Climate Data Forecasting -

Atmospheric CO_2 Concentration / Temperature / Precipitation

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Contents

1 Forecasting of Basel - Temperature and Precipitation Climate Analysis

1.1 Stationarity and differencing

Stationary time series is one whose properties do not depend on the time at which the series is observed. Thus, time series with trends, or with seasonality, are not stationary — the trend and seasonality will affect the value of the time series at different times. On the other hand, a white noise series is stationary — it does not matter when you observe it, it should look much the same at any point in time.

Stationary time series will have no predictable patterns in the long-term. Time plots will show the series to be roughly horizontal (although some cyclic behaviour is possible), with constant variance.

If y_t is a *stationary* time series, then for all s, the distribution of $(y_t, ..., y_{t+s})$ does not depend on t.

If Time Series data with seasonality are non-stationary

- => first take a seasonal difference
- if seasonally differenced data appear are still non-stationary
- => take an additional first seasonal difference

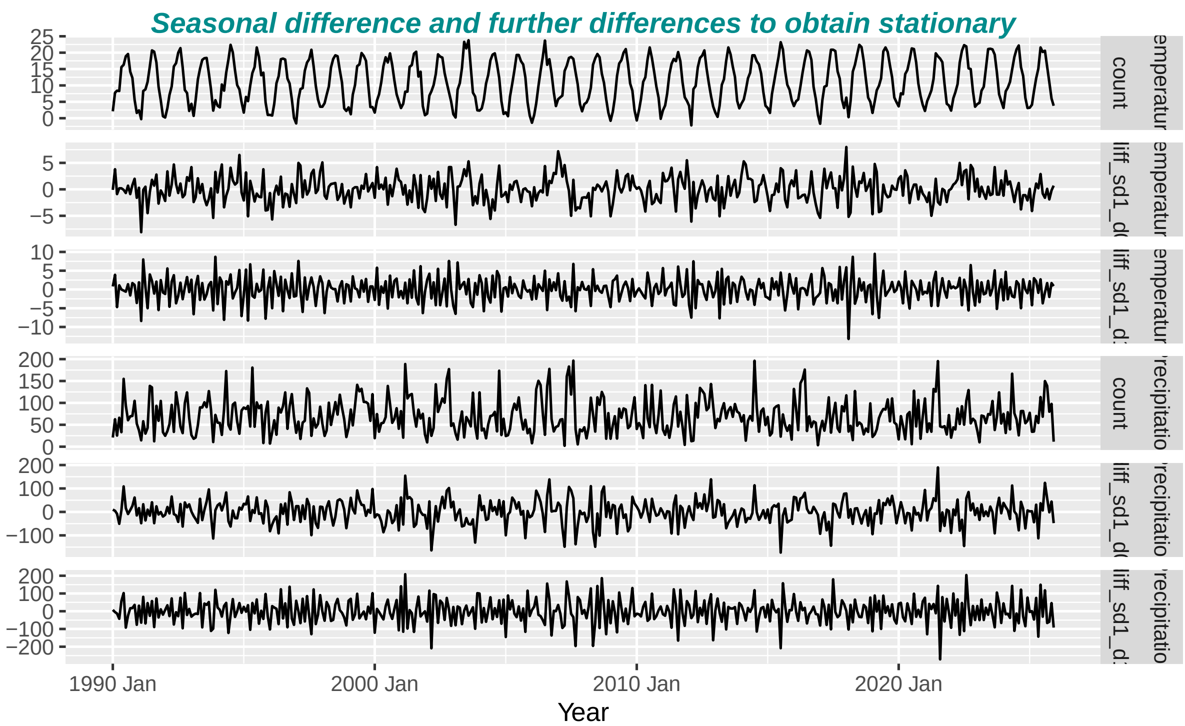
The model fit residuals have to be stationary. For good forecasting this has to be verified with residual diagnostics.

Essential:

- Residuals are uncorrelated
- The residuals have zero mean

Useful (but not necessary):

- The residuals have constant variance.
- The residuals are normally distributed.



1.1.1 Unitroot KPSS Test - fix number of seasonal differences/differences required

- For **ARIMA model** use Unitroot KPSS Test to fix number of differences
  - unitroot_nsdiffs() to determine D (the number of seasonal differences to use)
  - unitroot_ndiffs() to determine d (the number of ordinary differences to use)
  - The selection of the other model parameters (p,q,P and Q) are all determined by minimizing the AICc
- kpss test of stationary of the differentiated data (with seasonal and ordinary differences) used in the **ARIMA model**
  - stationary times series: the distribution of $(y_t, ..., y_{t-s})$ does not depend on t.

  - *Null Hypothesis* H_0: stationary is given in the time series: data are stationary and non seasonal
    - for $p < \alpha = 0.05$: reject H_0
    - for $p >> \alpha = 0.05$: conclude: differentiated data are stationary and non seasonal
- kpss test of stationary w/ unitroot_nsdiffs & unitroot_ndiff provides
  - minimum number of seasonal & ordinariel differences required for a stationary series
  - first fix required seasonal differences and then apply ndiffs to the seasonally differenced data
  - returns 1 => for stationarity one seasonal difference rsp. difference is required

```
#>   ndiffs gives the number of differences required and
#>  nsdiffs gives the number of seasonal differences required
#>  - to make a series stationary (test is based on the KPSS test
#> unitroot_kpss test to define seasonal (nsdiffs) and ordinary (ndiffs) differences
#> # A tibble: 2 x 5
#>   Measure       kpss_stat kpss_pvalue nsdiffs ndiffs
#>   <fct>             <dbl>       <dbl>   <int>  <int>
#> 1 Temperature       2.80       0.01         1      1
#> 2 Precipitation     0.371      0.0896       0      0
#> unitroot_kpss test on (difference(count, 12) + difference())
#> # A tibble: 2 x 3
#>   Measure       kpss_stat kpss_pvalue
#>   <fct>             <dbl>       <dbl>
#> 1 Temperature     0.00821         0.1
#> 2 Precipitation   0.00290         0.1
```

1.1.2 Ljung-Box Test - independence/white noise of the time series

The Ljung-Box Test becomes important when checking independence/white noise of the forecasts residuals of the fitted ETS rsp. ARIMA models. There we have to check whether the forecast errors are normally distributed with mean zero

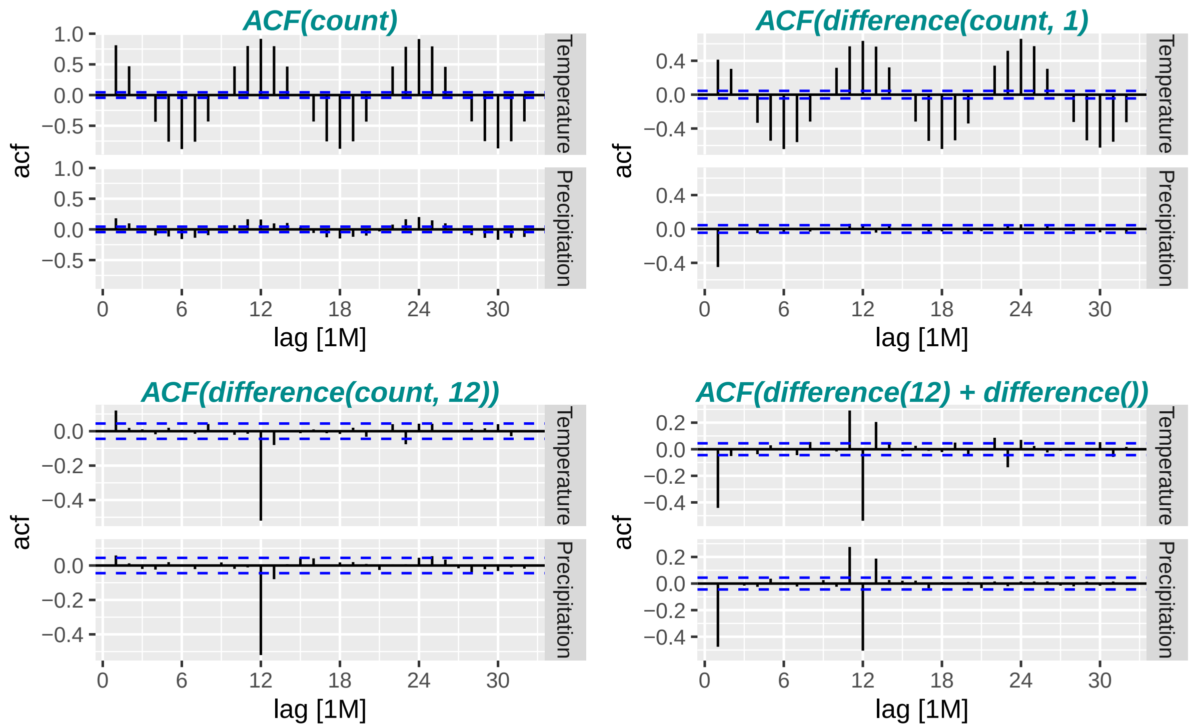
- augment(fit) |> features(.innov, ljung_box, lag=x, dof=y) (Ch. 5.4 Residaul diagnostics)
  - portmanteau test suggesting that the residuals are white noise
  - *Null Hypothesis* H_0: independence/white noise for residuals, i.e. each autocorrelation in the time series residuals of the model for lag l is close to zero.
    - for $p < \alpha = 0.05$: reject H_0
    - for $p >> \alpha = 0.05$: conclude: the residuals are not distinguishable from a white noise series
  - lag = 2*m (period of season, e.g. m=12 for monthly season) | no season: lag=10
  - dof = p + q + P + Q (for ARIMA models only, degree of freedom)

Time series with trend and/or seasonality is not stationary. Tests are to be taken on the residuals of the fitted models.

```
#> Ljung-Box test with (count), w/o differences
#> # A tibble: 2 x 3
#>   Measure       lb_stat lb_pvalue
#>   <fct>           <dbl>     <dbl>
#> 1 Temperature     6636.         0
#> 2 Precipitation    239.         0
#> Ljung-Box test on (difference(count, 12))
```

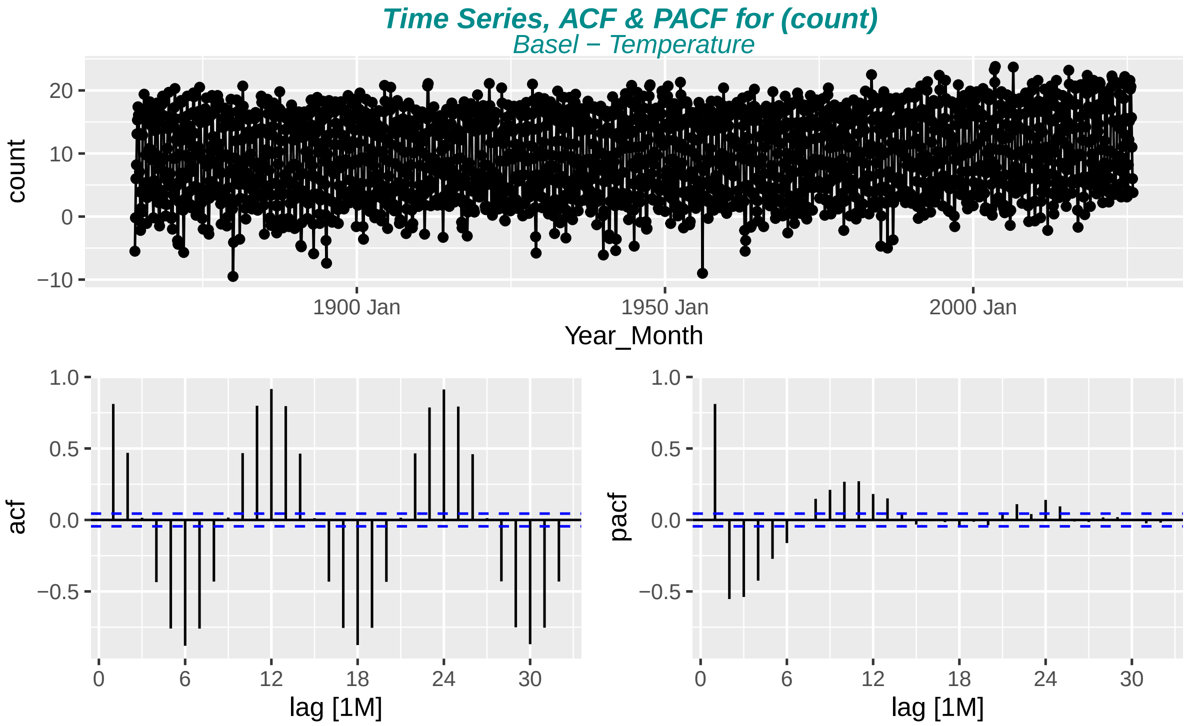
```
#> # A tibble: 2 x 3
#>   Measure       lb_stat lb_pvalue
#>   <fct>           <dbl>     <dbl>
#> 1 Temperature      35.1  0.000121
#> 2 Precipitation    11.9  0.294
#> Ljung-Box test on (difference(count, 12) + difference())
#> # A tibble: 2 x 3
#>   Measure       lb_stat lb_pvalue
#>   <fct>           <dbl>     <dbl>
#> 1 Temperature      397.         0
#> 2 Precipitation    445.         0
```

