

# Climate Data Forecasting - Atmospheric $CO_2$ Concentration / Temperature / Precipitation

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# 1 Forecasting of Davos - Temperature and Precipitation Climate Analysis

## 1.1 Stationarity and differencing

Stationary time series is one whose properties do not depend on the time at which the series is observed. Thus, time series with trends, or with seasonality, are not stationary — the trend and seasonality will affect the value of the time series at different times. On the other hand, a white noise series is stationary — it does not matter when you observe it, it should look much the same at any point in time.

Stationary time series will have no predictable patterns in the long-term. Time plots will show the series to be roughly horizontal (although some cyclic behaviour is possible), with constant variance.

If  $y_t$  is a *stationary* time series, then for all  $s$ , the distribution of  $(y_t, \dots, y_{t+s})$  does not depend on  $t$ .

If Time Series data with seasonality are non-stationary

- => first take a seasonal difference
- if seasonally differenced data appear are still non-stationary
- => take an additional first seasonal difference

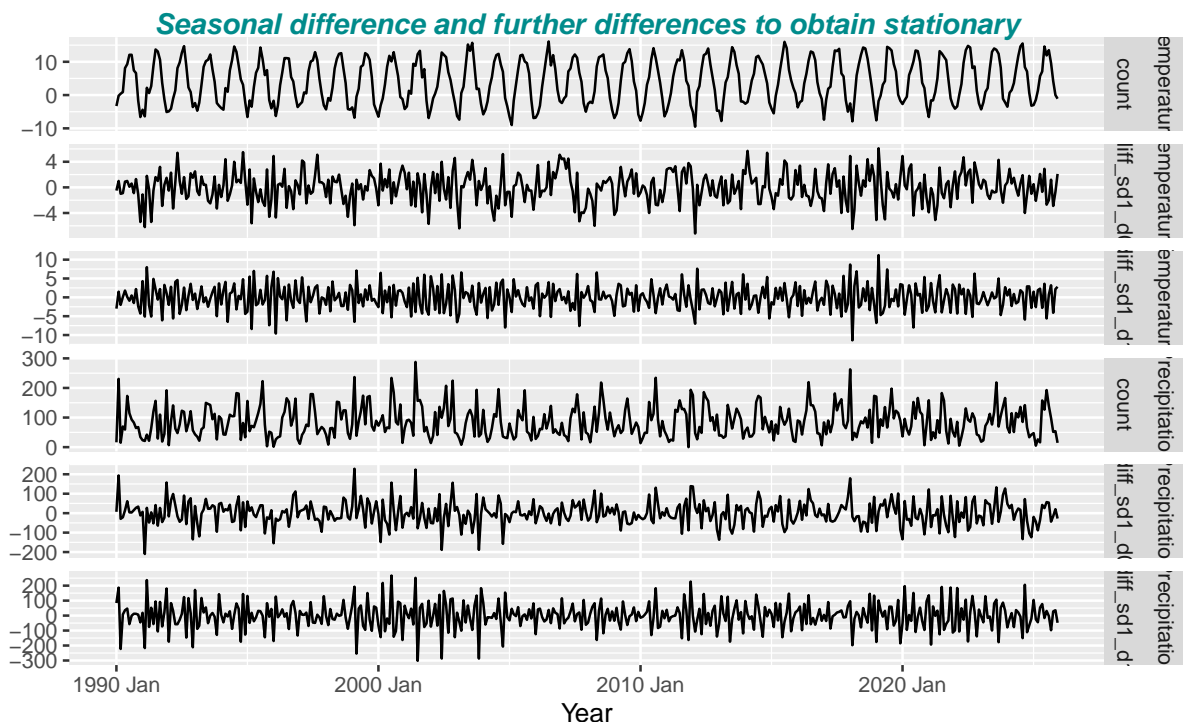
The model fit residuals have to be stationary. For good forecasting this has to be verified with residual diagnostics.

Essential:

- Residuals are uncorrelated
- The residuals have zero mean

Useful (but not necessary):

- The residuals have constant variance.
- The residuals are normally distributed.



### 1.1.1 Unitroot KPSS Test - fix number of seasonal differences/differences required

- For **ARIMA model** use Unitroot KPSS Test to fix number of differences
  - `unitroot_nsdiffs()` to determine  $D$  (the number of seasonal differences to use)
  - `unitroot_ndiffs()` to determine  $d$  (the number of ordinary differences to use)
  - The selection of the other model parameters ( $p, q, P$  and  $Q$ ) are all determined by minimizing the AICc
- kpss test of stationary of the differentiated data (with seasonal and ordinary differences) used in the **ARIMA model**
  - stationary times series: the distribution of  $(y_t, \dots, y_{t-s})$  does not depend on  $t$ .
  - *Null Hypothesis*  $H_0$ : stationary is given in the time series: data are stationary and non seasonal
    - \* for  $p < \alpha = 0.05$  : reject  $H_0$
    - \* for  $p >> \alpha = 0.05$  : conclude: differentiated data are stationary and non seasonal
- kpss test of stationary w/ `unitroot_nsdiffs` & `unitroot_ndiff` provides
  - minimum number of seasonal & ordinary differences required for a stationary series
  - first fix required seasonal differences and then apply `ndiffs` to the seasonally differenced data
  - returns 1 => for stationarity one seasonal difference resp. difference is required

```
#> ndiffs gives the number of differences required and
#> nsdifs gives the number of seasonal differences required
#> - to make a series stationary (test is based on the KPSS test
#> unitroot_kpss test to define seasonal (nsdifs) and ordinary (ndiffs) differences
#> # A tibble: 2 x 5
#>   Measure      kpss_stat kpss_pvalue nsdifs ndiffs
#>   <fct>         <dbl>      <dbl>    <int> <int>
#> 1 Temperature      2.68        0.01         1     1
#> 2 Precipitation    0.619      0.0209         0     1
#> unitroot_kpss test on (difference(count, 12) + difference())
#> # A tibble: 2 x 3
#>   Measure      kpss_stat kpss_pvalue
#>   <fct>         <dbl>      <dbl>
#> 1 Temperature    0.00592        0.1
#> 2 Precipitation  0.00525        0.1
```

### 1.1.2 Ljung-Box Test - independence/white noise of the time series

The Ljung-Box Test becomes important when checking independence/white noise of the forecasts residuals of the fitted ETS resp. ARIMA models. There we have to check whether the forecast errors are normally distributed with mean zero

- `augment(fit) |> features(.innov, ljung_box, lag=x, dof=y)` (Ch. 5.4 Residual diagnostics)
  - portmanteau test suggesting that the residuals are white noise
  - *Null Hypothesis*  $H_0$ : independence/white noise for residuals, i.e. each autocorrelation in the time series residuals of the model for lag  $l$  is close to zero.
    - \* for  $p < \alpha = 0.05$  : reject  $H_0$
    - \* for  $p >> \alpha = 0.05$  : conclude: the residuals are not distinguishable from a white noise series
  - `lag = 2*m` (period of season, e.g.  $m=12$  for monthly season) | no season: `lag=10`
  - `dof = p + q + P + Q` (for ARIMA models only, degree of freedom)

Time series with trend and/or seasonality is not stationary. Tests are to be taken on the residuals of the fitted models.

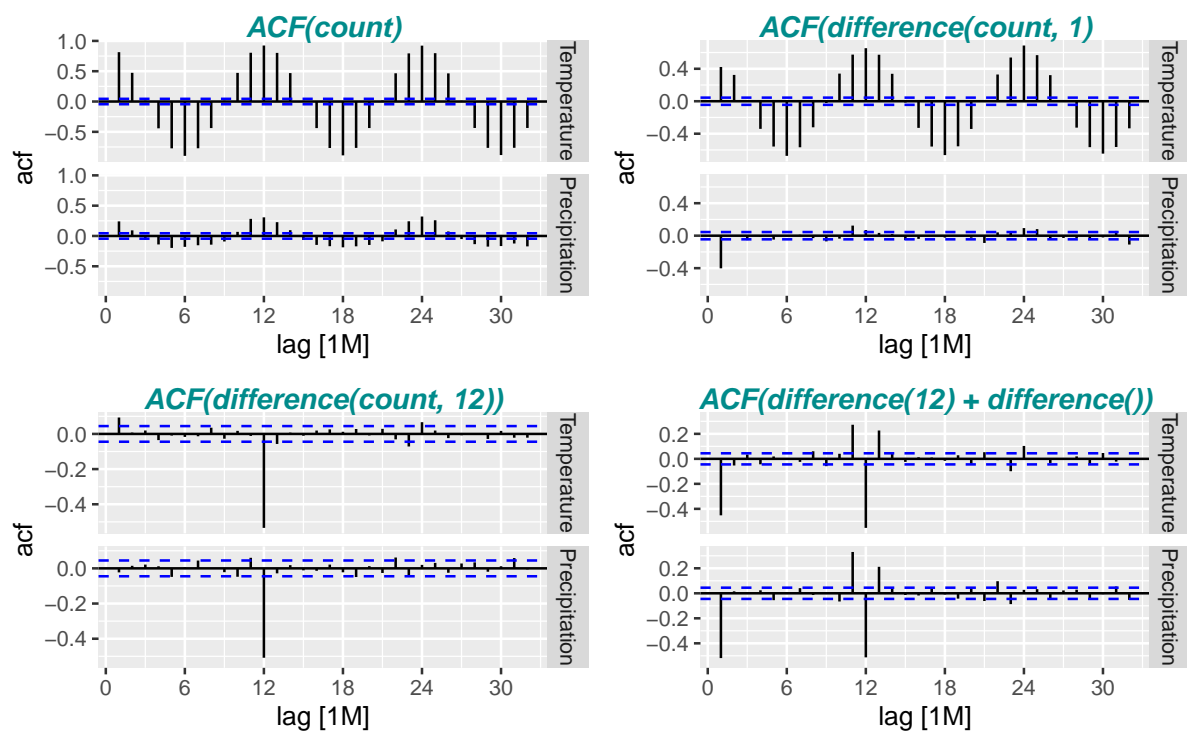
```
#> Ljung-Box test with (count), w/o differences
#> # A tibble: 2 x 3
#>   Measure      lb_stat lb_pvalue
#>   <fct>         <dbl>      <dbl>
#> 1 Temperature  6816.         0
#> 2 Precipitation  417.         0
#> Ljung-Box test on (difference(count, 12))
```

```

#> # A tibble: 2 x 3
#>   Measure      lb_stat lb_pvalue
#>   <fct>      <dbl>    <dbl>
#> 1 Temperature    25.5    0.00445
#> 2 Precipitation   16.1    0.0970
#> Ljung-Box test on (difference(count, 12) + difference())
#> # A tibble: 2 x 3
#>   Measure      lb_stat lb_pvalue
#>   <fct>      <dbl>    <dbl>
#> 1 Temperature   425.      0
#> 2 Precipitation  529.      0

```

### 1.1.3 ACF (Autocorrelation Function) Plots of Differences



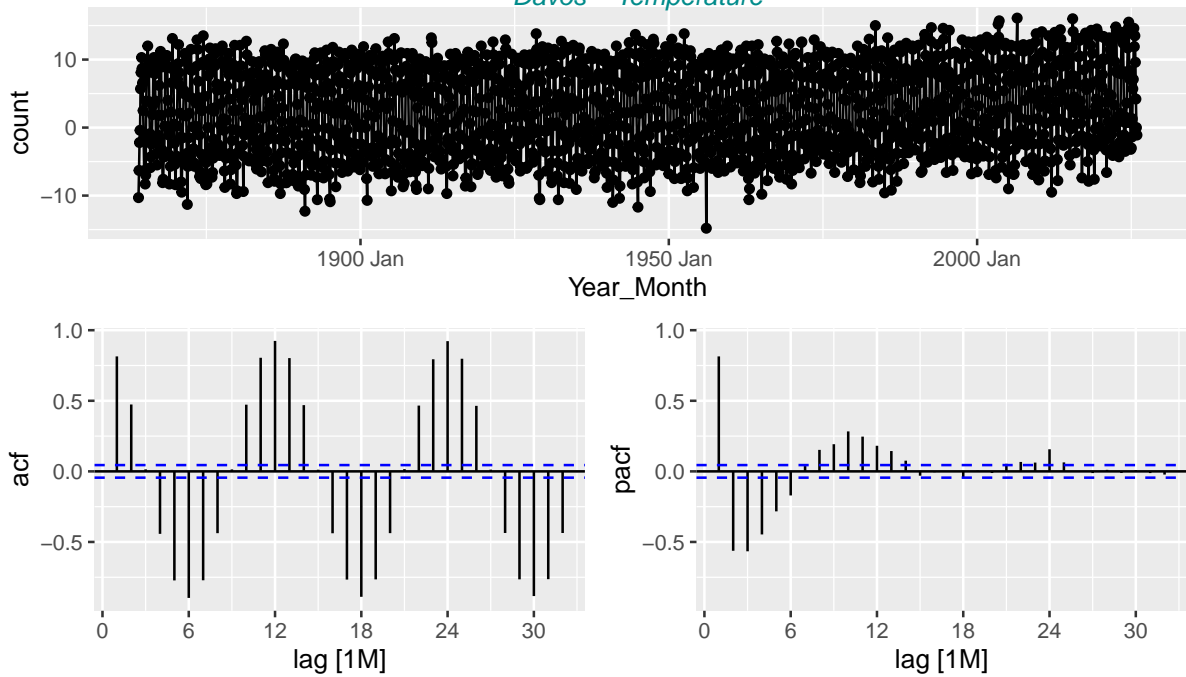
### 1.1.4 Time Series, ACF and PACF (Partial) Plots of Differences - for ARIMA p, q check

```

#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City Measure      Sum Mean
#>   <chr> <fct>      <dbl> <dbl>
#> 1 Davos Temperature 5276.  2.71

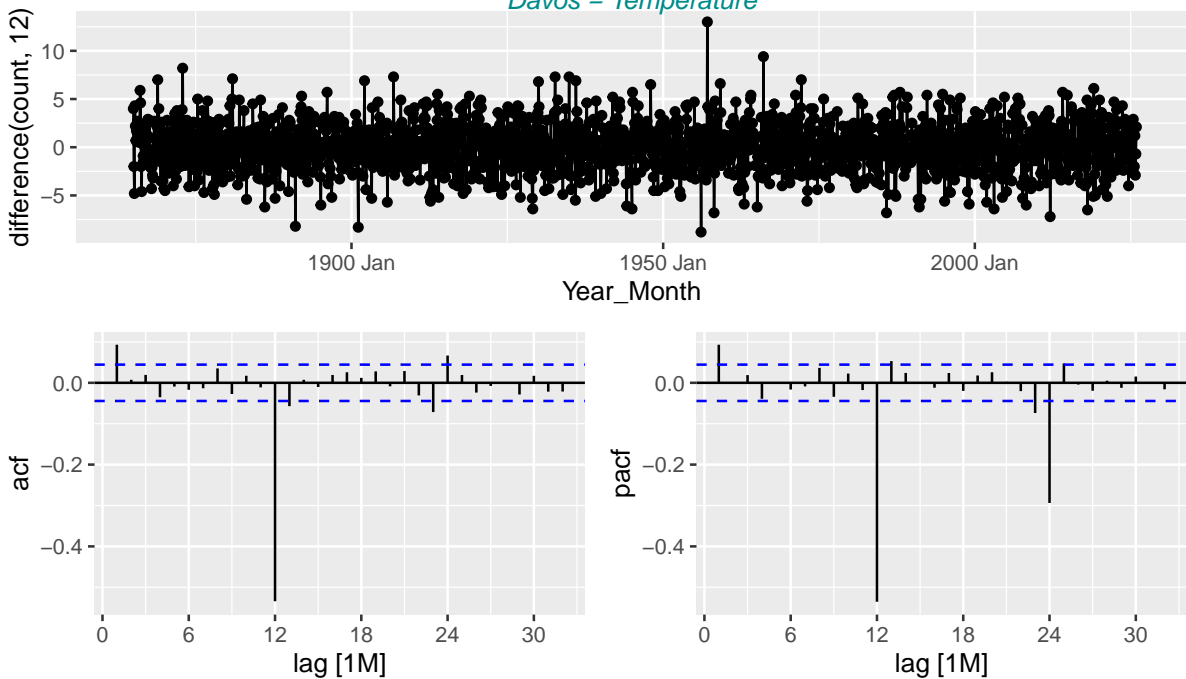
```

