

ASCC2022: GP+SOSP+Polynomial Controller

Consider a control affine dynamical system as follows,

$$\dot{x} = f(x) + g(x)u + d(x), \quad (4)$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$ and $u \in \mathcal{U} \subset \mathbb{R}^m$ denote the state and control of the system. The system is consisted of three Lipschitz continuous terms, $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ denotes a nonlinear term, $g : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$ denotes a polynomial term and $d : \mathbb{R}^n \rightarrow \mathbb{R}^n$ denotes an unknown term. We consider a polynomial control input u over the stabilization process in this paper.

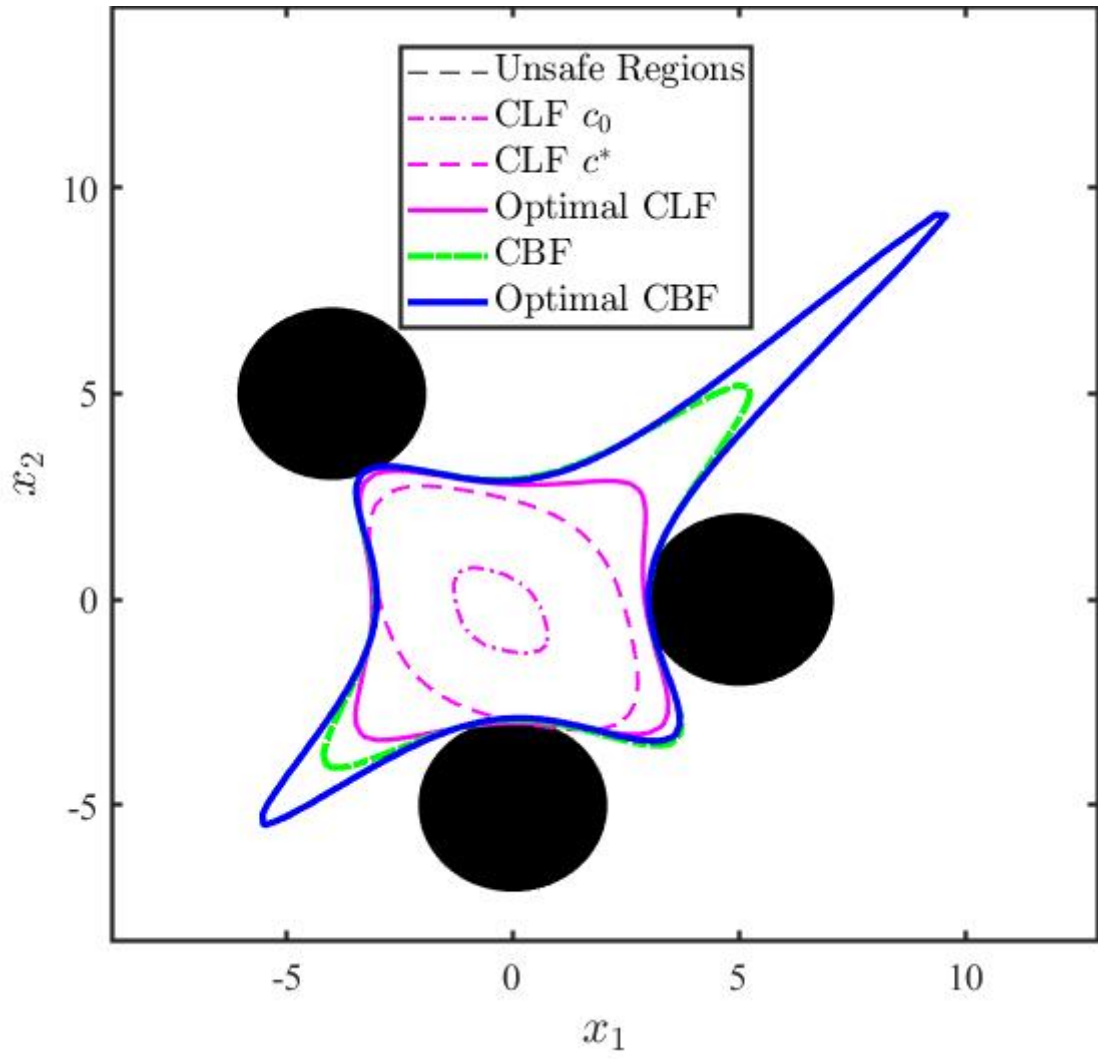
