RPIS - List 7 Task 6

Wojciech Adamiec

April 26, 2020

Prove that:

$$\sum_{k=1}^{n}(X_k-\mu)^2=\sum_{k=1}^{n}(X_k-\bar{X})^2+n(\bar{X}-\mu)^2$$

Proof.

$$\sum_{k=1}^{n}(X_k-\mu)^2=\sum_{k=1}^{n}(X_k-\bar{X}+\bar{X}-\mu)^2=$$

$$=\sum_{k=1}^{n}((X_k-\bar{X})^2+(\bar{X}-\mu)^2+2(X_k-\bar{X})(\bar{X}-\mu))=$$

$$=\sum_{k=1}^{n}(X_k-\bar{X})^2+n(\bar{X}-\mu)^2+2(\bar{X}-\mu)\sum_{k=1}^{n}(X_k-\bar{X})=$$

$$\sum_{k=1}^{n}(X_k-\bar{X})^2+n(\bar{X}-\mu)^2+2(\bar{X}-\mu)(\sum_{k=1}^{n}X_k-\sum_{k=1}^{n}\bar{X})=$$

$$=\sum_{k=1}^{n}(X_k-\bar{X})^2+n(\bar{X}-\mu)^2+2(\bar{X}-\mu)(n\cdot\frac{1}{n}\cdot\sum_{k=1}^{n}X_k-n\bar{X})=$$

$$=\sum_{k=1}^{n}(X_k-\bar{X})^2+n(\bar{X}-\mu)^2+2(\bar{X}-\mu)\cdot 0=$$

$$=\sum_{k=1}^{n}(X_k-\bar{X})^2+n(\bar{X}-\mu)^2$$

$\blacksquare$