### Time Series, ACF & PACF for (count) Davos – Temperature

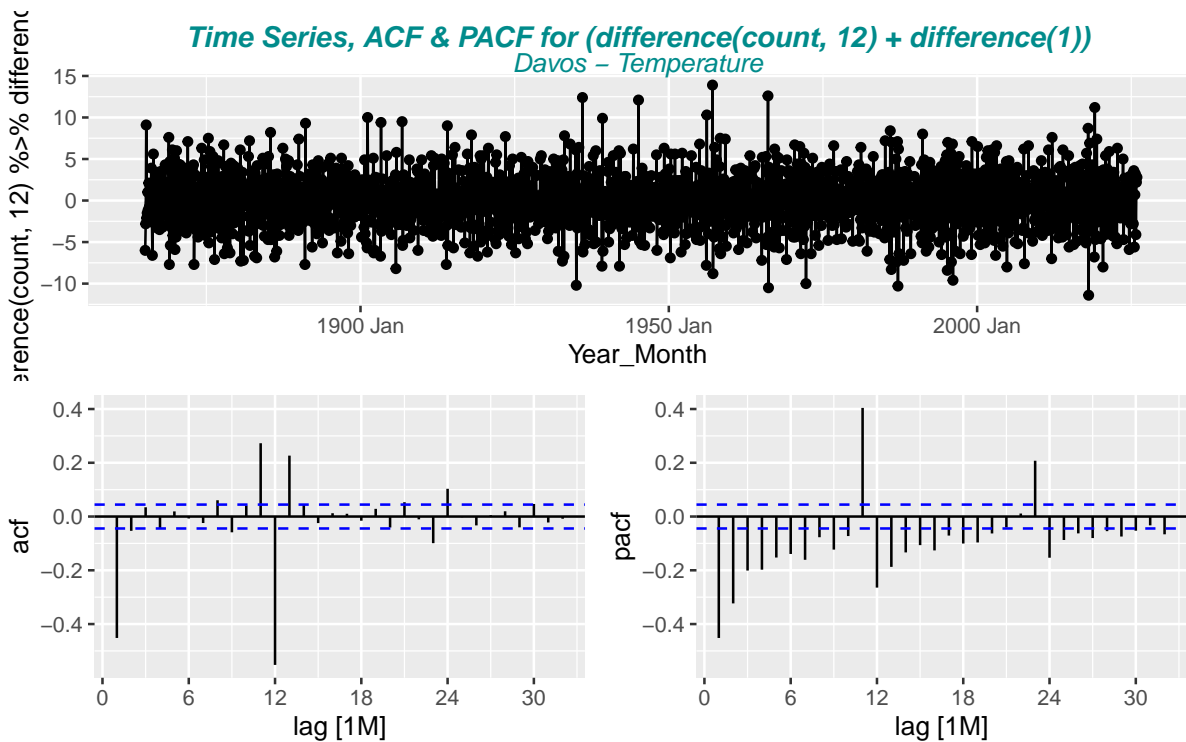


```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City Measure      Sum      Mean
#>   <chr> <fct>      <dbl>    <dbl>
#> 1 Davos Temperature 47.7 0.0247
```

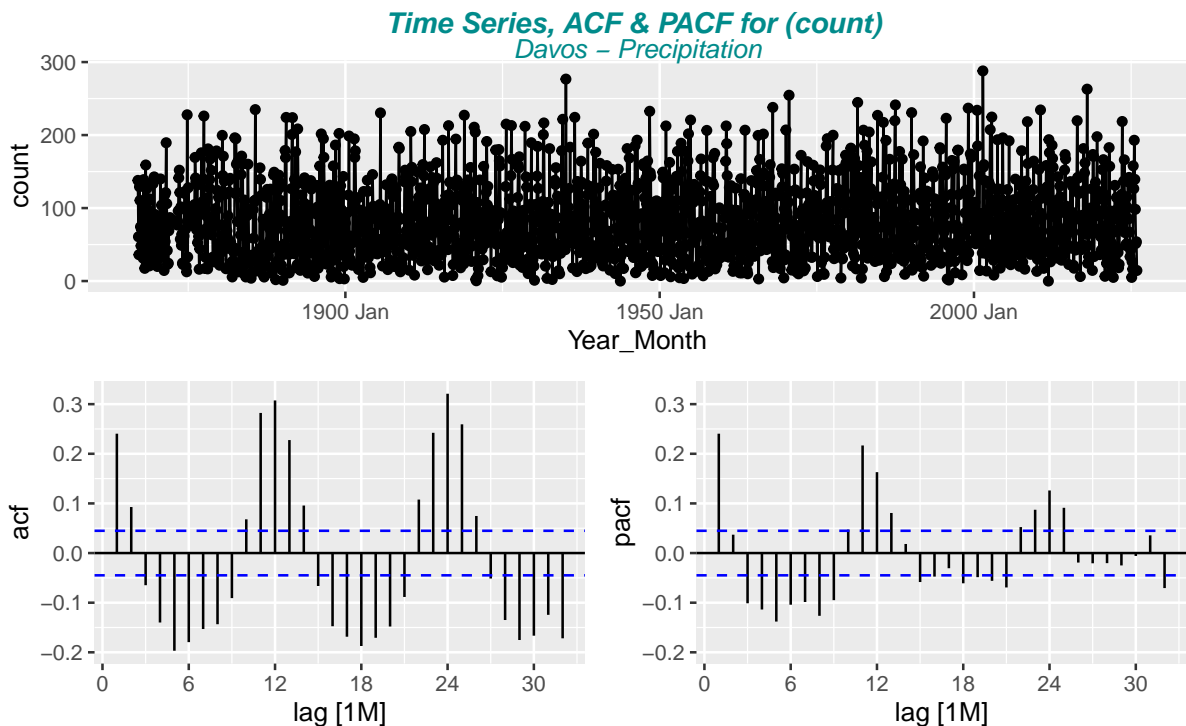
### Time Series, ACF & PACF for (difference(count, 12)) Davos – Temperature



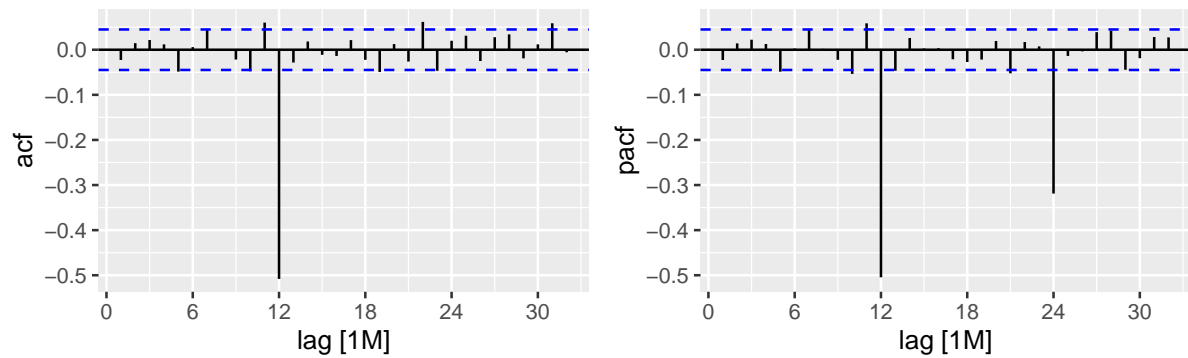
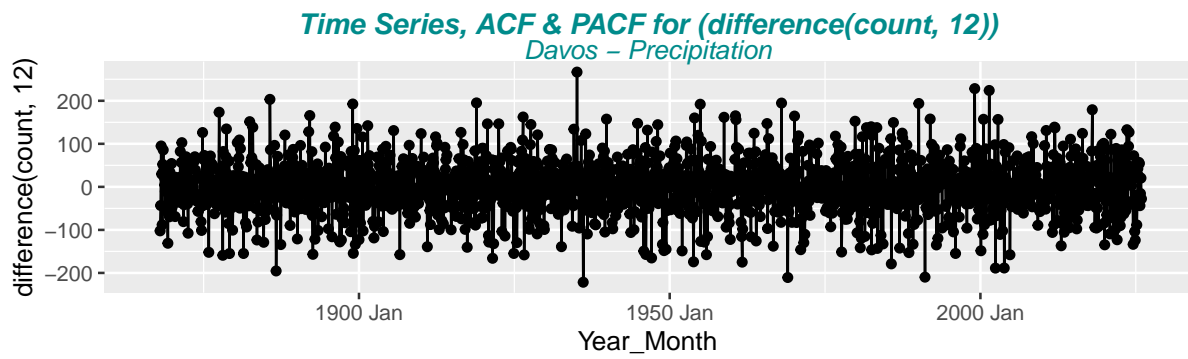
```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City Measure      Sum      Mean
#>   <chr> <fct>      <dbl>    <dbl>
#> 1 Davos Temperature -1.90 -0.000984
```



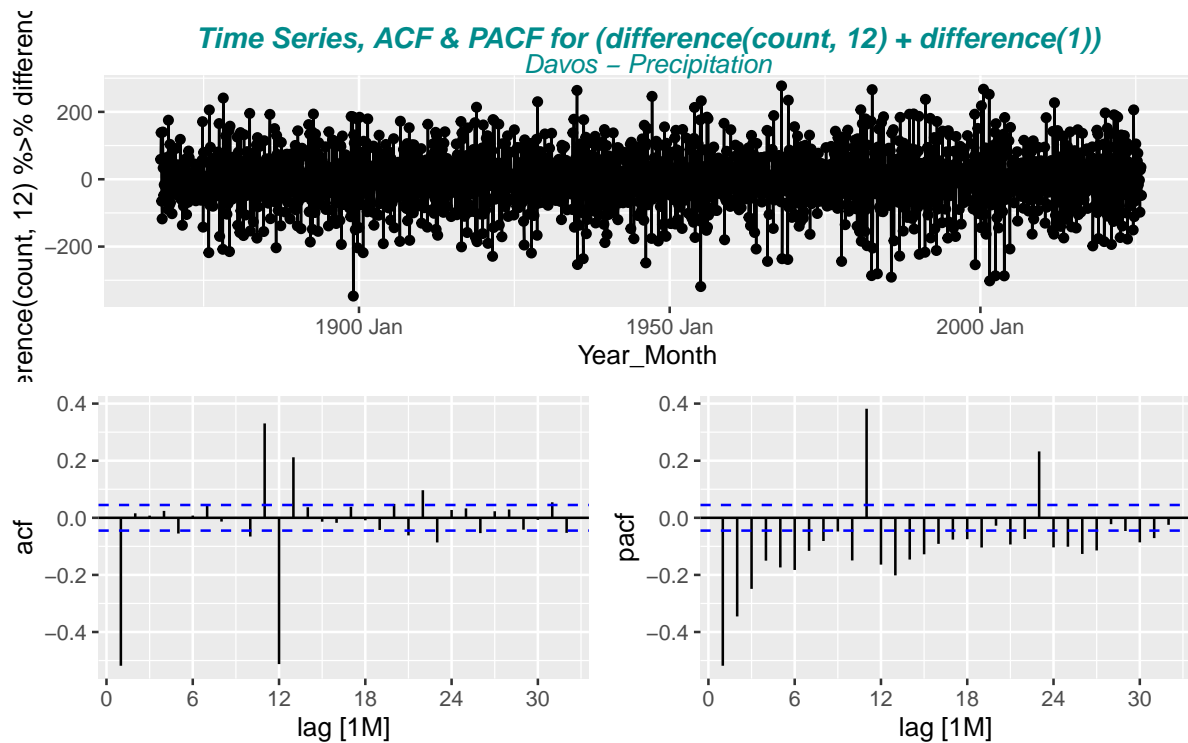
```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City Measure      Sum Mean
#>   <chr> <fct>      <dbl> <dbl>
#> 1 Davos Precipitation 156503.  82.0
```



```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City Measure      Sum Mean
#>   <chr> <fct>      <dbl> <dbl>
#> 1 Davos Precipitation -91.9 -0.0485
```



```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City Measure      Sum   Mean
#>   <chr> <fct>      <dbl> <dbl>
#> 1 Davos Precipitation 74.5 0.0393
```



## 2 Exponential Smoothing (ETS) Forecasting Models

Forecasts produced using exponential smoothing methods are weighted averages of past observations, with the weights decaying exponentially as the observations get older.

The parameters are estimated by maximising the “likelihood”. The likelihood is the probability of the data arising from the specified model. AIC, AICc and BIC can be used here to determine which of the ETS models is most appropriate for a given time series (see output `glance(fit_ets)`).

The model selection is based on recognising key components of the time series (trend and seasonal) and the way in which these enter the smoothing method (e.g., in an additive, damped or multiplicative manner).

- Mauna Loa  $CO_2$  data best Models: ETS(M,A,A) & ETS(A,A,A)
- Basel Temperature data best Models: ETS(A,N,A), ETS(A,A,A), ETS(A,Ad,A) (close together). Best Forecast accuracy is with ETS(A,A,A), ETS(A,Ad,A).
- Basel Precipitation data best Models: ETS(A,N,A), ETS(A,Ad,A), ETS(A,A,A) (close together). Best Forecast accuracy is with ETS(A,A,A), ETS(A,Ad,A), ETS(A,N,A),

Trend term “N” for Basel Temperature/Precipitation corresponds to a “pure” exponential smoothing which results in a slope  $\beta = 0$ . This results in a forecast predicting a constant level. This does not fit to the result of the STL decomposition. Therefore best model choice is **ETS(A,A,A)**.

### Method Selection

*Error term:* either additive (“A”) or multiplicative (“M”).

Both methods provide identical point forecasts, but different prediction intervals and different likelihoods. AIC & BIC are able to select between the error types because they are based on likelihood.

Nevertheless, difference is for

- Mauna Loa  $CO_2$  not relevant and AIC/AICc/BIC values are only a little bit smaller for multiplicative errors. The prediction interval plots are fully overlapping.
- Basel Temperature AIC/AICc/BIC of additive error types are much better than the multiplicative ones.
- Basel Precipitation AIC/AICc/BIC of additive error types are much better than the multiplicative ones.

Note: For Basel Temperature and Precipitation Forecast plots the models ETS\_MAdA, ETS\_MMA, ETS\_MMA, ETS\_MNA are to be taken out since forecasts with multiplicative errors are exploding (forecast > 3 years impossible !!)