1.1.3 ACF (Autocorrelation Function) Plots of Differences

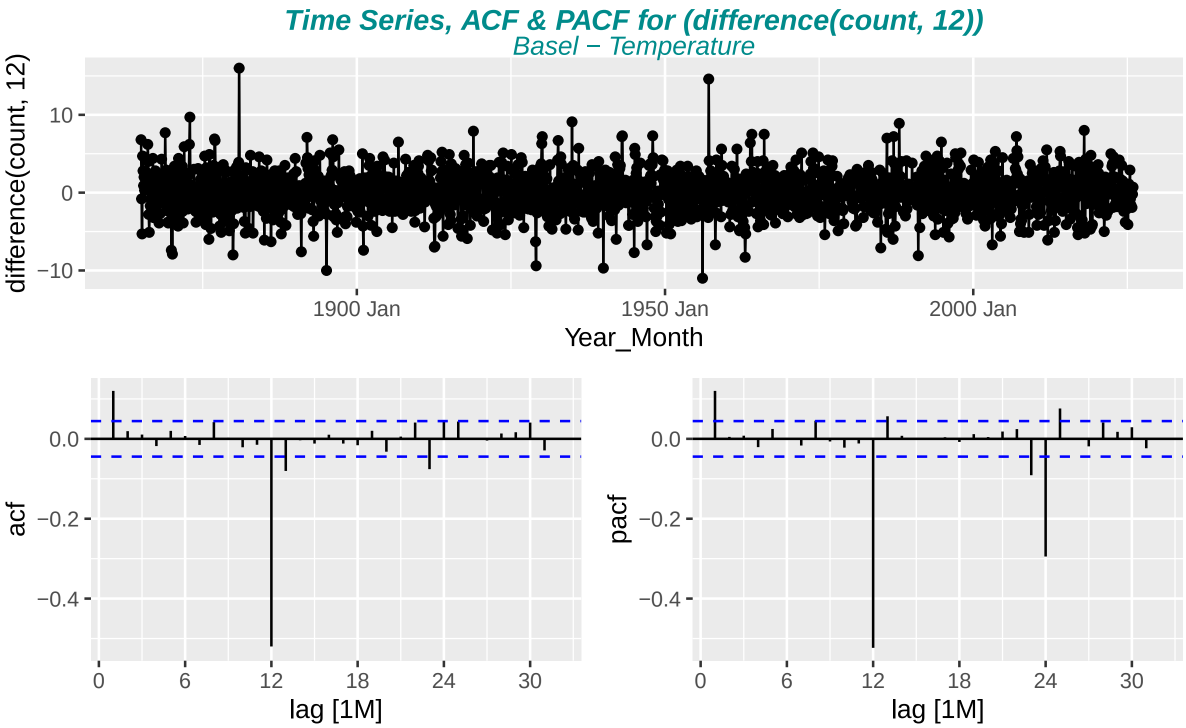


1.1.4 Time Series, ACF and PACF (Partial) Plots of Differences - for ARIMA p, q check

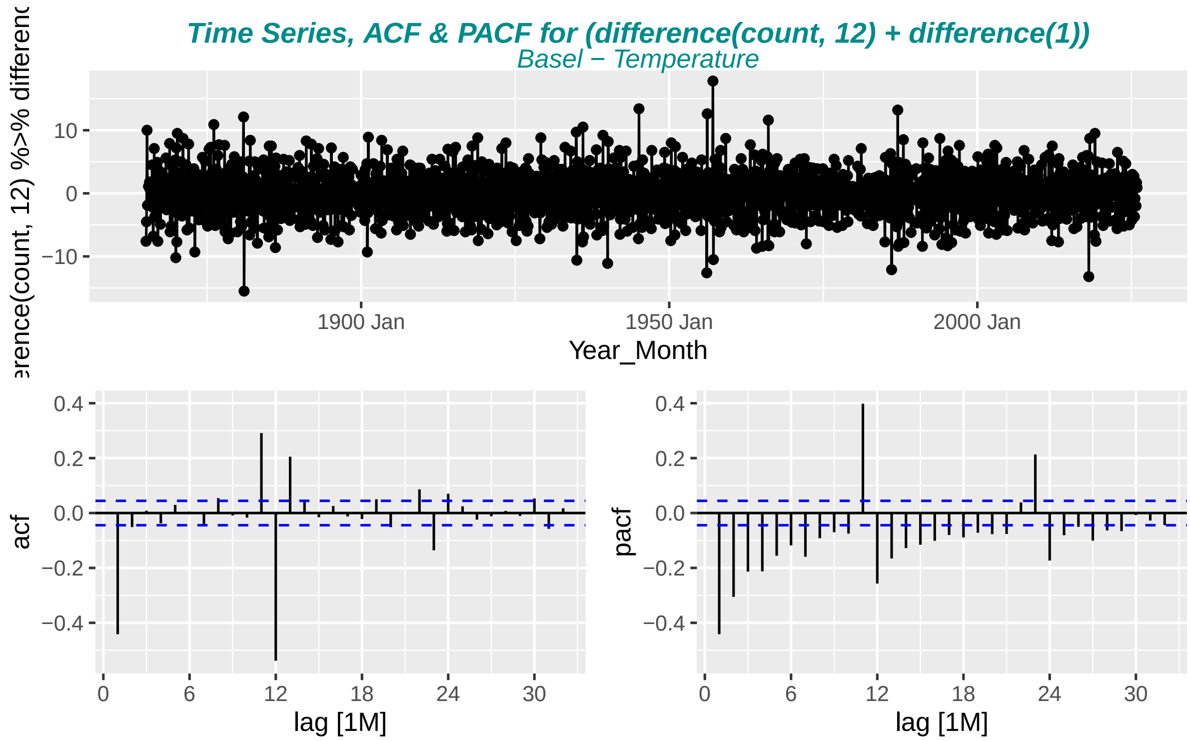
```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City  Measure       Sum  Mean
#>   <chr> <fct>       <dbl> <dbl>
#> 1 Basel Temperature 18653.  9.60
```



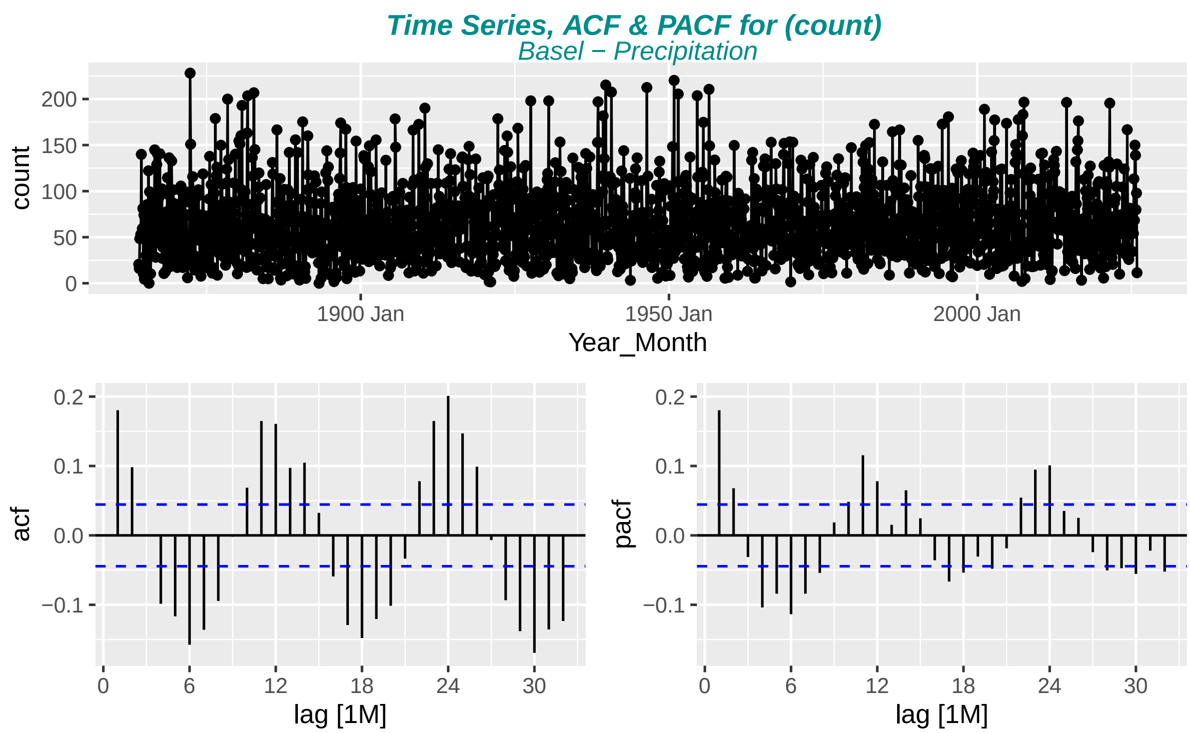

```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City  Measure       Sum   Mean
#>   <chr> <fct>       <dbl>  <dbl>
#> 1 Basel Temperature  49.3 0.0255
```



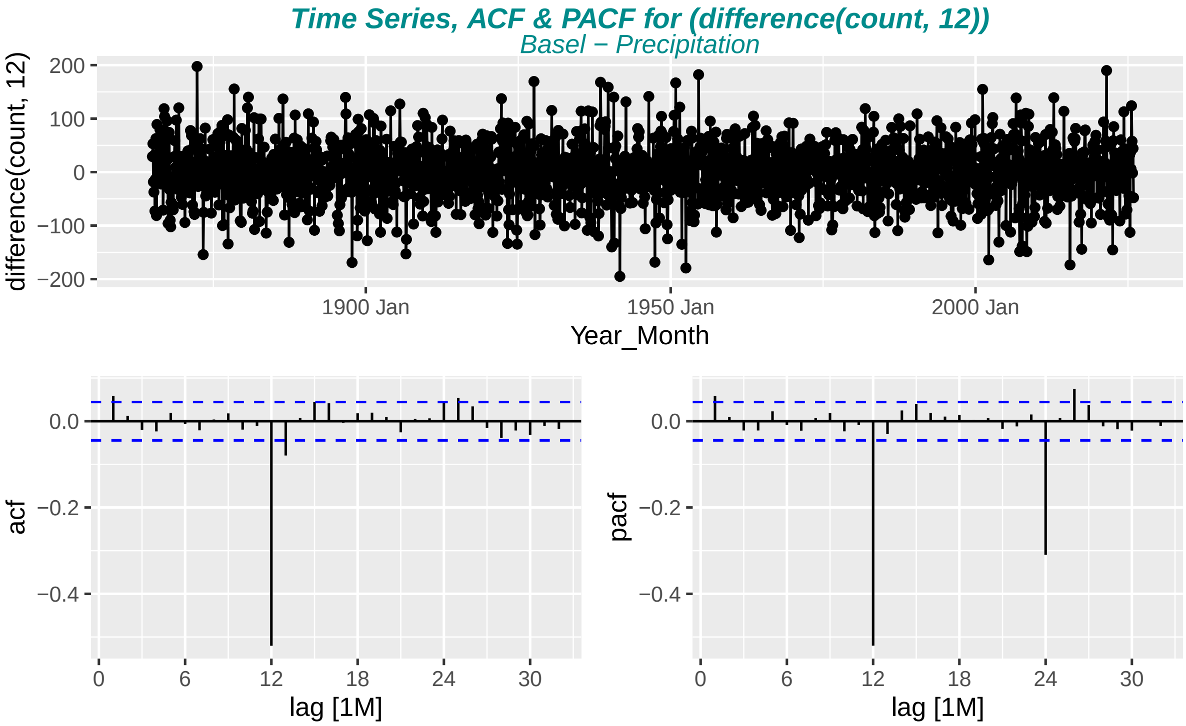

```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City  Measure       Sum     Mean
#>   <chr> <fct>       <dbl>    <dbl>
#> 1 Basel Temperature -6.10 -0.00316
```



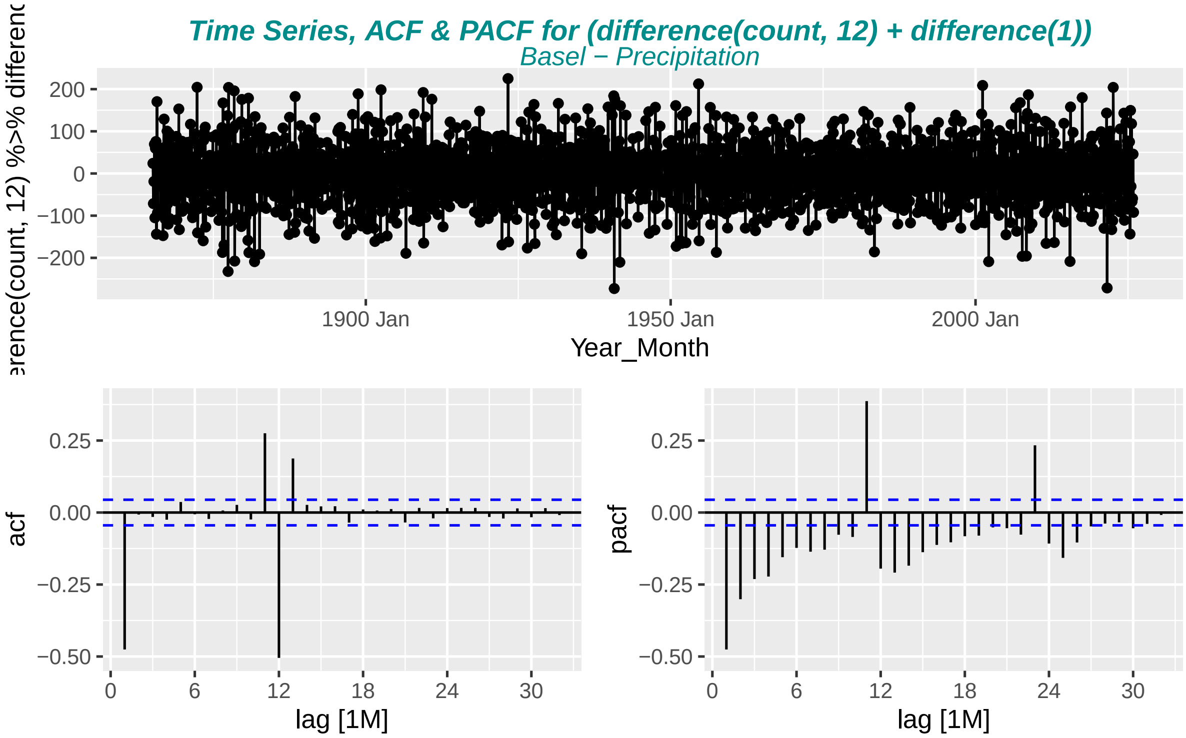

```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City  Measure             Sum  Mean
#>   <chr> <fct>             <dbl> <dbl>
#> 1 Basel Precipitation 128532.  66.1
```




```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City  Measure           Sum  Mean
#>   <chr> <fct>           <dbl> <dbl>
#> 1 Basel Precipitation  323. 0.167
```




```
#> # A tibble: 1 x 4
#> # Groups:   City [1]
#>   City  Measure          Sum    Mean
#>   <chr> <fct>          <dbl>   <dbl>
#> 1 Basel Precipitation -76.9 -0.0398
```



2 ExponenTial Smoothing (ETS) Forecasting Models

Forecasts produced using exponential smoothing methods are weighted averages of past observations, with the weights decaying exponentially as the observations get older.

The parameters are estimated by maximising the "likelihood". The likelihood is the probability of the data arising from the specified model. AIC, AICc and BIC can be used here to determine which of the ETS models is most appropriate for a given time series (see output glance(fit_ets)).

The model selection is based on recognising key components of the time series (trend and seasonal) and the way in which these enter the smoothing method (e.g., in an additive, damped or multiplicative manner).

- Mauna Loa CO_2 data best Models: ETS(M,A,A) & ETS(A,A,A)
- Basel Temperature data best Models: ETS(A,N,A), ETS(A,A,A), ETS(A,Ad,A) (close togehter). Best Forecast accuracy is with ETS(A,A,A), ETS(A,Ad,A).
- Basel Precipitation data best Models: ETS(A,N,A), ETS(A,Ad,A), ETS(A,A,A) (close togehter). Best Forecast accuracy is with ETS(A,A,A), ETS(A,Ad,A), ETS(A,N,A),

Trend term "N" for Basel Temperature/Precipitation correspondends to a "pure" exponential smooothing which results in a slope $\beta = 0$. This results in a forecast predicting a constant level. This does not fit to the result of the STL decomposition. Therefore best model choice is **ETS(A,A,A)**.

Method Selection

Error term: either additive ("A") or multiplicative ("M").

Both methods provide identical point forecasts, but different prediction intervals and different likelihoods. AIC & BIC are able to select between the error types because they are based on likelihood.

Nevertheless, difference is for

- Mauna Loa CO_2 not relevant and AIC/AICc/BIC values are only a little bit smaller for multiplicative errors. The prediction intervall plots are fully overlapping.
- Basel Temperature AIC/AICc/BIC of additive error types are much better than the multiplicative ones.
- Basel Precipitation AIC/AICc/BIC of additive error types are much better than the multiplicative ones.

Note: For Basel Temperature and Precipitation Forecast plots the models ETS_MAdA, ETS_MMA, ETS_MMA, ETS_MNA are to be taken out since forecasts with multiplicative errors are exploding (forecast > 3 years impossible !!)

Therefore finally **Error term = "A"** is chosen in general.

Trend term: either none ("N"), additive ("A"), multiplicative ("M") or damped variants ("Ad", "Md").

Note: Mauna Loa CO_2 model ETS(A,Ad,A) fit plot shows to strong damping. For Basel Temperature model ETS(A,N,A) and ETS(A,Ad,A) are providing more or less the same forecast. This means that forecast remains on constant level since Trend "N" means "pure" exponentiell smoothing without trend (see above).

