ETS_MAA     29.7 0.0991
#>  10 Cottbus Temperature   ETS_AMA     29.7 0.0975
#>  11 Cottbus Temperature   ETS_AAA     30.3 0.0862
#>  12 Cottbus Temperature   ETS_ANA     31.3 0.0691
#>  13 Cottbus Temperature   ETS_AAdA    31.9 0.0597
#>  14 Cottbus Temperature   ETS_MAdA    66.2 0.00000143
#>  15 Cottbus Temperature   ETS_MMA    215.   0
#>  16 Cottbus Temperature   ETS_MNA    154.   0
```

2.1.3 ETS Models - components of ETS(A,N,A), ETS(A,A,A), ETS(A,Ad,A), models



2.1.4 Forecast Accuracy with Training/Test Data

```
#> # A tibble: 10 x 7
#>    .model   City    Measure       .type       ME  RMSE   MAE
#>    <chr>    <chr>   <fct>         <chr>    <dbl> <dbl> <dbl>
#>  1 ETS_ANA  Cottbus Temperature   Test    0.0291  1.67  1.30
#>  2 ETS_AAdA Cottbus Temperature   Test   -0.0104  1.67  1.30
#>  3 ETS_AAA  Cottbus Temperature   Test   -0.377   1.69  1.32
#>  4 ETS_ANN  Cottbus Temperature   Test    3.82    7.89  6.50
#>  5 ETS_MAA  Cottbus Temperature   Test  -16.1    18.0  16.1
#>  6 ETS_AAdA Cottbus Precipitation Test   -4.68   25.2  20.2
#>  7 ETS_ANN  Cottbus Precipitation Test   -4.16   25.3  20.0
#>  8 ETS_ANA  Cottbus Precipitation Test   -4.57   25.4  20.5
#>  9 ETS_AAA  Cottbus Precipitation Test   -8.75   26.2  21.3
#> 10 ETS_MAA  Cottbus Precipitation Test   -9.73   26.4  21.7
```

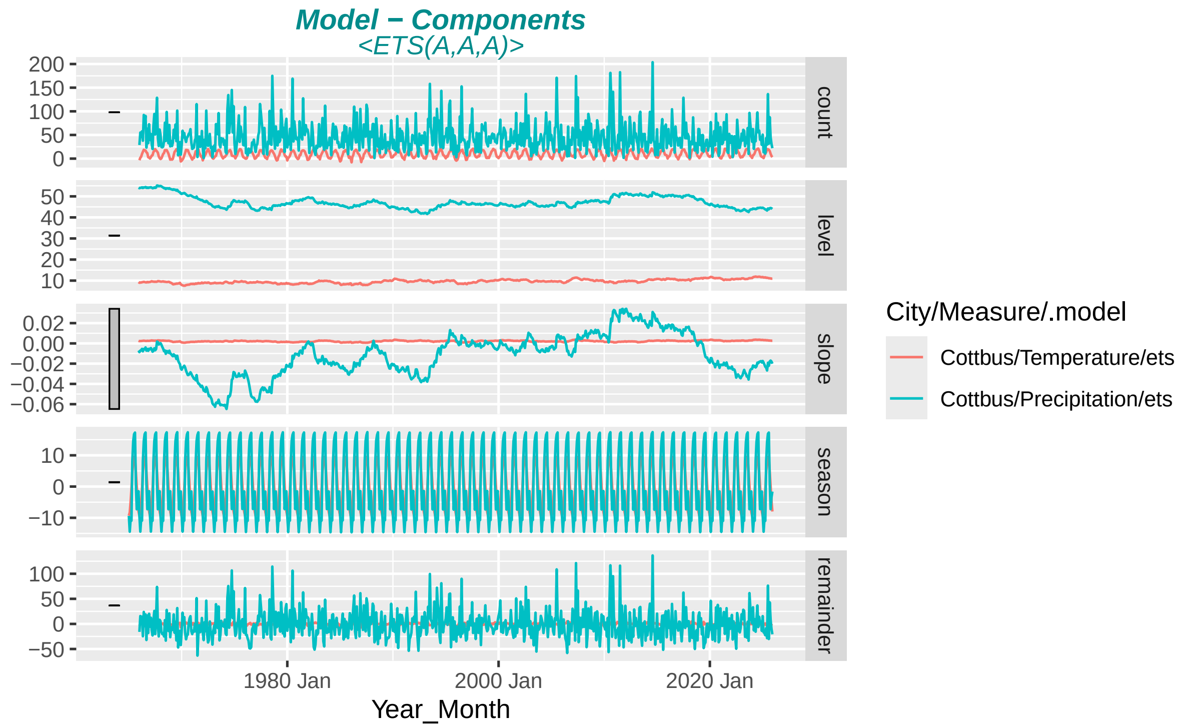


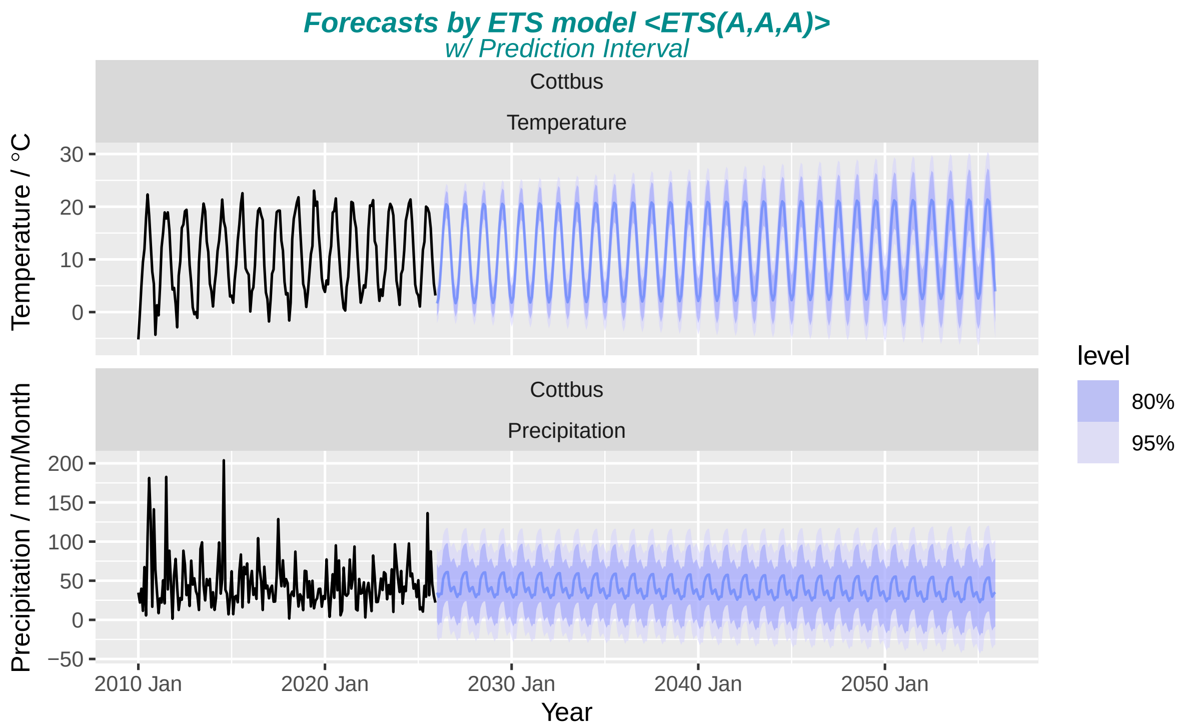
Accuracy of Monthly Forecasts
Cottbus − Temperature note: ET(Axy)/ETS(Mxy) are in general overlapping
Cottbus
Temperature
Cottbus
Precipitation
Temperature / °C
Precipitation / mm/Month
50
40
30
20
10
0
200
150
100
50
0
2000 Jan
2010 Jan
2020 Jan
Year
Forecast
ETS_AAA
ETS_AAdA
ETS_ANA
ETS_ANN
ETS_MAA

2.2 Forecasting with selected ETS model <ETS(A,A,A)>, <ETS(A,A,A)>

2.2.1 Forecast Plot of selected ETS model

