

$$\begin{array}{c}
\frac{}{u = wv + r \vdash s \mid u \ \& \ s \mid v \rightarrow s \mid r} \quad \frac{}{u = wv + r \vdash s \mid v \ \& \ s \mid r \rightarrow s \mid u} \quad (\vdash \ \&) \\
\hline
(**) \quad \frac{u = wv + r \vdash (s \mid u \ \& \ s \mid v \rightarrow s \mid r) \ \& \ (s \mid v \ \& \ s \mid r \rightarrow s \mid u)}{u = wv + r \vdash \forall z((z \mid u \ \& \ z \mid v \rightarrow z \mid r) \ \& \ (z \mid v \ \& \ z \mid r \rightarrow z \mid u))} \quad (\vdash \ \forall) \\
\hline
\frac{}{\vdash u = wv + r \rightarrow \forall z((z \mid u \ \& \ z \mid v \rightarrow z \mid r) \ \& \ (z \mid v \ \& \ z \mid r \rightarrow z \mid u))} \quad (\vdash \rightarrow) \\
(*) \quad \frac{}{\vdash \forall x \forall y \forall p \forall q (x = py + q \rightarrow \forall z((z \mid x \ \& \ z \mid y \rightarrow z \mid q) \ \& \ (z \mid y \ \& \ z \mid q \rightarrow z \mid x)))} \quad (\vdash \ \forall)
\end{array}$$

(5)

(6)

(7)

(8)

 $(\rightarrow\vdash)$ $(\forall\vdash)$

cut

 $(\&\vdash)$ $(\vdash\rightarrow)$

4

(1)

(2)

(3)

(4)

 $(\rightarrow\vdash)$ $(\forall\vdash)$

cut

 $(\&\vdash)$ $(\vdash\rightarrow)$

4

$$\begin{array}{c}
\frac{(s \mid u) \ (s \mid v) \ (u = wv + r) \vdash (s \mid r) \ (t_4 \mid t_2)}{(s \mid u) \ (s \mid v) \ (u = wv + r) \vdash (s \mid r) \ (t_4 \mid t_2 \ \& \ t_4 \mid t_3)} \quad \frac{(s \mid u) \ (s \mid v) \ (u = wv + r) \vdash (s \mid r) \ (t_4 \mid t_3)}{(s \mid u) \ (s \mid v) \ (u = wv + r) \vdash (s \mid r) \ (t_2 = t_3 + t_1)} \\
\frac{(s \mid u) \ (s \mid v) \ (u = wv + r) \vdash (s \mid r) \ (t_4 \mid t_2 \ \& \ t_4 \mid t_3 \ \& \ t_2 = t_3 + t_1)}{(t_4 \mid t_2 \ \& \ t_4 \mid t_3 \ \& \ t_2 = t_3 + t_1 \rightarrow t_4 \mid t_1) \ (s \mid u) \ (s \mid v) \ (u = wv + r) \vdash s \mid r} \quad \frac{(s \mid u) \ (s \mid v) \ (u = wv + r) \vdash (s \mid r) \ (t_2 = t_3 + t_1)}{(t_4 \mid t_1) \ (s \mid u) \ (s \mid v) \ (u = wv + r) \vdash s \mid r} \\
\frac{(t_4 \mid t_2 \ \& \ t_4 \mid t_3 \ \& \ t_2 = t_3 + t_1 \rightarrow t_4 \mid t_1) \ (s \mid u) \ (s \mid v) \ (u = wv + r) \vdash s \mid r}{\forall w \forall x \forall y \forall z (z \mid x \ \& \ z \mid y \ \& \ x = y + w \rightarrow z \mid w) \ (s \mid u) \ (s \mid v) \ (u = wv + r) \vdash s \mid r} \quad \frac{(s \mid u) \ (s \mid v) \ (u = wv + r) \vdash s \mid r}{(s \mid u \ \& \ s \mid v) \ (u = wv + r) \vdash s \mid r} \\
\frac{\forall w \forall x \forall y \forall z (z \mid x \ \& \ z \mid y \ \& \ x = y + w \rightarrow z \mid w) \ (s \mid u) \ (s \mid v) \ (u = wv + r) \vdash s \mid r \quad (s \mid u \ \& \ s \mid v) \ (u = wv + r) \vdash s \mid r}{u = wv + r \vdash s \mid u \ \& \ s \mid v \rightarrow s \mid r}
\end{array}$$