Therefore finally **Trend term = "A"** is chosen in general.

Seasonal term: either none ("N"), additive ("A") or multiplicative ("M").

For CO2 and Temperature Data we have a clear seasonal pattern and seasonal term adds always a (more or less) fix amount on level and trend component. Therefore "A" additve term is chosen. For Precipitation the seasonal patttern is only slight. Indead, a multiplicative seasonal term results in "exploding" forecasts.

Since monthly data are strongly seasonal **seasonal term "A"** is chosen.

2.1 ETS Models and their componentes

ETS model with automatically selected $ETS(A|M,N|A|M,N|A|M)$

```
#> [1] "model(ETS(count)) => provides default / best automatically chosen model"
#> # A mable: 2 x 3
#> # Key:     City, Measure [2]
#>   City  Measure                ETS
#>   <chr> <fct>              <model>
#> 1 Basel Temperature   <ETS(A,N,A)>
#> 2 Basel Precipitation <ETS(M,N,M)>
#> [1] "Basel Temperature"
#> Series: count
#> Model: ETS(A,N,A)
#>   Smoothing parameters:
#>     alpha = 0.02442208
#>     gamma = 0.0001015791
#>
#>   Initial states:
#>      l[0]      s[0]    s[-1]     s[-2]    s[-3]    s[-4]    s[-5]    s[-6]
#>  9.746802 -7.770647 -4.92664 0.3653624 4.747628 8.521807 9.107609 7.009985
#>     s[-7]      s[-8]     s[-9]    s[-10]    s[-11]
#>  3.735796 -0.6346741 -3.860148 -7.425381 -8.870697
#>
#>   sigma^2:  2.8661
#>
#>      AIC     AICc      BIC
#> 5511.052 5511.734 5579.741
#> [1] "Basel Precipitation"
#> Series: count
#> Model: ETS(M,N,M)
#>   Smoothing parameters:
#>     alpha = 0.01240284
#>     gamma = 0.0001026895
#>
#>   Initial states:
#>      l[0]      s[0]    s[-1]     s[-2]    s[-3]   s[-4]    s[-5]    s[-6]
#>  68.84995 0.9154611 0.951275 0.9554394 1.006745 1.22762 1.190632 1.234935
#>     s[-7]     s[-8]     s[-9]    s[-10]    s[-11]
#>  1.296867 0.9375161 0.7638363 0.7239218 0.7957513
#>
#>   sigma^2:  0.2738
#>
#>      AIC     AICc      BIC
#> 9864.476 9865.158 9933.164
#> # A tibble: 2 x 8
#>   City  Measure       .model   AIC  AICc   BIC     MSE   MAE
#>   <chr> <fct>         <chr>  <dbl> <dbl> <dbl>   <dbl> <dbl>
#> 1 Basel Temperature   ETS    5511. 5512. 5580.    2.81 1.33
#> 2 Basel Precipitation ETS    9864. 9865. 9933. 1216.   0.412
```

Fit of different pre-defined $ETS(A|M,N|A|M,N|A|M)$ models

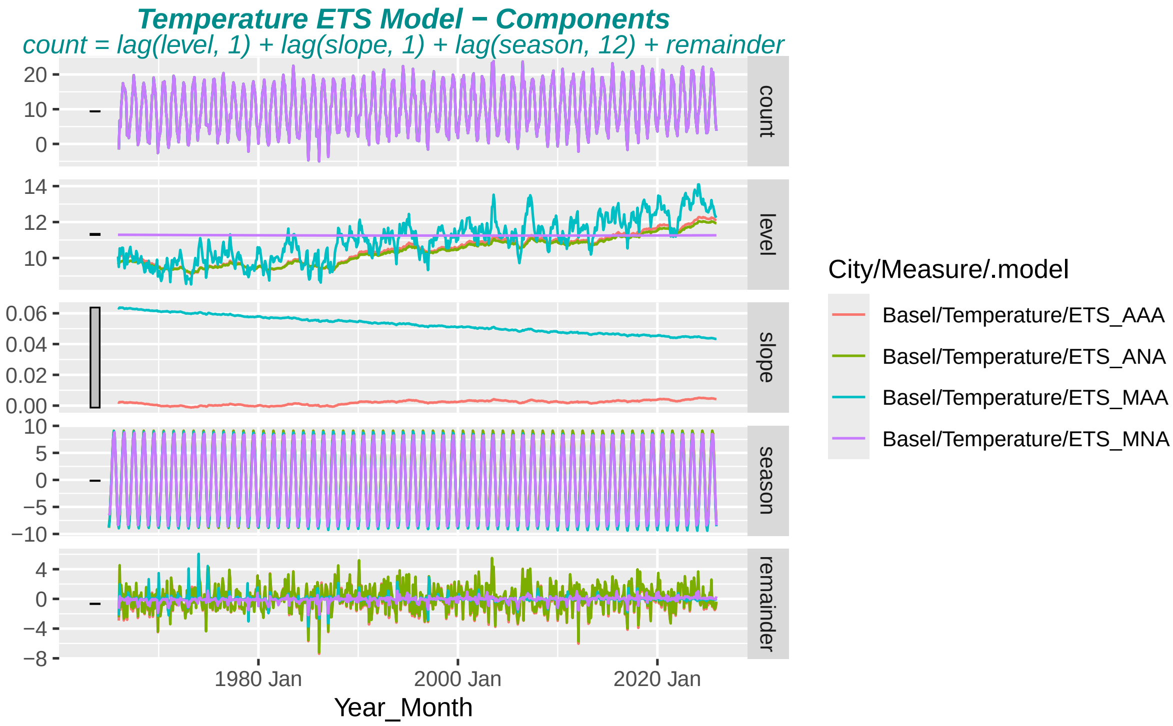
Model Selection by Information Criterion - lowest AIC, AICc, BIC

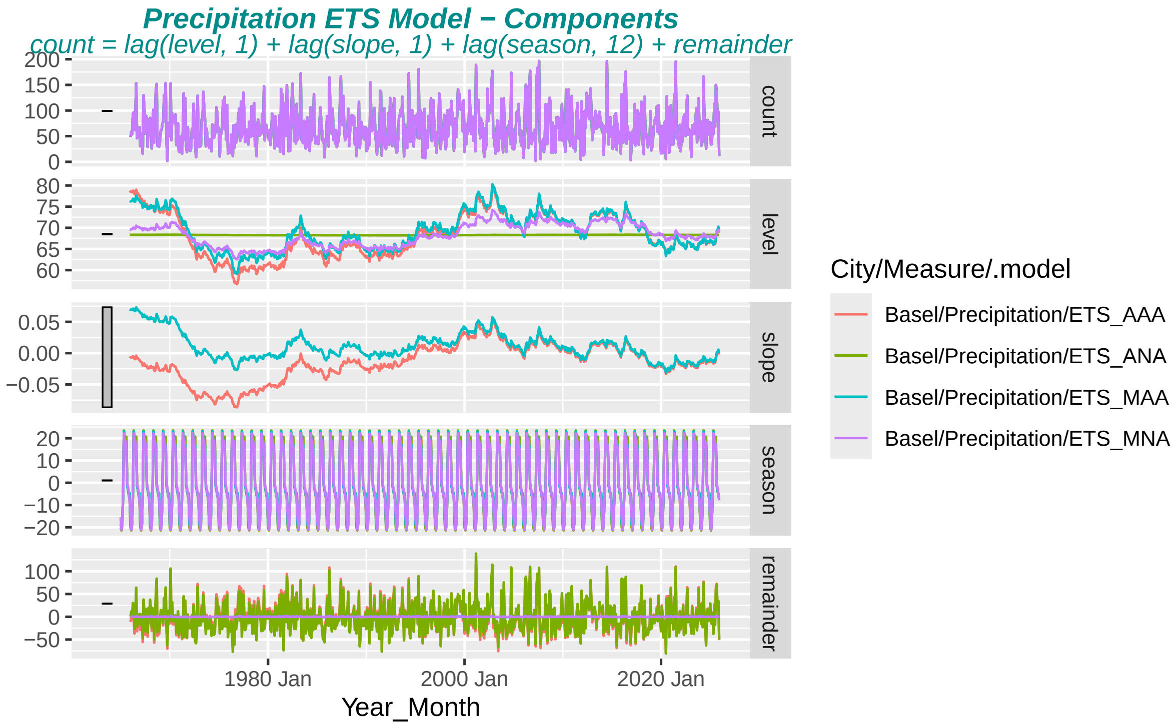
Best model fit with

- CV, AIC, AICc and BIC with the lowest values
- Adjusted R^2 the model with the highest value.

```
#> # A tibble: 16 x 9
#>    City  Measure       .model     AIC  AICc   BIC     MSE    AMSE    MAE
```

```
#>    <chr> <fct>         <chr>    <dbl> <dbl> <dbl>   <dbl>   <dbl>  <dbl>
#>  1 Basel Temperature   ETS_ANA  5511. 5512. 5580.    2.81    2.81  1.33
#>  2 Basel Temperature   ETS_AMA  5513. 5514. 5591.    2.80    2.81  1.33
#>  3 Basel Temperature   ETS_AAA  5513. 5514. 5591.    2.80    2.81  1.33
#>  4 Basel Temperature   ETS_AAdA 5514. 5514. 5596.    2.80    2.80  1.32
#>  5 Basel Temperature   ETS_MNA  6579. 6579. 6647.    4.19    4.15  0.241
#>  6 Basel Temperature   ETS_MMA  6673. 6674. 6751.    3.81    4.11  0.234
#>  7 Basel Temperature   ETS_MAdA 6805. 6806. 6887.    4.07    4.47  0.273
#>  8 Basel Temperature   ETS_MAA  7122. 7123. 7200.    3.14    3.23  0.288
#>  9 Basel Precipitation ETS_MNA  9869. 9870. 9938. 1211.   1213.    0.410
#> 10 Basel Precipitation ETS_ANA  9875. 9876. 9944. 1205.   1207.   27.3
#> 11 Basel Precipitation ETS_MAdA 9882. 9883. 9965. 1208.   1210.    0.414
#> 12 Basel Precipitation ETS_AAdA 9883. 9884. 9965. 1208.   1210.   27.3
#> 13 Basel Precipitation ETS_MAA  9889. 9890. 9967. 1225.   1227.    0.410
#> 14 Basel Precipitation ETS_AAA  9889. 9890. 9967. 1223.   1225.   27.6
#> 15 Basel Precipitation ETS_MMA  9890. 9891. 9968. 1226.   1229.    0.410
#> 16 Basel Precipitation ETS_AMA  9890. 9891. 9968. 1224.   1227.   27.6
```





2.1.1 Residual Accuracy with one-step-ahead fitted residuals - check RMSE, MAE

Residual accuracy can be computed directly from models as the one-step-ahead fitted residuals are available. Select forecast models that minimises for lowest

- MAE (Mean absolute error, will lead to forecasts of the median) and
- RMSE (Root mean squared error, lead to forecasts of the mean)

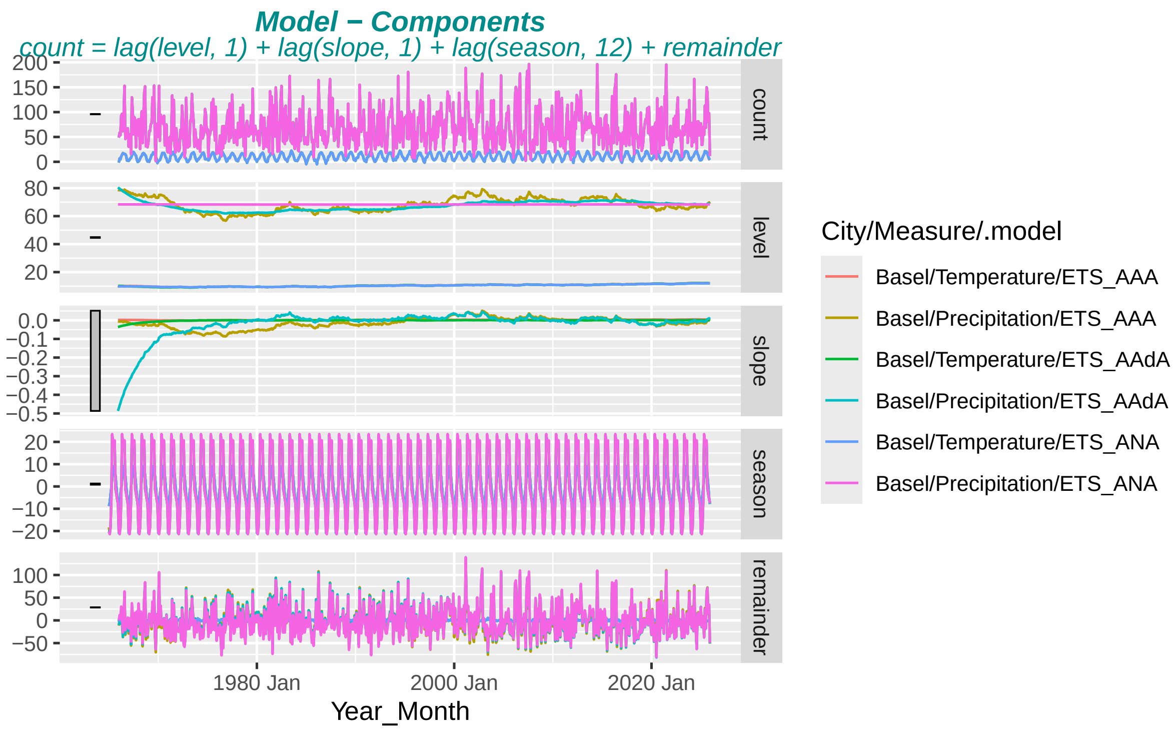
```
#> # A tibble: 16 x 7
#>    City  Measure       .model   .type          ME  RMSE   MAE
#>    <chr> <fct>         <chr>    <chr>       <dbl> <dbl> <dbl>
#>  1 Basel Temperature   ETS_AAdA Training  0.112    1.67  1.32
#>  2 Basel Temperature   ETS_AMA  Training  0.0214   1.67  1.33
#>  3 Basel Temperature   ETS_AAA  Training  0.0274   1.67  1.33
#>  4 Basel Temperature   ETS_ANA  Training  0.124    1.68  1.33
#>  5 Basel Temperature   ETS_MAA  Training -0.230    1.77  1.40
#>  6 Basel Temperature   ETS_MMA  Training -0.703    1.95  1.53
#>  7 Basel Temperature   ETS_MAdA Training -0.0351   2.02  1.56
#>  8 Basel Temperature   ETS_MNA  Training -0.402    2.05  1.60
#>  9 Basel Precipitation ETS_ANA  Training -0.113   34.7  27.3
#> 10 Basel Precipitation ETS_AAdA Training  0.777   34.8  27.3
#> 11 Basel Precipitation ETS_MAdA Training  0.338   34.8  27.4
#> 12 Basel Precipitation ETS_MNA  Training -0.0815 34.8  27.5
#> 13 Basel Precipitation ETS_AAA  Training  0.0463 35.0  27.6
#> 14 Basel Precipitation ETS_AMA  Training  0.216   35.0  27.6
#> 15 Basel Precipitation ETS_MAA  Training -0.968   35.0  27.7
#> 16 Basel Precipitation ETS_MMA  Training -1.12    35.0  27.8
```