Therefore finally **Error term = “A”** is chosen in general.

*Trend term:* either none (“N”), additive (“A”), multiplicative (“M”) or damped variants (“Ad”, “Md”).

Note: Mauna Loa  $CO_2$  model ETS(A,Ad,A) fit plot shows to strong damping. For Basel Temperature model ETS(A,N,A) and ETS(A,Ad,A) are providing more or less the same forecast. This means that forecast remains on constant level since Trend “N” means “pure” exponential smoothing without trend (see above).

Therefore finally **Trend term = “A”** is chosen in general.

*Seasonal term:* either none (“N”), additive (“A”) or multiplicative (“M”).

For  $CO_2$  and Temperature Data we have a clear seasonal pattern and seasonal term adds always a (more or less) fix amount on level and trend component. Therefore “A” additive term is chosen. For Precipitation the seasonal pattern is only slight. Indeed, a multiplicative seasonal term results in “exploding” forecasts.

Since monthly data are strongly seasonal **seasonal term “A”** is chosen.

## 2.1 ETS Models and their componentes

ETS model with automatically selected  $ETS(A|M, N|A|M, N|A|M)$

```
#> [1] "model(ETS(count)) => provides default / best automatically chosen model"
#> # A mable: 2 x 3
#> # Key:      City, Measure [2]
#>   City Measure      ETS
#>   <chr> <fct>      <model>
#> 1 Davos Temperature <ETS(A,A,A)>
#> 2 Davos Precipitation <ETS(A,N,A)>
#> [1] "Davos Temperature"
#> Series: count
#> Model: ETS(A,A,A)
#> Smoothing parameters:
#>   alpha = 0.01609448
#>   beta  = 0.000100032
#>   gamma = 0.0001000532
#>
#> Initial states:
#>   l[0]      b[0]      s[0]      s[-1]      s[-2]      s[-3]      s[-4]      s[-5]
#> 2.635736 0.002361262 -7.373144 -4.009896 1.529508 5.035256 8.350951 8.70623
#>   s[-6]      s[-7]      s[-8]      s[-9]      s[-10]      s[-11]
#> 6.747244 3.232402 -1.378519 -4.74598 -7.703309 -8.390743
#>
#> sigma^2: 2.9466
#>
#>   AIC      AICc      BIC
#> 5532.955 5533.827 5610.802
#> [1] "Davos Precipitation"
#> Series: count
#> Model: ETS(A,N,A)
#> Smoothing parameters:
#>   alpha = 0.0001013012
#>   gamma = 0.0001001011
#>
#> Initial states:
#>   l[0]      s[0]      s[-1]      s[-2]      s[-3]      s[-4]      s[-5]      s[-6]
#> 86.23715 -21.76854 -16.94973 -16.82367 9.496136 56.02326 53.57759 38.50998
#>   s[-7]      s[-8]      s[-9]      s[-10]      s[-11]
#> 6.407523 -31.56906 -25.54687 -33.42778 -17.92885
#>
#> sigma^2: 1881.72
#>
#>   AIC      AICc      BIC
#> 10181.68 10182.36 10250.37
#> # A tibble: 2 x 8
#>   City Measure      .model      AIC      AICc      BIC      MSE      MAE
#>   <chr> <fct>      <chr>    <dbl>    <dbl>    <dbl>    <dbl> <dbl>
#> 1 Davos Temperature ETS      5533.   5534.   5611.     2.88  1.36
#> 2 Davos Precipitation ETS      10182.  10182.  10250.  1845.   32.8
```

Fit of different pre-defined  $ETS(A|M, N|A|M, N|A|M)$  models

Model Selection by Information Criterion - lowest AIC, AICc, BIC

Best model fit with

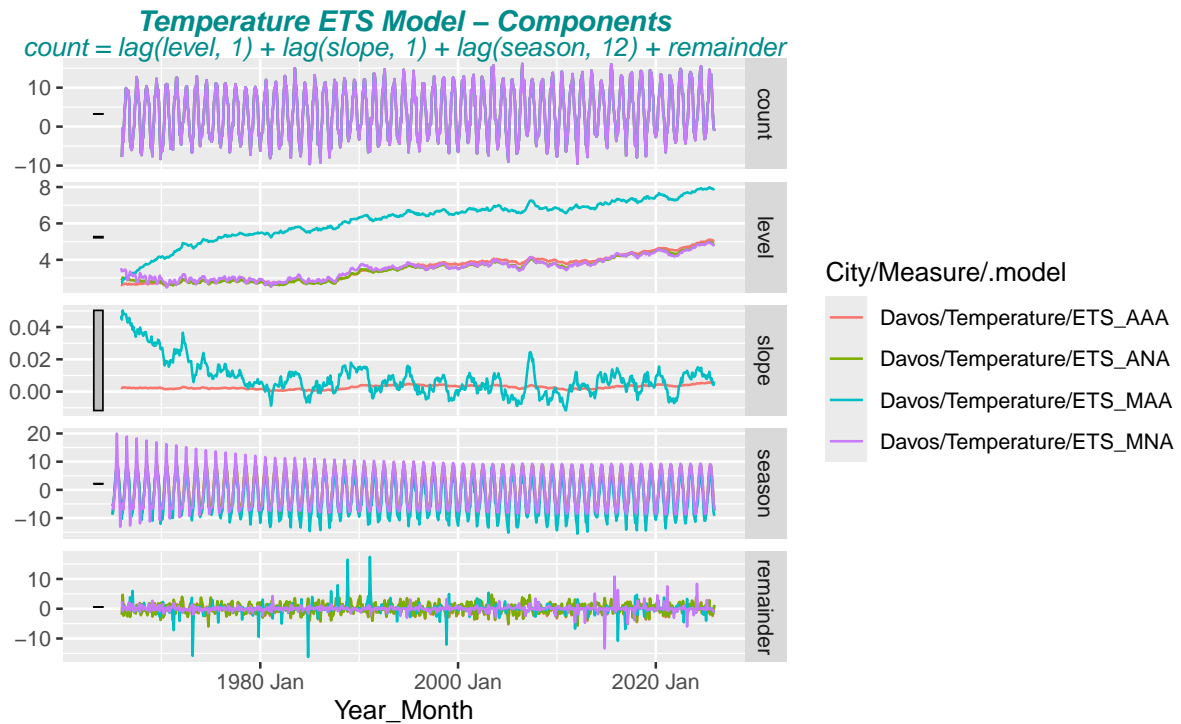
- CV, AIC, AICc and BIC with the lowest values
- Adjusted  $R^2$  the model with the highest value.

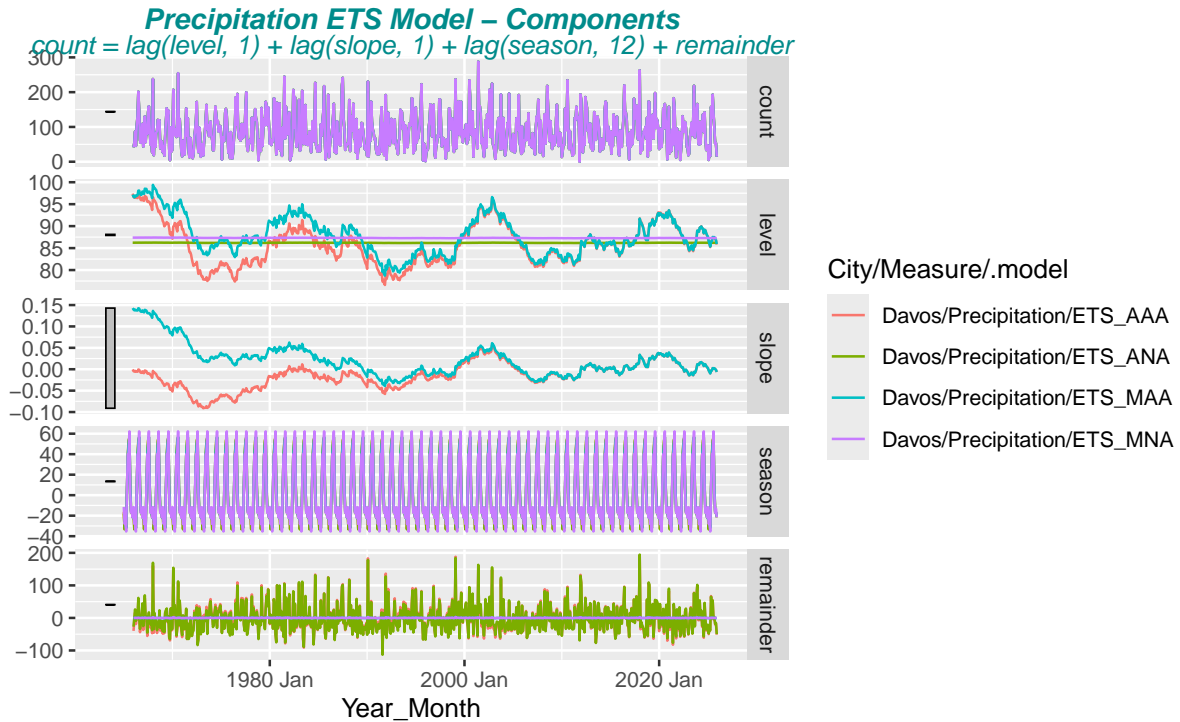
```
#> # A tibble: 16 x 9
```

```

#>   City Measure      .model      AIC      AICc      BIC      MSE      AMSE      MAE
#>   <chr> <fct>      <chr>      <dbl>      <dbl>      <dbl>      <dbl>      <dbl>      <dbl>
#> 1 Davos Temperature ETS_AAA  5533.  5534.  5611.    2.88    2.88    1.36
#> 2 Davos Temperature ETS_AMA  5533.  5534.  5611.    2.88    2.88    1.36
#> 3 Davos Temperature ETS_AAdA 5537.  5538.  5619.    2.89    2.88    1.37
#> 4 Davos Temperature ETS_ANA  5545.  5546.  5614.    2.95    2.94    1.38
#> 5 Davos Temperature ETS_MNA  7199.  7200.  7268.    5.77    5.77    0.573
#> 6 Davos Temperature ETS_MAdA 7658.  7659.  7740.    5.60    5.61    0.772
#> 7 Davos Temperature ETS_MMA  7799.  7800.  7877.    5.44    5.46    0.802
#> 8 Davos Temperature ETS_MAA  7805.  7806.  7883.    5.15    5.15    0.779
#> 9 Davos Precipitation ETS_ANA 10182. 10182. 10250.   1845.   1847.   32.8
#>10 Davos Precipitation ETS_AAdA 10188. 10189. 10271.   1847.   1848.   32.7
#>11 Davos Precipitation ETS_AAA  10198. 10199. 10276.   1877.   1875.   33.3
#>12 Davos Precipitation ETS_AMA  10198. 10199. 10276.   1878.   1876.   33.3
#>13 Davos Precipitation ETS_MNA 10269. 10270. 10338.   1863.   1865.    0.413
#>14 Davos Precipitation ETS_MAdA 10298. 10299. 10381.   1863.   1864.    0.415
#>15 Davos Precipitation ETS_MAA 10314. 10315. 10391.   1897.   1895.    0.420
#>16 Davos Precipitation ETS_MMA 10316. 10317. 10394.   1898.   1896.    0.421

```





### 2.1.1 Residual Accuracy with one-step-ahead fitted residuals - check RMSE, MAE

Residual accuracy can be computed directly from models as the one-step-ahead fitted residuals are available. Select forecast models that minimises for lowest

- MAE (Mean absolute error, will lead to forecasts of the median) and
- RMSE (Root mean squared error, lead to forecasts of the mean)

```
#> # A tibble: 16 x 7
#>   City Measure      .model .type      ME RMSE  MAE
#>   <chr> <fct>      <chr> <chr>    <dbl> <dbl> <dbl>
#> 1 Davos Temperature ETS_AAA Training 0.0412 1.70 1.36
#> 2 Davos Temperature ETS_AMA Training 0.0183 1.70 1.36
#> 3 Davos Temperature ETS_AAdA Training 0.0967 1.70 1.37
#> 4 Davos Temperature ETS_ANA Training 0.121 1.72 1.38
#> 5 Davos Temperature ETS_MAA Training -0.0534 2.27 1.79
#> 6 Davos Temperature ETS_MMA Training -0.0481 2.33 1.84
#> 7 Davos Temperature ETS_MAdA Training 0.0177 2.37 1.87
#> 8 Davos Temperature ETS_MNA Training 0.0588 2.41 1.77
#> 9 Davos Precipitation ETS_ANA Training -0.471 43.0 32.8
#> 10 Davos Precipitation ETS_AAdA Training 0.0259 43.0 32.7
#> 11 Davos Precipitation ETS_MNA Training -1.48 43.1 33.3
#> 12 Davos Precipitation ETS_MAdA Training -1.41 43.1 33.2
#> 13 Davos Precipitation ETS_AAA Training -0.0837 43.3 33.3
#> 14 Davos Precipitation ETS_AMA Training -0.139 43.3 33.3
#> 15 Davos Precipitation ETS_MAA Training -1.98 43.5 33.8
#> 16 Davos Precipitation ETS_MMA Training -1.85 43.5 33.7
```

### 2.1.2 Ljung-Box Test - independence/white noise of the forecasts residuals

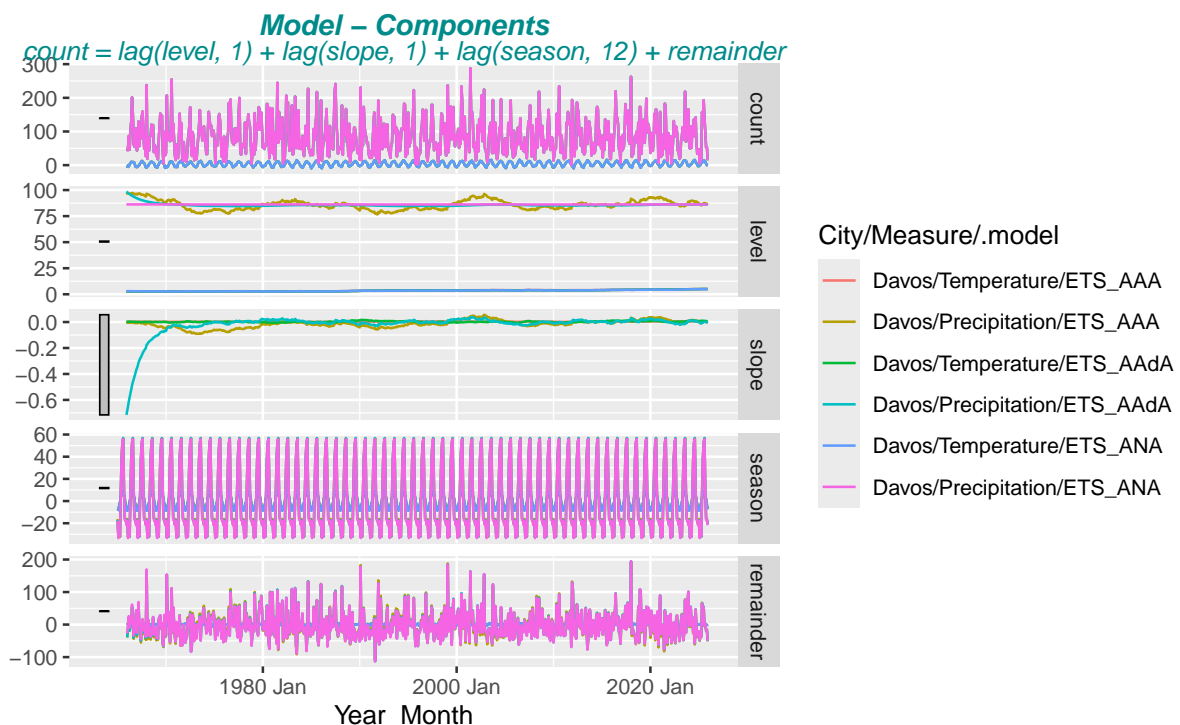
```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 16 x 5
#>   City Measure      .model lb_stat lb_pvalue
#>   <chr> <fct>      <chr>    <dbl>    <dbl>
#> 1 Davos Precipitation ETS_MNA      28.6     0.125
```

```

#> 2 Davos Precipitation ETS_MAdA 30.0 0.0912
#> 3 Davos Precipitation ETS_MMA 30.1 0.0906
#> 4 Davos Precipitation ETS_MAA 30.3 0.0863
#> 5 Davos Precipitation ETS_ANA 31.8 0.0607
#> 6 Davos Precipitation ETS_AAdA 32.2 0.0564
#> 7 Davos Precipitation ETS_AMA 32.2 0.0559
#> 8 Davos Precipitation ETS_AAA 32.3 0.0546
#> 9 Davos Temperature ETS_AAA 33.1 0.0456
#> 10 Davos Temperature ETS_AAdA 33.3 0.0434
#> 11 Davos Temperature ETS_AMA 33.4 0.0416
#> 12 Davos Temperature ETS_ANA 34.8 0.0301
#> 13 Davos Temperature ETS_MAA 190. 0
#> 14 Davos Temperature ETS_MAdA 217. 0
#> 15 Davos Temperature ETS_MMA 187. 0
#> 16 Davos Temperature ETS_MNA 565. 0

