ASCC2022: GP+SOSP+Polynomial Controller

Consider a control affine dynamical system as follows,

$$\dot{x} = f(x) + g(x)u + d(x), \tag{1}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$ and $u \in \mathcal{U} \subset \mathbb{R}^m$ denote the state and control of the system. The system is consisted of three Lipschitz continuous terms, $f : \mathbb{R}^n \to \mathbb{R}^n$ denotes a nonlinear term, $g : \mathbb{R}^n \to \mathbb{R}^{n \times m}$ denotes a polynomial term and $d : \mathbb{R}^n \to \mathbb{R}^n$ denotes an unknown term. We consider a polynomial control input u over the stabilization process in this paper.

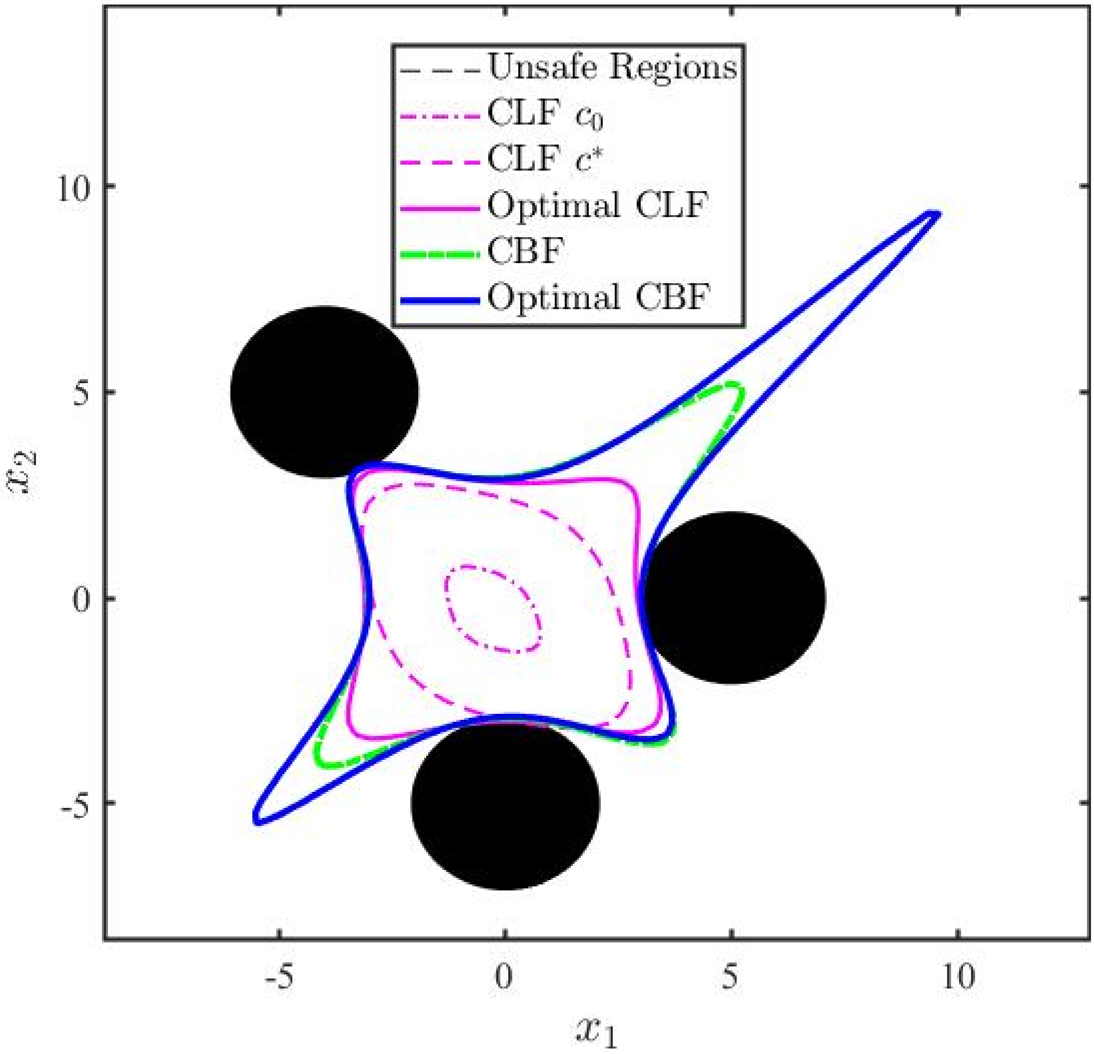
In this repo, we use

- Chebfun Toolbox: To approximate nonlinear terms by Chebyshev Interpolants,
- GPML Toolbox: Expressed the Gaussian processes mean function of this unknown term $d(x)$ into the polynomial form,
- SOSOPT+Mosek: To solve some sum-of-squares programmings in this learned polynomial system.