In this repo, we use

- Chebfun Toolbox: To approximate nonlinear terms by Chebyshev Interpolants,
- GPML Toolbox: Expressed the Gaussian processes mean function of this unknown term $d(x)$ into the polynomial form,
- SOSOPT+Mosek: To solve some sum-of-squares programmings in this learned polynomial system.

Note that, please run *sosaddpath.m* at the beginning and Do not forget to install the Mosek Solver in advance.

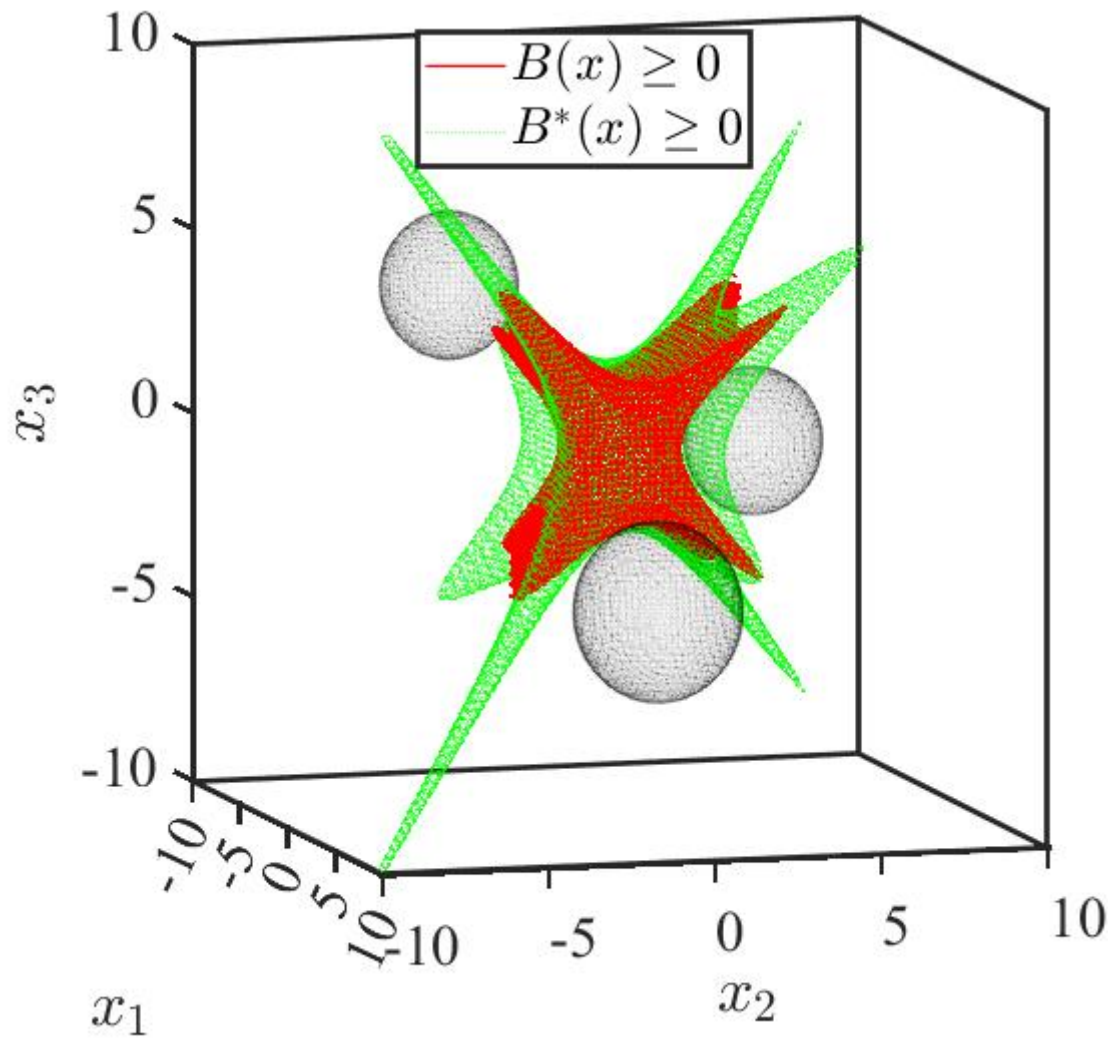
The final ROA with polynomial controller of the 2D system is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 + u_1 \\ x_1^2 x_2 + 1 - \sqrt{|\exp(x_1) \cos(x_1)|} + u_2 + d(x) \end{bmatrix}, \quad (5)$$



