Set 10 Problem 3

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Task:

Find MLE of k in Pareto distribution.

Solution:

$$f(x; a, k) = \frac{k a^k}{x^{k+1}}, \quad a \text{ known}$$

$$L(a,k) = \prod_{i=1}^{n} \frac{k a^k}{x_i^{k+1}} = k^n \cdot a^{k \cdot n} \cdot \prod_{i=1}^{n} \frac{1}{x_i^{k+1}}$$

$$\log L(a,k) = \log k^n + \log a^{kn} + \underline{\log \prod_{i=1}^{n} \frac{1}{x_i^{k+1}}}$$

$$\underline{\log \prod_{i=1}^{n} \frac{1}{x_i^{k+1}}} = \sum_{i=1}^{n} \log \frac{1}{x_i^{k+1}} = \sum_{i=1}^{n} \log x_i^{-(k+1)}$$

$$= -(k+1) \sum_{i=1}^{n} \log x_i, \quad \text{so}$$

$$\log L(a,k) = n \cdot \log k + nk \cdot \log a - (k+1) \sum_{i=1}^{n} \log x_i$$

$$\frac{d \log L(a,k)}{dk} = \frac{n}{k} + n \log a - \sum_{i=1}^{n} \log x_i = 0$$

$$\frac{n}{k} + \sum_{i=1}^{n} \log a - \sum_{i=1}^{n} \log x_i = 0$$

$$\frac{n}{k} + \sum_{i=1}^{n} \log \frac{a}{x_i} = 0 \Rightarrow \quad k = \frac{n}{\sum_{i=1}^{n} \log \frac{x_i}{a}}$$