

$$\begin{array}{c}
\frac{(t = t) \ ((t_1 t_2)v = t_1(t_2 v)) \vdash B(u) \ (klv + kl = klv + kl)}{\forall x(x = x) \ ((t_1 t_2)v = t_1(t_2 v)) \vdash B(u) \ (klv + kl = klv + kl)} \\
\frac{\forall x \forall y \forall z(x(y + z) = xy + xz) \vdash \forall x(x = x)}{\forall x \forall y \forall z(x(y + z) = xy + xz) \ ((t_1 t_2)v = t_1(t_2 v)) \vdash B(u) \ (klv + kl = klv + kl)} \text{ cut} \\
\frac{\forall x \forall y \forall z(x(y + z) = xy + xz) \ ((t_1 t_2)v = t_1(t_2 v)) \vdash B(u) \ (klv + kl = k(lv + l))}{\forall x \forall y \forall z(x(y + z) = xy + xz) \ ((t_1 t_2)v = t_1(t_2 v)) \vdash B(u) \ ((kl)(v + 1) = k(l(v + 1)))} \\
\frac{\vdash \forall x \forall y \forall z(x(y + z) = xy + xz)}{\vdash \forall x \forall y \forall z(x(y + z) = xy + xz) \ ((t_1 t_2)v = t_1(t_2 v)) \vdash B(u) \ ((kl)(v + 1) = k(l(v + 1)))} \text{ cut} \\
\frac{\frac{\frac{(t_1 t_2)v = t_1(t_2 v) \vdash B(u) \ ((kl)(v + 1) = k(l(v + 1)))}{\forall x \forall y((xy)v = x(yv)) \vdash B(u) \ ((kl)(v + 1) = k(l(v + 1)))} (\forall \vdash)}{\forall x \forall y((xy)v = x(yv)) \vdash B(u) \ \forall x \forall y((xy)(v + 1) = x(y(v + 1)))} (\vdash \forall)} \\
\frac{\vdash \forall x \forall y((xy)v = x(yv)) \vdash B(u) \ \forall x \forall y((xy)(v + 1) = x(y(v + 1)))}{\frac{B(v) \vdash B(v + 1) \ B(u)}{\vdash (B(v) \rightarrow B(v + 1)) \ B(u)} (\vdash \rightarrow)} \text{ def} \\
\frac{\vdash (B(v) \rightarrow B(v + 1)) \ B(u)}{\vdash \forall z(B(z) \rightarrow B(z + 1)) \ B(u)} (\vdash \forall) \\
\frac{\vdash \forall z(B(z) \rightarrow B(z + 1)) \ B(u)}{\vdash (B(0) \& \forall z(B(z) \rightarrow B(z + 1))) \ B(u)} \\
\frac{\vdash (B(0) \& \forall z(B(z) \rightarrow B(z + 1))) \ B(u)}{\frac{B(0) \& \forall z(B(z) \rightarrow B(z + 1)) \rightarrow \forall z B(z) \vdash B(u)}{B(0) \& \forall z(B(z) \rightarrow B(z + 1)) \rightarrow \forall z B(z) \vdash \forall z B(z)} (\vdash \forall)
\end{array}$$

$$\begin{array}{c}
\frac{t = t \vdash B(u) \ (0 = 0)}{\forall x(x = x) \vdash B(u) \ (0 = 0)} \\
\frac{\forall x(x = x) \vdash \forall x(x \cdot 0 = 0)}{\forall x(x \cdot 0 = 0) \vdash B(u) \ (0 = 0)} \text{ cut} \\
\frac{\forall x(x \cdot 0 = 0) \vdash B(u) \ (0 = 0)}{\forall x(x \cdot 0 = 0) \vdash B(u) \ (0 = v \cdot 0)} \\
\frac{\forall x(x \cdot 0 = 0) \vdash B(u) \ ((vw) \cdot 0 = v(w \cdot 0))}{\vdash \forall x(x \cdot 0 = 0)} \text{ cut} \\
\frac{\vdash \forall x(x \cdot 0 = 0)}{\vdash B(u) \ ((vw) \cdot 0 = v(w \cdot 0))} (\vdash \forall) \\
\frac{\vdash B(u) \ \forall x \forall y((xy) \cdot 0 = x(y \cdot 0))}{\vdash B(0) \ B(u)} \text{ def} \\
\frac{\vdash B(0) \ B(u)}{\vdash B(0) \& \forall z(B(z) \rightarrow B(z + 1)) \ B(u)} (\vdash \&) \\
\frac{\vdash B(0) \& \forall z(B(z) \rightarrow B(z + 1)) \ B(u)}{\frac{B(t) \vdash B(u)}{\forall z B(z) \vdash B(u)} (\forall \vdash)} (\rightarrow \vdash)
\end{array}$$