```
#> Provide model coefficients by report(fit_model)
#> # A tibble: 2 x 11
#>   City    Measure    .model sigma2 log_lik   AIC  AICc   BIC    MSE   AMSE   MAE
#>   <chr>   <fct>      <chr>   <dbl>   <dbl> <dbl> <dbl> <dbl>  <dbl>  <dbl> <dbl>
#> 1 Cottbus Temperatu~ ets      3.73  -2834. 5703. 5703. 5780.   3.65   3.72  1.48
#> 2 Cottbus Precipita~ ets    825.    -4778. 9590. 9591. 9668. 807.   808.   21.2
```

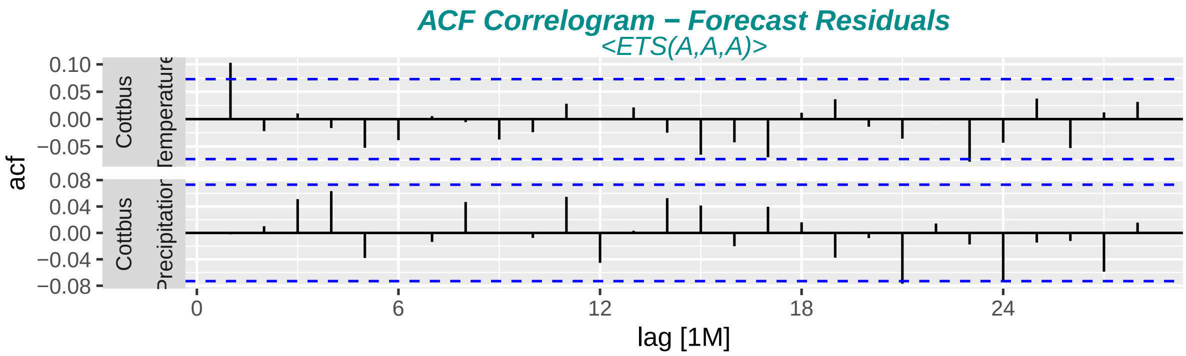


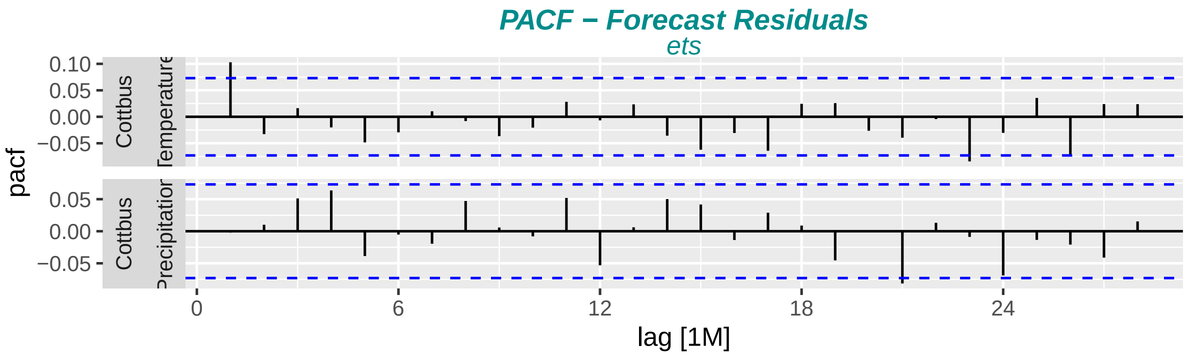


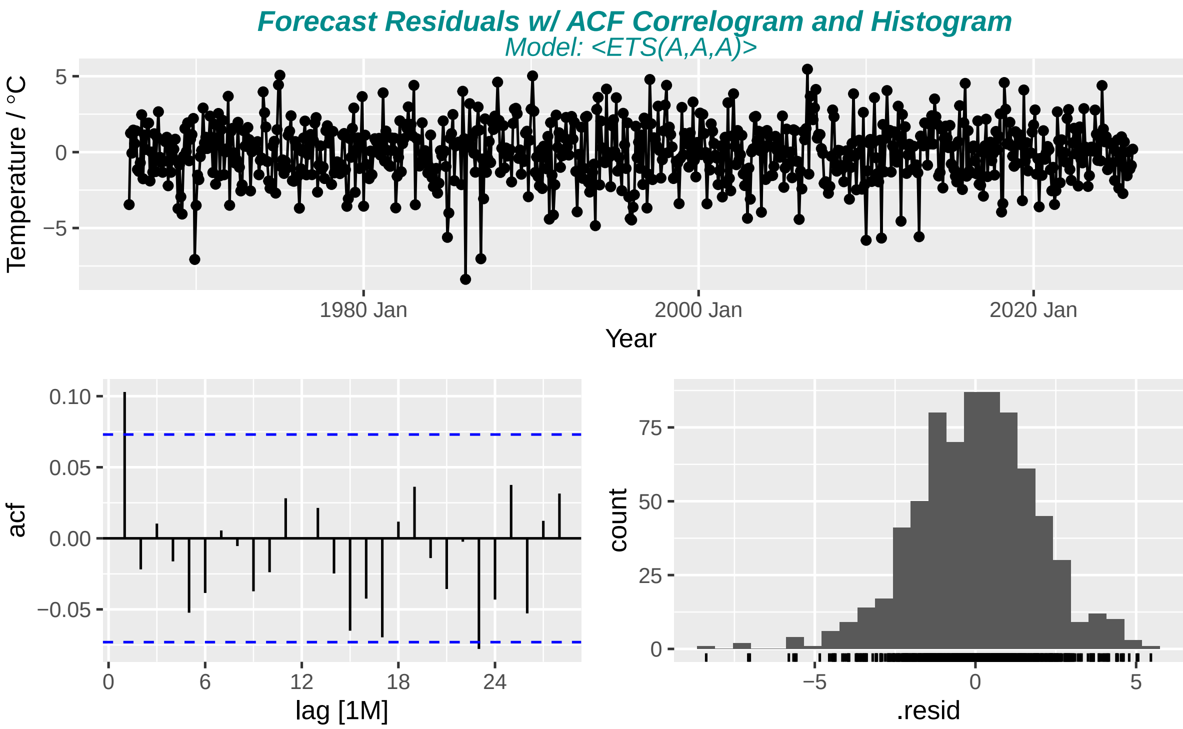
2.2.2 Residual Stationarity

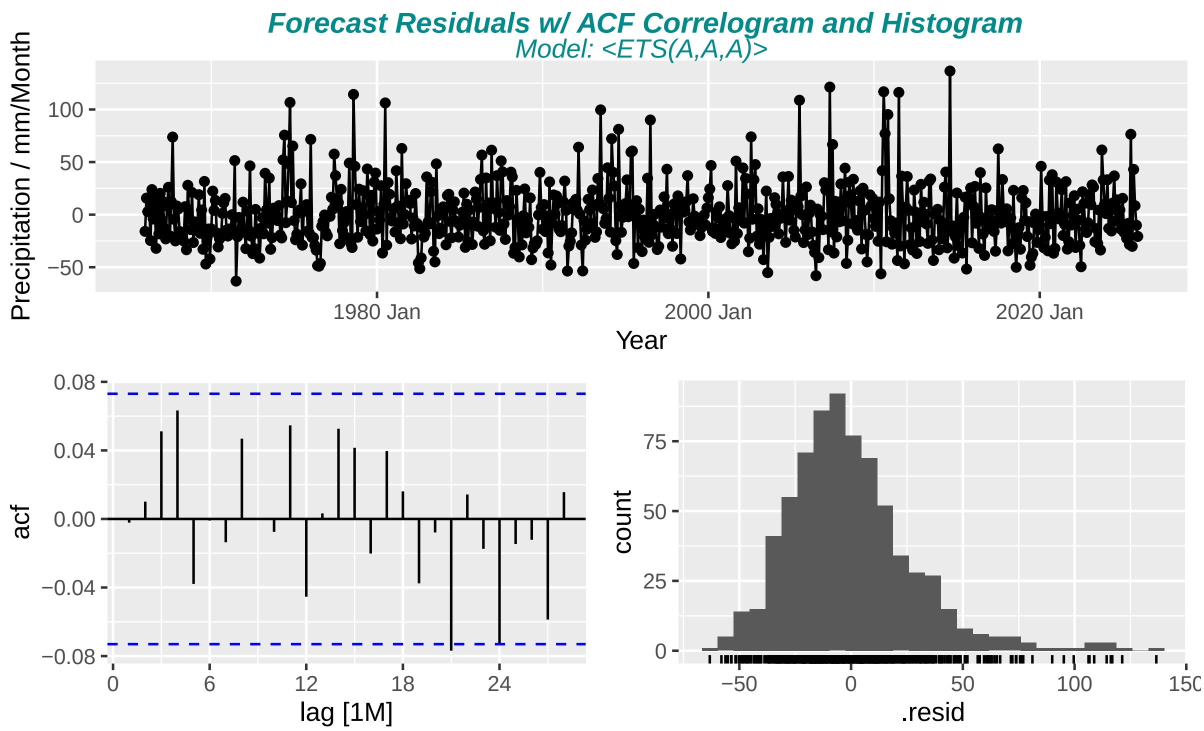
Required checks to be ready for forecasting:

- ACF Forecast Residual: all spikes are within the significance limits, so the residuals appear to be white noise
- The Ljung-Box test also shows that the residuals have no remaining autocorrelations
- Forecast Residuals are more or less normally distributed with roughly centred on zero



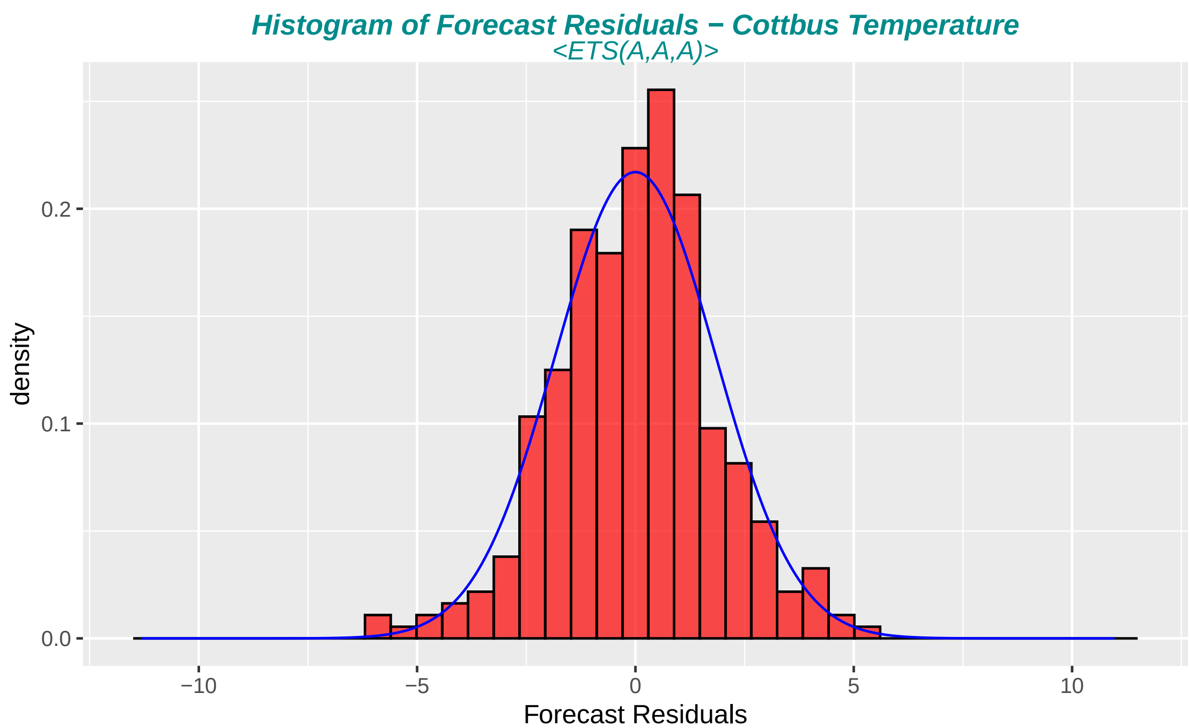


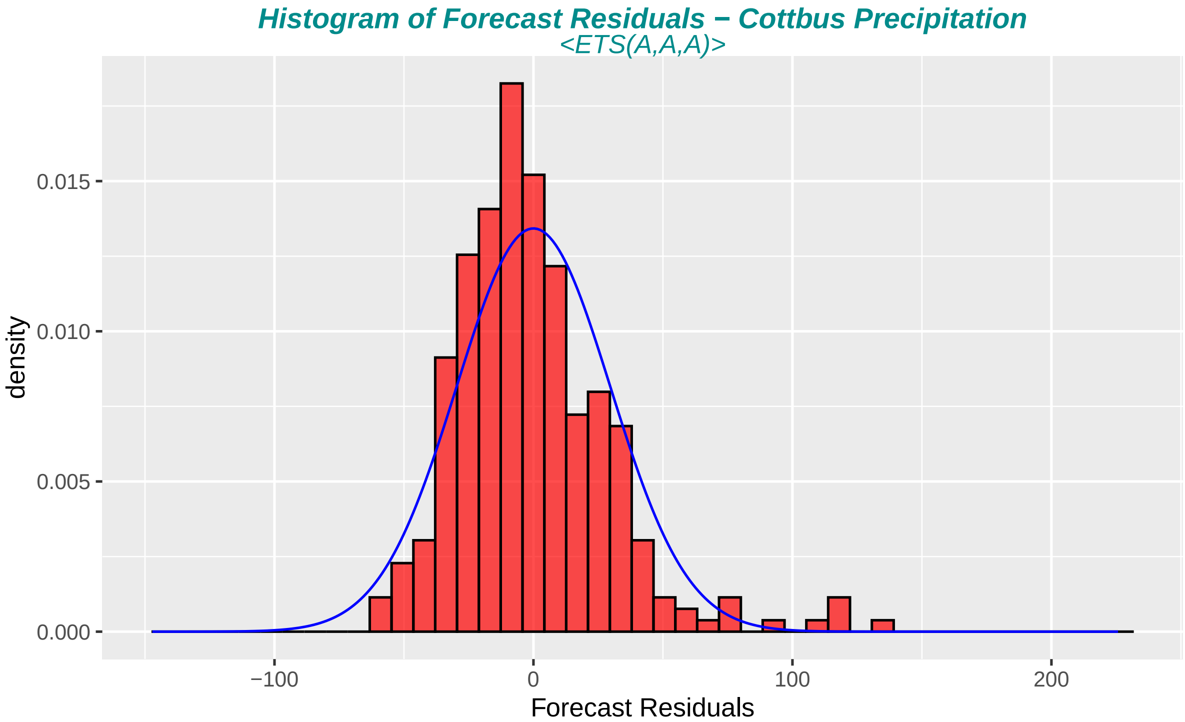




2.2.3 Ljung-Box Test and Histogram of forecast residuals with overlaid normal curve

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 2 x 5
#>   City    Measure       .model lb_stat lb_pvalue
#>   <chr>   <fct>         <chr>    <dbl>     <dbl>
#> 1 Cottbus Temperature   ets       18.5     0.778
#> 2 Cottbus Precipitation ets       25.5     0.381
```