```

### 2.1.3 ETS Models - components of ETS(A,N,A), ETS(A,A,A), ETS(A,Ad,A), models

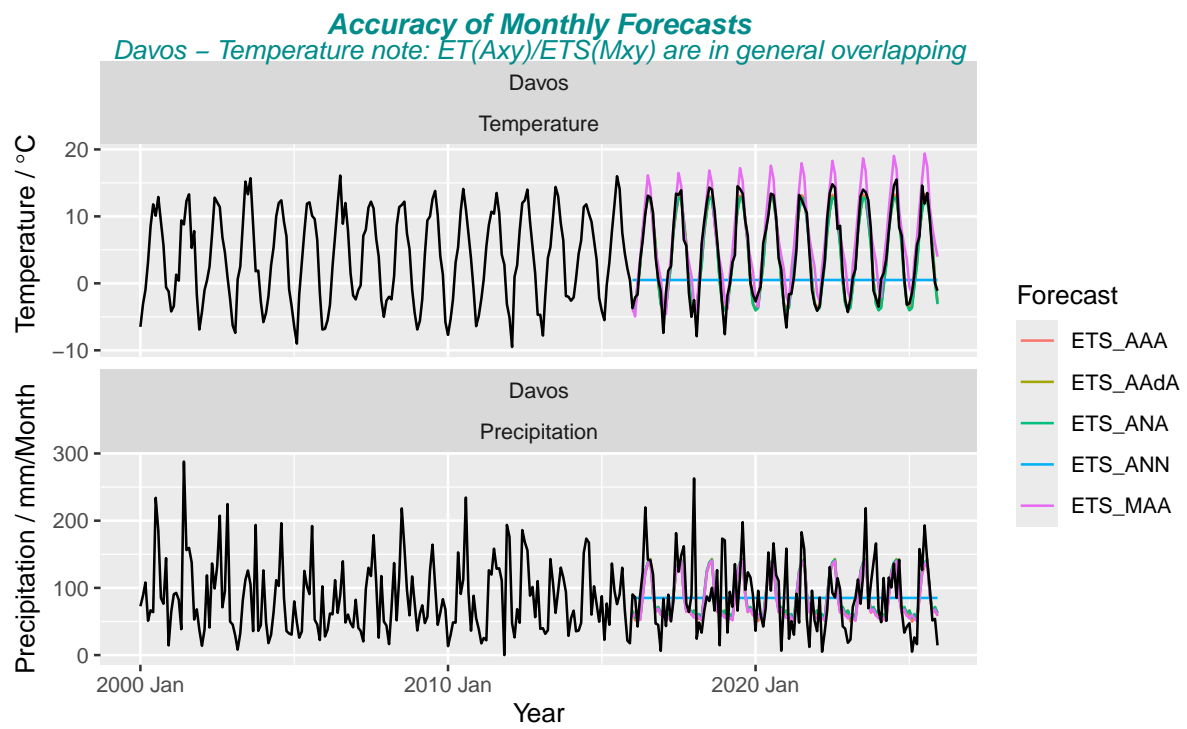


### 2.1.4 Forecast Accuracy with Training/Test Data

```

#> # A tibble: 10 x 7
#>   .model City Measure .type ME RMSE MAE
#>   <chr>   <chr> <fct>   <chr> <dbl> <dbl> <dbl>
#> 1 ETS_AAA Davos Temperature Test 0.243 1.63 1.30
#> 2 ETS_AAdA Davos Temperature Test 0.472 1.69 1.37
#> 3 ETS_ANA Davos Temperature Test 0.591 1.72 1.40
#> 4 ETS_MAA Davos Temperature Test -2.05 3.11 2.60
#> 5 ETS_ANN Davos Temperature Test 4.27 7.80 6.35
#> 6 ETS_AAdA Davos Precipitation Test 4.06 44.0 34.1
#> 7 ETS_ANA Davos Precipitation Test 3.71 44.1 34.3
#> 8 ETS_AAA Davos Precipitation Test 7.44 44.4 34.0
#> 9 ETS_MAA Davos Precipitation Test 6.82 45.1 34.5
#> 10 ETS_ANN Davos Precipitation Test 4.10 52.1 41.4

```



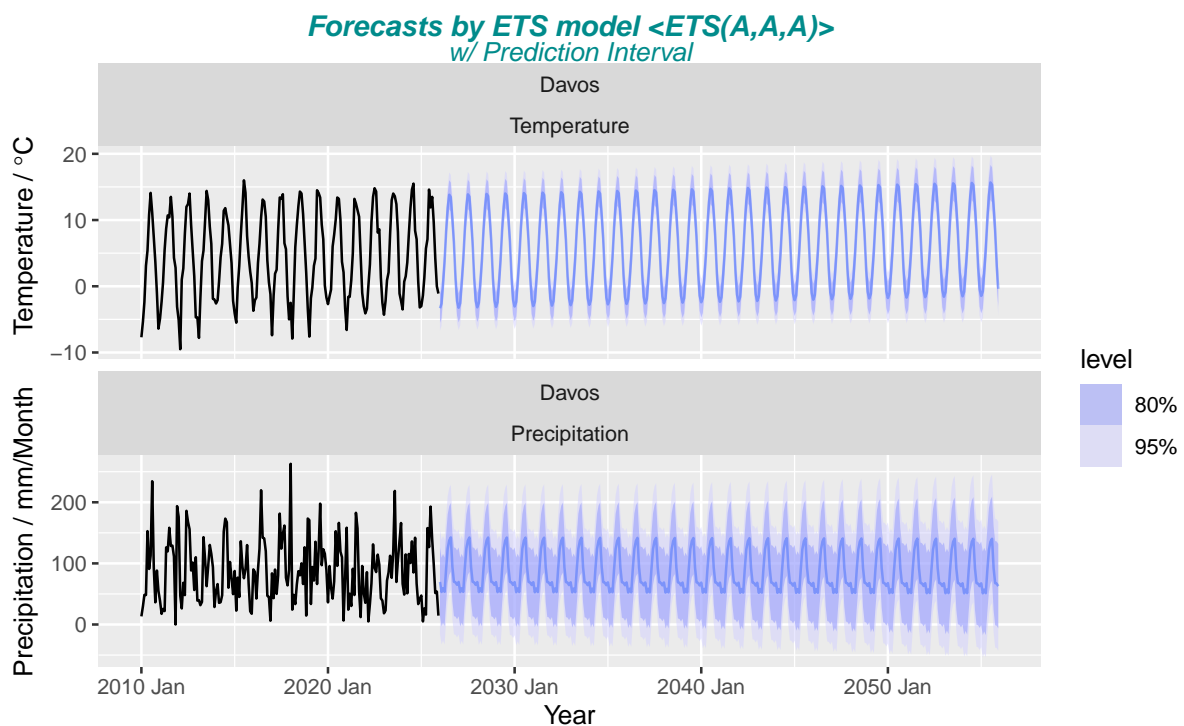
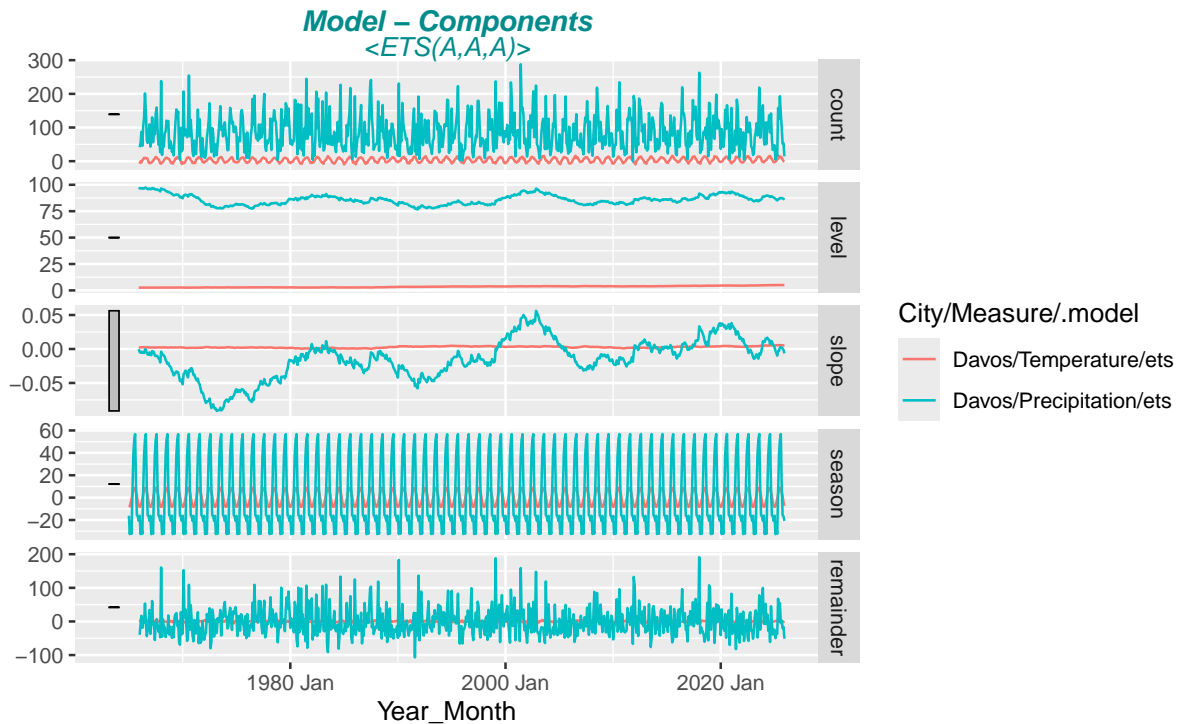
## 2.2 Forecasting with selected ETS model $\langle \text{ETS}(A,A,A) \rangle$ , $\langle \text{ETS}(A,A,A) \rangle$

### 2.2.1 Forecast Plot of selected ETS model

```
#> Provide model coefficients by report(fit_model)
```

```
#> # A tibble: 2 x 11
```

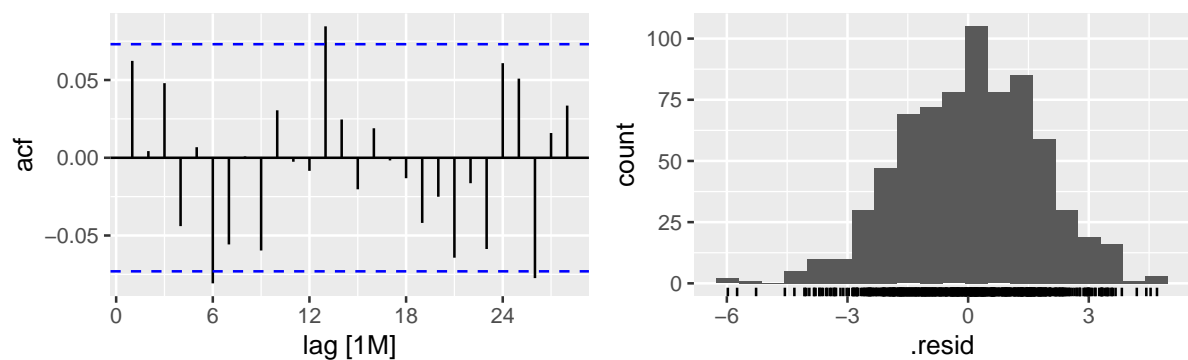
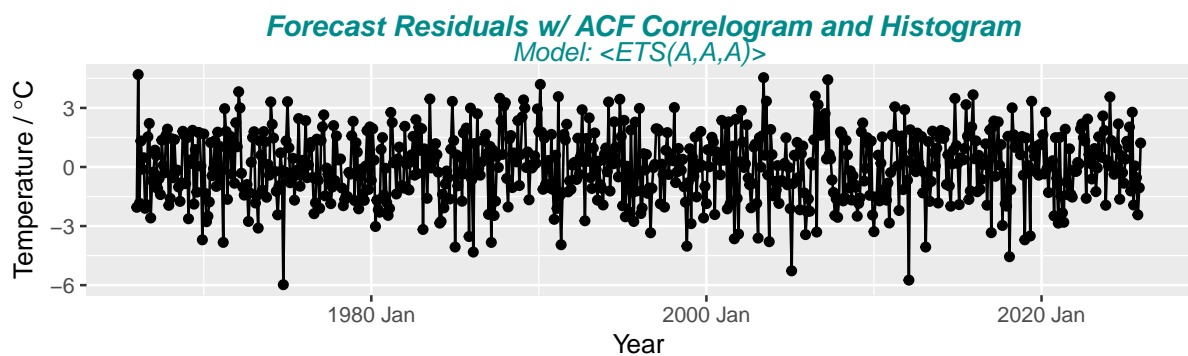
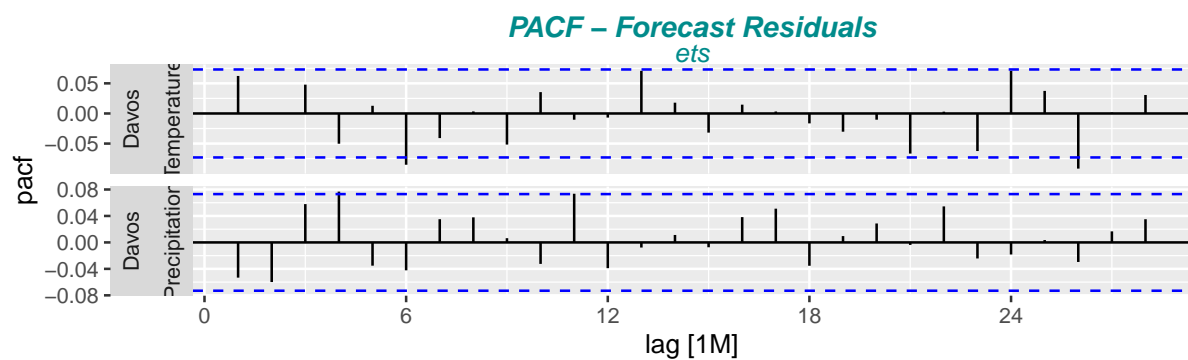
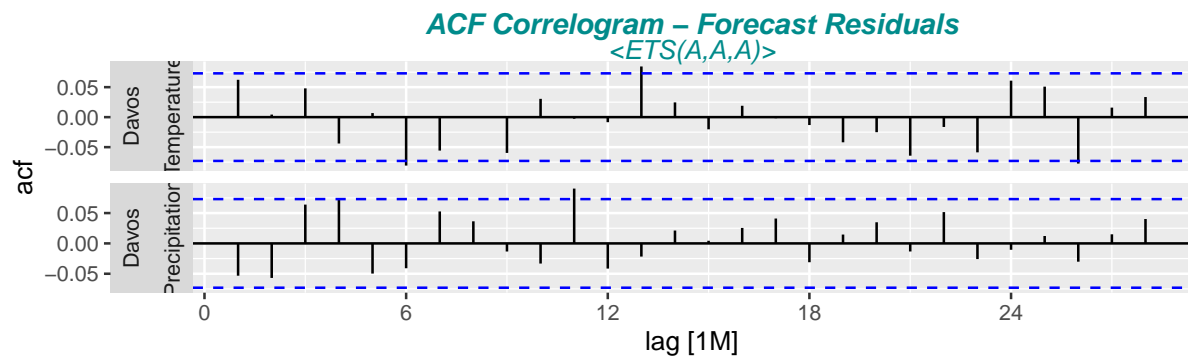
#>	City	Measure	.model	sigma2	log_lik	AIC	AICc	BIC	MSE	AMSE	MAE
#>	<chr>	<fct>	<chr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
#> 1	Davos	Temperat~	ets	2.95e0	-2749.	5533.	5534.	5611.	2.88e0	2.88e0	1.36
#> 2	Davos	Precipit~	ets	1.92e3	-5082.	10198.	10199.	10276.	1.88e3	1.88e3	33.3