2.1.2 Ljung-Box Test - independence/white noise of the forecasts residuals

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 16 x 5
#>    City  Measure       .model   lb_stat lb_pvalue
#>    <chr> <fct>         <chr>      <dbl>     <dbl>
#>  1 Basel Precipitation ETS_MAdA    26.4  1.90e- 1
```

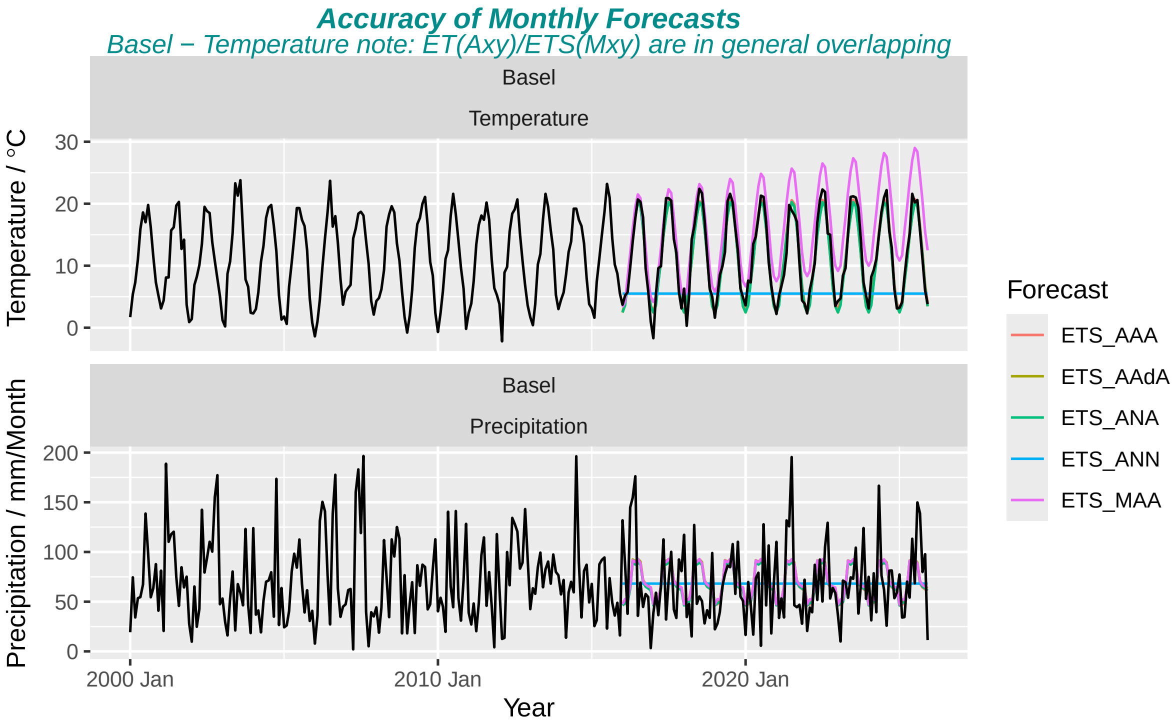
```
#>  2 Basel Precipitation ETS_AAdA    26.7  1.80e- 1
#>  3 Basel Precipitation ETS_AMA     27.1  1.69e- 1
#>  4 Basel Precipitation ETS_MAA     27.2  1.64e- 1
#>  5 Basel Precipitation ETS_AAA     27.2  1.64e- 1
#>  6 Basel Precipitation ETS_MNA     27.2  1.63e- 1
#>  7 Basel Precipitation ETS_MMA     27.5  1.56e- 1
#>  8 Basel Precipitation ETS_ANA     27.6  1.51e- 1
#>  9 Basel Temperature   ETS_ANA     29.1  1.11e- 1
#> 10 Basel Temperature   ETS_AAA     29.2  1.10e- 1
#> 11 Basel Temperature   ETS_AMA     29.2  1.09e- 1
#> 12 Basel Temperature   ETS_AAdA    30.2  8.83e- 2
#> 13 Basel Temperature   ETS_MMA     32.5  5.15e- 2
#> 14 Basel Temperature   ETS_MAA     44.0  2.35e- 3
#> 15 Basel Temperature   ETS_MAdA   110.   4.16e-14
#> 16 Basel Temperature   ETS_MNA    815.   0
```

2.1.3 ETS Models - components of ETS(A,N,A), ETS(A,A,A), ETS(A,Ad,A), models



2.1.4 Forecast Accuracy with Training/Test Data

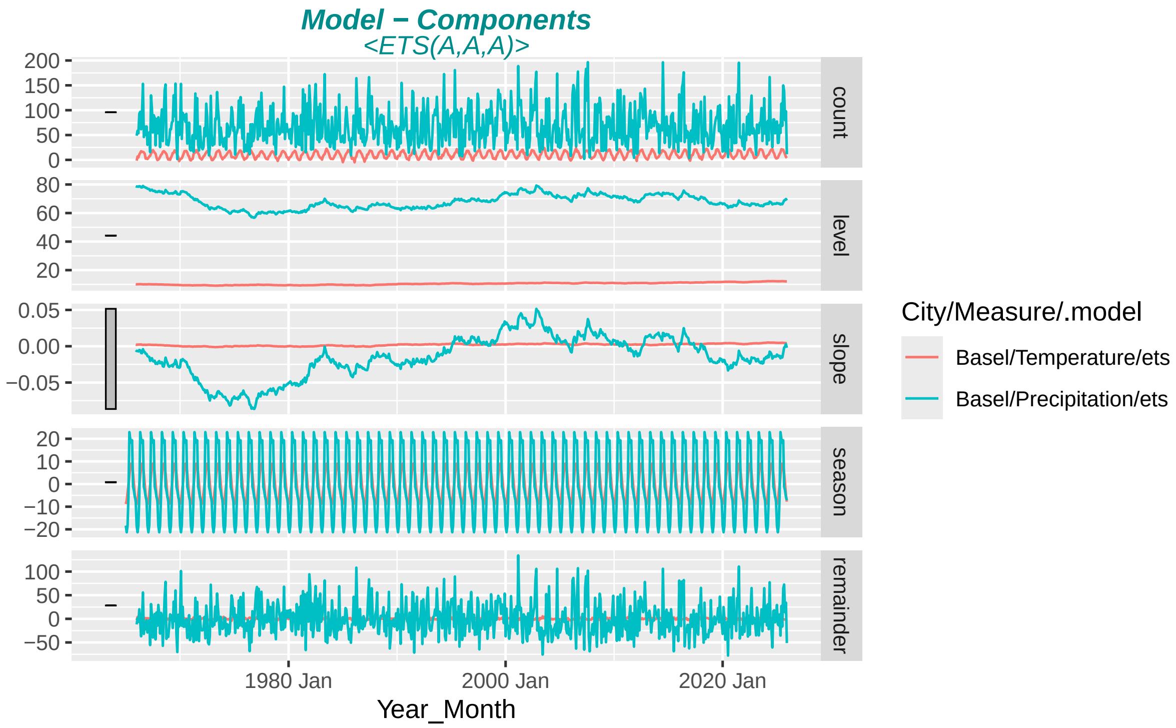
```
#> # A tibble: 10 x 7
#>    .model   City  Measure       .type      ME  RMSE   MAE
#>    <chr>    <chr> <fct>         <chr>   <dbl> <dbl> <dbl>
#>  1 ETS_AAA  Basel Temperature   Test    0.219  1.57  1.24
#>  2 ETS_AAdA Basel Temperature   Test    0.533  1.66  1.32
#>  3 ETS_ANA  Basel Temperature   Test    0.605  1.70  1.36
#>  4 ETS_MAA  Basel Temperature   Test   -4.32   5.06  4.41
#>  5 ETS_ANN  Basel Temperature   Test    6.30   9.13  7.21
#>  6 ETS_ANA  Basel Precipitation Test   -0.342  35.6  28.1
#>  7 ETS_AAA  Basel Precipitation Test   -0.0237 35.6  28.1
#>  8 ETS_AAdA Basel Precipitation Test   -2.53   35.7  28.6
#>  9 ETS_MAA  Basel Precipitation Test   -2.48   35.8  28.7
#> 10 ETS_ANN  Basel Precipitation Test   -0.0790 38.2  30.0
```

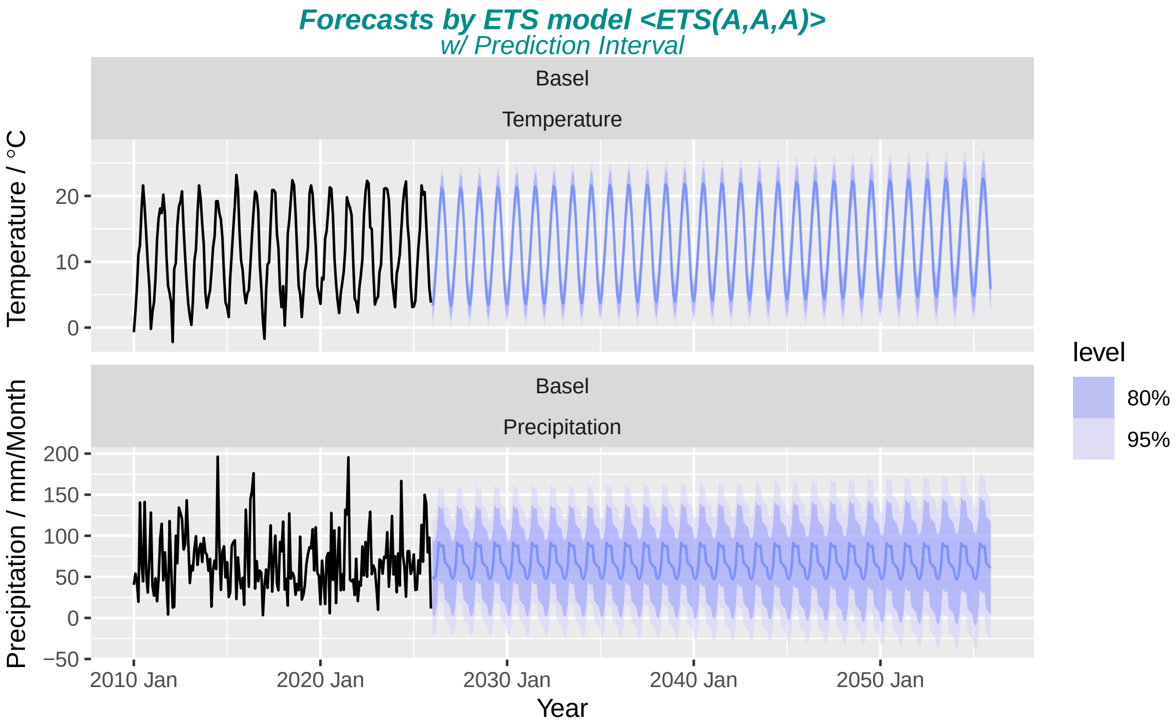

Accuracy of Monthly Forecasts
Basel − Temperature note: ET(Axy)/ETS(Mxy) are in general overlapping
Basel
Temperature
Basel
Precipitation
Temperature / °C
Precipitation / mm/Month
30
20
10
0
200
150
100
50
0
2000 Jan
2010 Jan
2020 Jan
Year
Forecast
ETS_AAA
ETS_AAdA
ETS_ANA
ETS_ANN
ETS_MAA

2.2 Forecasting with selected ETS model <ETS(A,A,A)>, <ETS(A,A,A)>

2.2.1 Forecast Plot of selected ETS model