Histogram of Forecast Residuals − Cottbus Precipitation
<ETS(A,A,A)>
density
0.015
0.010
0.005
0.000
−100
0
100
200
Forecast Residuals

3 ARIMA Forecasting Models - AutoRegressive-Integrated Moving Average

Exponential smoothing and ARIMA (AutoRegressive-Integrated Moving Average)models are the two most widely used approaches to time series forecasting, and provide complementary approaches to the problem.

While exponential smoothing models are based on a description of the trend and seasonality in the data, ARIMA models aim to describe the autocorrelations in the data.

3.1 Seasonal ARIMA models

Non-seasonal ARIMA models are generally denoted ARIMA(p,d,q) where parameters p, d, and q are non-negative integers, * p is the order (number of time lags) of the autoregressive model * d is the degree of differencing (number of times the data have had past values subtracted) * q is the order of the moving-average model of past forecast errors .

The value of d has an effect on the prediction intervals — the higher the value of d, the more rapidly the prediction intervals increase in size. For d=0, the point forecasts are equal to the mean of the data and the long-term forecast standard deviation will go to the standard deviation of the historical data, so the prediction intervals will all be essentially the same.

Seasonal ARIMA models are usually denoted ARIMA(p,d,q)(P,D,Q)m, where m refers to the number of periods in each season, and the uppercase P,D,Q refer to the autoregressive, differencing, and moving average terms for the seasonal part of the ARIMA model.

$ARIMA(pdq)(PDQ)$ model with automatically selected $(pdq)(PDQ)$ values

Fit of different pre-defined $ARIMA(pdq)(PDQ)$ models

```
#> Model Selection by Information Criterion - lowest AIC, AICc, BIC
#>  choose p, q parameter accordingly - but only for same d, D values
#> # A tibble: 8 x 8
#>   City    Measure     .model          sigma2 log_lik   AIC  AICc   BIC
#>   <chr>   <fct>       <chr>            <dbl>   <dbl> <dbl> <dbl> <dbl>
#> 1 Cottbus Temperature arima_111_011     3.62  -1484. 2976. 2976. 2995.
#> 2 Cottbus Temperature arima_012_011     3.63  -1485. 2978. 2978. 2996.
#> 3 Cottbus Temperature arima_111_012     3.62  -1484. 2978. 2978. 3000.
#> 4 Cottbus Temperature arima_211_011     3.62  -1484. 2978. 2978. 3001.
#> 5 Cottbus Temperature arima_012_112     3.64  -1484. 2981. 2981. 3008.
#> 6 Cottbus Temperature arima_100_210     4.96  -1573. 3153. 3153. 3171.
#> 7 Cottbus Temperature arima_200_011     5.41  -1602. 3213. 3213. 3231.
#> 8 Cottbus Temperature arima_100_110_c   5.42  -1602. 3214. 3214. 3237.
#> # A tibble: 8 x 8
#>   City    Measure       .model          sigma2 log_lik   AIC  AICc   BIC
#>   <chr>   <fct>         <chr>            <dbl>   <dbl> <dbl> <dbl> <dbl>
#> 1 Cottbus Precipitation arima_111_011     826.  -3399. 6807. 6807. 6825.
#> 2 Cottbus Precipitation arima_012_011     826.  -3399. 6807. 6807. 6825.
#> 3 Cottbus Precipitation arima_111_012     827.  -3399. 6808. 6808. 6830.
#> 4 Cottbus Precipitation arima_012_112     825.  -3398. 6808. 6808. 6836.
#> 5 Cottbus Precipitation arima_211_011     827.  -3399. 6809. 6809. 6832.
#> 6 Cottbus Precipitation arima_001_002     902.  -3469. 6948. 6948. 6971.
#> 7 Cottbus Precipitation arima_200_011    1278.  -3537. 7082. 7082. 7100.
#> 8 Cottbus Precipitation arima_100_110_c  1280.  -3537. 7084. 7084. 7107.
```

Good models are obtained by minimising the AIC, AICc or BIC (see glance(fit_arima) output). The preference is to use the AICc to selec p and q.

These information criteria tend not to be good guides to selecting the appropriate order of differencing (d) of a model, but only for selecting the values of p and q. This is because the differencing changes the data on which the likelihood is computed, making the AIC values between models with different orders of differencing not comparable.