Note that, please run *sosaddpath.m* at the beginning and Do not forget to install the Mosek Solver in advance.

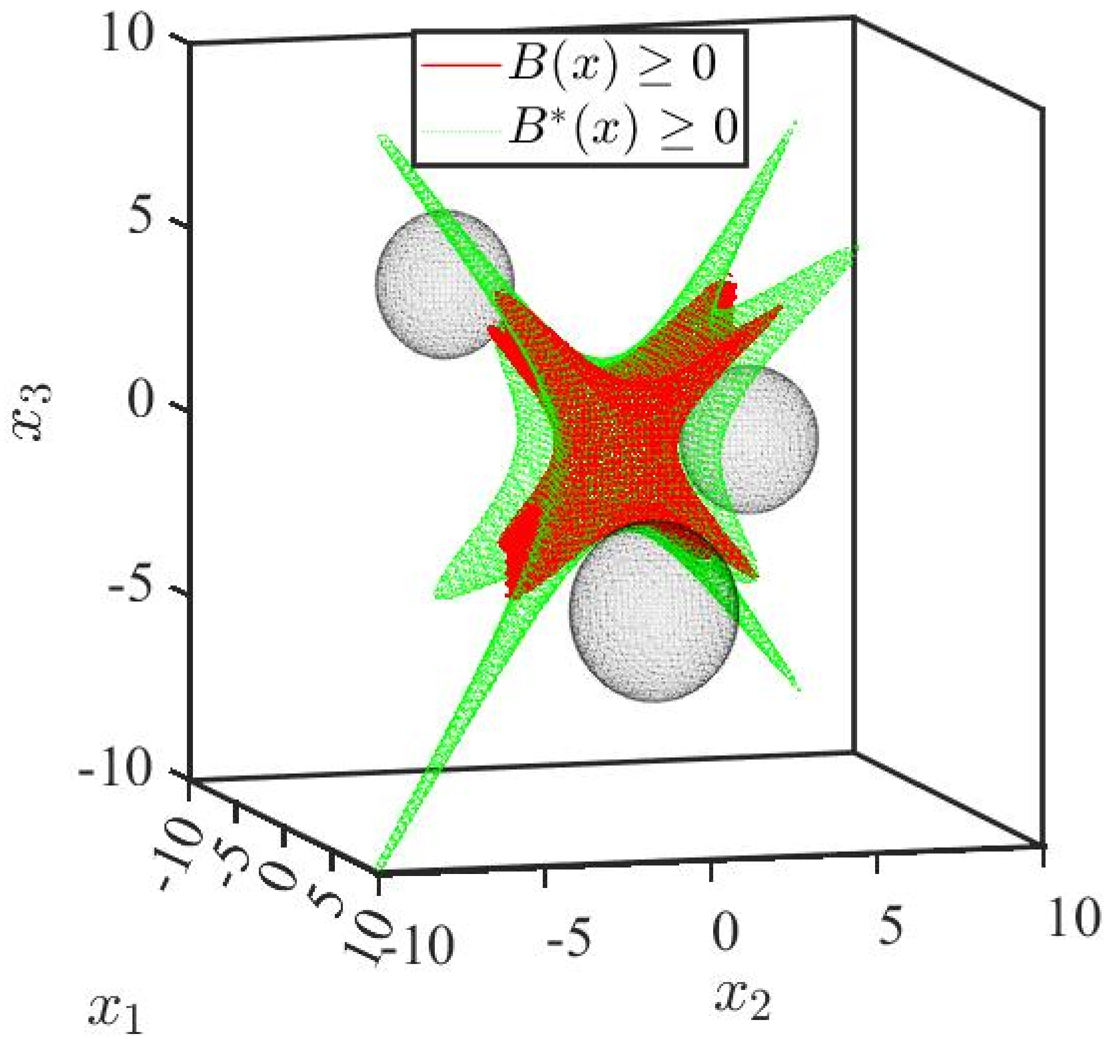
The final ROA with polynomial controller of the 2D system is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 + u_1 \\ x_1^2 x_2 + 1 - \sqrt{|\exp(x_1)\cos(x_1)|} + u_2 + d(x) \end{bmatrix}. \tag{2}$$

