

# ASCC2022: GP+SOSP+Polynomial Controller

---

Consider a control affine dynamical system as follows,

$$\dot{x} = f(x) + g(x)u + d(x), \quad (1)$$

where  $x \in \mathcal{X} \subset \mathbb{R}^n$  and  $u \in \mathcal{U} \subset \mathbb{R}^m$  denote the state and control of the system. The system is consisted of three Lipschitz continuous terms,  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  denotes a nonlinear term,  $g : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$  denotes a polynomial term and  $d : \mathbb{R}^n \rightarrow \mathbb{R}^n$  denotes an unknown term. We consider a polynomial control input  $u$  over the stabilization process in this paper.

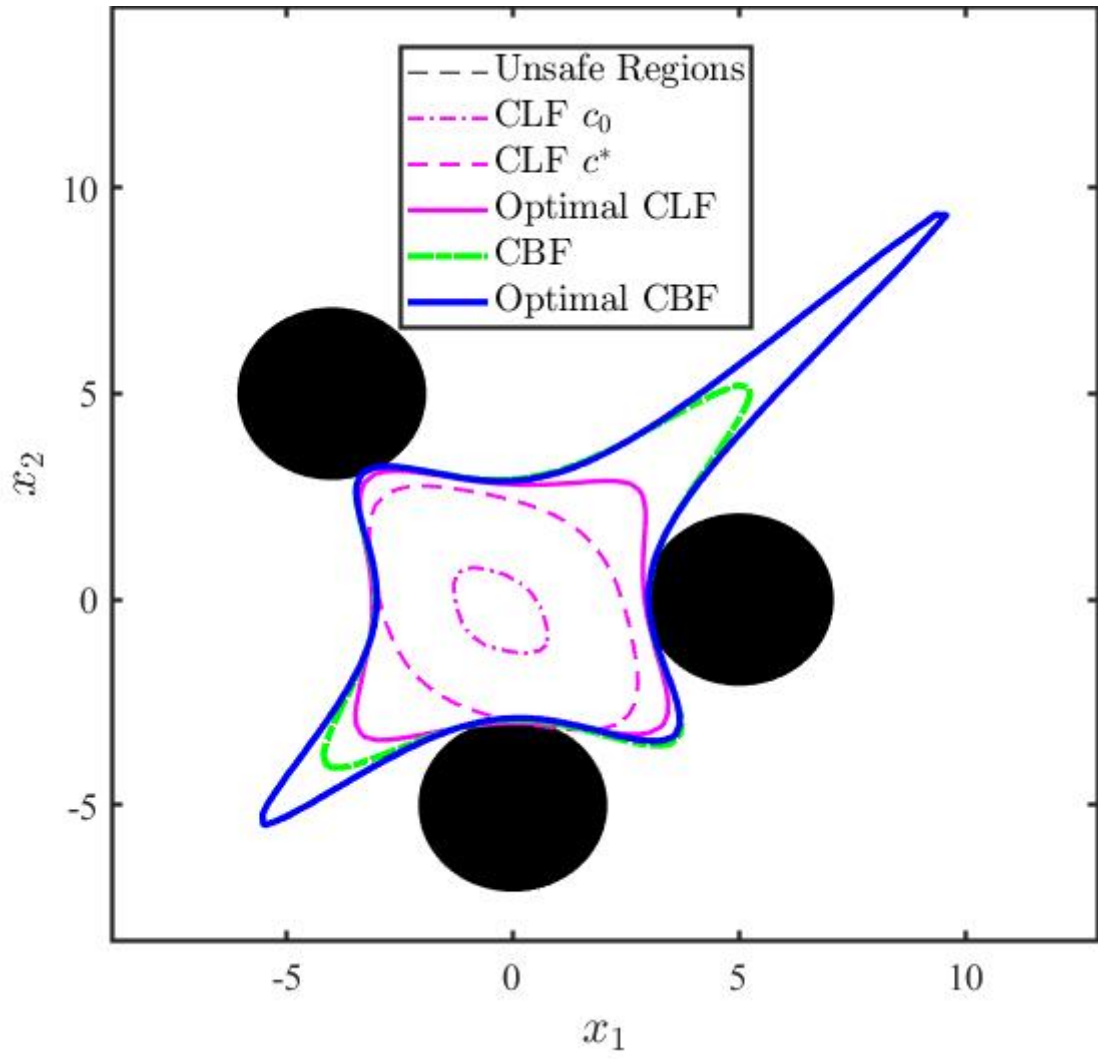
In this repo, we use

- Chebfun Toolbox: To approximate nonlinear terms by Chebyshev Interpolants,
- GPML Toolbox: Expressed the Gaussian processes mean function of this unknown term  $d(x)$  into the polynomial form,
- SOSOPT+Mosek: To solve some sum-of-squares programmings in this learned polynomial system.

Note that, please run *sosaddpath.m* at the beginning and Do not forget to install the Mosek Solver in advance.

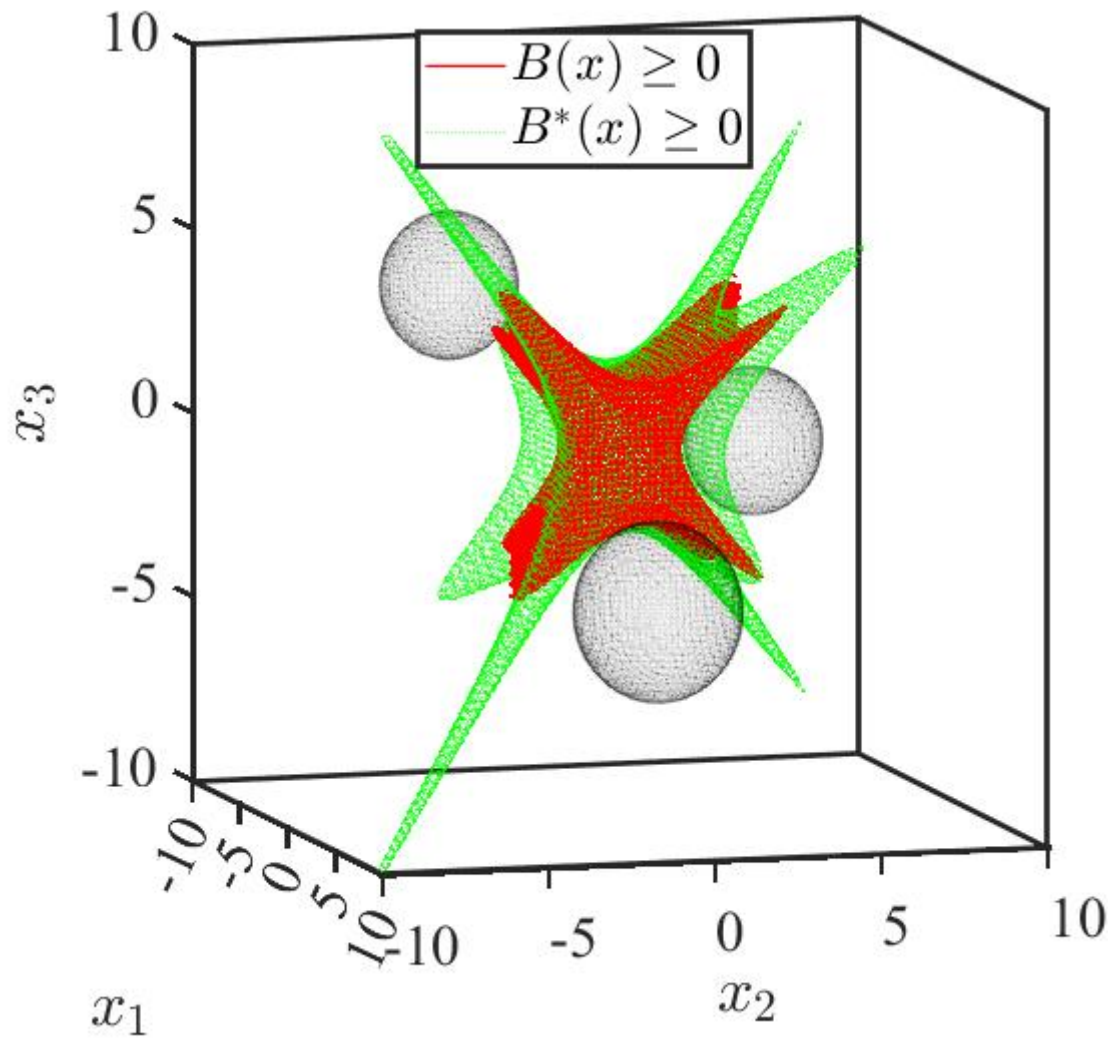
The final ROA with polynomial controller of the 2D system is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 + u_1 \\ x_1^2 x_2 + 1 - \sqrt{|\exp(x_1) \cos(x_1)|} + u_2 + d(x) \end{bmatrix}. \quad (2)$$