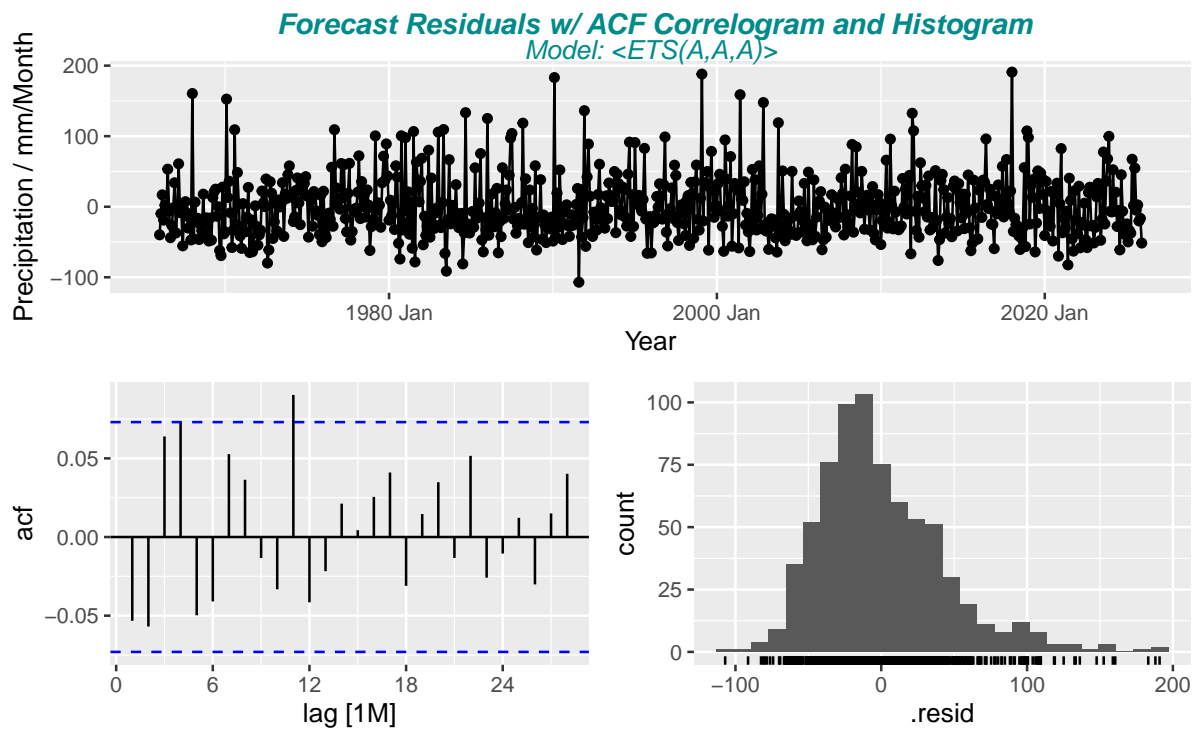


### 2.2.2 Residual Stationarity

Required checks to be ready for forecasting:

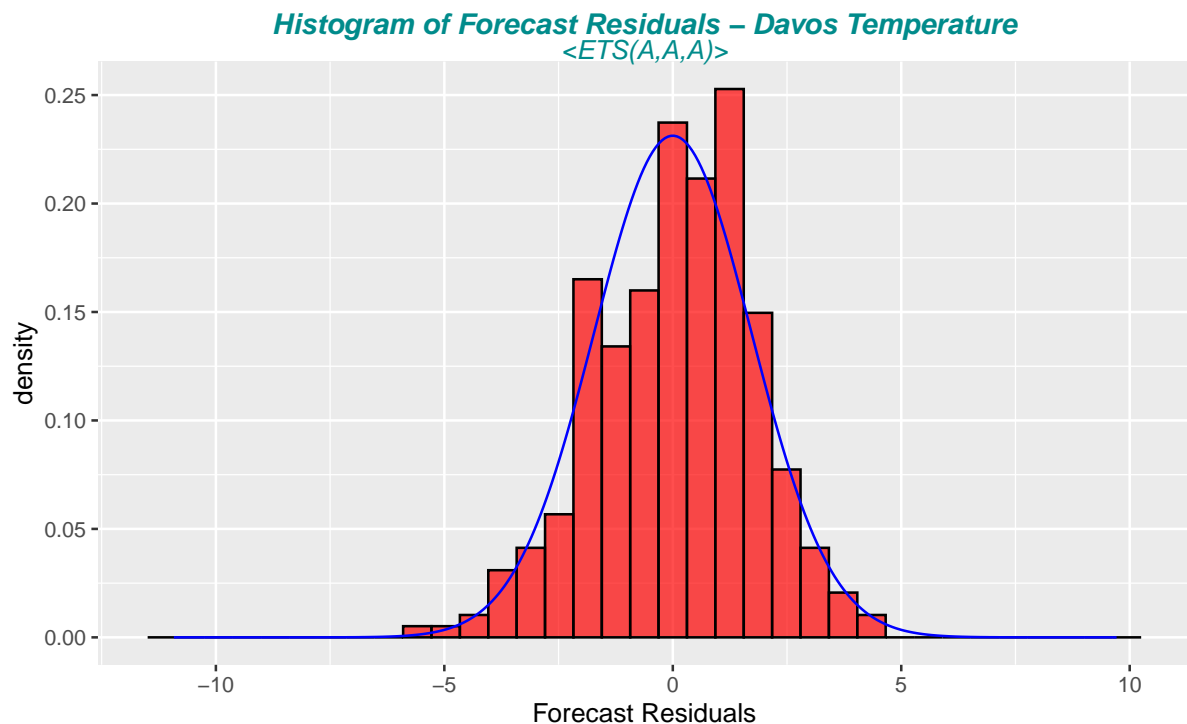
- ACF Forecast Residual: all spikes are within the significance limits, so the residuals appear to be white noise
- The Ljung-Box test also shows that the residuals have no remaining autocorrelations
- Forecast Residuals are more or less normally distributed with roughly centred on zero

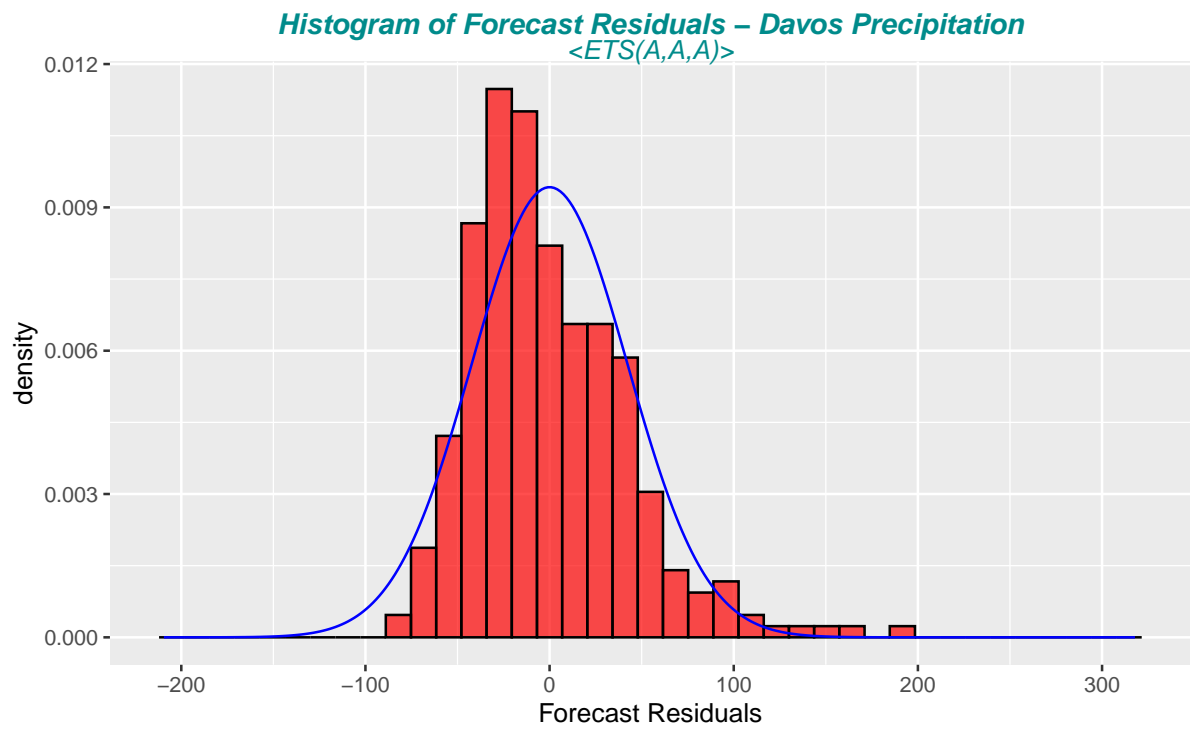




### 2.2.3 Ljung-Box Test and Histogram of forecast residuals with overlaid normal curve

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 2 x 5
#>   City Measure      .model lb_stat lb_pvalue
#>   <chr> <fct>      <chr>   <dbl>   <dbl>
#> 1 Davos Temperature ets       20.8   0.652
#> 2 Davos Precipitation ets       33.9   0.0861
```





### 3 ARIMA Forecasting Models - AutoRegressive-Integrated Moving Average

Exponential smoothing and ARIMA (AutoRegressive-Integrated Moving Average) models are the two most widely used approaches to time series forecasting, and provide complementary approaches to the problem.

While exponential smoothing models are based on a description of the trend and seasonality in the data, ARIMA models aim to describe the autocorrelations in the data.

#### 3.1 Seasonal ARIMA models

Non-seasonal ARIMA models are generally denoted  $ARIMA(p,d,q)$  where parameters  $p$ ,  $d$ , and  $q$  are non-negative integers, \*  $p$  is the order (number of time lags) of the autoregressive model \*  $d$  is the degree of differencing (number of times the data have had past values subtracted) \*  $q$  is the order of the moving-average model of past forecast errors .

The value of  $d$  has an effect on the prediction intervals — the higher the value of  $d$ , the more rapidly the prediction intervals increase in size. For  $d=0$ , the point forecasts are equal to the mean of the data and the long-term forecast standard deviation will go to the standard deviation of the historical data, so the prediction intervals will all be essentially the same.

Seasonal ARIMA models are usually denoted  $ARIMA(p,d,q)(P,D,Q)m$ , where  $m$  refers to the number of periods in each season, and the uppercase  $P,D,Q$  refer to the autoregressive, differencing, and moving average terms for the seasonal part of the ARIMA model.

*ARIMA(pdq)(PDQ) model with automatically selected (pdq)(PDQ) values*

**Fit of different pre-defined *ARIMA(pdq)(PDQ)* models**

```
#> Model Selection by Information Criterion - lowest AIC, AICc, BIC
#> choose p, q parameter accordingly - but only for same d, D values
#> # A tibble: 8 x 8
#>   City Measure      .model      sigma2 log_lik    AIC  AICc    BIC
#>   <chr> <fct>      <chr>      <dbl>    <dbl> <dbl> <dbl> <dbl>
#> 1 Davos Temperature arima_012_011    2.99  -1406. 2821. 2821. 2839.
#> 2 Davos Temperature arima_111_011    2.99  -1406. 2821. 2821. 2839.
#> 3 Davos Temperature arima_111_012    2.99  -1406. 2823. 2823. 2845.
#> 4 Davos Temperature arima_211_011    2.99  -1406. 2823. 2823. 2846.
#> 5 Davos Temperature arima_012_112    2.99  -1406. 2824. 2824. 2851.
#> 6 Davos Temperature arima_300_111    3.11  -1413. 2838. 2838. 2865.
#> 7 Davos Temperature arima_102_211    3.13  -1414. 2842. 2842. 2874.
#> 8 Davos Temperature arima_100_200    3.13  -1414. 2842. 2842. 2874.
#> # A tibble: 8 x 8
#>   City Measure      .model      sigma2 log_lik    AIC  AICc    BIC
#>   <chr> <fct>      <chr>      <dbl>    <dbl> <dbl> <dbl> <dbl>
#> 1 Davos Precipitation arima_211_011   1874.  -3696. 7402. 7402. 7425.
#> 2 Davos Precipitation arima_012_011   1877.  -3697. 7402. 7402. 7420.
#> 3 Davos Precipitation arima_111_011   1878.  -3697. 7402. 7402. 7420.
#> 4 Davos Precipitation arima_111_012   1878.  -3697. 7404. 7404. 7427.
#> 5 Davos Precipitation arima_012_112   1881.  -3697. 7406. 7406. 7433.
#> 6 Davos Precipitation arima_100_210   2559.  -3784. 7576. 7576. 7594.
#> 7 Davos Precipitation arima_200_011   2835.  -3819. 7646. 7646. 7664.
#> 8 Davos Precipitation arima_100_110_c 2838.  -3819. 7648. 7648. 7671.
```

Good models are obtained by minimising the AIC, AICc or BIC (see `glance(fit_arima)` output). The preference is to use the AICc to select  $p$  and  $q$ .

These information criteria tend not to be good guides to selecting the appropriate order of differencing ( $d$ ) of a model, but only for selecting the values of  $p$  and  $q$ . This is because the differencing changes the data on which the likelihood is computed, making the AIC values between models with different orders of differencing not comparable.

### 3.1.1 Residual Accuracy with one-step-ahead fitted residuals - check RMSE, MAE

Residual accuracy can be computed directly from models as the one-step-ahead fitted residuals are available. Select forecast models that minimises for lowest

- MAE (Mean absolute error, will lead to forecasts of the median) and
- RMSE (Root mean squared error, lead to forecasts of the mean)