```
#> Provide model coefficients by report(fit_model)
#> # A tibble: 2 x 11
#>   City  Measure      .model sigma2 log_lik   AIC  AICc   BIC    MSE   AMSE   MAE
#>   <chr> <fct>        <chr>   <dbl>   <dbl> <dbl> <dbl> <dbl>  <dbl>  <dbl> <dbl>
#> 1 Basel Temperature  ets    2.87e0  -2740. 5513. 5514. 5591. 2.80e0 2.81e0  1.33
#> 2 Basel Precipitati~ ets    1.25e3  -4928. 9889. 9890. 9967. 1.22e3 1.22e3 27.6
```

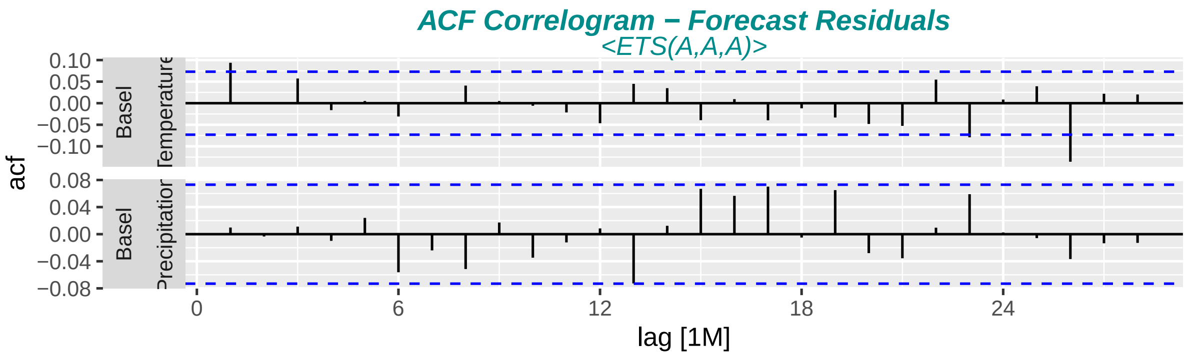


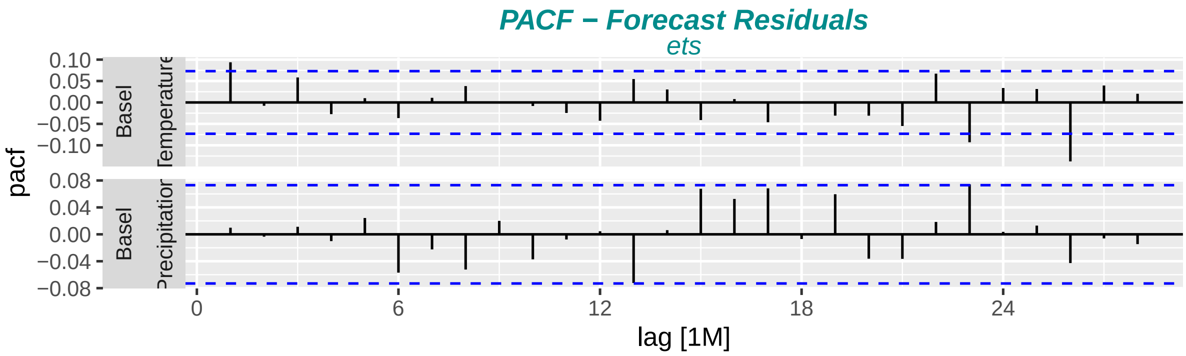


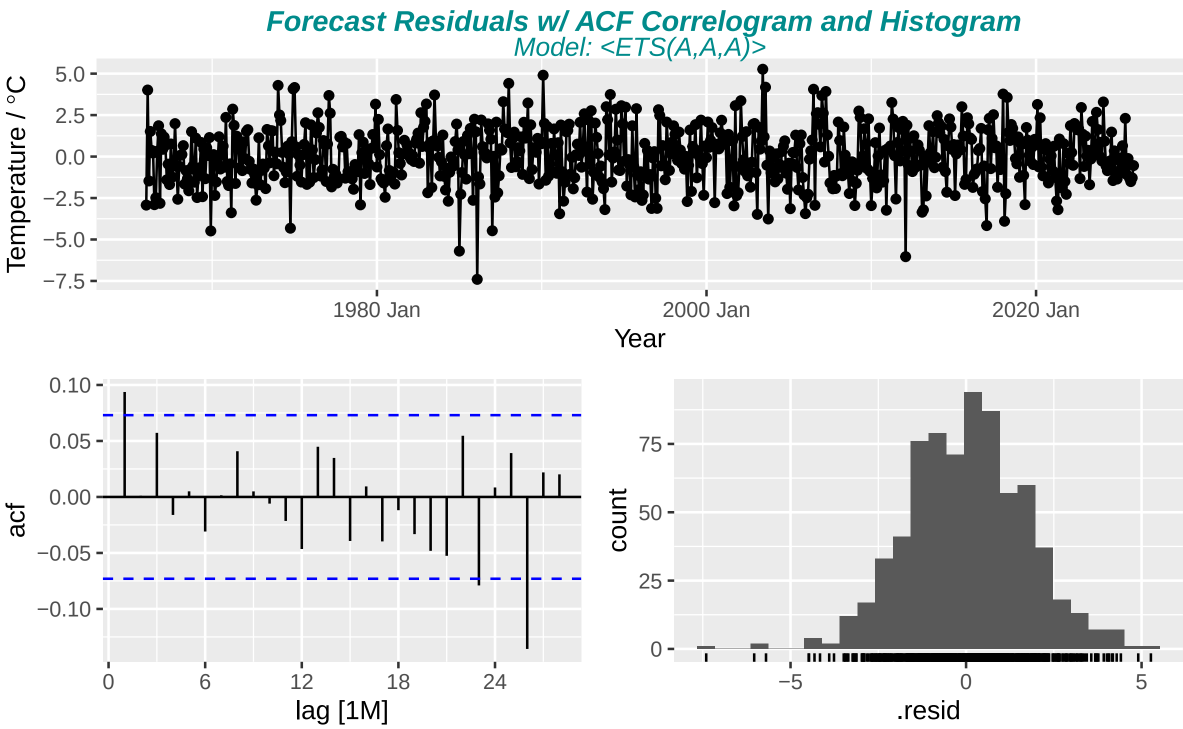
2.2.2 Residual Stationarity

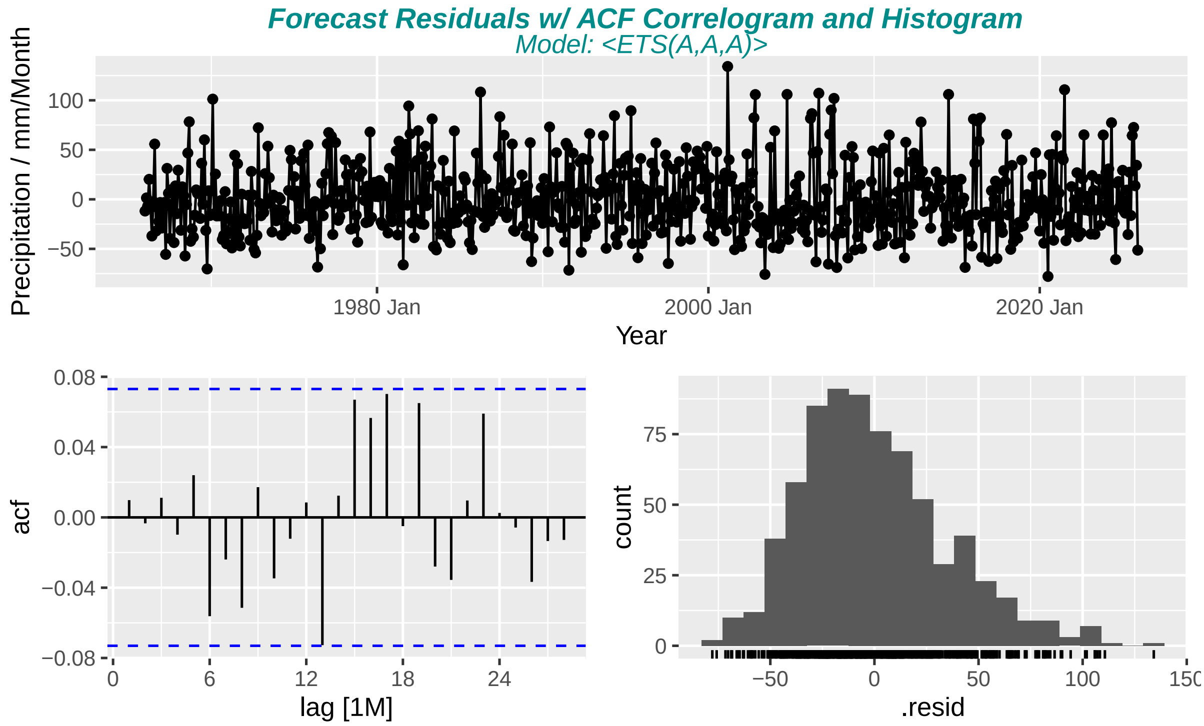
Required checks to be ready for forecasting:

- ACF Forecast Residual: all spikes are within the significance limits, so the residuals appear to be white noise
- The Ljung-Box test also shows that the residuals have no remaining autocorrelations
- Forecast Residuals are more or less normally distributed with roughly centred on zero



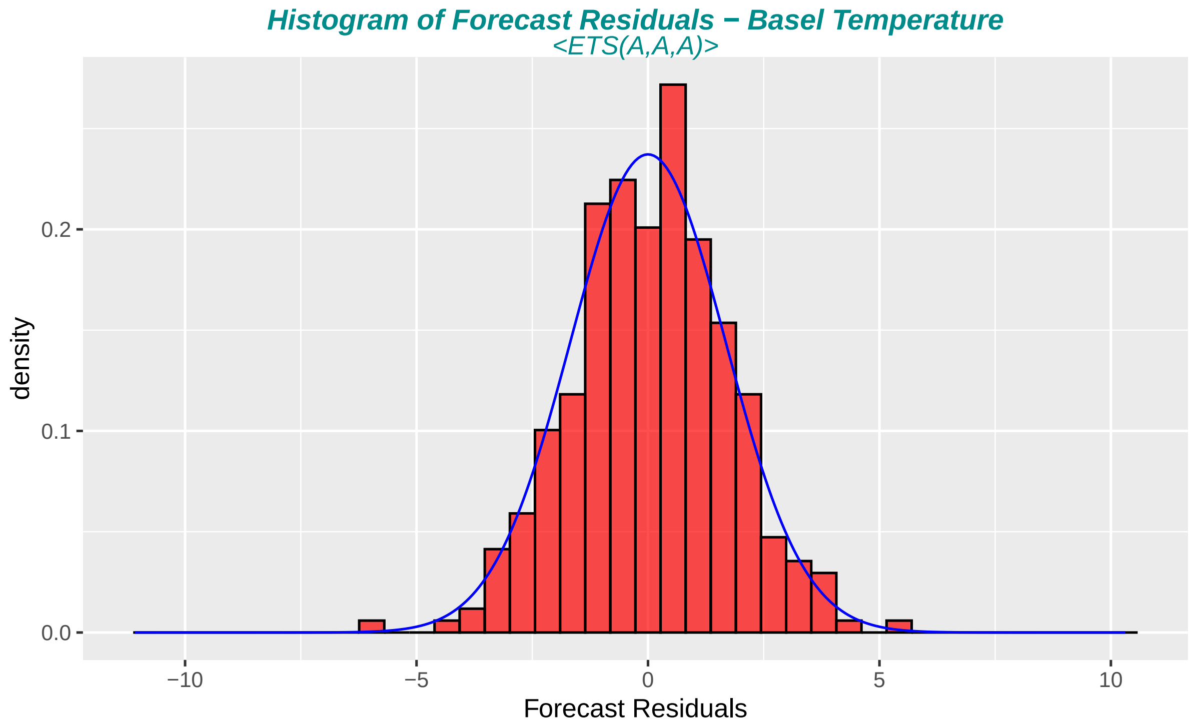


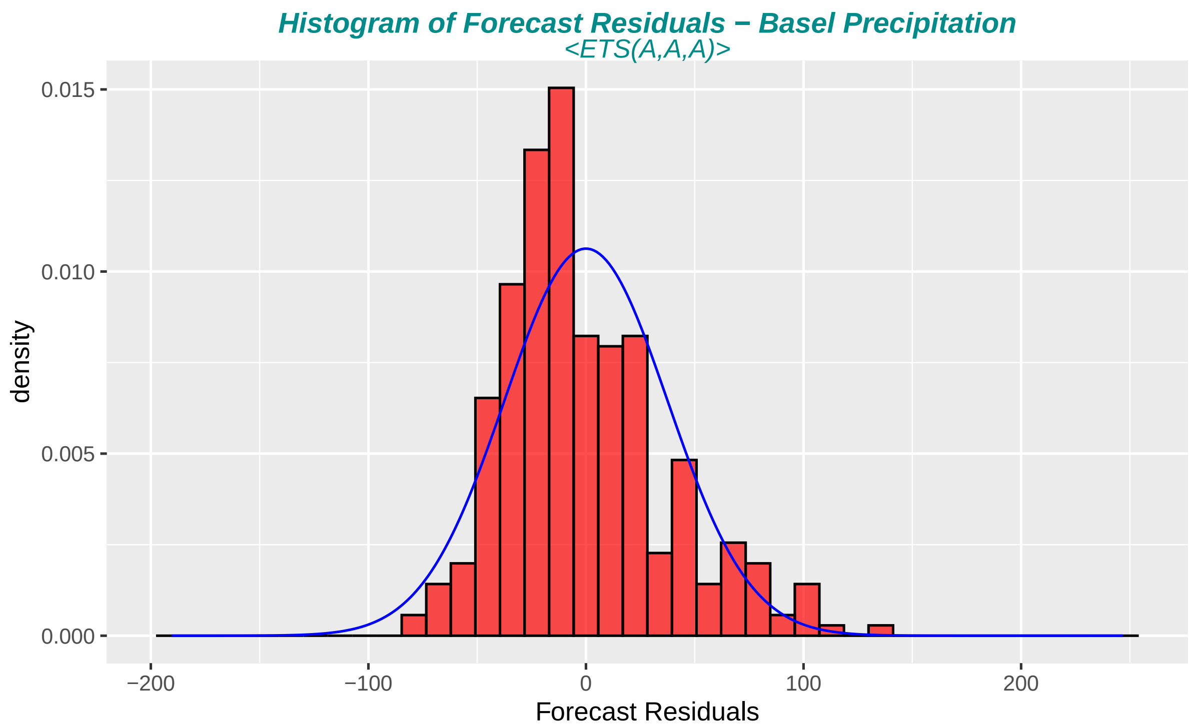




2.2.3 Ljung-Box Test and Histogram of forecast residuals with overlaid normal curve

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 2 x 5
#>   City  Measure       .model lb_stat lb_pvalue
#>   <chr> <fct>         <chr>    <dbl>     <dbl>
#> 1 Basel Temperature   ets       36.6    0.0484
#> 2 Basel Precipitation ets       29.7    0.194
```




Histogram of Forecast Residuals − Basel Precipitation
<ETS(A,A,A)>
0.015
0.010
0.005
0.000
density
−200
−100
0
100
200
Forecast Residuals

3 ARIMA Forecasting Models - AutoRegressive-Integrated Moving Average

Exponential smoothing and ARIMA (AutoRegressive-Integrated Moving Average)models are the two most widely used approaches to time series forecasting, and provide complementary approaches to the problem.

While exponential smoothing models are based on a description of the trend and seasonality in the data, ARIMA models aim to describe the autocorrelations in the data.

3.1 Seasonal ARIMA models

Non-seasonal ARIMA models are generally denoted ARIMA(p,d,q) where parameters p, d, and q are non-negative integers, * p is the order (number of time lags) of the autoregressive model * d is the degree of differencing (number of times the data have had past values subtracted) * q is the order of the moving-average model of past forecast errors .

The value of d has an effect on the prediction intervals — the higher the value of d, the more rapidly the prediction intervals increase in size. For d=0, the point forecasts are equal to the mean of the data and the long-term forecast standard deviation will go to the standard deviation of the historical data, so the prediction intervals will all be essentially the same.

Seasonal ARIMA models are usually denoted ARIMA(p,d,q)(P,D,Q)m, where m refers to the number of periods in each season, and the uppercase P,D,Q refer to the autoregressive, differencing, and moving average terms for the seasonal part of the ARIMA model.

$ARIMA(pdq)(PDQ)$ **model with automatically selected** $(pdq)(PDQ)$ **values**

Fit of different pre-defined $ARIMA(pdq)(PDQ)$ **models**

```
#> Model Selection by Information Criterion - lowest AIC, AICc, BIC
#>  choose p, q parameter accordingly - but only for same d, D values
#> # A tibble: 8 x 8
#>   City  Measure     .model        sigma2 log_lik   AIC  AICc   BIC
#>   <chr> <fct>       <chr>          <dbl>   <dbl> <dbl> <dbl> <dbl>
#> 1 Basel Temperature arima_111_011   2.78  -1394. 2796. 2796. 2814.
#> 2 Basel Temperature arima_012_011   2.78  -1394. 2796. 2796. 2814.
#> 3 Basel Temperature arima_111_012   2.82  -1394. 2797. 2797. 2820.
#> 4 Basel Temperature arima_211_011   2.79  -1394. 2798. 2798. 2821.
#> 5 Basel Temperature arima_012_112   2.83  -1394. 2799. 2799. 2826.
#> 6 Basel Temperature arima_300_111   3.03  -1404. 2820. 2820. 2847.
#> 7 Basel Temperature arima_102_211   3.07  -1407. 2829. 2829. 2861.
#> 8 Basel Temperature arima_100_200   3.07  -1407. 2829. 2829. 2861.
#> # A tibble: 8 x 8
#>   City  Measure       .model          sigma2 log_lik   AIC  AICc   BIC
#>   <chr> <fct>         <chr>            <dbl>   <dbl> <dbl> <dbl> <dbl>
#> 1 Basel Precipitation arima_012_011    1233.  -3547. 7101. 7102. 7120.
#> 2 Basel Precipitation arima_111_011    1233.  -3547. 7101. 7102. 7120.
#> 3 Basel Precipitation arima_111_012    1234.  -3546. 7103. 7103. 7126.
#> 4 Basel Precipitation arima_211_011    1235.  -3547. 7103. 7104. 7126.
#> 5 Basel Precipitation arima_012_112    1235.  -3546. 7105. 7105. 7132.
#> 6 Basel Precipitation arima_001_002    1379.  -3622. 7254. 7255. 7277.
#> 7 Basel Precipitation arima_200_011    1811.  -3660. 7329. 7329. 7347.
#> 8 Basel Precipitation arima_100_110_c  1813.  -3660. 7331. 7331. 7353.
```

Good models are obtained by minimising the AIC, AICc or BIC (see glance(fit_arima) output). The preference is to use the AICc to selec p and q.

These information criteria tend not to be good guides to selecting the appropriate order of differencing (d) of a model, but only for selecting the values of p and q. This is because the differencing changes the data on which the likelihood is computed, making the AIC values between models with different orders of differencing not comparable.