3.1.1 Residual Accuracy with one-step-ahead fitted residuals - check RMSE, MAE

Residual accuracy can be computed directly from models as the one-step-ahead fitted residuals are available. Select forecast models that minimises for lowest

- MAE (Mean absolute error, will lead to forecasts of the median) and
- RMSE (Root mean squared error, lead to forecasts of the mean)

```
#> # A tibble: 8 x 7
#>   City    Measure     .model          .type         ME  RMSE   MAE
#>   <chr>   <fct>       <chr>           <chr>      <dbl> <dbl> <dbl>
#> 1 Cottbus Temperature arima_211_011   Training 0.0946   1.88  1.43
#> 2 Cottbus Temperature arima_111_012   Training 0.0928   1.88  1.43
#> 3 Cottbus Temperature arima_111_011   Training 0.0902   1.88  1.43
#> 4 Cottbus Temperature arima_012_112   Training 0.0832   1.88  1.44
#> 5 Cottbus Temperature arima_012_011   Training 0.0781   1.88  1.44
#> 6 Cottbus Temperature arima_100_210   Training 0.0508   2.20  1.69
#> 7 Cottbus Temperature arima_100_110_c Training 0.00223  2.30  1.76
#> 8 Cottbus Temperature arima_200_110_c Training 0.00223  2.30  1.76
#> # A tibble: 8 x 7
#>   City    Measure       .model          .type         ME  RMSE   MAE
#>   <chr>   <fct>         <chr>           <chr>      <dbl> <dbl> <dbl>
#> 1 Cottbus Precipitation arima_012_112   Training  0.648   28.4  20.7
#> 2 Cottbus Precipitation arima_211_011   Training  0.637   28.4  20.7
#> 3 Cottbus Precipitation arima_012_011   Training  0.645   28.4  20.7
#> 4 Cottbus Precipitation arima_111_011   Training  0.644   28.4  20.7
#> 5 Cottbus Precipitation arima_111_012   Training  0.656   28.4  20.7
#> 6 Cottbus Precipitation arima_001_002   Training -0.0100  29.9  22.2
#> 7 Cottbus Precipitation arima_100_110_c Training  0.0216  35.4  25.9
#> 8 Cottbus Precipitation arima_200_110_c Training  0.0216  35.4  25.9
```

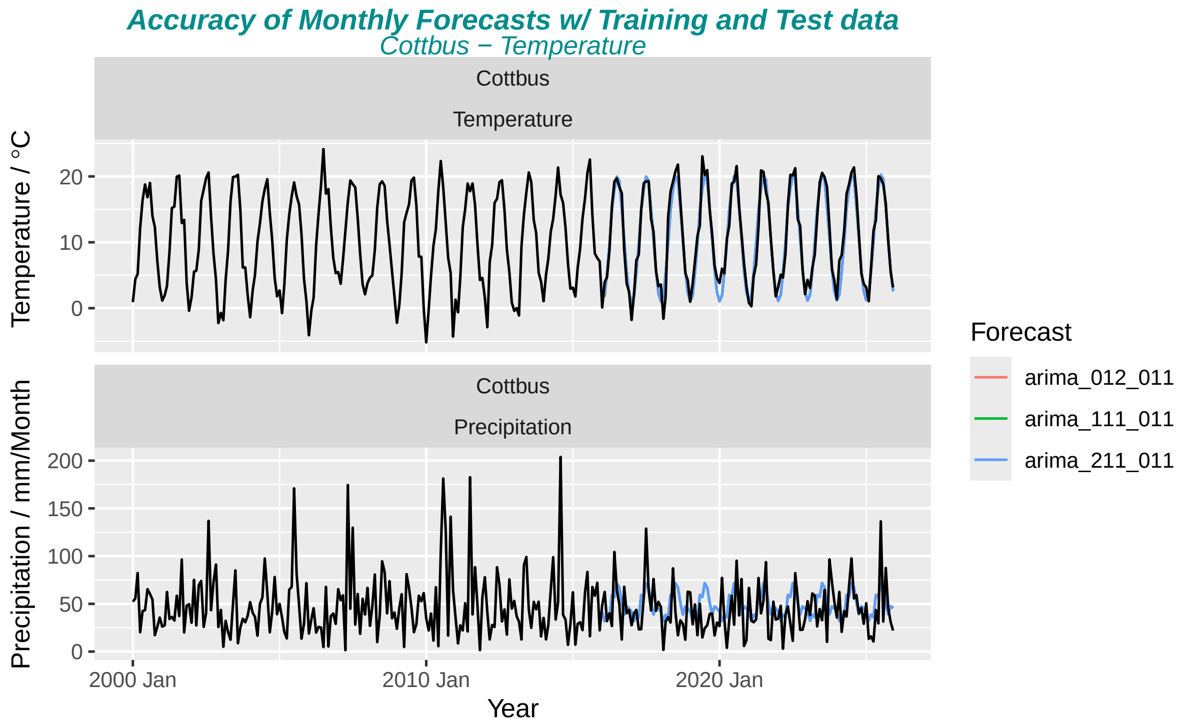
3.1.2 Ljung-Box Test - independence/white noise of the forecasts residuals

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 8 x 5
#>   City    Measure     .model          lb_stat lb_pvalue
#>   <chr>   <fct>       <chr>             <dbl>     <dbl>
#> 1 Cottbus Temperature arima_211_011      20.5  4.93e- 1
#> 2 Cottbus Temperature arima_111_011      20.5  4.93e- 1
#> 3 Cottbus Temperature arima_012_011      20.6  4.84e- 1
#> 4 Cottbus Temperature arima_111_012      20.6  4.80e- 1
#> 5 Cottbus Temperature arima_012_112      21.0  4.61e- 1
#> 6 Cottbus Temperature arima_100_210      51.2  2.44e- 4
#> 7 Cottbus Temperature arima_200_011     106.   2.33e-13
#> 8 Cottbus Temperature arima_100_110_c   106.   2.25e-13
#> # A tibble: 8 x 5
#>   City    Measure       .model        lb_stat lb_pvalue
#>   <chr>   <fct>         <chr>           <dbl>     <dbl>
#> 1 Cottbus Precipitation arima_111_012    25.0  0.246
#> 2 Cottbus Precipitation arima_012_011    25.8  0.213
#> 3 Cottbus Precipitation arima_111_011    25.8  0.213
#> 4 Cottbus Precipitation arima_211_011    25.8  0.213
#> 5 Cottbus Precipitation arima_012_112    27.3  0.162
#> 6 Cottbus Precipitation arima_001_002    48.4  0.000611
#> 7 Cottbus Precipitation arima_010_110   367.   0
#> 8 Cottbus Precipitation arima_012_010   200.   0
```

3.1.3 Forecast Accuracy with Training/Test Data