```
#> # A tibble: 8 x 7
#>   City Measure .model .type ME RMSE MAE
#>   <chr> <fct>   <chr>   <chr> <dbl> <dbl> <dbl>
#> 1 Davos Temperature arima_012_112 Training 0.0590 1.71 1.35
#> 2 Davos Temperature arima_111_011 Training 0.0597 1.71 1.35
#> 3 Davos Temperature arima_211_011 Training 0.0596 1.71 1.35
#> 4 Davos Temperature arima_012_011 Training 0.0595 1.71 1.35
#> 5 Davos Temperature arima_111_012 Training 0.0591 1.71 1.35
#> 6 Davos Temperature arima_300_111 Training 0.228 1.74 1.38
#> 7 Davos Temperature arima_102_211 Training 0.249 1.75 1.38
#> 8 Davos Temperature arima_100_200 Training 0.249 1.75 1.38
#> # A tibble: 8 x 7
#>   City Measure .model .type ME RMSE MAE
#>   <chr> <fct>   <chr>   <chr> <dbl> <dbl> <dbl>
#> 1 Davos Precipitation arima_211_011 Training 0.334 42.8 32.9
#> 2 Davos Precipitation arima_111_012 Training 0.280 42.8 32.9
#> 3 Davos Precipitation arima_012_112 Training 0.276 42.8 32.9
#> 4 Davos Precipitation arima_012_011 Training 0.294 42.8 32.9
#> 5 Davos Precipitation arima_111_011 Training 0.286 42.8 32.9
#> 6 Davos Precipitation arima_001_002 Training -0.0462 49.6 39.0
#> 7 Davos Precipitation arima_100_210 Training -0.274 50.1 37.9
#> 8 Davos Precipitation arima_100_110_c Training -0.0459 52.7 40.0
```

### 3.1.2 Ljung-Box Test - independence/white noise of the forecasts residuals

#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H<sub>0</sub>

```
#> # A tibble: 8 x 5
#>   City Measure .model lb_stat lb_pvalue
#>   <chr> <fct>   <chr>   <dbl>   <dbl>
#> 1 Davos Temperature arima_300_111 23.5 0.317
#> 2 Davos Temperature arima_100_200 25.1 0.242
#> 3 Davos Temperature arima_102_211 25.1 0.242
#> 4 Davos Temperature arima_012_112 29.3 0.106
#> 5 Davos Temperature arima_111_012 29.7 0.0977
#> 6 Davos Temperature arima_111_011 29.8 0.0959
#> 7 Davos Temperature arima_211_011 29.8 0.0954
#> 8 Davos Temperature arima_012_011 29.9 0.0948
#> # A tibble: 8 x 5
#>   City Measure .model lb_stat lb_pvalue
#>   <chr> <fct>   <chr>   <dbl>   <dbl>
#> 1 Davos Precipitation arima_211_011 25.2 2.38e- 1
#> 2 Davos Precipitation arima_012_112 29.3 1.07e- 1
#> 3 Davos Precipitation arima_012_011 29.7 9.90e- 2
#> 4 Davos Precipitation arima_111_012 29.8 9.68e- 2
#> 5 Davos Precipitation arima_111_011 30.2 8.80e- 2
#> 6 Davos Precipitation arima_100_210 46.9 9.58e- 4
#> 7 Davos Precipitation arima_100_110_c 105. 3.76e-13
#> 8 Davos Precipitation arima_200_110_c 105. 3.76e-13
```

### 3.1.3 Forecast Accuracy with Training/Test Data

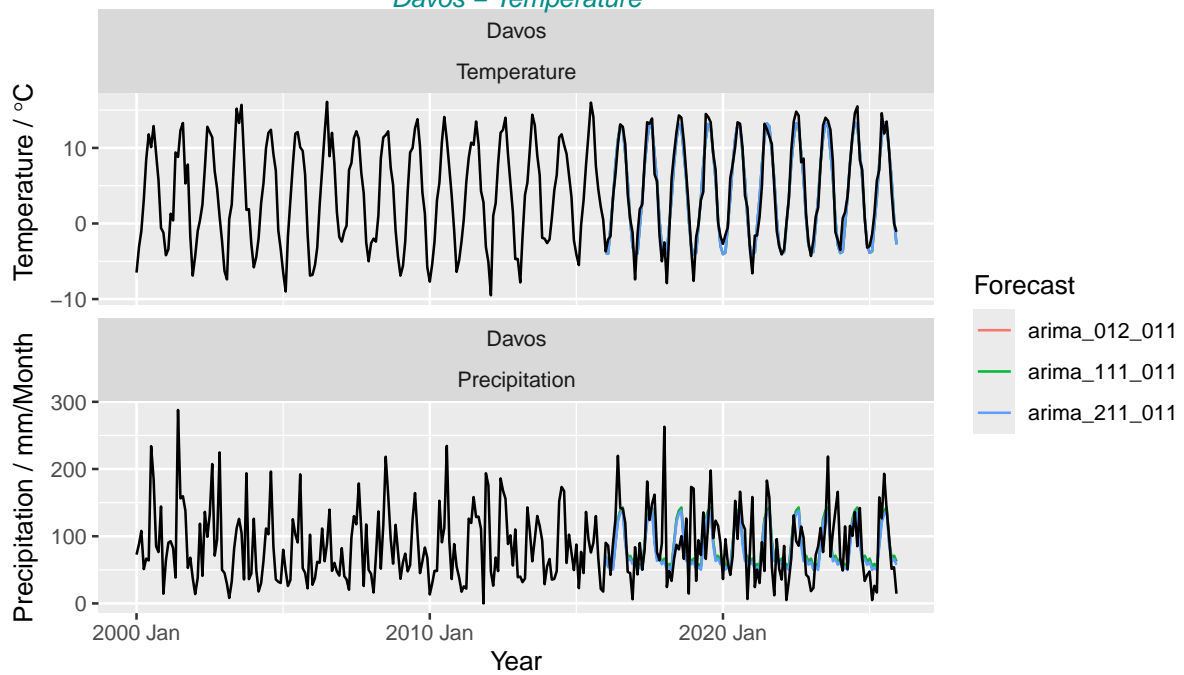
```
#> # A tibble: 6 x 7
```

