

# ASCC2022

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Consider a control affine dynamical system as follows,

$$\dot{x} = f(x) + g(x)u + d(x),$$

where  $x \in \mathcal{X} \subset \mathbb{R}^n$  and  $u \in \mathcal{U} \subset \mathbb{R}^m$  denote the state and control of the system. The system is consisted of three Lipschitz continuous terms,  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  denotes a nonlinear term,  $g : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$  denotes a polynomial term and  $d : \mathbb{R}^n \rightarrow \mathbb{R}^n$  denotes an unknown term. We consider a polynomial control input  $u$  over the stabilization process in this paper.

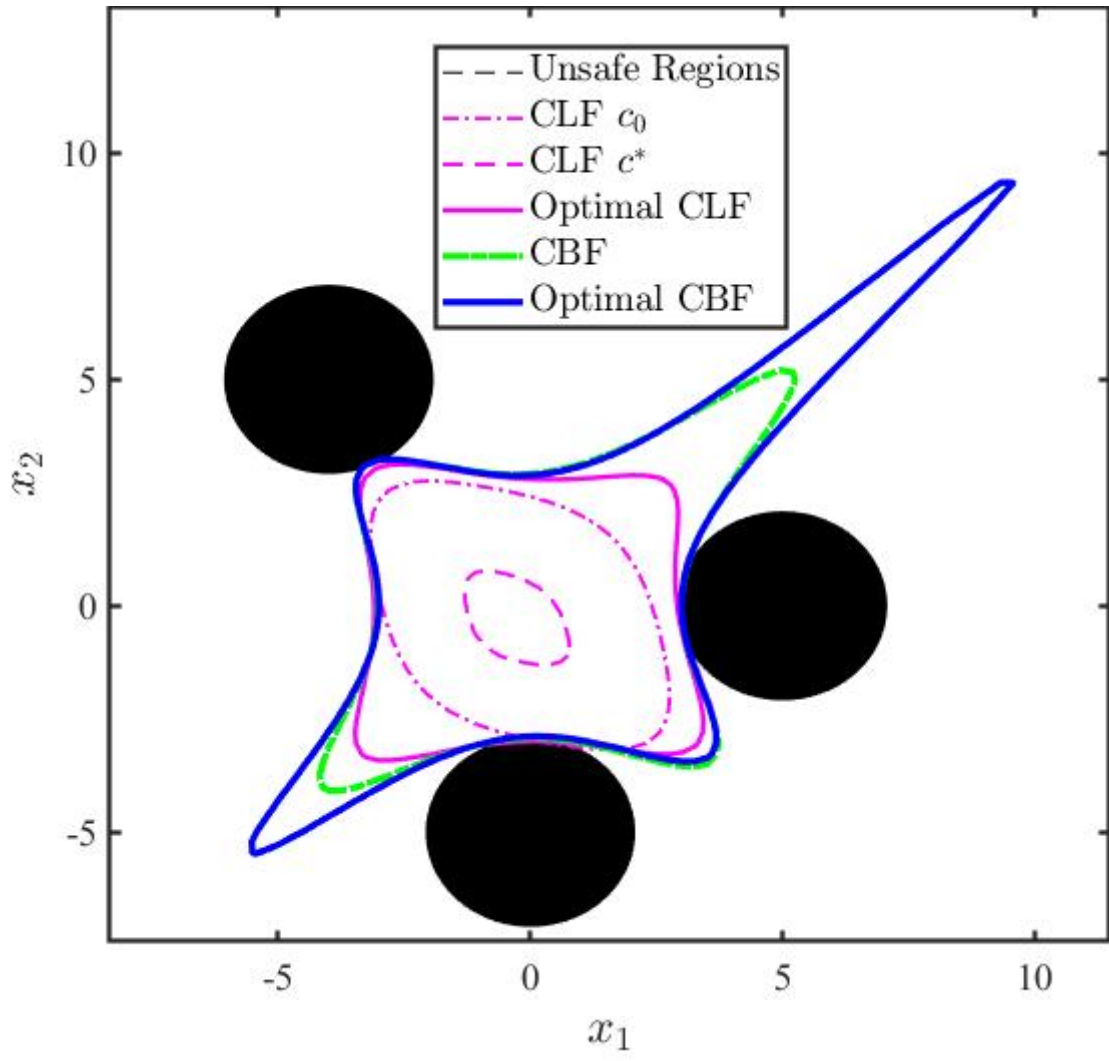
In this repo, we use

- Chebfun Toolbox: To approximate nonlinear terms by Chebyshev Interpolants,
- GPML Toolbox: Expressed the Gaussian processes mean function of this unknown term  $d(x)$  into the polynomial form,
- SOSOPT+Mosek: To solve some sum-of-squares programmings in this learned polynomial system.