```
#> # A tibble: 6 x 7
```

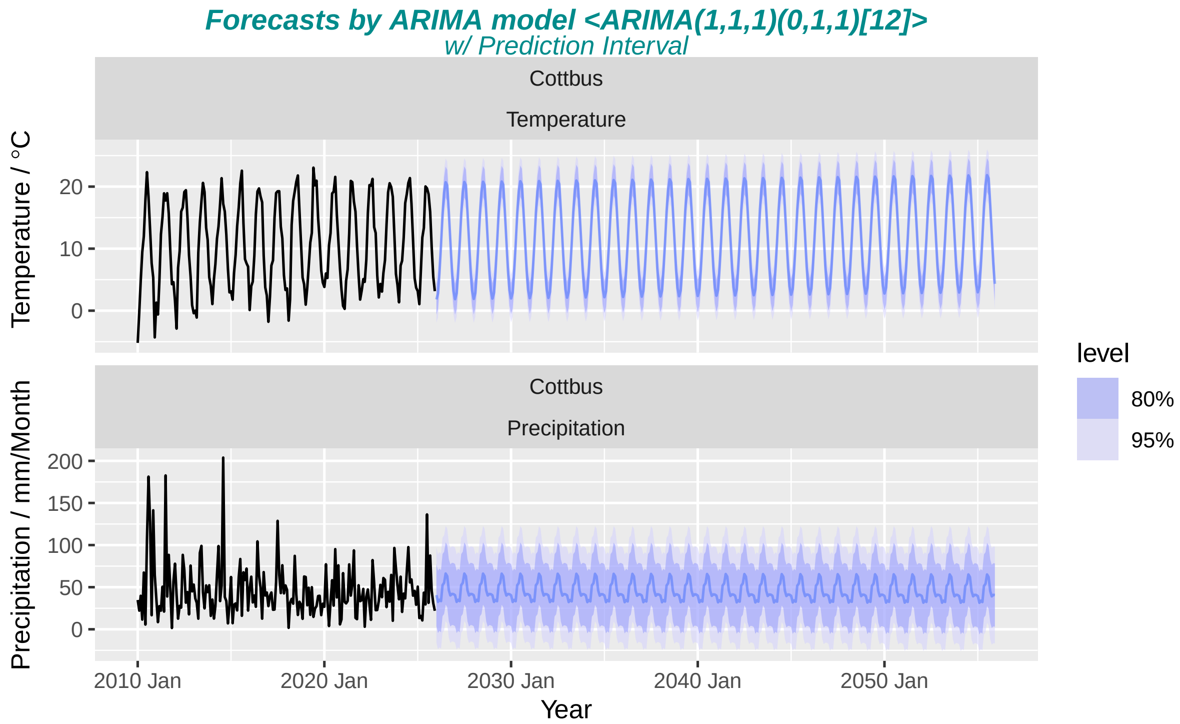
```
#>    .model         City     Measure        .type     ME  RMSE   MAE
#>    <chr>          <chr>    <fct>          <chr>  <dbl> <dbl> <dbl>
#> 1 arima_211_011 Cottbus Temperature   Test   0.495  1.74  1.38
#> 2 arima_111_011 Cottbus Temperature   Test   0.501  1.74  1.38
#> 3 arima_012_011 Cottbus Temperature   Test   0.508  1.75  1.38
#> 4 arima_012_011 Cottbus Precipitation Test  -5.36  25.1  20.3
#> 5 arima_111_011 Cottbus Precipitation Test  -5.36  25.1  20.3
#> 6 arima_211_011 Cottbus Precipitation Test  -5.36  25.1  20.3
```



3.2 Temperature, Precipitation - Forecasting with selected ARIMA model <ARIMA(1,1,1)(0,1,1)[12]>, <ARIMA(1,1,1)(0,1,1)[12]>

3.2.1 Forecast Plot of selected ARIMA model

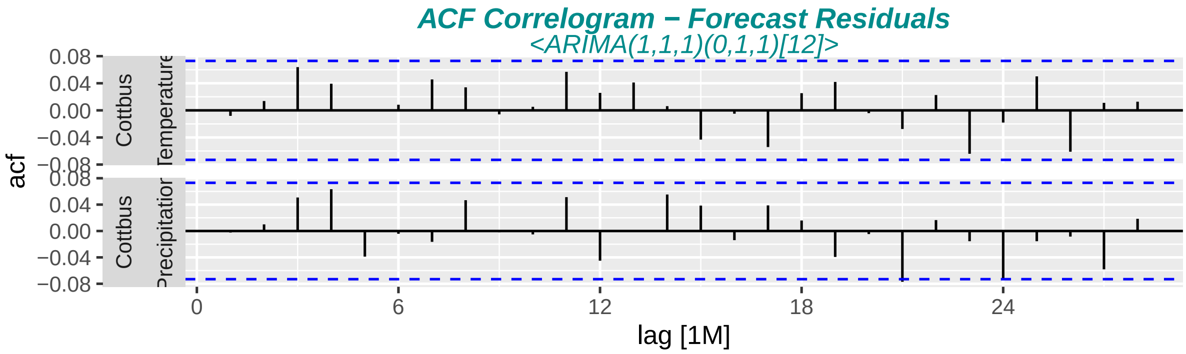
```
#> Provide model coefficients by report(fit_model)
#> # A tibble: 2 x 10
#>   City     Measure      .model sigma2 log_lik   AIC  AICc   BIC ar_roots ma_roots
#>   <chr>    <fct>        <chr>   <dbl>   <dbl> <dbl> <dbl> <dbl> <list>   <list>
#> 1 Cottbus Temperature  arima    3.62  -1484. 2976. 2976. 2995. <cpl>    <cpl>
#> 2 Cottbus Precipitati~ arima  826.    -3399. 6807. 6807. 6825. <cpl>    <cpl>
```

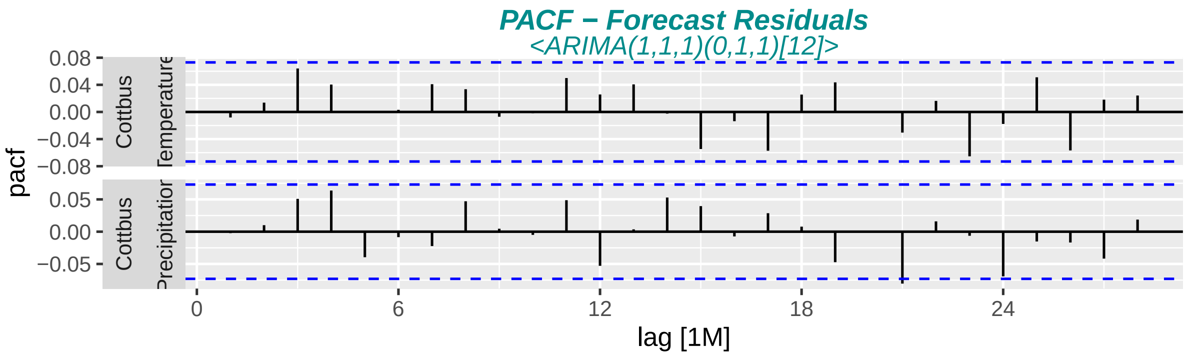


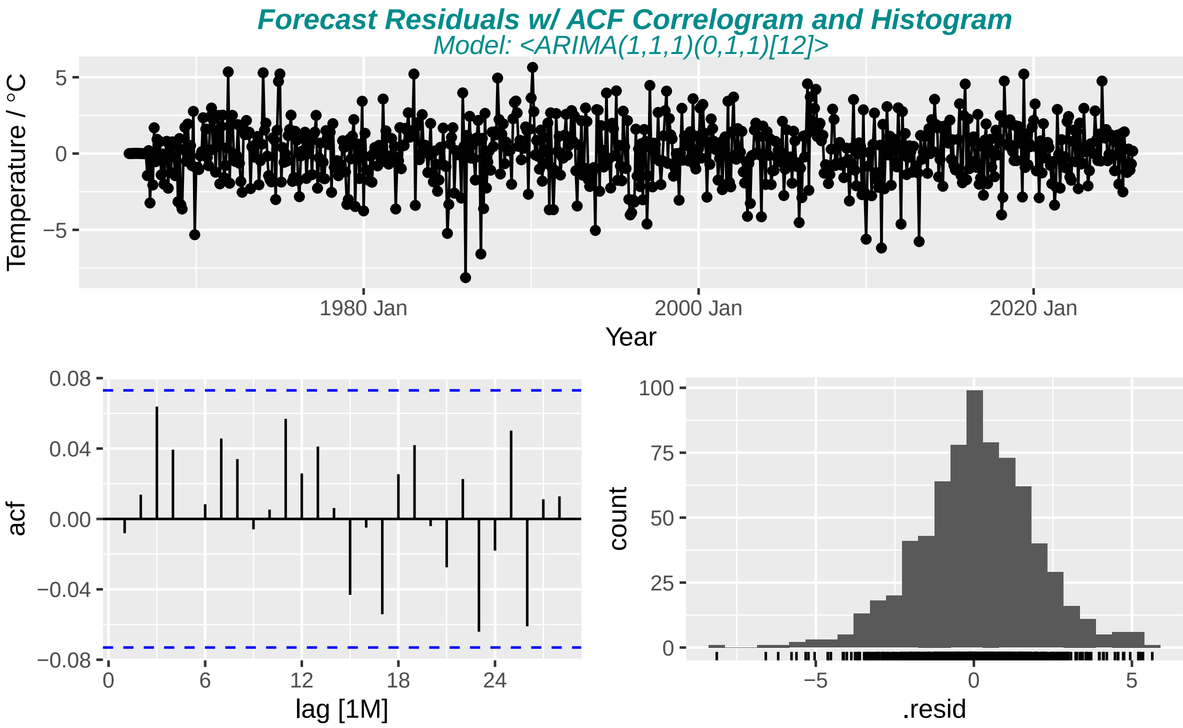
3.2.2 Residual Stationarity

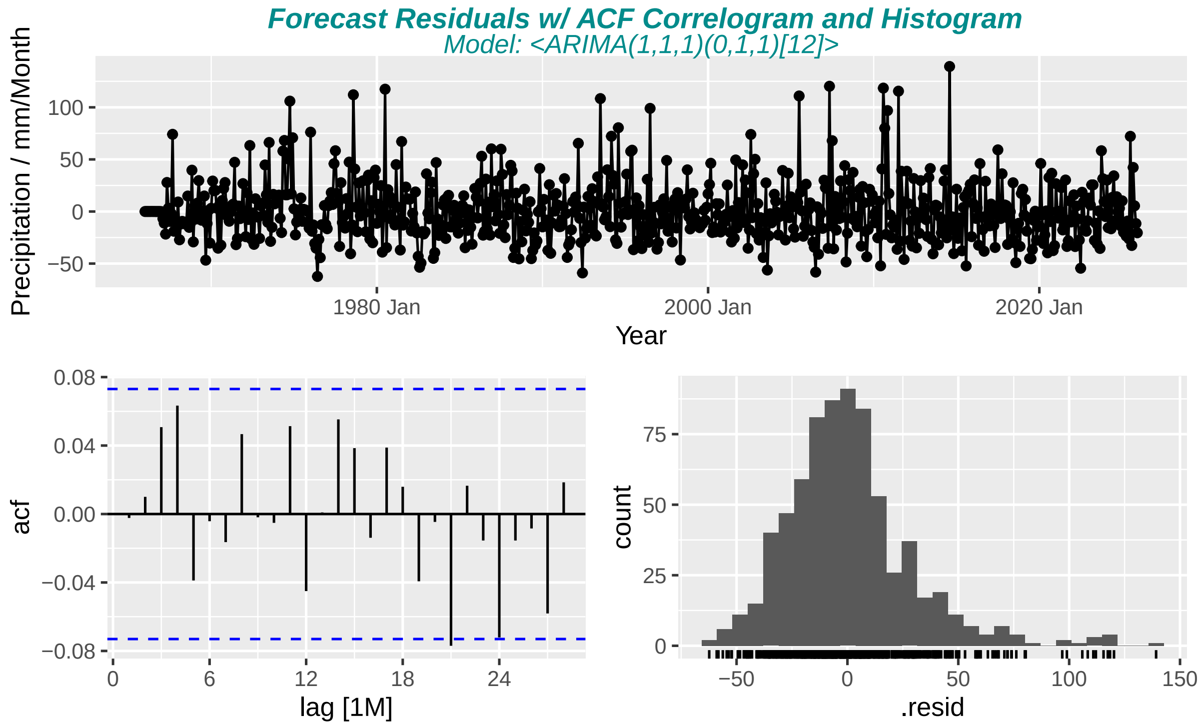
Required checks to be ready for forecasting:

- ACF Forecast Residual: all spikes are within the significance limits, so the residuals appear to be white noise
- The Ljung-Box test also shows that the residuals have no remaining autocorrelations
- Forecast Residuals are more or less normally distributed with roughly centred on zero


ACF Correlogram − Forecast Residuals
<ARIMA(1,1,1)(0,1,1)[12]>
acf
Cottbus
Temperature
Precipitation
lag [1M]

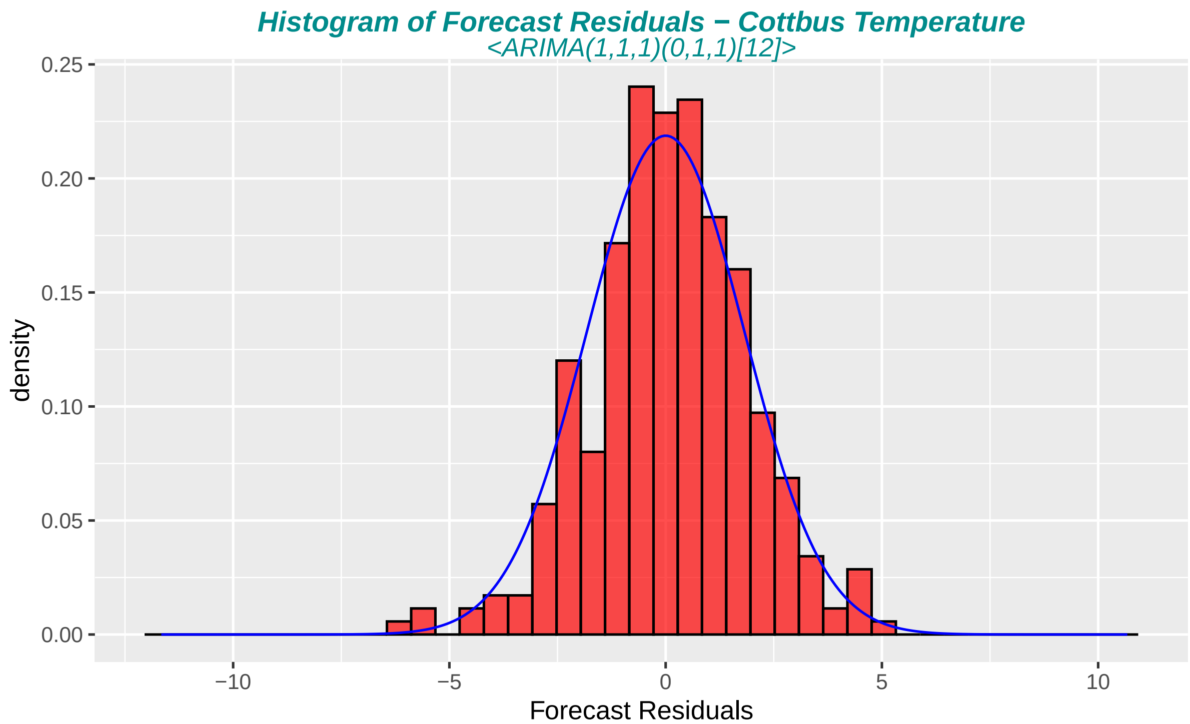

PACF − Forecast Residuals
<ARIMA(1,1,1)(0,1,1)[12]>
pacf
Cottbus
Temperature
Precipitation
lag [1M]

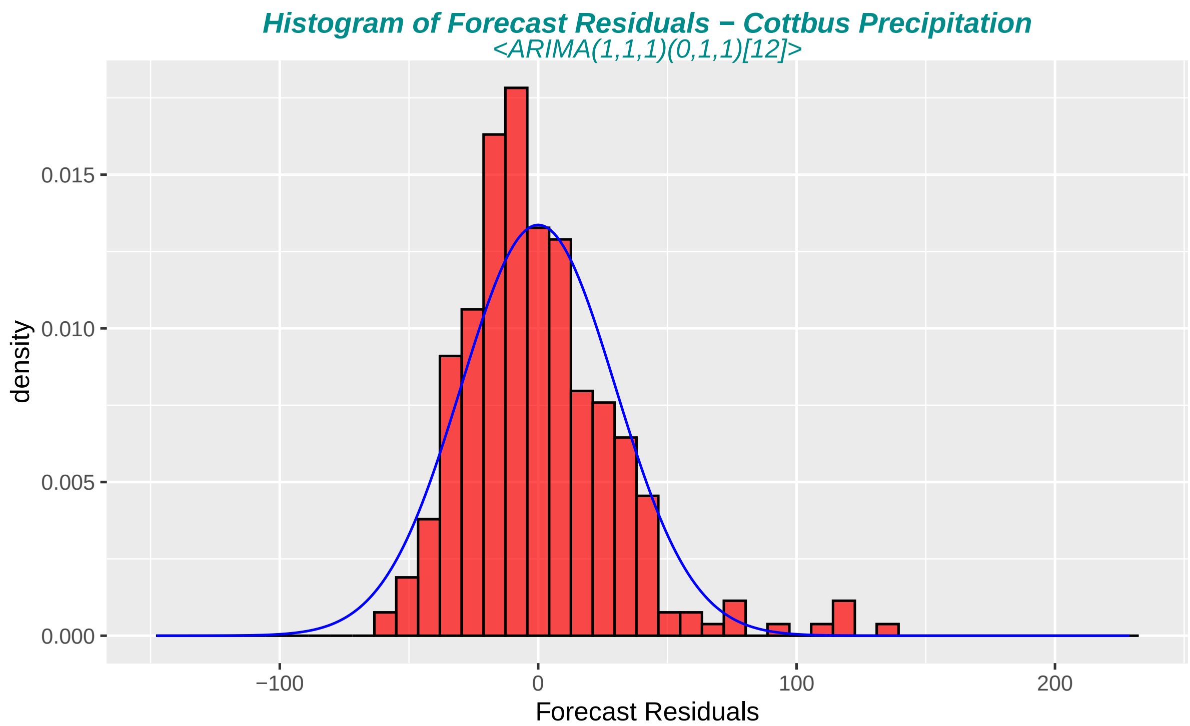

Forecast Residuals w/ ACF Correlogram and Histogram
Model: <ARIMA(1,1,1)(0,1,1)[12]>
Temperature / °C
1980 Jan
2000 Jan
2020 Jan
Year
acf
lag [1M]
count
.resid



3.2.3 Ljung-Box Test and Histogram of forecast residuals with overlaid normal curve

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 2 x 5
#>   City    Measure       .model lb_stat lb_pvalue
#>   <chr>   <fct>         <chr>    <dbl>     <dbl>
#> 1 Cottbus Temperature   arima     16.5     0.742
#> 2 Cottbus Precipitation arima     25.1     0.244
```