```

#>   .model      City Measure      .type    ME  RMSE  MAE
#>   <chr>      <chr> <fct>      <chr> <dbl> <dbl> <dbl>
#> 1 arima_111_011 Davos Temperature Test  0.338  1.64  1.32
#> 2 arima_211_011 Davos Temperature Test  0.339  1.64  1.32
#> 3 arima_012_011 Davos Temperature Test  0.339  1.64  1.32
#> 4 arima_012_011 Davos Precipitation Test  3.79 44.0 34.1
#> 5 arima_111_011 Davos Precipitation Test  3.80 44.0 34.1
#> 6 arima_211_011 Davos Precipitation Test  8.34 44.5 33.9

```

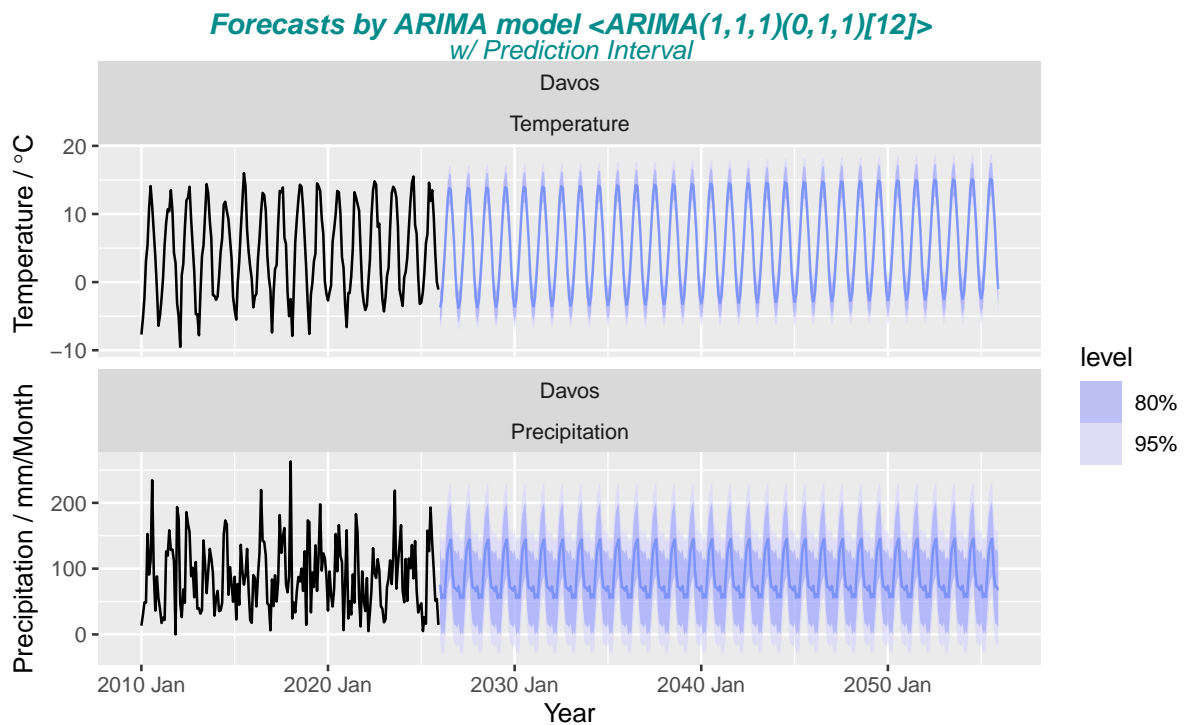
### Accuracy of Monthly Forecasts w/ Training and Test data Davos – Temperature



## 3.2 Temperature, Precipitation - Forecasting with selected ARIMA model <ARIMA(1,1,1)(0,1,1)[12]>, <ARIMA(1,1,1)(0,1,1)[12]>

### 3.2.1 Forecast Plot of selected ARIMA model

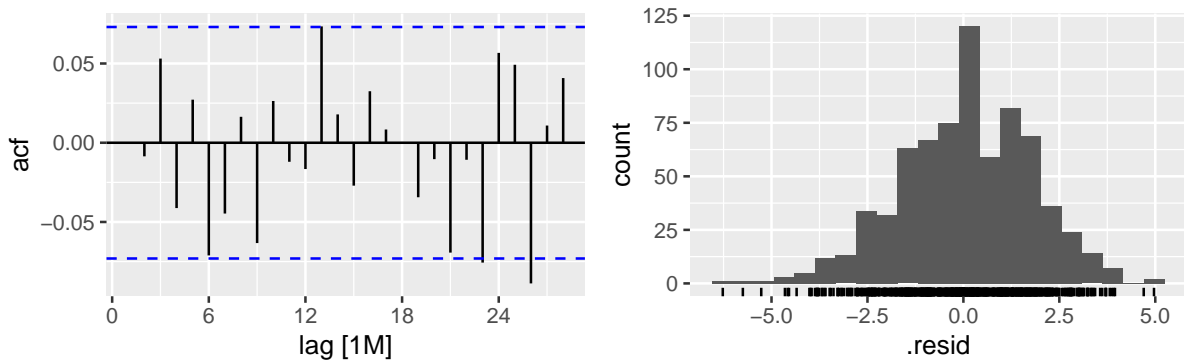
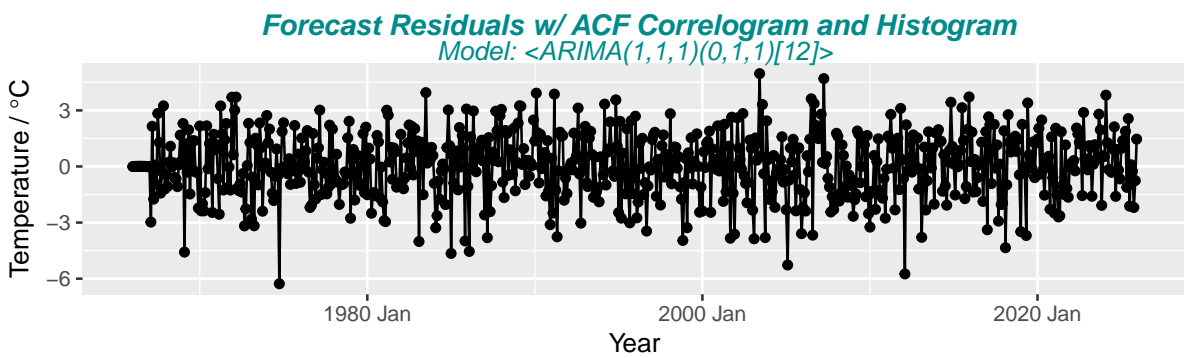
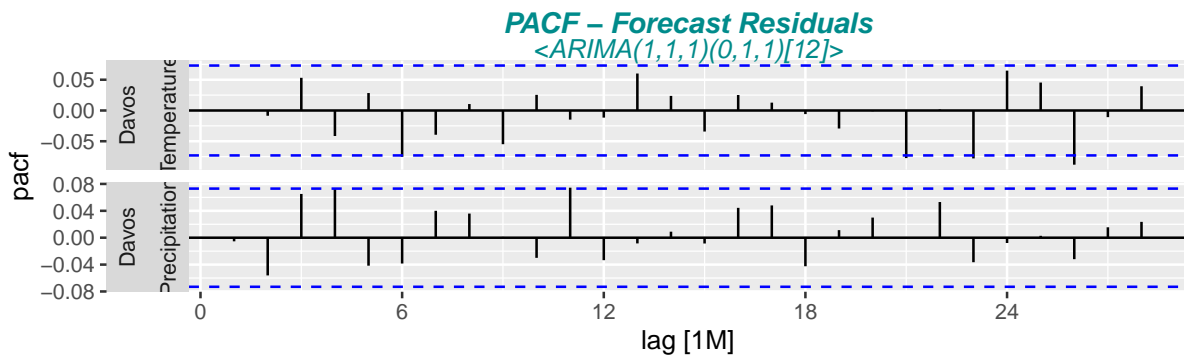
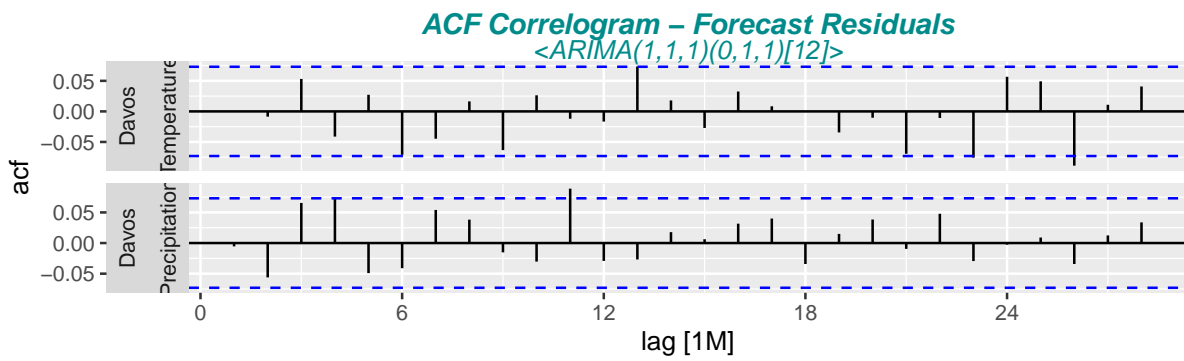
```
#> Provide model coefficients by report(fit_model)
#> # A tibble: 2 x 10
#>   City Measure      .model sigma2 log_lik  AIC  AICc  BIC ar_roots ma_roots
#>   <chr> <fct>      <chr>   <dbl>  <dbl> <dbl> <dbl> <dbl> <list>  <list>
#> 1 Davos Temperature  arima    2.99 -1406. 2821. 2821. 2839. <cpl>  <cpl>
#> 2 Davos Precipitation arima  1878. -3697. 7402. 7402. 7420. <cpl>  <cpl>
```

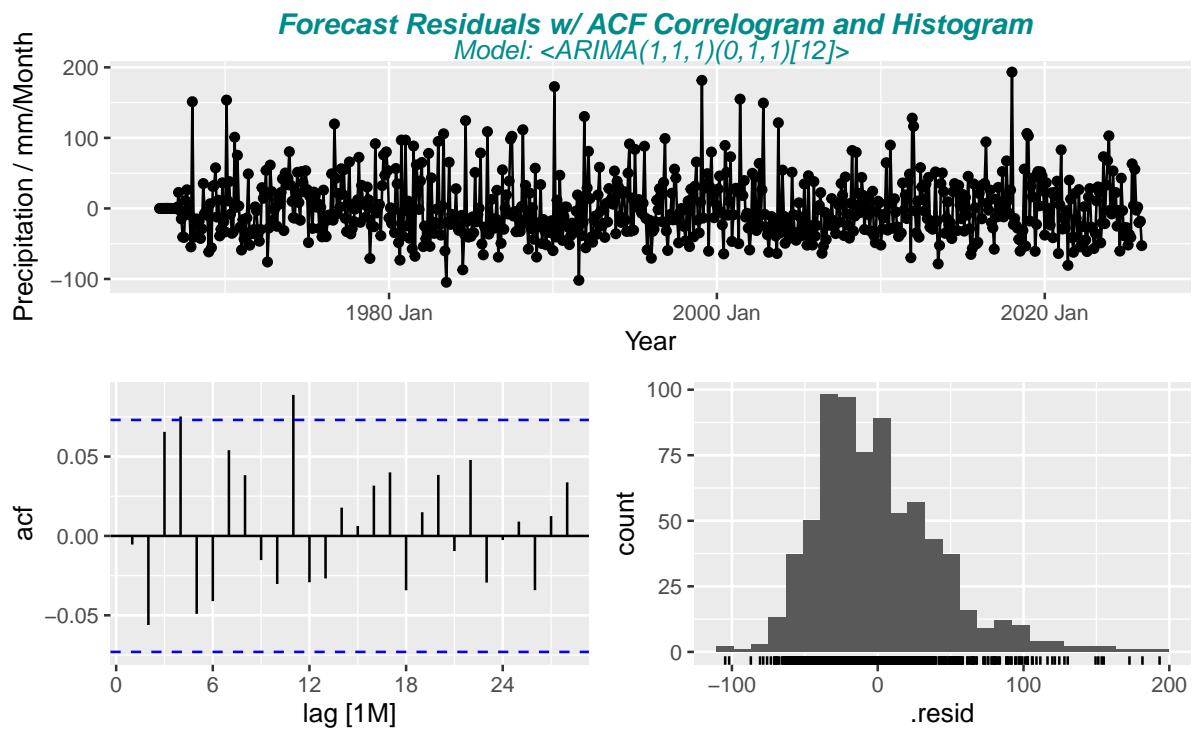


### 3.2.2 Residual Stationarity

Required checks to be ready for forecasting:

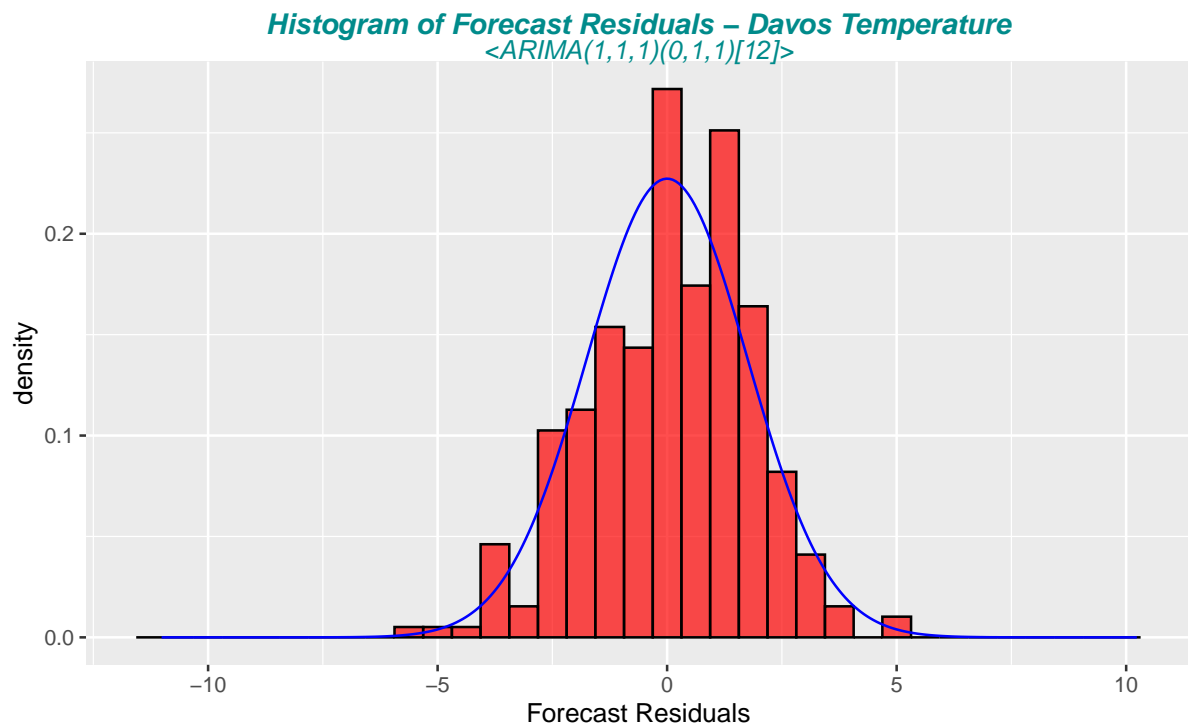
- ACF Forecast Residual: all spikes are within the significance limits, so the residuals appear to be white noise
- The Ljung-Box test also shows that the residuals have no remaining autocorrelations
- Forecast Residuals are more or less normally distributed with roughly centred on zero



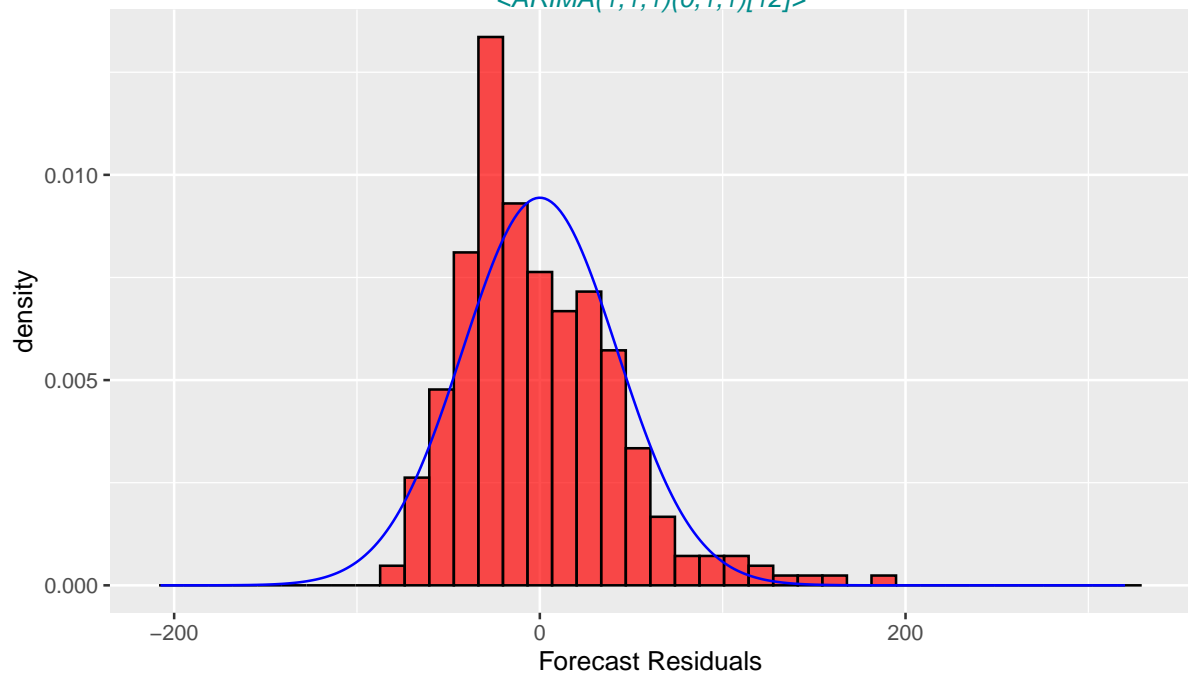


### 3.2.3 Ljung-Box Test and Histogram of forecast residuals with overlaid normal curve

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 2 x 5
#>   City Measure      .model lb_stat lb_pvalue
#>   <chr> <fct>      <chr>   <dbl>   <dbl>
#> 1 Davos Temperature arima      18.9   0.594
#> 2 Davos Precipitation arima     32.7   0.0491
```



***Histogram of Forecast Residuals – Davos Precipitation***  
*<ARIMA(1,1,1)(0,1,1)[12]>*



## 4 ARIMA vs ETS

In particular, all ETS models are non-stationary, while some ARIMA models are stationary.

The ETS models with seasonality or non-damped trend or both have two unit roots (i.e., they need two levels of differencing to make them stationary). All other ETS models have one unit root (they need one level of differencing to make them stationary).

We compare for the chosen ETS resp. ARIMA model the RMSE / MAE values. Lower values indicate a more accurate model based on the test set RMSE, ..., MASE.

- Residual Accuracy with one-step-ahead fitted residuals
- Forecast Accuracy with Training/Test Data

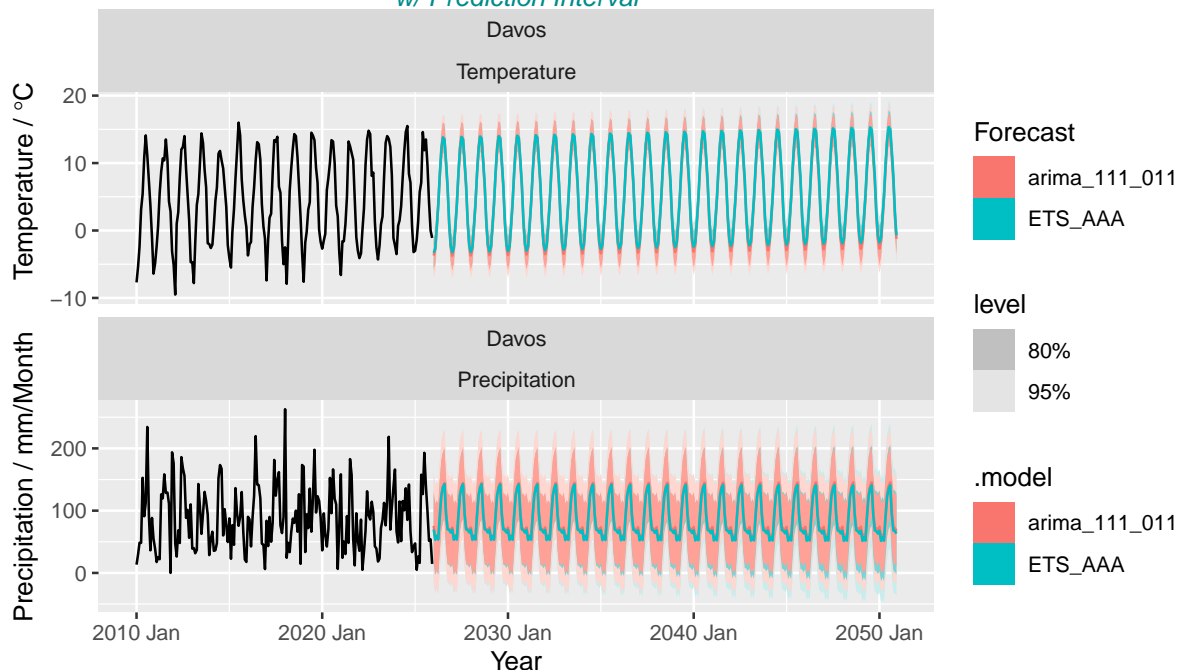
Note: a good fit to training data is never an indication that the model will forecast well. Therefore the values of the Forecast Accuracy are the more relevant one.