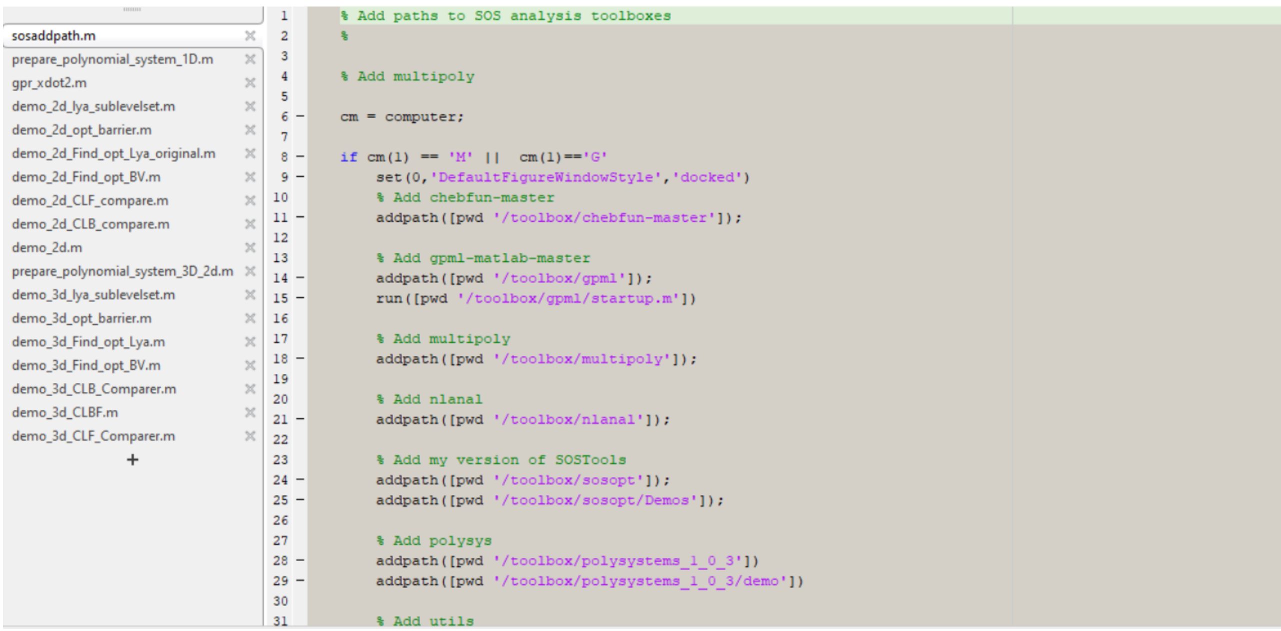


The final ROA of the 3D demo:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix} -x_1^2 - \cos(x_1^2)\sin(x_1) + u_1(x) + d_1(x) \\ -x_2 - x_1^3 x_2 + u_2(x) \\ -x_1^2 x_3 + 1 - \sqrt{|\exp(x_1)\cos(x_1)} + u_3(x) + d_3(x) \end{bmatrix}. \tag{3}$$



The related files are concluded in the figure below



To verify the **2D demo**, please run these files in a sequent.

- prepare_polynomial_system_1D.m
- demo_2d_lya_sublevelset.m

- demo_2d_opt_barrier.m
- demo_2d_Find_opt_Lya_original.m
- demo_2d_Find_opt_BV.m
- demo_2d_CLF_compare.m
- demo_2d_CLB_compare.m

To verify the **3D demo**, please run these files in a sequent.

- prepare_polynomial_system_3D_2d.m
- demo_3d_lya_sublevelset.m
- demo_3d_opt_barrier.m
- demo_3d_Find_opt_Lya.m
- demo_3d_Find_opt_BV.m
- demo_3d_CLB_Comparer.m
- demo_3d_CLF_Comparer.m
- demo_3d_CLBF.m

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