The final ROA of the 3D demo:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -x_1^2 - \cos(x_1^2) \sin(x_1) + u_1(x) + d_1(x) \\ -x_2 - x_1^3 x_2 + u_2(x) \\ -x_1^2 x_3 + 1 - \sqrt{|\exp(x_1) \cos(x_1) + u_3(x) + d_3(x)|} \end{bmatrix}. \quad (3)$$



The related files are concluded in the figure below

<pre> sosaddpath.m prepare_polynomial_system_1D.m gpr_xdot2.m demo_2d_lya_sublevelset.m demo_2d_opt_barrier.m demo_2d_Find_opt_lya_original.m demo_2d_Find_opt_BV.m demo_2d_CLF_compare.m demo_2d_CLB_compare.m demo_2d.m prepare_polynomial_system_3D_2d.m demo_3d_lya_sublevelset.m demo_3d_opt_barrier.m demo_3d_Find_opt_lya.m demo_3d_Find_opt_BV.m demo_3d_CLB_Comparer.m demo_3d_CLBF.m demo_3d_CLF_Comparer.m + </pre>	<pre> 1 % Add paths to SOS analysis toolboxes 2 % 3 4 % Add multipoly 5 6 cm = computer; 7 8 if cm(1) == 'M'    cm(1) == 'G' 9     set(0, 'DefaultFigureWindowStyle', 'docked') 10    % Add chebfun-master 11    addpath([pwd '/toolbox/chebfun-master']); 12 13    % Add gpml-matlab-master 14    addpath([pwd '/toolbox/gpml']); 15    run([pwd '/toolbox/gpml/startup.m']); 16 17    % Add multipoly 18    addpath([pwd '/toolbox/multipoly']); 19 20    % Add nlanal 21    addpath([pwd '/toolbox/nlanal']); 22 23    % Add my version of SOSTools 24    addpath([pwd '/toolbox/sosopt']); 25    addpath([pwd '/toolbox/sosopt/Demos']); 26 27    % Add polysys 28    addpath([pwd '/toolbox/polysystems_1_0_3']); 29    addpath([pwd '/toolbox/polysystems_1_0_3/demo']); 30 31    % Add utils </pre>
--	--

To verify the **2D demo**, please run these files in a sequent.

- prepare\_polynomial\_system\_1D.m
- demo\_2d\_lya\_sublevelset.m

- demo\_2d\_opt\_barrier.m
- demo\_2d\_Find\_opt\_Lya\_original.m
- demo\_2d\_Find\_opt\_BV.m
- demo\_2d\_CLF\_compare.m
- demo\_2d\_CLB\_compare.m

To verify the **3D demo**, please run these files in a sequent.

- prepare\_polynomial\_system\_3D\_2d.m
- demo\_3d\_lya\_sublevelset.m
- demo\_3d\_opt\_barrier.m
- demo\_3d\_Find\_opt\_Lya.m
- demo\_3d\_Find\_opt\_BV.m
- demo\_3d\_CLB\_Comparer.m
- demo\_3d\_CLF\_Comparer.m
- demo\_3d\_CLBF.m

Feel free to contact [hejunhuang@cuhk.edu.hk](mailto:hejunhuang@cuhk.edu.hk) for more details.