3.1.1 Residual Accuracy with one-step-ahead fitted residuals - check RMSE, MAE

Residual accuracy can be computed directly from models as the one-step-ahead fitted residuals are available. Select forecast models that minimises for lowest

- MAE (Mean absolute error, will lead to forecasts of the median) and
- RMSE (Root mean squared error, lead to forecasts of the mean)

```
#> # A tibble: 8 x 7
#>   City  Measure     .model        .type       ME  RMSE   MAE
#>   <chr> <fct>       <chr>         <chr>    <dbl> <dbl> <dbl>
#> 1 Basel Temperature arima_111_011 Training 0.104  1.65  1.27
#> 2 Basel Temperature arima_211_011 Training 0.104  1.65  1.27
#> 3 Basel Temperature arima_012_011 Training 0.105  1.65  1.27
#> 4 Basel Temperature arima_111_012 Training 0.106  1.66  1.28
#> 5 Basel Temperature arima_012_112 Training 0.105  1.66  1.28
#> 6 Basel Temperature arima_300_111 Training 0.222  1.72  1.35
#> 7 Basel Temperature arima_102_211 Training 0.240  1.73  1.35
#> 8 Basel Temperature arima_100_200 Training 0.240  1.73  1.35
#> # A tibble: 8 x 7
#>   City  Measure       .model          .type          ME  RMSE   MAE
#>   <chr> <fct>         <chr>           <chr>       <dbl> <dbl> <dbl>
#> 1 Basel Precipitation arima_012_112   Training  0.676    34.7  27.3
#> 2 Basel Precipitation arima_111_012   Training  0.662    34.7  27.3
#> 3 Basel Precipitation arima_012_011   Training  0.680    34.7  27.3
#> 4 Basel Precipitation arima_111_011   Training  0.680    34.7  27.3
#> 5 Basel Precipitation arima_211_011   Training  0.682    34.7  27.3
#> 6 Basel Precipitation arima_001_002   Training  0.00318  37.0  29.6
#> 7 Basel Precipitation arima_100_110_c Training -0.0561   42.1  32.6
#> 8 Basel Precipitation arima_200_110_c Training -0.0561   42.1  32.6
```

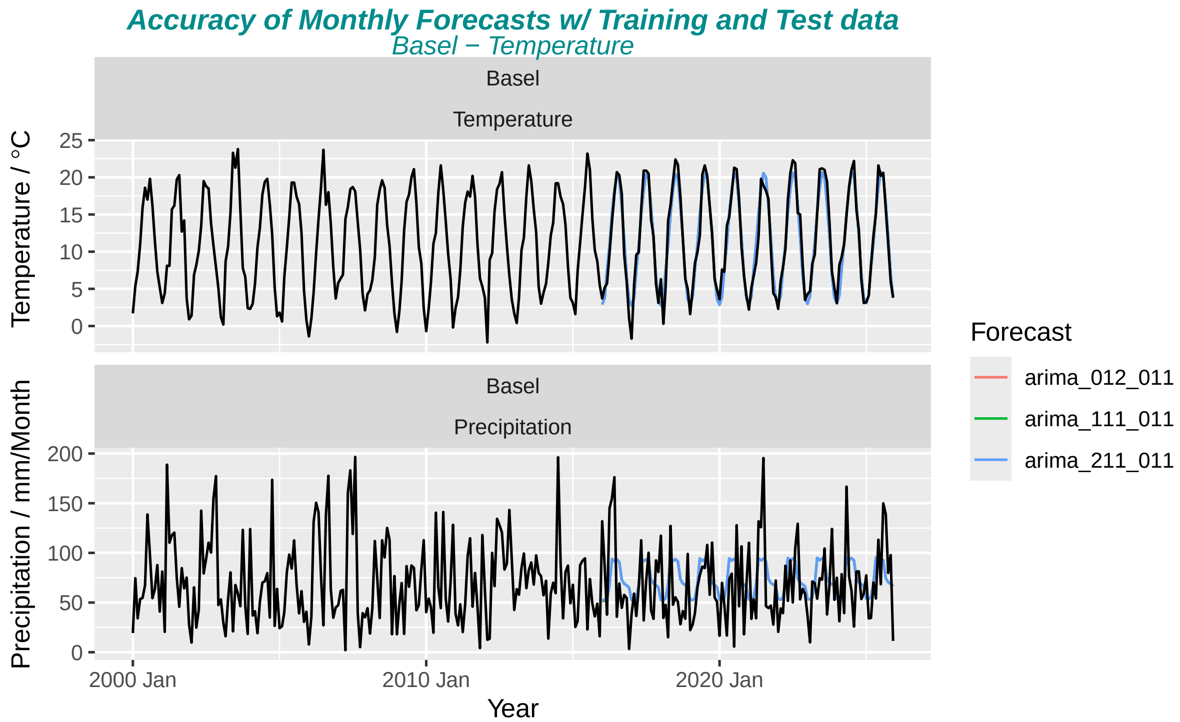
3.1.2 Ljung-Box Test - independence/white noise of the forecasts residuals

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 8 x 5
#>   City  Measure     .model        lb_stat lb_pvalue
#>   <chr> <fct>       <chr>           <dbl>     <dbl>
#> 1 Basel Temperature arima_300_111    25.5    0.228
#> 2 Basel Temperature arima_012_112    28.1    0.138
#> 3 Basel Temperature arima_111_012    28.1    0.137
#> 4 Basel Temperature arima_211_011    28.9    0.116
#> 5 Basel Temperature arima_111_011    28.9    0.115
#> 6 Basel Temperature arima_012_011    29.1    0.112
#> 7 Basel Temperature arima_100_200    29.7    0.0989
#> 8 Basel Temperature arima_102_211    29.7    0.0989
#> # A tibble: 8 x 5
#>   City  Measure       .model          lb_stat lb_pvalue
#>   <chr> <fct>         <chr>             <dbl>     <dbl>
#> 1 Basel Precipitation arima_111_012      27.2  1.63e- 1
#> 2 Basel Precipitation arima_012_112      27.5  1.55e- 1
#> 3 Basel Precipitation arima_111_011      27.6  1.51e- 1
#> 4 Basel Precipitation arima_012_011      27.6  1.51e- 1
#> 5 Basel Precipitation arima_211_011      27.6  1.51e- 1
#> 6 Basel Precipitation arima_001_002      52.6  1.55e- 4
#> 7 Basel Precipitation arima_100_110_c   113.   1.45e-14
#> 8 Basel Precipitation arima_200_011     113.   1.45e-14
```

3.1.3 Forecast Accuracy with Training/Test Data

