

$$\begin{array}{c}
\frac{\forall x \forall y (x(y+1) = xy + x) \quad \forall x \forall y (x + (y+1) = (x+y) + 1) \quad (t_1(t_2 + v) = t_1 t_2 + t_1 v) \vdash (k(v + (l+1)) = k(l+1) + kv) \quad A(u)}{\forall x \forall y (x(y+1) = xy + x) \quad \forall x \forall y (x + (y+1) = (x+y) + 1) \quad (t_1(t_2 + v) = t_1 t_2 + t_1 v) \vdash (k((v+l) + 1) = k(l+1) + kv) \quad A(u)} \\
\frac{\forall x \forall y (x(y+1) = xy + x) \quad \forall x \forall y (x + (y+1) = (x+y) + 1) \quad (t_1(t_2 + v) = t_1 t_2 + t_1 v) \vdash (k((l+v) + 1) = kl + kv + k) \quad A(u)}{\forall x \forall y (x(y+1) = xy + x) \quad \forall x \forall y (x + (y+1) = (x+y) + 1) \quad (t_1(t_2 + v) = t_1 t_2 + t_1 v) \vdash (k(l + (v+1)) = kl + k(v+1)) \quad A(u)} \quad \vdash \forall x \forall y (x(y+1) = xy + x) \quad \forall x \forall y (x + (y+1) = (x+y) + 1) \quad \text{cut} \\
\frac{\frac{\frac{t_1(t_2 + v) = t_1 t_2 + t_1 v \vdash (k(l + (v+1)) = kl + k(v+1)) \quad A(u)}{\forall x \forall y (x(y+v) = xy + xv) \vdash (k(l + (v+1)) = kl + k(v+1)) \quad A(u)} \quad (\forall \vdash)}{\forall x \forall y (x(y+v) = xy + xv) \vdash \forall x \forall y (x(y + (v+1)) = xy + x(v+1)) \quad A(u)} \quad (\vdash \forall)} \quad \text{def} \\
\frac{\frac{\frac{A(v) \vdash A(v+1) \quad A(u)}{\vdash (A(v) \rightarrow A(v+1)) \quad A(u)} \quad (\vdash \rightarrow)}{\vdash \forall z (A(z) \rightarrow A(z+1)) \quad A(u)} \quad (\vdash \forall)} \\
\frac{\vdash (A(0) \& \forall z (A(z) \rightarrow A(z+1))) \quad A(u)}{\frac{A(0) \& \forall z (A(z) \rightarrow A(z+1)) \rightarrow \forall z A(z) \vdash A(u)}{A(0) \& \forall z (A(z) \rightarrow A(z+1)) \rightarrow \forall z A(z) \vdash \forall z A(z)} \quad (\vdash \forall)}
\end{array}$$

$$\begin{array}{c}
\frac{t = t \vdash vw = vw \quad A(u)}{\forall x (x = x) \vdash (vw = vw) \quad A(u)} \quad \frac{\forall x (x + 0 = x) \quad \forall x (x \cdot 0 = 0) \vdash \forall x (x = x)}{\forall x (x + 0 = x) \quad \forall x (x \cdot 0 = 0) \vdash (vw = vw) \quad A(u)} \quad \text{cut} \\
\frac{\forall x (x + 0 = x) \quad \forall x (x \cdot 0 = 0) \vdash (vw = vw + 0) \quad A(u)}{\forall x (x + 0 = x) \quad \forall x (x \cdot 0 = 0) \vdash (v(w+0) = vw + v \cdot 0) \quad A(u)} \quad \vdash \forall x (x + 0 = x) \quad \forall x (x \cdot 0 = 0) \quad \text{cut} \\
\frac{\vdash (v(w+0) = vw + v \cdot 0) \quad A(u)}{\vdash \forall x \forall y (x(y+0) = xy + x \cdot 0) \quad A(u)} \quad (\vdash \forall) \quad \text{def} \\
\frac{\vdash A(0) \quad A(u)}{\vdash A(0) \& \forall z (A(z) \rightarrow A(z+1)) \quad A(u)} \quad (\vdash \&) \quad \frac{A(t) \vdash A(u)}{\forall z A(z) \vdash A(u)} \quad (\forall \vdash) \\
\frac{\vdash (A(0) \& \forall z (A(z) \rightarrow A(z+1))) \quad A(u)}{\frac{A(0) \& \forall z (A(z) \rightarrow A(z+1)) \rightarrow \forall z A(z) \vdash A(u)}{A(0) \& \forall z (A(z) \rightarrow A(z+1)) \rightarrow \forall z A(z) \vdash \forall z A(z)} \quad (\vdash \rightarrow)}
\end{array}$$