The final ROA of the 3D demo:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -x_1^2 - \cos(x_1^2) \sin(x_1) + u_1(x) + d_1(x) \\ -x_2 - x_1^3 x_2 + u_2(x) \\ -x_1^2 x_3 + 1 - \sqrt{|\exp(x_1) \cos(x_1)|} + u_3(x) + d_3(x) \end{bmatrix}. \quad (6)$$



The related files are concluded in the figure below

<pre> sosaddpath.m prepare_polynomial_system_1D.m gpr_xdot2.m demo_2d_lya_sublevelset.m demo_2d_opt_barrier.m demo_2d_Find_opt_lya_original.m demo_2d_Find_opt_BV.m demo_2d_CLF_compare.m demo_2d_CLB_compare.m demo_2d.m prepare_polynomial_system_3D_2d.m demo_3d_lya_sublevelset.m demo_3d_opt_barrier.m demo_3d_Find_opt_lya.m demo_3d_Find_opt_BV.m demo_3d_CLB_Comparer.m demo_3d_CLBF.m demo_3d_CLF_Comparer.m + </pre>	<pre> 1 % Add paths to SOS analysis toolboxes 2 % 3 4 % Add multipoly 5 6 cm = computer; 7 8 if cm(1) == 'M' cm(1) == 'G' 9 set(0, 'DefaultFigureWindowStyle', 'docked') 10 % Add chebfun-master 11 addpath([pwd '/toolbox/chebfun-master']); 12 13 % Add gpml-matlab-master 14 addpath([pwd '/toolbox/gpml']); 15 run([pwd '/toolbox/gpml/startup.m']) 16 17 % Add multipoly 18 addpath([pwd '/toolbox/multipoly']); 19 20 % Add nlanal 21 addpath([pwd '/toolbox/nlanal']); 22 23 % Add my version of SOSTools 24 addpath([pwd '/toolbox/sosopt']); 25 addpath([pwd '/toolbox/sosopt/Demos']); 26 27 % Add polysys 28 addpath([pwd '/toolbox/polysystems_1_0_3']); 29 addpath([pwd '/toolbox/polysystems_1_0_3/demo']); 30 31 % Add utils </pre>
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To verify the **2D demo**, please run these files in a sequent.

- prepare_polynomial_system_1D.m
- demo_2d_lya_sublevelset.m

- demo_2d_opt_barrier.m
- demo_2d_Find_opt_Lya_original.m
- demo_2d_Find_opt_BV.m
- demo_2d_CLF_compare.m
- demo_2d_CLB_compare.m

To verify the **3D demo**, please run these files in a sequent.

- prepare_polynomial_system_3D_2d.m
- demo_3d_lya_sublevelset.m
- demo_3d_opt_barrier.m
- demo_3d_Find_opt_Lya.m
- demo_3d_Find_opt_BV.m
- demo_3d_CLB_Comparer.m
- demo_3d_CLF_Comparer.m
- demo_3d_CLBF.m

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