Note that, please run *sosaddpath.m* at the beginning and Do not forget to install the Mosek Solver in advance.

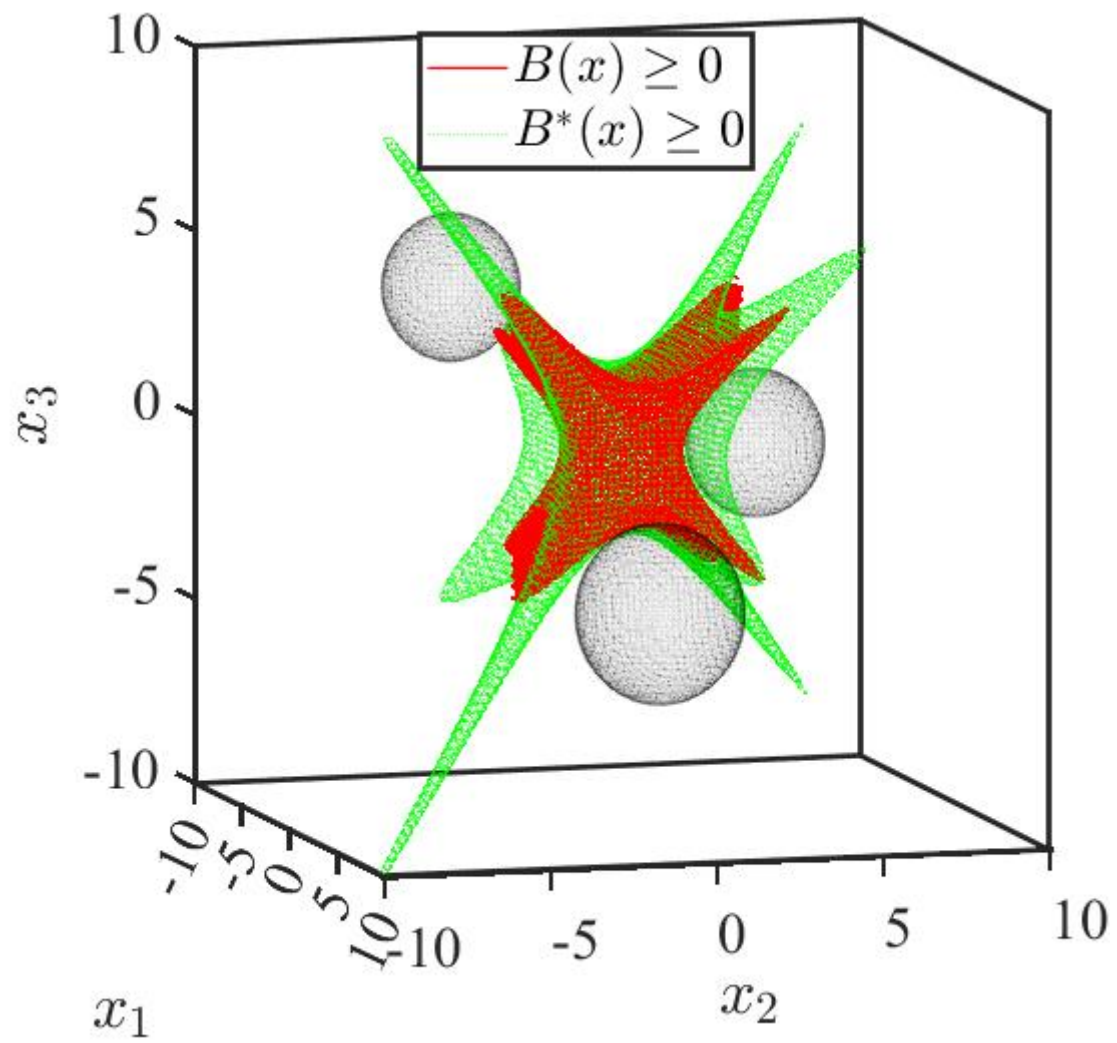
The final ROA with polynomial controller of the 2D system is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 + u_1 \\ x_1^2 x_2 + 1 - \sqrt{|\exp(x_1) \cos(x_1)|} + u_2 + d(x) \end{bmatrix}, \quad (1)$$



The final ROA of the 3D demo:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -x_1^2 - \cos(x_1^2) \sin(x_1) + u_1(x) + d_1(x) \\ -x_2 - x_1^3 x_2 + u_2(x) \\ -x_1^2 x_3 + 1 - \sqrt{|\exp(x_1) \cos(x_1)|} + u_3(x) + d_3(x) \end{bmatrix}. \quad (2)$$



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