Histogram of Forecast Residuals − Cottbus Precipitation
<ARIMA(1,1,1)(0,1,1)[12]>
density
0.015
0.010
0.005
0.000
−100
0
100
200
Forecast Residuals

4 ARIMA vs ETS

In particular, all ETS models are non-stationary, while some ARIMA models are stationary.

The ETS models with seasonality or non-damped trend or both have two unit roots (i.e., they need two levels of differencing to make them stationary). All other ETS models have one unit root (they need one level of differencing to make them stationary).

We compare for the chosen ETS rsp. ARIMA model the RMSE / MAE values. Lower values indicate a more accurate model based on the test set RMSE, . . . , MASE.

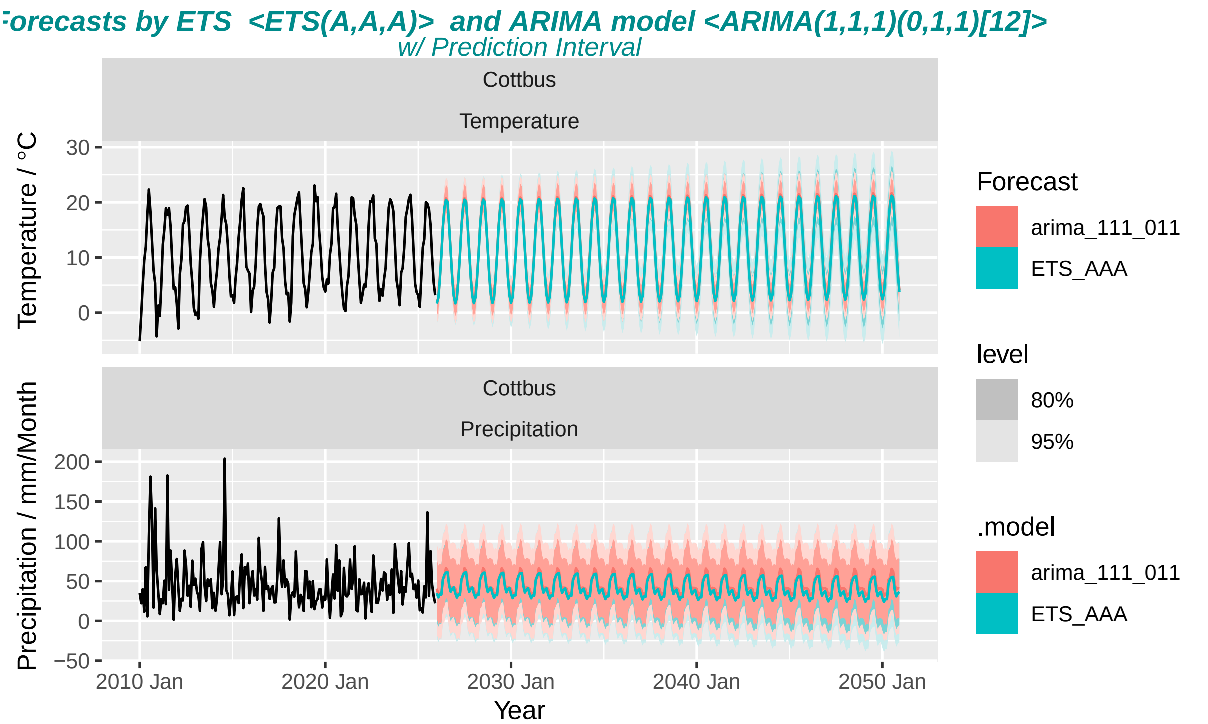
- Residual Accuracy with one-step-ahead fitted residuals
- Forecast Accuracy with Training/Test Data

Note: a good fit to training data is never an indication that the model will forecast well. Therefore the values of the Forecast Accuracy are the more relevant one.