```
#> # A tibble: 6 x 7
```

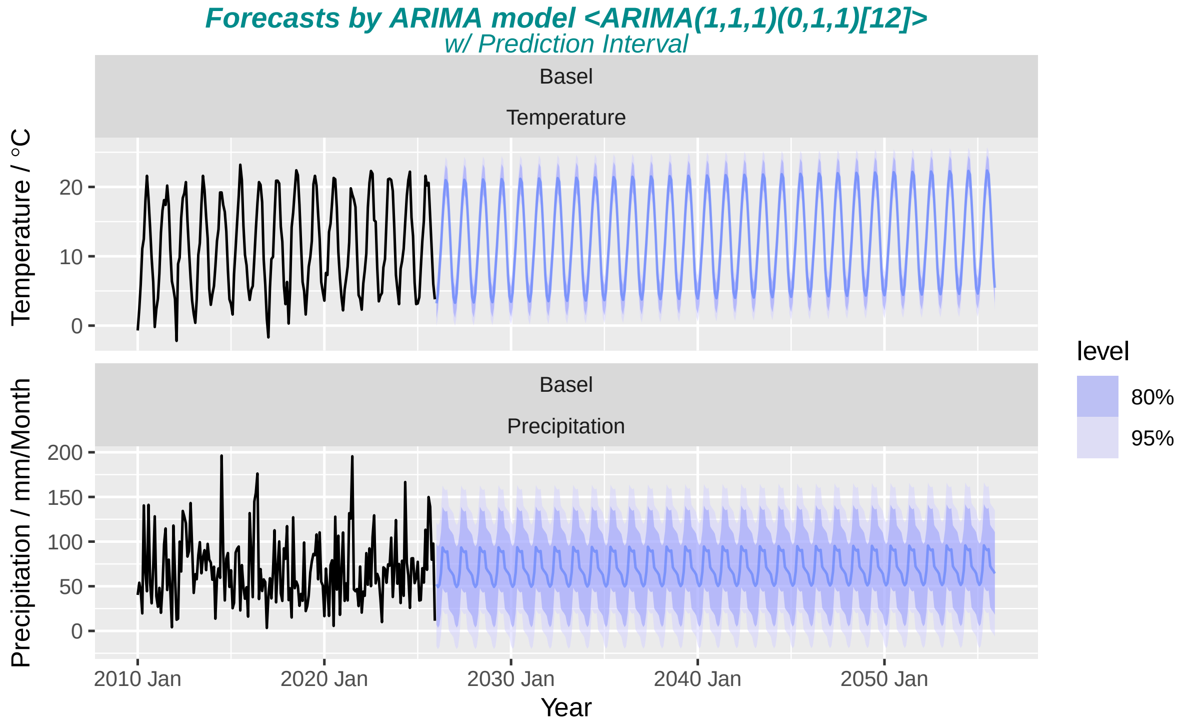
```
#>   .model         City  Measure       .type     ME  RMSE   MAE
#>   <chr>          <chr> <fct>         <chr>  <dbl> <dbl> <dbl>
#> 1 arima_211_011 Basel Temperature   Test   0.277  1.58  1.25
#> 2 arima_111_011 Basel Temperature   Test   0.277  1.58  1.25
#> 3 arima_012_011 Basel Temperature   Test   0.278  1.58  1.25
#> 4 arima_211_011 Basel Precipitation Test  -5.15  35.9  29.1
#> 5 arima_111_011 Basel Precipitation Test  -5.13  35.9  29.1
#> 6 arima_012_011 Basel Precipitation Test  -5.13  35.9  29.1
```



3.2 Temperature, Precipitation - Forecasting with selected ARIMA model <ARIMA(1,1,1)(0,1,1)[12]>, <ARIMA(1,1,1)(0,1,1)[12]>

3.2.1 Forecast Plot of selected ARIMA model

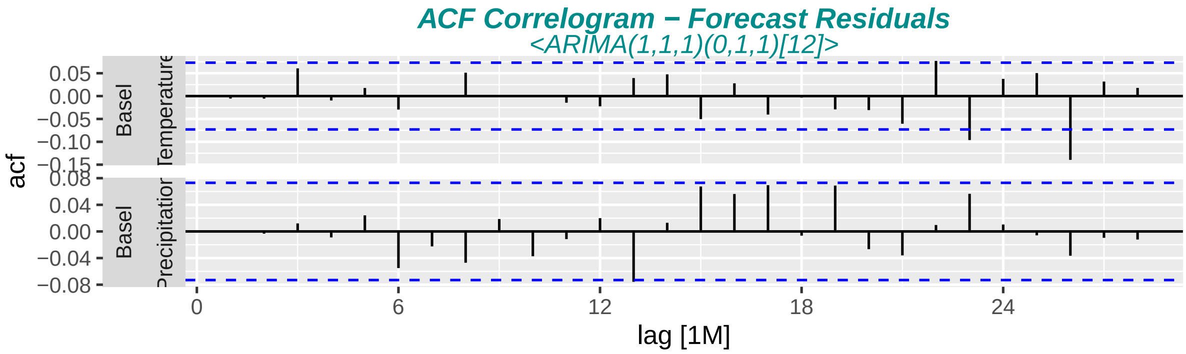
```
#> Provide model coefficients by report(fit_model)
#> # A tibble: 2 x 10
#>   City  Measure       .model  sigma2 log_lik   AIC  AICc   BIC ar_roots ma_roots
#>   <chr> <fct>         <chr>    <dbl>   <dbl> <dbl> <dbl> <dbl> <list>   <list>
#> 1 Basel Temperature   arima     2.78  -1394. 2796. 2796. 2814. <cpl>    <cpl>
#> 2 Basel Precipitation arima  1233.    -3547. 7101. 7102. 7120. <cpl>    <cpl>
```

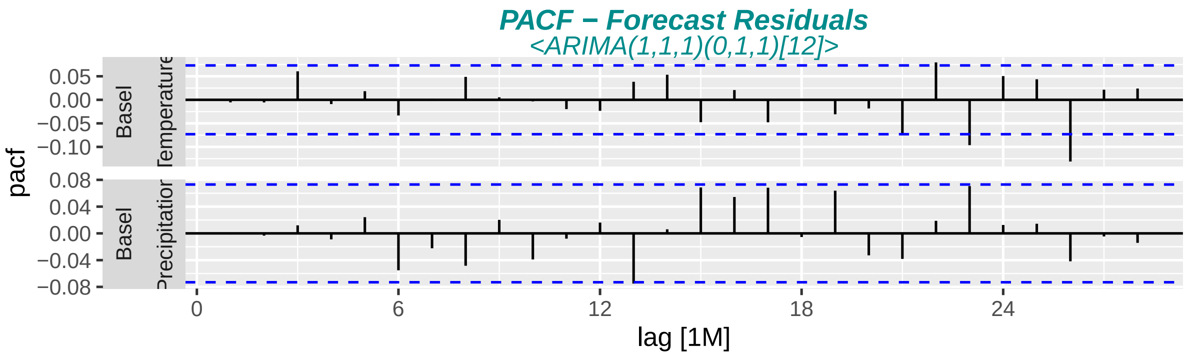


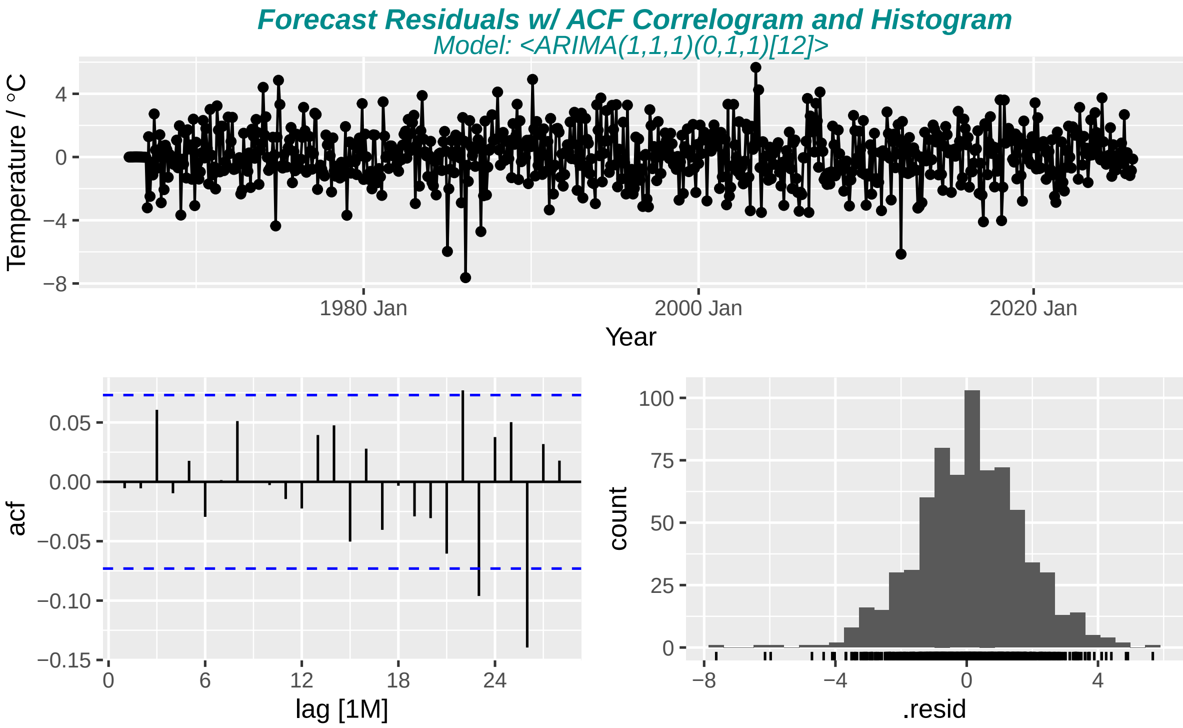
3.2.2 Residual Stationarity

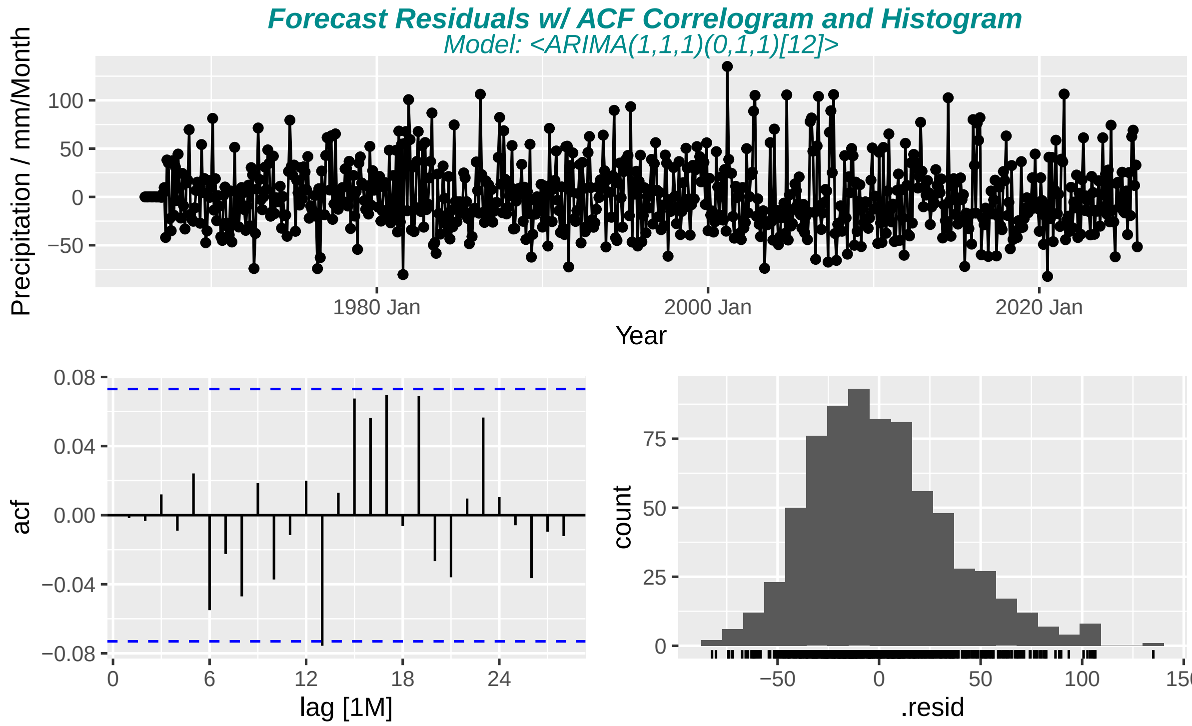
Required checks to be ready for forecasting:

- ACF Forecast Residual: all spikes are within the significance limits, so the residuals appear to be white noise
- The Ljung-Box test also shows that the residuals have no remaining autocorrelations
- Forecast Residuals are more or less normally distributed with roughly centred on zero


ACF Correlogram − Forecast Residuals
<ARIMA(1,1,1)(0,1,1)[12]>
acf
Basel
Temperature
Basel
Precipitation
lag [1M]

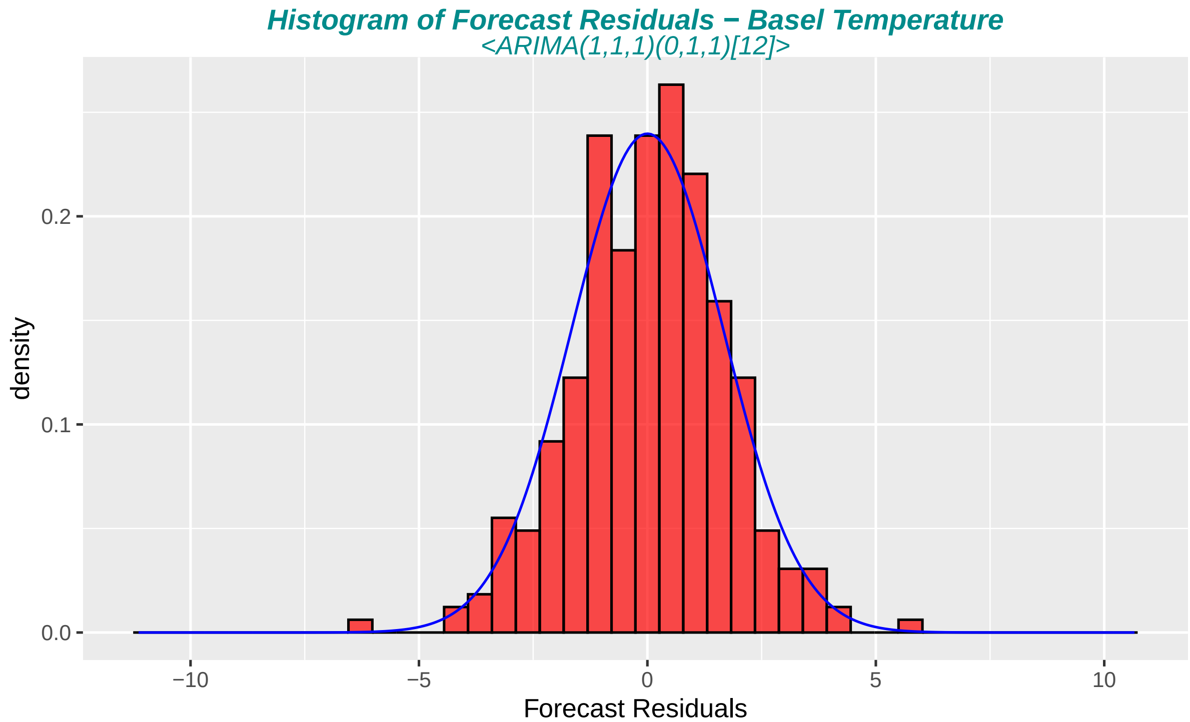

PACF − Forecast Residuals
<ARIMA(1,1,1)(0,1,1)[12]>
pacf
Basel
Temperature
Basel
Precipitation
lag [1M]

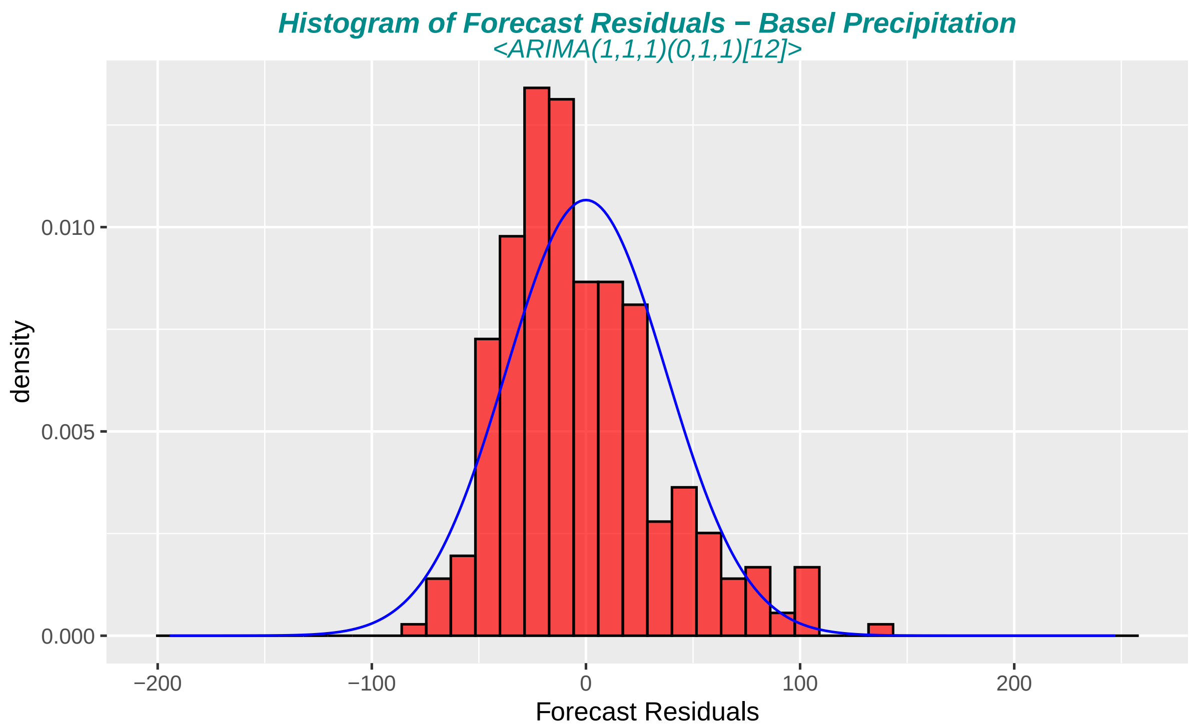

Forecast Residuals w/ ACF Correlogram and Histogram
Model: <ARIMA(1,1,1)(0,1,1)[12]>
Temperature / °C
1980 Jan
2000 Jan
2020 Jan
Year
acf
lag [1M]
count
.resid



3.2.3 Ljung-Box Test and Histogram of forecast residuals with overlaid normal curve

```
#> Null Hypothesis of independence/white noise for residuals - for p < 0.05: reject H_0
#> # A tibble: 2 x 5
#>   City  Measure       .model lb_stat lb_pvalue
#>   <chr> <fct>         <chr>    <dbl>     <dbl>
#> 1 Basel Temperature   arima     26.8     0.176
#> 2 Basel Precipitation arima     27.7     0.150
```




Histogram of Forecast Residuals − Basel Precipitation
<ARIMA(1,1,1)(0,1,1)[12]>
density
0.010
0.005
0.000
−200
−100
0
100
200
Forecast Residuals

4 ARIMA vs ETS

In particular, all ETS models are non-stationary, while some ARIMA models are stationary.

The ETS models with seasonality or non-damped trend or both have two unit roots (i.e., they need two levels of differencing to make them stationary). All other ETS models have one unit root (they need one level of differencing to make them stationary).

We compare for the chosen ETS rsp. ARIMA model the RMSE / MAE values. Lower values indicate a more accurate model based on the test set RMSE, . . . , MASE.

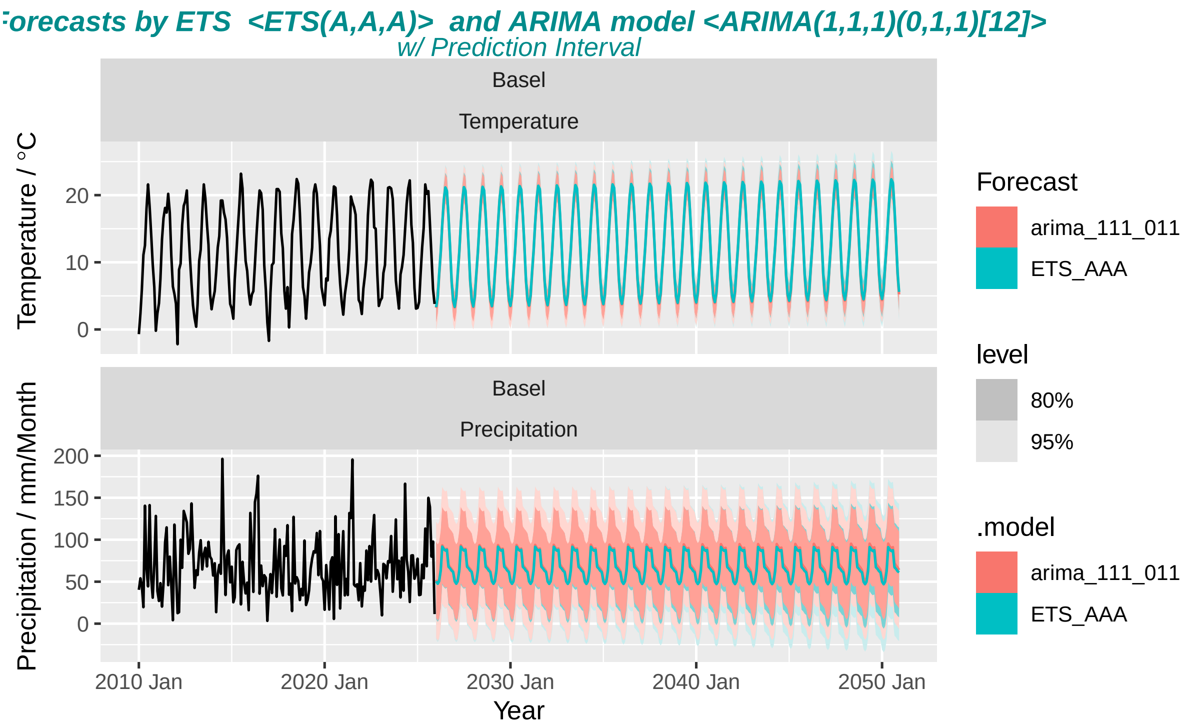
- Residual Accuracy with one-step-ahead fitted residuals
- Forecast Accuracy with Training/Test Data

Note: a good fit to training data is never an indication that the model will forecast well. Therefore the values of the Forecast Accuracy are the more relevant one.

4.0.1 Comparing Residual and Forecast Accuracy of selected ETS and ARIMA model