### 4.0.1 Comparing Residual and Forecast Accuracy of selected ETS and ARIMA model

```
#> # A tibble: 8 x 9
#>   City Measure      .model      .type      RMSE      MAE      MAPE      MASE      RMSSE
#>   <chr> <fct>      <chr>      <chr>      <dbl>    <dbl>    <dbl>    <dbl>    <dbl>
#> 1 Davos Temperature ETS_AAA      Test        1.63     1.30    Inf      0.696    0.688
#> 2 Davos Temperature arima_111_011 Test        1.64     1.32    Inf      0.705    0.691
#> 3 Davos Temperature ets           Training    1.70     1.36    Inf      0.722    0.711
#> 4 Davos Temperature arima           Training    1.71     1.35    Inf      0.714    0.716
#> 5 Davos Precipitation arima           Training   42.8    32.9    Inf      0.698    0.689
#> 6 Davos Precipitation ets           Training   43.3    33.3    Inf      0.706    0.696
#> 7 Davos Precipitation arima_111_011 Test       44.0    34.1   83.1     0.736    0.709
#> 8 Davos Precipitation ETS_AAA      Test       44.4    34.0   78.3     0.735    0.715
```

### 4.0.2 Forecast Plot of selected ETS and ARIMA model

**Forecasts by ETS <ETS(A,A,A)> and ARIMA model <ARIMA(1,1,1)(0,1,1)[12]>**  
w/ Prediction Interval



Forecasts by ETS  $\langle ETS(A,A,A) \rangle$  and ARIMA model  $\langle ARIMA(1,1,1)(0,1,1)[12] \rangle$   
w/ Prediction Interval

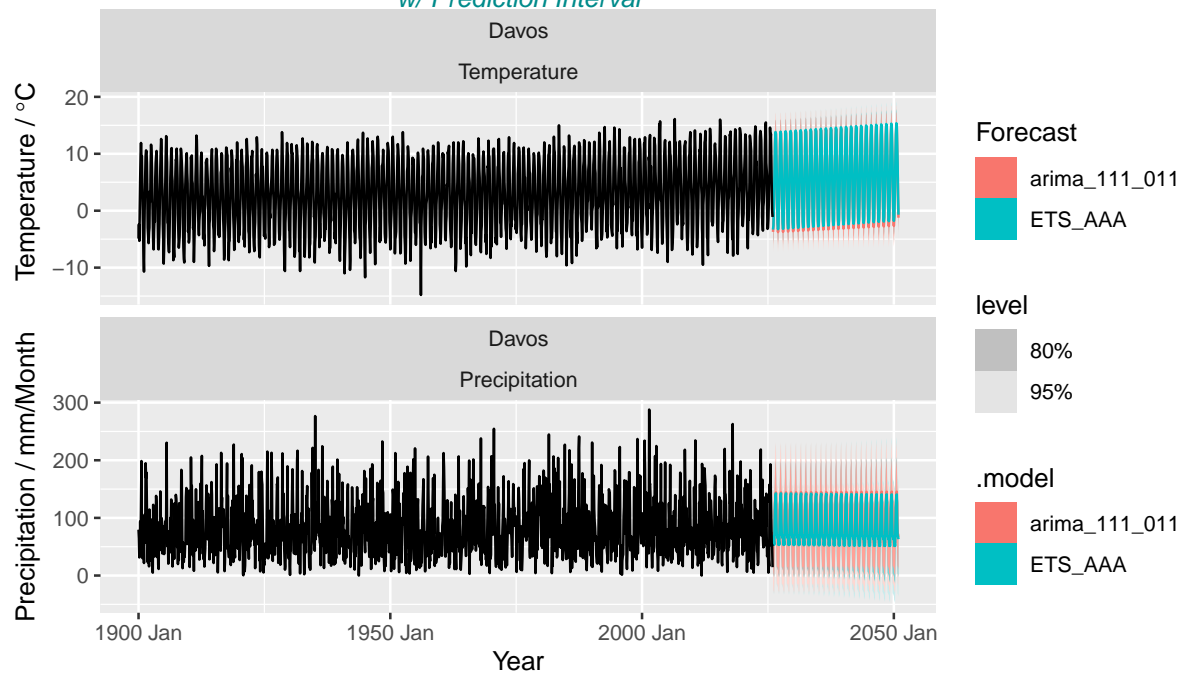


Table 1: Mean values for the given time periods; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Period_Time	Temperature	Precipitation
1841-1870	2.1	76.2
1871-1900	1.9	78.9
1901-1930	2.2	80.2
1931-1960	2.4	81.4
1961-1990	2.8	83.2
1991-2020	3.9	87.2
2021-2025	5.0	82.1

Table 2: Mean Yearly ARIMA and ETS Forecast values (next 25 years); Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

City	Measure	Year	ETS_AAA	arma_111_011
Davos	Temperature	2026	5.11	5.01
Davos	Temperature	2030	5.37	5.20
Davos	Temperature	2035	5.69	5.43
Davos	Temperature	2040	6.00	5.67
Davos	Temperature	2045	6.32	5.91
Davos	Temperature	2050	6.64	6.15
Davos	Precipitation	2026	85.77	87.55
Davos	Precipitation	2030	85.46	87.55
Davos	Precipitation	2035	85.07	87.81
Davos	Precipitation	2040	84.68	88.07
Davos	Precipitation	2045	84.29	88.34
Davos	Precipitation	2050	83.90	88.60

Table 3: Forecast increase/decrease over the next 25 years; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Measure	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETs	Delta_ARIMA
Temperature	2026	2050	5.11	5.01	6.64	6.15	1.53	1.14
Precipitation	2026	2050	85.77	87.55	83.90	88.60	-1.87	1.05

Table 4: Forecast increase/decrease over the next 25 years; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Measure	Month	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETs	Delta_ARIMA
Temperature	Jan	2026	2050	-3.31	-3.72	-1.78	-2.66	1.53	1.06
Temperature	Feb	2026	2050	-2.62	-2.70	-1.08	-1.56	1.53	1.14
Temperature	Mrz	2026	2050	0.34	0.27	1.88	1.41	1.53	1.14
Temperature	Apr	2026	2050	3.72	3.89	5.25	5.04	1.53	1.14
Temperature	Mai	2026	2050	8.33	8.07	9.87	9.22	1.53	1.14
Temperature	Jun	2026	2050	11.85	12.27	13.39	13.41	1.53	1.14
Temperature	Jul	2026	2050	13.82	13.78	15.35	14.92	1.53	1.14
Temperature	Aug	2026	2050	13.47	13.56	15.00	14.70	1.53	1.14
Temperature	Sep	2026	2050	10.16	9.88	11.69	11.02	1.53	1.14
Temperature	Okt	2026	2050	6.66	6.33	8.19	7.47	1.53	1.14
Temperature	Nov	2026	2050	1.12	0.90	2.66	2.04	1.53	1.14

Measure	Month	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETTS	Delta_ARIMA
Temperature	Dez	2026	2050	-2.23	-2.37	-0.70	-1.23	1.53	1.14
Precipitation	Jan	2026	2050	69.67	75.65	67.80	74.25	-1.87	-1.40
Precipitation	Feb	2026	2050	53.19	54.70	51.32	56.10	-1.87	1.40
Precipitation	Mrz	2026	2050	59.38	60.38	57.51	61.63	-1.87	1.26
Precipitation	Apr	2026	2050	53.48	55.24	51.62	56.50	-1.87	1.26
Precipitation	Mai	2026	2050	91.10	93.14	89.24	94.40	-1.87	1.26
Precipitation	Jun	2026	2050	124.22	125.76	122.36	127.02	-1.87	1.26
Precipitation	Jul	2026	2050	137.51	137.75	135.64	139.01	-1.87	1.26
Precipitation	Aug	2026	2050	142.54	144.25	140.67	145.52	-1.87	1.26
Precipitation	Sep	2026	2050	94.12	95.87	92.26	97.13	-1.87	1.26
Precipitation	Okt	2026	2050	70.00	71.34	68.13	72.61	-1.87	1.26
Precipitation	Nov	2026	2050	69.04	70.62	67.17	71.88	-1.87	1.26
Precipitation	Dez	2026	2050	64.95	65.87	63.09	67.13	-1.87	1.26

## 5 Yearly Data Forecasts with ARIMA and ETS

For yearly data the seasonal monthly data are replaced by the yearly average data. Therefore the seasonal component of the ETS and ARIMA model are to be taken out.

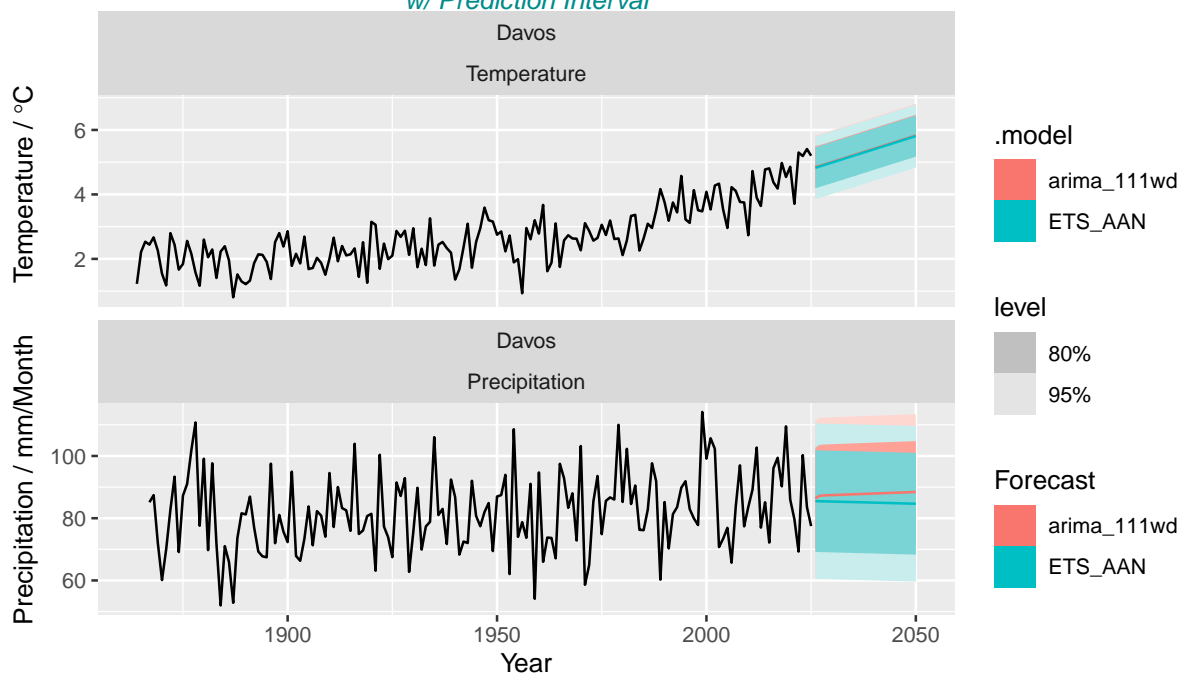
The ETS model  $\langle ETS(A, A, N) \rangle$  with seasonal term change “A”  $\rightarrow$  “N” is chosen. For ARIMA models the seasonal term (P,D,Q)<sub>m</sub> has to be taken out and an optimal ARIMA(p,1,q) with one differencing (d=1) is selected. However, for Mauna Loa two times differencing had to be selected  $\langle CO_2 \rangle \langle ARIMA(0,2,1) \text{ w/ poly} \rangle$ . For Temperature and Precipitation the same model as for monthly data can be taken by leaving out the seasonal term  $\langle ARIMA(0,1,2) \text{ w/ drift} \rangle$ .

### 5.0.1 Comparing Residual and Forecast Accuracy of selected ETS and ARIMA model

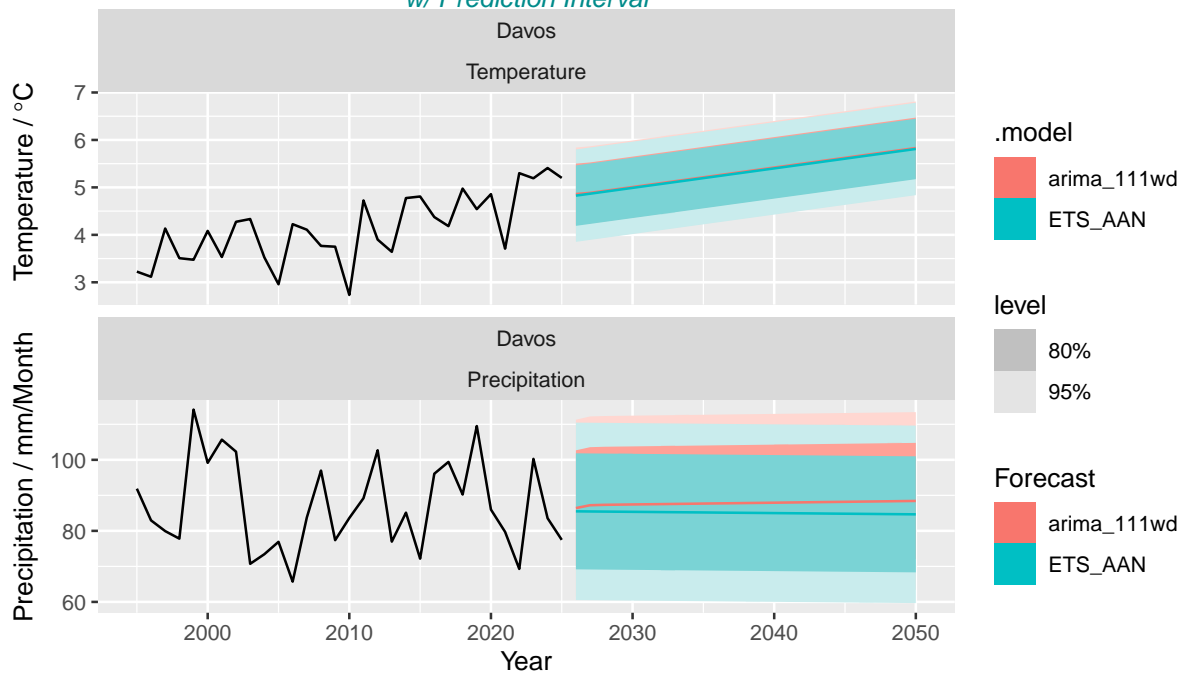
### 5.0.2 Forecast Plot of selected ETS and ARIMA model

```
#> selected: ETS_AAN and arima_111 w/ drift
```

Yearly Forecasts by ETS  $\langle ETS(A,A,N) \rangle$  and ARIMA model  $\langle ARIMA(1,1,1) \text{ w/ drift} \rangle$   
w/ Prediction Interval



## Early Forecasts by ETS <ETS(A,A,N)> and ARIMA model <ARIMA(1,1,1) w/ drift> w/ Prediction Interval



```
#> # A tibble: 4 x 13
#>   City Measure .model sigma2 log_lik AIC AICc BIC MSE AMSE MAE
#>   <chr> <fct>   <chr>   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1 Davos Tempera~ arima~ 0.246 -42.8 93.5 94.3 102. NA NA NA
#> 2 Davos Tempera~ ETS_A~ 0.247 -78.8 168. 169. 178. 0.230 0.233 0.393
#> 3 Davos Precipi~ arima~ 158. -234. 475. 476. 483. NA NA NA
#> 4 Davos Precipi~ ETS_A~ 162. -273. 557. 558. 567. 152. 154. 9.75
#> # i 2 more variables: ar_roots <list>, ma_roots <list>
```

Ljung-Box Test - independence/white noise of the forecasts residuals

```
#> # A tibble: 2 x 5
#>   City Measure .model lb_stat lb_pvalue
#>   <chr> <fct>   <chr>   <dbl>   <dbl>
#> 1 Davos Temperature ETS_AAN 23.2 0.508
#> 2 Davos Precipitation ETS_AAN 29.3 0.208
#> # A tibble: 2 x 5
#>   City Measure .model lb_stat lb_pvalue
#>   <chr> <fct>   <chr>   <dbl>   <dbl>
#> 1 Davos Temperature arima_111wd 23.2 0.332
#> 2 Davos Precipitation arima_111wd 30.6 0.0798
```

Table 5: Mean Yearly ARIMA and ETS Forecast values of mean yearly data (next 25 years); Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

City	Measure	Year	ETS_AAN	arima_111wd
Davos	Temperature	2026	4.83	4.86
Davos	Temperature	2030	4.99	5.01
Davos	Temperature	2035	5.20	5.21
Davos	Temperature	2040	5.40	5.42
Davos	Temperature	2045	5.61	5.62
Davos	Temperature	2050	5.81	5.83
Davos	Precipitation	2026	85.50	86.41
Davos	Precipitation	2030	85.36	87.44
Davos	Precipitation	2035	85.18	87.68
Davos	Precipitation	2040	85.00	87.93
Davos	Precipitation	2045	84.83	88.18
Davos	Precipitation	2050	84.65	88.43

Table 6: Forecast increase/decrease over the next 25 years; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Measure	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETS	Delta_ARIMA
Temperature	2026	2050	4.83	4.86	5.81	5.83	0.98	0.97
Precipitation	2026	2050	85.50	86.41	84.65	88.43	-0.85	2.01