4.0.1 Comparing Residual and Forecast Accuracy of selected ETS and ARIMA model

```
#> # A tibble: 8 x 9
#>   City    Measure       .model        .type     RMSE   MAE  MAPE  MASE RMSSE
#>   <chr>   <fct>         <chr>         <chr>    <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1 Cottbus Temperature   ETS_AAA       Test      1.69  1.32  46.0 0.647 0.630
#> 2 Cottbus Temperature   arima_111_011 Test      1.74  1.38  42.2 0.676 0.650
#> 3 Cottbus Temperature   arima         Training  1.88  1.43 133.  0.709 0.708
#> 4 Cottbus Temperature   ets           Training  1.91  1.48 125.  0.733 0.719
#> 5 Cottbus Precipitation arima_111_011 Test     25.1  20.3  104.  0.660 0.592
#> 6 Cottbus Precipitation ETS_AAA       Test     26.2  21.3  113.  0.695 0.616
#> 7 Cottbus Precipitation ets           Training 28.4  21.2  100.0 0.712 0.695
#> 8 Cottbus Precipitation arima         Training 28.4  20.7   95.6 0.694 0.696
```

4.0.2 Forecast Plot of selected ETS and ARIMA model



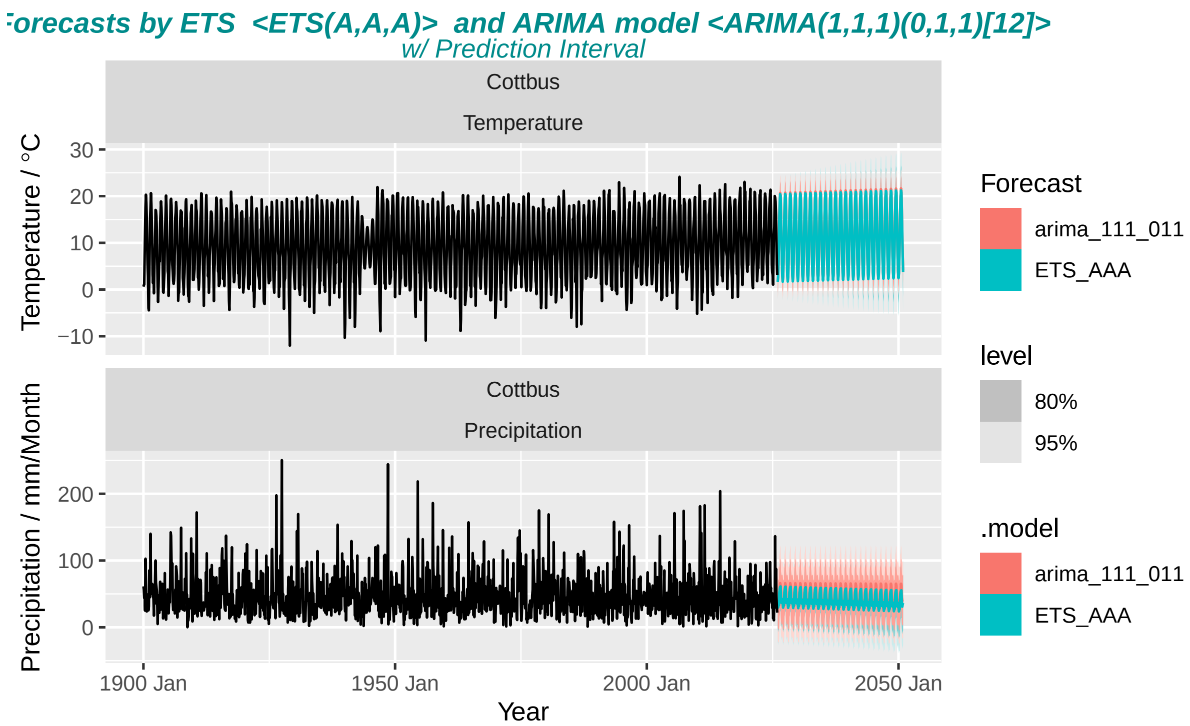

Forecasts by ETS <ETS(A,A,A)> and ARIMA model <ARIMA(1,1,1)(0,1,1)[12]>
w/ Prediction Interval
Cottbus
Temperature
Temperature / °C
30
20
10
0
−10
Cottbus
Precipitation
Precipitation / mm/Month
200
100
0
1900 Jan
1950 Jan
2000 Jan
2050 Jan
Year
Forecast
arima_111_011
ETS_AAA
level
80%
95%
.model
arima_111_011
ETS_AAA

Table 1: Mean values for the given time periods; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Period_Time	Temperature	Precipitation
1871-1900	8.8	48.2
1901-1930	8.9	48.5
1931-1960	9.0	48.1
1961-1990	8.9	46.9
1991-2020	10.0	47.2
2021-2025	11.0	44.8

Table 2: Mean Yearly ARIMA and ETS Forecast values (next 25 years); Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

City	Measure	Year	ETS_AAA	arima_111_011
Cottbus	Temperature	2026	10.98	11.11
Cottbus	Temperature	2030	11.11	11.26
Cottbus	Temperature	2035	11.26	11.46
Cottbus	Temperature	2040	11.42	11.66
Cottbus	Temperature	2045	11.58	11.86
Cottbus	Temperature	2050	11.74	12.05
Cottbus	Precipitation	2026	43.89	45.75
Cottbus	Precipitation	2030	42.92	45.60
Cottbus	Precipitation	2035	41.72	45.41
Cottbus	Precipitation	2040	40.51	45.22
Cottbus	Precipitation	2045	39.31	45.03
Cottbus	Precipitation	2050	38.11	44.84

Table 3: Forecast increase/decrease over the next 25 years; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Measure	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETS	Delta_ARIMA
Temperature	2026	2050	10.98	11.11	11.74	12.05	0.76	0.95
Precipitation	2026	2050	43.89	45.75	38.11	44.84	-5.78	-0.91

Table 4: Forecast increase/decrease over the next 25 years; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Measure	Month	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETS	Delta_ARIMA
Temperature	Jan	2026	2050	1.65	1.82	2.41	2.76	0.76	0.95
Temperature	Feb	2026	2050	2.67	2.76	3.43	3.71	0.76	0.95
Temperature	Mrz	2026	2050	5.89	6.04	6.65	6.98	0.76	0.95
Temperature	Apr	2026	2050	10.29	10.48	11.06	11.42	0.76	0.95
Temperature	Mai	2026	2050	15.48	15.57	16.24	16.52	0.76	0.95
Temperature	Jun	2026	2050	19.02	19.02	19.78	19.97	0.76	0.95
Temperature	Jul	2026	2050	20.45	20.71	21.22	21.65	0.76	0.95
Temperature	Aug	2026	2050	20.05	20.19	20.82	21.14	0.76	0.95
Temperature	Sep	2026	2050	15.77	15.89	16.53	16.84	0.76	0.95
Temperature	Okt	2026	2050	11.15	11.27	11.92	12.22	0.76	0.95
Temperature	Nov	2026	2050	6.29	6.35	7.06	7.29	0.76	0.95
Temperature	Dez	2026	2050	3.01	3.19	3.77	4.14	0.76	0.95

Measure	Month	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETS	Delta_ARIMA
Precipitation	Jan	2026	2050	34.82	40.59	29.04	39.67	-5.78	-0.92
Precipitation	Feb	2026	2050	29.53	32.56	23.75	31.64	-5.78	-0.91
Precipitation	Mrz	2026	2050	33.36	35.09	27.58	34.18	-5.78	-0.91
Precipitation	Apr	2026	2050	32.99	33.43	27.21	32.52	-5.78	-0.91
Precipitation	Mai	2026	2050	52.02	52.48	46.24	51.57	-5.78	-0.91
Precipitation	Jun	2026	2050	58.16	55.14	52.38	54.22	-5.78	-0.91
Precipitation	Jul	2026	2050	60.79	66.23	55.01	65.32	-5.78	-0.91
Precipitation	Aug	2026	2050	61.16	63.09	55.38	62.18	-5.78	-0.91
Precipitation	Sep	2026	2050	44.67	46.42	38.89	45.51	-5.78	-0.91
Precipitation	Okt	2026	2050	36.58	40.20	30.80	39.28	-5.78	-0.91
Precipitation	Nov	2026	2050	40.37	42.03	34.59	41.12	-5.78	-0.91
Precipitation	Dez	2026	2050	42.18	41.78	36.40	40.87	-5.78	-0.91