```
#> # A tibble: 8 x 9
#>   City  Measure       .model        .type     RMSE   MAE  MAPE  MASE RMSSE
#>   <chr> <fct>         <chr>         <chr>    <dbl> <dbl> <dbl> <dbl> <dbl>
#> 1 Basel Temperature   ETS_AAA       Test      1.57  1.24  27.8 0.669 0.658
#> 2 Basel Temperature   arima_111_011 Test      1.58  1.25  27.1 0.675 0.664
#> 3 Basel Temperature   arima         Training  1.65  1.27  52.8 0.683 0.690
#> 4 Basel Temperature   ets           Training  1.67  1.33  53.3 0.712 0.700
#> 5 Basel Precipitation arima         Training 34.7  27.3   75.9 0.730 0.711
#> 6 Basel Precipitation ets           Training 35.0  27.6   79.9 0.738 0.716
#> 7 Basel Precipitation ETS_AAA       Test     35.6  28.1   81.7 0.756 0.739
#> 8 Basel Precipitation arima_111_011 Test     35.9  29.1   89.3 0.784 0.746
```

4.0.2 Forecast Plot of selected ETS and ARIMA model



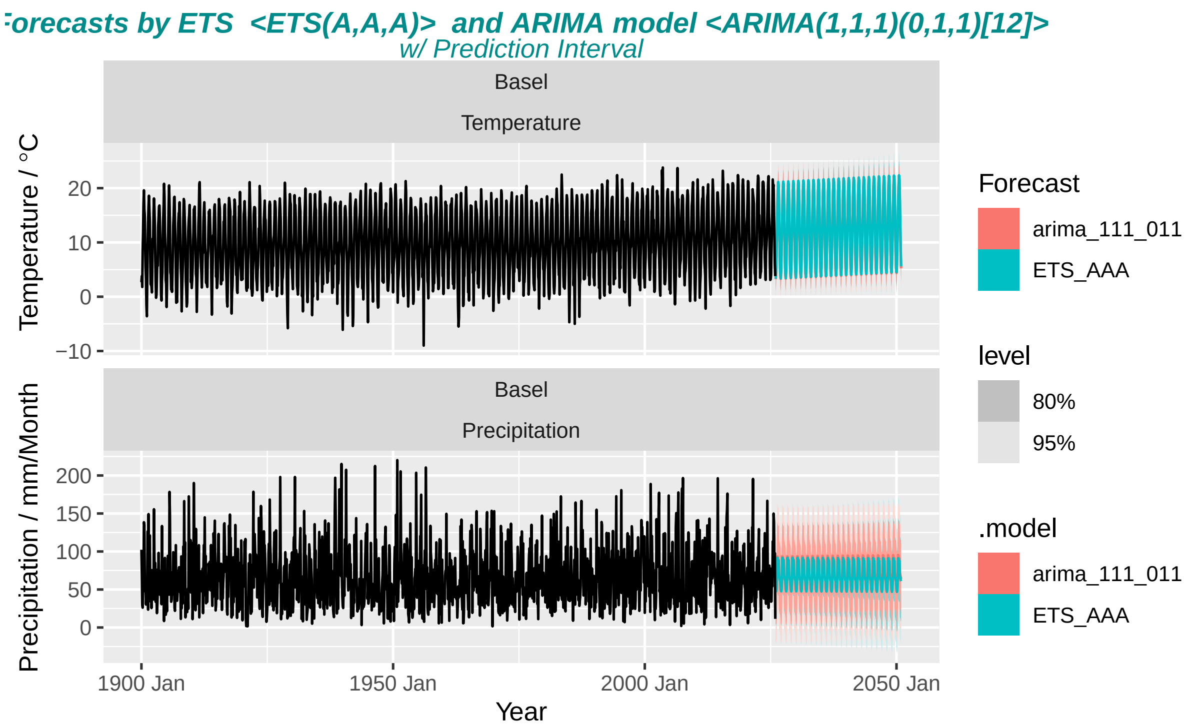

Forecasts by ETS <ETS(A,A,A)> and ARIMA model <ARIMA(1,1,1)(0,1,1)[12]>
w/ Prediction Interval
Basel
Temperature
Temperature / °C
20
10
0
−10
Basel
Precipitation
Precipitation / mm/Month
200
150
100
50
0
1900 Jan
1950 Jan
2000 Jan
2050 Jan
Year
Forecast
arima_111_011
ETS_AAA
level
80%
95%
.model
arima_111_011
ETS_AAA

Table 1: Mean values for the given time periods; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Period_Time	Temperature	Precipitation
1841-1870	9.0	62.4
1871-1900	8.7	64.3
1901-1930	9.0	65.9
1931-1960	9.4	65.5
1961-1990	9.6	64.8
1991-2020	10.9	70.2
2021-2025	12.0	70.9

Table 2: Mean Yearly ARIMA and ETS Forecast values (next 25 years); Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

City	Measure	Year	ETS_AAA	arima_111_011
Basel	Temperature	2026	12.13	11.93
Basel	Temperature	2030	12.33	12.12
Basel	Temperature	2035	12.59	12.36
Basel	Temperature	2040	12.84	12.60
Basel	Temperature	2045	13.09	12.84
Basel	Temperature	2050	13.35	13.07
Basel	Precipitation	2026	68.81	70.21
Basel	Precipitation	2030	68.71	70.59
Basel	Precipitation	2035	68.58	71.01
Basel	Precipitation	2040	68.45	71.42
Basel	Precipitation	2045	68.32	71.84
Basel	Precipitation	2050	68.20	72.25

Table 3: Forecast increase/decrease over the next 25 years; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Measure	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETS	Delta_ARIMA
Temperature	2026	2050	12.13	11.93	13.35	13.07	1.22	1.14
Precipitation	2026	2050	68.81	70.21	68.20	72.25	-0.61	2.04

Table 4: Forecast increase/decrease over the next 25 years; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Measure	Month	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETS	Delta_ARIMA
Temperature	Jan	2026	2050	3.32	3.21	4.54	4.38	1.22	1.17
Temperature	Feb	2026	2050	4.87	4.58	6.09	5.73	1.22	1.14
Temperature	Mrz	2026	2050	8.23	8.03	9.45	9.17	1.22	1.14
Temperature	Apr	2026	2050	11.50	11.34	12.71	12.48	1.22	1.14
Temperature	Mai	2026	2050	15.60	15.45	16.82	16.59	1.22	1.14
Temperature	Jun	2026	2050	19.24	19.01	20.46	20.15	1.22	1.14
Temperature	Jul	2026	2050	21.15	20.99	22.37	22.13	1.22	1.14
Temperature	Aug	2026	2050	20.63	20.44	21.84	21.58	1.22	1.14
Temperature	Sep	2026	2050	16.90	16.68	18.12	17.82	1.22	1.14
Temperature	Okt	2026	2050	12.46	12.30	13.67	13.44	1.22	1.14
Temperature	Nov	2026	2050	7.28	7.04	8.50	8.18	1.22	1.14

Measure	Month	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETS	Delta_ARIMA
Temperature	Dez	2026	2050	4.38	4.08	5.60	5.22	1.22	1.14
Precipitation	Jan	2026	2050	50.46	52.19	49.85	54.79	-0.61	2.61
Precipitation	Feb	2026	2050	47.50	49.02	46.89	51.02	-0.61	2.00
Precipitation	Mrz	2026	2050	50.41	51.65	49.80	53.64	-0.61	1.99
Precipitation	Apr	2026	2050	63.22	65.10	62.61	67.09	-0.61	1.99
Precipitation	Mai	2026	2050	91.75	93.41	91.14	95.40	-0.61	1.99
Precipitation	Jun	2026	2050	88.80	89.77	88.19	91.76	-0.61	1.99
Precipitation	Jul	2026	2050	87.03	88.04	86.42	90.03	-0.61	1.99
Precipitation	Aug	2026	2050	88.12	89.09	87.51	91.08	-0.61	1.99
Precipitation	Sep	2026	2050	67.69	70.17	67.08	72.16	-0.61	1.99
Precipitation	Okt	2026	2050	65.94	67.20	65.33	69.19	-0.61	1.99
Precipitation	Nov	2026	2050	63.09	64.92	62.48	66.92	-0.61	1.99
Precipitation	Dez	2026	2050	61.66	61.94	61.05	63.93	-0.61	1.99

5 Yearly Data Forecasts with ARIMA and ETS

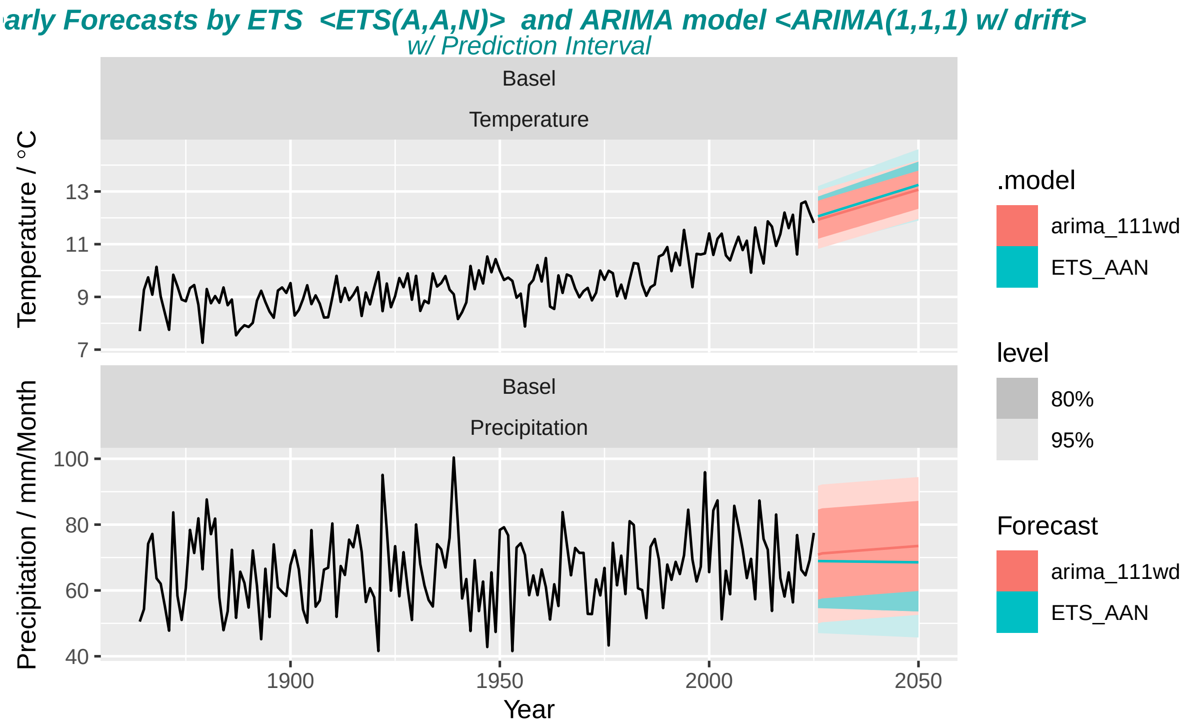
For yearly data the seasonal monthly data are replaced by the yearly average data. Therefore the seasonal component of the ETS and ARIMA model are to be taken out.

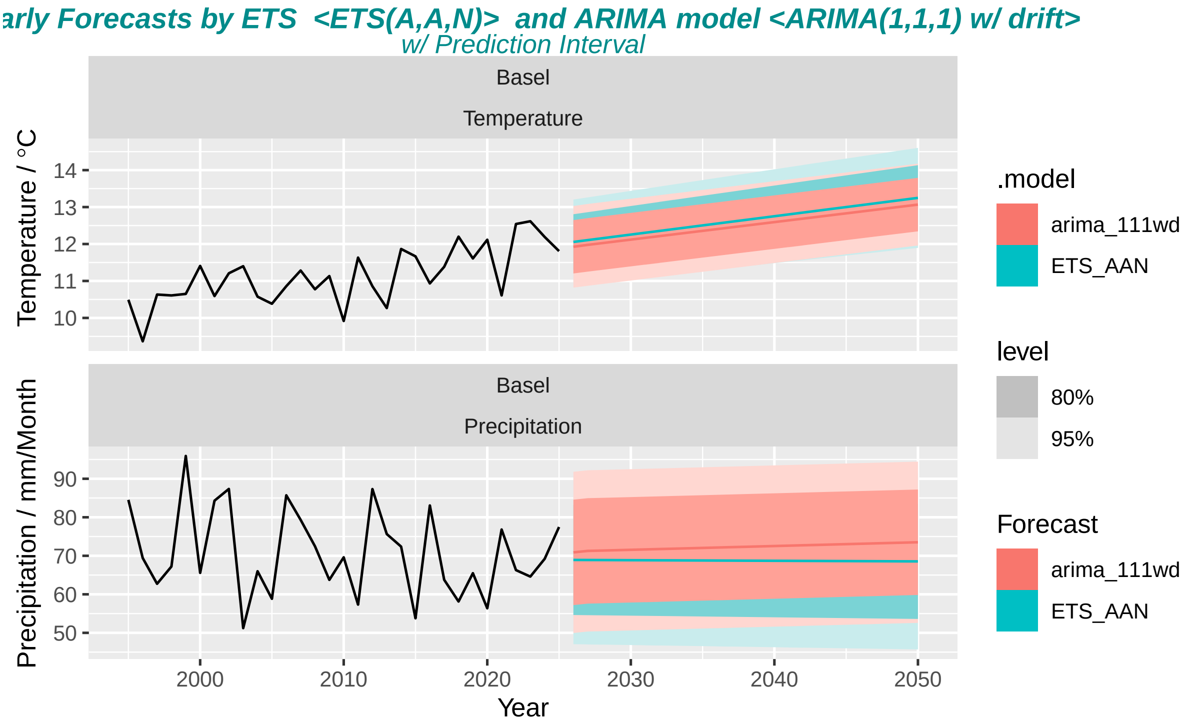
The ETS model $< ETS(A, A, N) >$ with seasonal term change “A” -> “N” is chosen. For ARIMA models the seasonal term (P,D,Q)m has to be taken out and an optimal ARIMA(p,1,q) with one differencing (d=1) is selected. However, for Mauna Loa two times differncing had to be selected \$CO_2 <ARIMA(0,2,1) w/ poly>. For Temperature and Precipitation the same model as for monthly data can be taken by leaving out the seasonal term $< ARIMA(0, 1, 2)w/drift >$.

5.0.1 Comparing Residual and Forecast Accuracy of selected ETS and ARIMA model

5.0.2 Forecast Plot of selected ETS and ARIMA model

```
#> selected: ETS_AAN and arima_111 w/ drift
```






```
#> # A tibble: 4 x 13
#>   City  Measure  .model  sigma2 log_lik   AIC  AICc   BIC     MSE    AMSE    MAE
#>   <chr> <fct>    <chr>    <dbl>   <dbl> <dbl> <dbl> <dbl>   <dbl>   <dbl>  <dbl>
#> 1 Basel Tempera~ arima~   0.312   -49.8  108.  108.  116.  NA      NA      NA
#> 2 Basel Tempera~ ETS_A~   0.342   -88.6  187.  188.  198.   0.319   0.317  0.463
#> 3 Basel Precipi~ arima~ 112.     -223.   455.  456.  463.  NA      NA      NA
#> 4 Basel Precipi~ ETS_A~ 124.     -265.   541.  542.  551. 116.    118.     8.46
#> # i 2 more variables: ar_roots <list>, ma_roots <list>
```

Ljung-Box Test - independence/white noise of the forecasts residuals

```
#> # A tibble: 2 x 5
#>   City  Measure       .model  lb_stat lb_pvalue
#>   <chr> <fct>         <chr>     <dbl>     <dbl>
#> 1 Basel Temperature   ETS_AAN    21.2     0.626
#> 2 Basel Precipitation ETS_AAN    28.2     0.253
#> # A tibble: 2 x 5
#>   City  Measure       .model      lb_stat lb_pvalue
#>   <chr> <fct>         <chr>         <dbl>     <dbl>
#> 1 Basel Temperature   arima_111wd    19.7     0.540
#> 2 Basel Precipitation arima_111wd    29.3     0.108
```

Table 5: Mean Yearly ARIMA and ETS Forecast values of mean yearly data (next 25 years); Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

City	Measure	Year	ETS_AAN	arima_111wd
Basel	Temperature	2026	12.06	11.92
Basel	Temperature	2030	12.26	12.12
Basel	Temperature	2035	12.50	12.36
Basel	Temperature	2040	12.75	12.59
Basel	Temperature	2045	13.00	12.83
Basel	Temperature	2050	13.25	13.07
Basel	Precipitation	2026	68.92	70.88
Basel	Precipitation	2030	68.86	71.55
Basel	Precipitation	2035	68.78	72.04
Basel	Precipitation	2040	68.70	72.53
Basel	Precipitation	2045	68.62	73.02
Basel	Precipitation	2050	68.55	73.51

Table 6: Forecast increase/decrease over the next 25 years; Units: Temperature (degree C), Precipitation (mm/Month), CO2 (ppm)

Measure	Year.x	Year.y	ETS.x	ARIMA.x	ETS.y	ARIMA.y	Delta_ETS	Delta_ARIMA
Temperature	2026	2050	12.06	11.92	13.25	13.07	1.19	1.14
Precipitation	2026	2050	68.92	70.88	68.55	73.51	-0.38	2.63