5 Yearly Data Forecasts with ARIMA and ETS

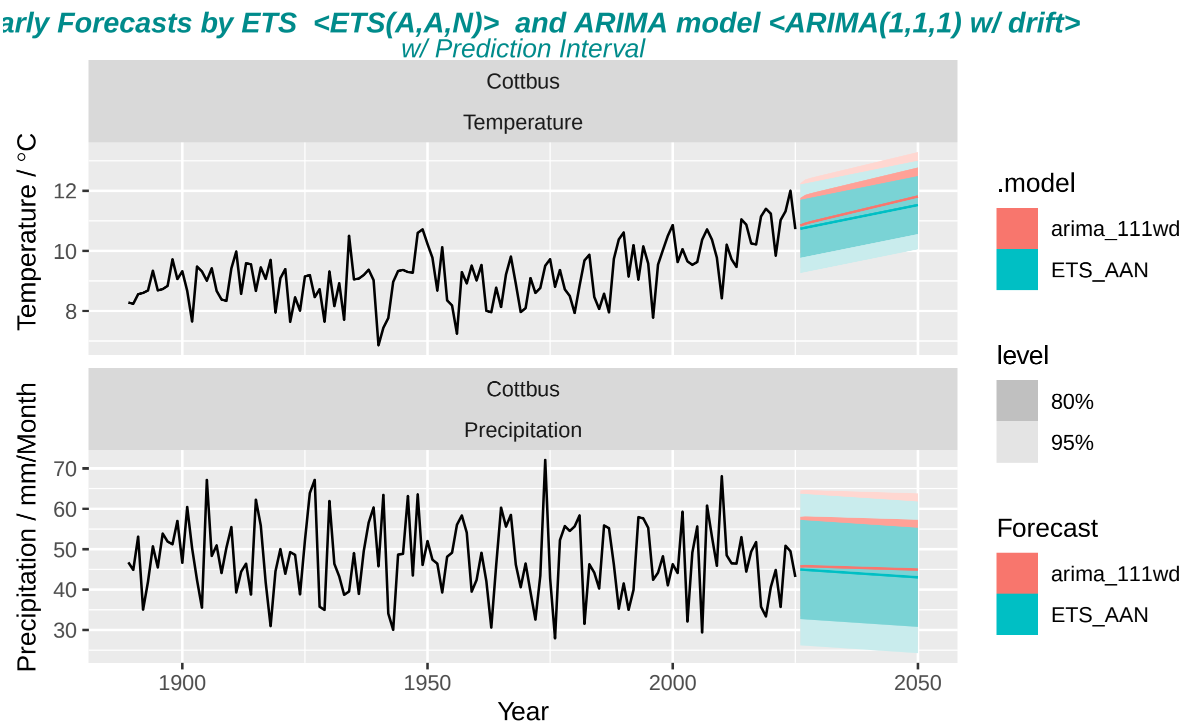
For yearly data the seasonal monthly data are replaced by the yearly average data. Therefore the seasonal component of the ETS and ARIMA model are to be taken out.

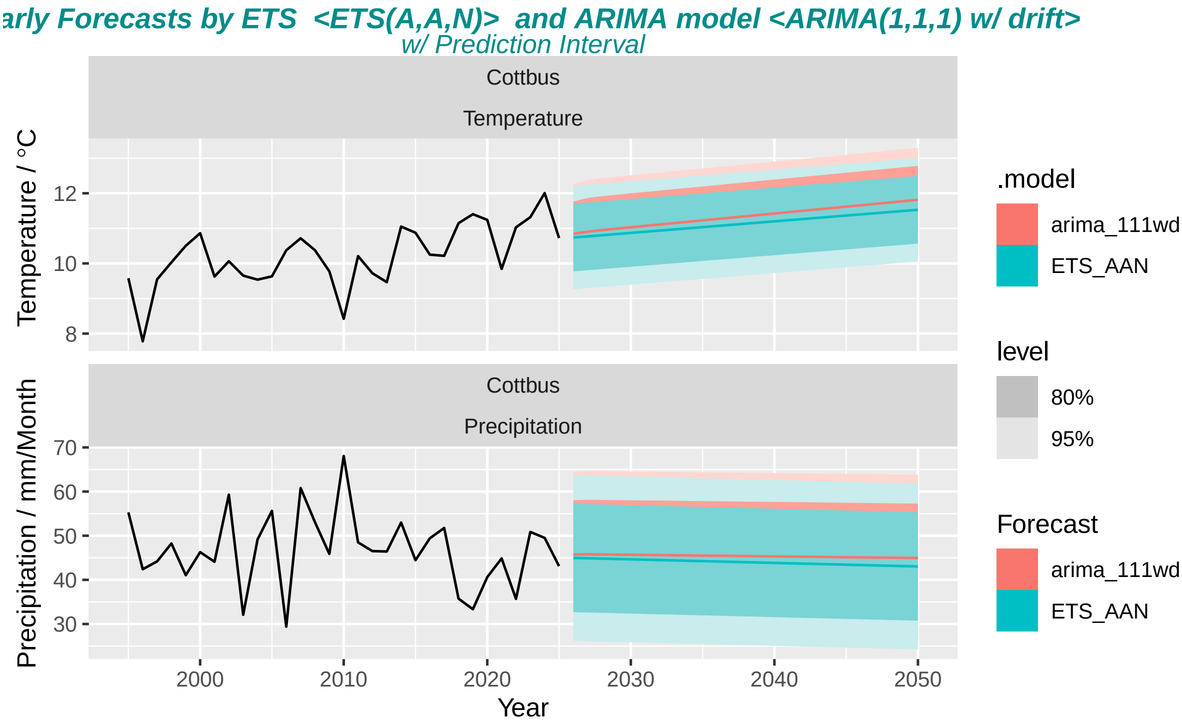
The ETS model $< ETS(A, A, N) >$ with seasonal term change "A" -> "N" is chosen. For ARIMA models the seasonal term (P,D,Q)m has to be taken out and an optimal ARIMA(p,1,q) with one differencing (d=1) is selected. However, for Mauna Loa two times differncing had to be selected $CO_2 <ARIMA(0,2,1) w/ poly>. For Temperature and Precipitation the same model as for monthly data can be taken by leaving out the seasonal term $< ARIMA(0, 1, 2)w/drift >$.

5.0.1 Comparing Residual and Forecast Accuracy of selected ETS and ARIMA model

5.0.2 Forecast Plot of selected ETS and ARIMA model

```
#> selected: ETS_AAN and arima_111 w/ drift
```






```
#> # A tibble: 4 x 13
#>   City    Measure    .model sigma2 log_lik   AIC  AICc   BIC    MSE   AMSE    MAE
#>   <chr>   <fct>      <chr>   <dbl>   <dbl> <dbl> <dbl> <dbl>  <dbl>  <dbl>  <dbl>
#> 1 Cottbus Temperat~ arima~  0.510   -64.1  136.  137.  144. NA     NA     NA
#> 2 Cottbus Temperat~ ETS_A~  0.567  -104.   217.  219.  228.  0.529  0.530  0.599
#> 3 Cottbus Precipit~ arima~ 90.9    -217.   442.  443.  451. NA     NA     NA
#> 4 Cottbus Precipit~ ETS_A~ 91.8    -256.   523.  524.  533. 85.7   86.4    7.50
#> # i 2 more variables: ar_roots <list>, ma_roots <list>
```

Ljung-Box Test - independence/white noise of the forecasts residuals

```
#> # A tibble: 2 x 5
#>   City    Measure       .model  lb_stat lb_pvalue
#>   <chr>   <fct>         <chr>     <dbl>     <dbl>
#> 1 Cottbus Temperature   ETS_AAN    40.9    0.0169
#> 2 Cottbus Precipitation ETS_AAN    28.2    0.251
#> # A tibble: 2 x 5
#>   City    Measure       .model      lb_stat lb_pvalue
#>   <chr>   <fct>         <chr>         <dbl>     <dbl>
#> 1 Cottbus Temperature   arima_111wd    24.8     0.256
#> 2 Cottbus Precipitation arima_111wd    28.8     0.120
```

Table 5: Mean Yearly ARIMA and ETS Forecast values of mean yearly data (next 25 years); Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

City	Measure	Year	ETS_AAN	arima_111wd
Cottbus	Temperature	2026	10.74	10.84
Cottbus	Temperature	2030	10.87	11.03
Cottbus	Temperature	2035	11.03	11.23
Cottbus	Temperature	2040	11.20	11.42
Cottbus	Temperature	2045	11.36	11.62
Cottbus	Temperature	2050	11.53	11.81
Cottbus	Precipitation	2026	44.97	45.72
Cottbus	Precipitation	2030	44.65	45.68
Cottbus	Precipitation	2035	44.24	45.50
Cottbus	Precipitation	2040	43.84	45.32
Cottbus	Precipitation	2045	43.44	45.13
Cottbus	Precipitation	2050	43.03	44.95

Table 6: Forecast increase/decrease over the next 25 years; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Measure	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETS	Delta_ARIMA
Temperature	2026	2050	10.74	10.84	11.53	11.81	0.79	0.97
Precipitation	2026	2050	44.97	45.72	43.03	44.95	-1.94	-0.77